**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: false

Employees older than 44 years of age is :

import numpy as np

import pandas as pd

import scipy.stats as stats

p= stats.norm.cdf(44,loc=38,scale=6)

=0.8413

1-p

=0.15865525393145707

Employees between 38 to 44 years of age:

p2=stats.norm.cdf(44,loc=38,scale=6)

p2=0.84134

p1=stats.norm.cdf(38,loc=38,scale=6)

p1 =0.5

p2-p1

=0.3413447460685429

The above statement is false, employees between is 0.158 and employees between 38 and 44 is 0.3413.so, there is less employees at the processing centre are older than 44 than between 38 and 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True

Employees under age 30 is:

stats.norm.cdf(30,loc=38,scale=6)

=0.09121121972586788

Number of employees attending training program from 400members is:

400\*0.09121121972586788

=36.484487890347154

The above statement is true, number of employees under the age of 30 expected to attending the training is 36 employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

The Normal Distribution has its link with the Central Limit Theorem, which states that Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal .only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2).

Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D

np.round(stats.norm.interval(0.99,loc=100,scale=20),1)

Output:

=array([ 48.5, 151.5])

The two values of a and b,symmetric about the mean,are such that the probability of the random variable taking a value between them is 0.99:

array([ 48.5, 151.5]).

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

import numpy as np

import scipy.stats as stats

Mean:

mean=5+7

print("Mean Profit is Rs",mean\*45,"Million")

Output:

Mean Profit is Rs 540 Million

Standard deviation:

stand\_dev=np.sqrt((9)+(16))

print("Standard Deviation is RS",stand\_dev\*45,"Million")

Output:

Standard Deviation is RS 225.0 Million

A:

print("Range is Rs",(stats.norm.interval(0.95,540,225)),"in Millions")

Output:

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

The above is values is containing 95% probability for the annual profitof the company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

The 5th percentile of profit for the company is:

X=mean+z(std\_dev)

X=540+(-1.645)\*(225)

print("5th Perecentile of profit(in Million Rupees )is",np.round(x))

Output:

5th Percentile of profit(in Million Rupees )is 170

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Probability of division 1 making a loss p(X<0):

stats.norm.cdf(0,5,3)

=0.0477903522728147

Probability of division 2 making a loss p(X<0):

stats.norm.cdf(0,7,4)

=0.0400591156863817086

Therefore, both the divisions are very similar,but the division 2 is having a probability of making a loss in a given year.