MAP OF FUNDAMENTAL PARTICLES

THE STANDARD MODEL

FERMIONS

QUARKS: UP (u), CHARM (c), TOP (t), DOWN (d), STRANGE (s), BOTTOM (b)

LEPTONS: ELECTRON (e), MUON (μ), TAU (τ), ELECTRON NEUTRINO (ν_e), MUON NEUTRINO (ν_μ), TAU NEUTRINO (ν_τ)

GAUGE BOSONS: GLUON (g), PHOTON (γ), Z BOSON (Z), W BOSON (W)

HIGGS BOSON (H) — SCALAR BOSON

CONSERVATION LAWS

WEAK FORCE: ENERGY, LINEAR MOMENTUM, ANGULAR MOMENTUM, CHARGE, COLOR, LEPTON NUMBERS (ELECTRON, MUON, TAU), BARYON NUMBER

ELECTROMAGNETIC FORCE: PARITY, ISOSPIN Z COMPONENT, TIME REVERSAL, STRANGENESS, CHARGE CONJUGATION

STRONG FORCE: ISOSPIN MAGNITUDE

FIELD INTERACTIONS

INTERACT WITH THE ELECTROMAGNETIC FIELD

INTERACT WITH THE STRONG FIELD

INTERACT WITH THE WEAK FIELD

INTERACT WITH THE HIGGS FIELD

ALL THE FUNDAMENTAL PARTICLES

STANDARD MODEL INTERACTIONS

X IS ANY FERMION IN THE STANDARD MODEL

X IS ELECTRICALLY CHARGED

L IS A LEPTON AND ν IS THE CORRESPONDING NEUTRINO

X IS ANY QUARK

X IS A PHOTON OR Z-BOSON

U IS AN UP-TYPE QUARK
D IS A DOWN-TYPE QUARK

X AND Y ARE ANY TWO ELECTROWEAK BOSONS SUCH THAT CHARGE IS CONSERVED

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QUANTUM

QUANDARIES (24')

Shyla Agrawal NCHS

relativity

Relativity

γ is "gamma factor" or "lorentz factor/term" } is a measurement that expresses how time, length, and other factors* change for a moving object

↳ coined by Hendrick Lorentz (Dutch)

* mass, momentum, energy

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

The Lorentz Transformations describe how spacetime coords change frame one frame to another:

$$x' = \gamma(x - vt) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

~~x is interchangeable with y and z~~

Time Dilation

$$\Delta t' = \gamma \Delta t$$

$$\left(t = \frac{t_0}{\sqrt{1-(v^2)}}\right)$$

on a moving clock, time appears slower

Δt = normal time between ticks

$\Delta t'$ = slower time

t_0 = slower

t = regular

Length Contraction

$$\Delta l' = \frac{1}{\gamma}\Delta l$$

on a moving object, length appears smaller

Δl = normal length

$\Delta l'$ = changed length

Relavistic Mass

$$m = \gamma m_0$$

on a moving object, mass changes

m = changed mass

m_0 = mass at rest

Relavistic Momentum

$$\vec{p} = m\vec{v} = \gamma m_0 \vec{v}$$

$$\vec{p} = \gamma mc$$

Relavistic kinetic Energy

$$\mathcal{E}_k = \mathcal{E} - \mathcal{E}_0 = (\gamma - 1)m_0c^2$$

$$\mathcal{E} = \gamma \mathcal{E}_0$$

$\lim_{v/c \to 0}$

$$E_K = \frac{1}{2}m_0 v^2$$

If an object is at such a low speed that γ is appracting 1, you use:

$$p = mv \quad \text{and} \quad \mathcal{E} = mc^2 + \frac{1}{2}mv^2 \quad \text{(newtonian)}$$

When two objects are coming together/moving apart, use this to calculate relavistic speed:

DOPPLER:

$$\gamma_{obs} = \gamma_s \sqrt{\frac{1 \pm \frac{u}{c}}{1 \pm \frac{u}{c}}}$$

bottom is negative if coming closer :)

- u and v are speeds of objects from outside view

$$u' = \frac{u - v}{1 - \frac{vu}{c^2}}$$

- u' and v' are speed from the other objects view

The twin paradox

The faster an object moves through space, the slower it moves through time. (This idea is made possible by the fact einstein said that light travels the same speed no matter the frame)

↳ a still object seems to not be moving at all, but it's still moving through time. (all motion would be purely in the time direction). If the object starts moving through space, it diverts some of the previous motion into a different direction, slowing down time for that object. :)

This relationship can be quantified with the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

γ = the lorentz factor

v = speed of moving observer

→ $\gamma = \frac{T_a}{T_b}$ time observed by stationary object over relative

Time Dilation is when time appears to pass differently for observers in different frames of reference.

↳ when an object relative to a stationary object reaches speeds of 86.6% of light (0.866 • c), the lorentz factor is 2, meaning times passes twice as slow for the moving object.

The **Twin Paradox** is the idea where if one twin flew on a rocket and came back to earth, both twins would not be the same age anymore.

↳ the twin moving really fast would age slower and come back looking younger than the stationary twin would be "older"

↳ the "paradox" comes from the idea that since everything is relative, the whole universe could be moving around the rocket, making time twice as slow for them. The space twin would end up older.

the fact that the space twin turns around and breaks the symmetry is why this is not really a paradox. that big acceleration (turning around) is what makes the twin at home older when the space twin returns.

Angular Momentum

momentum:

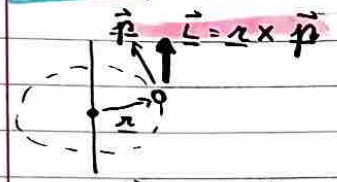
$$\vec{p} = m\vec{v}$$

angular momentum:

$$\vec{\omega} = \vec{p} \cdot r$$

$$\hookrightarrow \vec{\omega} = p_\theta = mr^2\dot{\theta}$$

CLASSICAL



$$\vec{L} = (L_x, L_y, L_z) \rightarrow (L_1, L_2, L_3)$$

$$\underline{r} = (x, y, z) \rightarrow (r_1, r_2, r_3)$$

$$\vec{p} = (p_x, p_y, p_z) \rightarrow (p_1, p_2, p_3)$$

$$\underbrace{\vec{L} = \underline{r} \times \vec{p} = (r_2 p_3 - r_3 p_2,\ r_3 p_1 - r_1 p_3,\ r_1 p_2 - r_2 p_1)}_{\text{angular momentum}}$$

QUANTUM

$$\hat{L} = (\hat{L}_1, \hat{L}_2, \hat{L}_3)$$

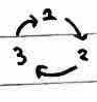
$$\hat{L}_1 = \hat{r}_2\hat{p}_3 - \hat{r}_3\hat{p}_2 \qquad \hat{L}_2 = \hat{r}_3\hat{p}_1 - \hat{r}_1\hat{p}_3 \qquad \hat{L}_3 = \hat{r}_1\hat{p}_2 - \hat{r}_2\hat{p}_1$$

these angular momentum components are hermitian because $\hat{L}_1^\dagger = \hat{L}_1$

↳ meaning that this vector is observable

commutation relations: $[\hat{L}_1, \hat{L}_2] = i\hbar\hat{L}_3$ $\quad [\hat{L}_2, \hat{L}_3] = i\hbar\hat{L}_1$

$$\boxed{[\hat{L}_i, \hat{L}_j] = i\hbar \sum_k \varepsilon_{ijk}\hat{L}_k} \qquad [\hat{L}_3, \hat{L}_1] = i\hbar\hat{L}_2$$



where k is a dummy index and ε_{ijk} is Levi-Civita

Dot vs Cross Product (vectors)

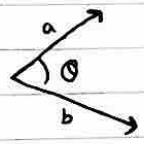
dot vs cross

dot

$a = \langle a_1, a_2, a_3 \rangle \qquad b = \langle b_1, b_2, b_3 \rangle$

$a \cdot b = (a_1 b_1) + (a_2 b_2) + (a_3 b_3)$

you will end up with one number so $a \cdot b = b \cdot a$



$a \cdot b = |a||b| \cos\theta$

$\hookrightarrow \cos\theta = \dfrac{a \cdot b}{|a||b|}$

$\boxed{|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}}$

cross

$a = \langle a_1, a_2, a_3 \rangle \qquad b = \langle b_1, b_2, b_3 \rangle$

$a = a_1 i, a_2 j, a_3 k \qquad b = b_1 i, b_2 j, b_3 k$

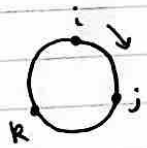
$$a \times b = \begin{vmatrix} j & i & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= (a_2 b_3 - a_3 b_2) i - (a_1 b_3 - a_3 b_1) j + (a_1 b_2 - a_2 b_1) k$$

you will end up with a vector which is orthogonal to the initial ones

$\boxed{a \times a = 0}$ $\qquad |a \times b| = |a||b| \sin\theta$

if a and b are parallel, then $a \times b = 0$



if cross product is in counter clockwise order, the answer will be −

if cross product is in clockwise order, answer will be +

ex. $i \times j = k$ but $j \times i = -k$

Levi-Civita* symbol: $\quad \varepsilon_{ijk} = \begin{cases} 1, & (ijk) = (123) \\ -1, & (ijk) = (321) \\ 0, & \text{anything else} \end{cases}$

$\hookrightarrow (a \times b)_i = \sum_{j,k=1}^{3} \varepsilon_{ijk} a_j b_k$

*also said as Tensor, pseudotensor, totally antisymmetric symbol

Dirac Notation

(V) State space
↳ where quantum systems apply
- quantum
- infinite D's
  ↳ complex numbers

Euclidean space ($\mathbb{R}^3$)
↳ where the coord system applies
- classical
- 3D at most
  ↳ real numbers

Both use inner product
↓
HILBERT SPACE, which uses
dirac notation

	$\mathbb{R}^3$	V
Denoting an element:	r "vector"	$\lvert\psi\rangle$ "ket"
Adding elements:	$r_1 + r_2 = r_3 \in \mathbb{R}^3$	$\lvert\psi_1\rangle + \lvert\psi_2\rangle = \lvert\psi\rangle_3 \in V$
Scalar multiplication:	$a \cdot r \in \mathbb{R}^3$	$a \cdot \lvert\psi\rangle \in V$

dirac notation

Scalar Product: $\boxed{SP(r_1, r_2) = r_1 \cdot r_2 = c}$ ⇨ $c \in \mathbb{R}$

"scalar belongs to a real number"

($\mathbb{R}^3$)

① conjugation
$SP(r_1, r_2) = SP(r_2, r_1)$
↳ $r_1 \cdot r_2 = r_2 \cdot r_1$

② linear
$SP(r_1, ar_2) = a \cdot SP(r_1, r_2)$
↳ $r_1 \cdot ar_2 = a(r_1 \cdot r_2)$

③ positivity (when squared)
$SP(r_1, r_1) \geq 0 \rightarrow r_1 \cdot r_1 \geq 0 \rightarrow |r_1|^2 \geq 0$

Scalar Product: $\boxed{SP(|\psi\rangle, |\varphi\rangle) = |\psi\rangle \cdot |\varphi\rangle = c}$ $c \in \mathbb{C}$

"scalar belongs to a complex number"

(V)

① conjugation
$SP(|\psi\rangle, |\varphi\rangle) = [SP(|\varphi\rangle, |\psi\rangle)]^*$
↳ $[\,]^*$ means complex conjugate

② linear in second argument
$SP(|\psi\rangle, a|\varphi\rangle) = a\,SP(|\psi\rangle, |\varphi\rangle)$

③ antilinear in first argument
$SP(a|\psi\rangle, |\varphi\rangle) = a^* SP(|\psi\rangle, |\varphi\rangle)$
↳ this combines the two

④ positivity (when squared)
$SP(|\psi\rangle_1, |\psi\rangle_1) \geq 0$

wave function

Wave Function - denoted by $|\psi\rangle$ - "eigen state"

$|\psi\rangle = (\text{probability} \cdot \text{state})_{\text{dim } 1} + (\text{probability} \cdot \text{state})_{\text{dim } 2}$

$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$ probability is a^2

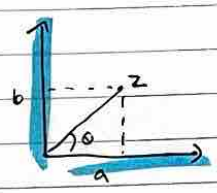
R C

$\boxed{|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle}$

ψ - can mean any property if it stays the same - "observables"

Complex

$z = a + ib \quad z^* = a - ib$



$z = \cos\theta + i\sin\theta$

$\hookrightarrow z = e^{i\theta}$

$|z| = \sqrt{a^2+b^2} = z \cdot z^*$

Time dependent: multiply states by $e^{\frac{-iE_n t}{\hbar}}$

$|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle$

$\hookrightarrow \langle\psi| = |\psi^*\rangle$

$a^2 + b^2 =$

$\langle\psi| \cdot |\psi\rangle = \langle\psi|\psi\rangle$

$= \boxed{1}$

Example:

$|\psi\rangle_1 = a|\uparrow\rangle + b|\downarrow\rangle \qquad |\psi\rangle_2 = c|\rightarrow\rangle + d|\leftarrow\rangle$

$a|\uparrow\rangle = a\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle\right)$

$b|\downarrow\rangle = b\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow\rangle\right)$

$\boxed{|\psi\rangle_1 = |\rightarrow\rangle}$

$\hbar = \frac{h}{2\pi} \approx 1.06 \times 10^{-34} \ \frac{m^2 kg}{s}$

degree of freedom = how many dimensions it can move in ($\uparrow$ and $\downarrow$)

Orthogonal States

$$\left.\begin{aligned} |\psi\rangle &= a|+\rangle + b|-\rangle \\ \langle\varphi| &= a^*\langle +| + b^*\langle -| \end{aligned}\right\} \quad \langle\varphi|\psi\rangle = 0$$

Dirac → Matrix

$|+\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad \langle +| = \begin{pmatrix}1 & 0\end{pmatrix}$ $\quad \langle +|+\rangle = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 1$

$|-\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\quad \langle -| = \begin{pmatrix}0 & 1\end{pmatrix}$ $\quad |+\rangle\langle +| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$

Hermitian

$\langle\psi|A^\dagger|\varphi\rangle = \langle\varphi|A|\psi\rangle^*$ $\quad$ Hermitian Adjoint : $A^\dagger$

- means to transpose, and conjugate
- if $A = A^\dagger$, then A is **Hermitian!**

$A = \begin{pmatrix}0 & 1 & 2\\ 1 & i & i\\ -i & 0 & 0\end{pmatrix}$

$A^\dagger = \begin{pmatrix}0 & 1 & i\\ 1 & -i & 0\\ 2 & -i & 0\end{pmatrix}$ "A dagger"

If A is hermitian, then A has real λ_i (eigen values) AND Eigen vectors ($|\lambda\rangle$) are complete

$E = \frac{hc}{\lambda}$ $\quad$ h = planks constant $\quad h = 6.62607015 \times 10^{-34}$
c = speed of light

↳ $E = hf$

Time dependancy

↳ multiplying the eigen state by $e^{\frac{-i\mathcal{E}_n t}{\hbar}}$ $\quad$ $\mathcal{E}_n$ = energy at n level

$\langle x \rangle$ for position $\Rightarrow x$

$\langle p \rangle$ for momentum $\Rightarrow -i\hbar \frac{d}{dx}$

(operator) $\longrightarrow$

$$\left(\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx \right)$$

Expectation value. → how much energy in "A"

$$\langle A \rangle = \langle \psi | A | \psi \rangle$$

ψ will be given

A is a matrix

ex.

$$\psi = a|\uparrow\rangle + b|\downarrow\rangle$$

$$\downarrow \qquad \downarrow$$

$$\begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\langle \psi | A | \psi \rangle = \psi^* \cdot A \cdot \psi$$

$\langle A \rangle =$ ← calculate

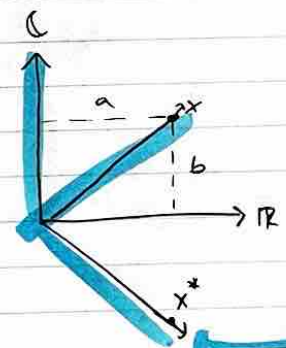
Special Case: (Momentum expectation value)

$$\langle L \rangle = \frac{\hbar}{2} \langle \psi | L | \psi \rangle$$

→ answer will be • with $\frac{\hbar}{2}$

expectation value

Complex Numbers



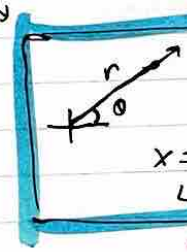
$x = a + ib$

$x = \cos\theta + i\sin\theta$

$x = e^{i\theta}$

$x = a + ib$

1. $|x| = \sqrt{a^2 + b^2}$
2. $x^* = a - ib$
3. $x \cdot x^* = a^2 + b^2 = |x|^2$



$x = a + ib$

↳ $x = re^{i\theta}$

$x^* = re^{-i\theta}$

Multiplying Matrecies

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad A_{12} = 1 \qquad A \cdot B \neq B \cdot A$$

$$A \cdot B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

don't add if the end dimension doesn't need it

Identity matrecies

- diagnols are all 1
- $A \cdot A^{-1} = 1$
- when you take * of a matrix, the dimensions swap and i becomes negative

$A = \begin{pmatrix} 1 & 0 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Bra and Ket

ket → $|B\rangle$ → column

bra → $\langle A|$ → row

braket → $\langle A|B\rangle$

$\langle A| = [1\ 0\ 1]$

$|B\rangle = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\langle A|B\rangle = [2]$

bra and ket are complex conjugets

↳ $|B\rangle^* = \langle A|$

↳ $\langle A|B\rangle^* = \langle B|A\rangle$

$\langle +|+\rangle = 1$

$\langle +|-\rangle = 0$

$\langle -|-\rangle = 1$

$\langle -|+\rangle = 0$

normalization

Normalization

$\left(\int_{x_0}^{x_1} |\psi(x)|^2 dx = 1 \right)$

↳ is when all probabilities $= 1$

TO NORMALIZE

$\frac{1}{\sqrt{|a|^2 + |b|^2}} (a|\psi\rangle + b|\psi\rangle)$

$\langle \psi|\psi\rangle = 1$ → $a^2 + b^2 = 1$

"condition" "limitation"

(find the conjugate and multiply)

Determenint

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det A = ad - bc$

$\det A = |A|$ NOT MAGNITUDE

Eigen state / value

$$|\psi\rangle = a|\psi_1\rangle + b|\psi_2\rangle$$

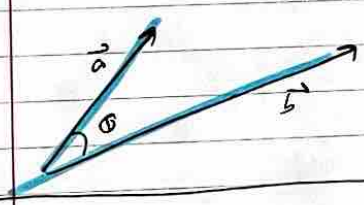
a = eigen value
$|\psi\rangle$ = eigen state
} both can be matrecies

$\boxed{\text{FOR ELECTRON} \rightarrow \text{"spin"} = 1/2 \quad \text{eigen value} = \hbar/2}$

Vectors / matrecies

- two orthoganol vectors "dot product" $= 0$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = |\vec{a}||\vec{b}| \cos\theta$$



Hooke's Law

$$F_{spring} = -kx$$

k is the (stiffness)
x is the displacement

force exerted by the spring is opposite to motion

$\sin(z)^* = -\sin(z^*)$
$\cos(z)^* = -\cos(z^*)$

Hookes + probabilities

Probabilities (must be normalized)

$$P\left(\frac{\hbar}{2}\right) = |\langle + | \psi \rangle|^2$$

probability of ↑ spin in state $|\psi\rangle$

Complex / Hermitian Conjugates

- $(aA)^\dagger = a^* A^\dagger$
- $(A^\dagger)^\dagger = A$
- $(A+B)^\dagger = A^\dagger + B^\dagger$
- $(AB)^\dagger = B^\dagger A^\dagger$
- $(AB|\psi\rangle)^\dagger = \langle\psi| B^\dagger A^\dagger$

Eigen Value of a Matrix

$\vec{v}$ = eigen vector

$A\vec{v} = \lambda\vec{v}$ $\quad$ λ = eigen value of A (scalar)

↳ iff $\boxed{\det(\lambda I_n - A) = 0}$ $\quad$ I_n = identity matrix

EXAMPLE:

Eigen Values

$A = \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}$ $\quad$ $\det\left(\lambda I_n - \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}\right) = 0$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}\right) = 0$

$\det\begin{pmatrix} \lambda+3 & +4 \\ -5 & \lambda-6 \end{pmatrix} = 0 \rightarrow \lambda = \{1, 2\}$

Eigen Vectors

$A\vec{v} = \lambda\vec{v}$

$\begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}\vec{v} = I_n \lambda \vec{v} \Rightarrow \vec{v}\left(\begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$

$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow$ find relationship $\Rightarrow$ solve!

- If A is 3×3, when calculating det, you write first 3 lines then repeat the first two again!

↳ $\det\begin{bmatrix} \lambda-1 & -1 & -1 \\ -2 & \lambda-1 & 0 \\ -3 & -1 & \lambda \end{bmatrix} \Rightarrow \begin{matrix} \lambda-1 & -1 & -1 & \lambda-1 & -1 \\ -2 & \lambda-1 & 0 & -2 & \lambda-1 \\ -3 & -1 & \lambda & -3 & -1 \end{matrix}$

when right diagnol, + $\Rightarrow$ (vice versa)

Diagnalization:

↳ use eigen values $\rightarrow$ $D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

Matrix inverse:

↳ use A $\rightarrow$ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow A^{-1} = \dfrac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{\det A} \rightarrow$ switch diagnal, inverse left diagnol

Eigen Process

Diagnalization Pt. 2

$A = X D X^{-1}$

↳ or... $D = X^{-1} A X$

* operators only apply to the D *

WHOLE PROCESS (with examples)

1. matrix: $A = \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}$
2. find eigen values: $\lambda = \{1, 2\}$
3. find eigen vectors: $\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4/5 \\ 1 \end{bmatrix}$
4. use λ and diagnalization: $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
5. use vectors for X: $\begin{bmatrix} -1 & -4/5 \\ 1 & 1 \end{bmatrix}$
6. find $(\cdot)^{-1}$ of X: $\begin{bmatrix} -5 & -4 \\ 5 & 5 \end{bmatrix}$
7. PLUG IN $A = X D X^{-1}$! $\rightarrow \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix}$

BRAKETS → VECTORS → MATRIX

$|1\rangle \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ NOT $\rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|0\rangle \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Hadamard (H) $\rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Pauli-Z (Z) $\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Phase (S, P) $\rightarrow \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

Example Eigen

$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ $\quad A\vec{v} = \lambda\vec{v} \rightarrow \det(\lambda \cdot I_n - A) = 0$

$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\right) = 0$

$\det\begin{bmatrix} \lambda-1 & -1 \\ -4 & \lambda-1 \end{bmatrix} = 0$

$(1-\lambda)(1-\lambda) - (4)(1) = 0$

$1 - 2\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1) = 0$

eigen values $\rightarrow$ $\boxed{\lambda = \{3, -1\}}$

$X = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$ $\quad X^{-1} = \begin{bmatrix} -2 & -1 \\ -2 & 1 \end{bmatrix}$

$A = X \cdot D \cdot X^{-1}$

$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

good luck from here...

too lazy to do it now

$(A - \lambda I)\vec{x} = 0$

$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$

$-2x_1 + x_2 = 0$

x_1 is ALWAYS $\boxed{1}$

$x_2 = 2$

vector $\hookrightarrow$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$

$2x_1 + x_2 = 0$

x_1 is 1

$x_2 = -2(1)$

vector $\hookrightarrow$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Example Eigen

$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ $\quad A\vec{v} = \lambda\vec{v} \rightarrow \det(\lambda \cdot I_n - A) = 0$

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$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

good luck from here...

too lazy to do it now

$(A - \lambda I)\vec{x} = 0$

$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}$

$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \downarrow \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$

$-2x_1 + x_2 = 0$

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$x_2 = 2$

vector $\hookrightarrow$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \downarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$

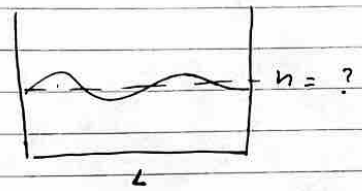
$2x_1 + x_2 = 0$

x_1 is 1

$x_2 = -2(1)$

vector $\hookrightarrow$ $\boxed{\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}}$ $\quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Particle in a Box



SCHRODINGER:

$$\Psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

n is the energy level

↳ n = 1 →

n = 2 →

Probability of finding a particle here:

$$P = \int_0^L |\Psi|^2 \, dx \Rightarrow P = \int_0^L \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

L doesn't matter, make it 1

expectation is the average position of a particle

Heisenburg's Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

as certainty of x grows, certainty of p shrinks

$$\Delta E \cdot \Delta t > \frac{h}{4\pi}$$

heisen's uncertainty + particle in box

Hermitian Dagger ($(\cdot)^\dagger$)

$|\psi\rangle = a|0\rangle + b|1\rangle$

$\hookrightarrow |\psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$

$|\psi\rangle^\dagger = a^*\langle 0| + b^*\langle 1|$

$\hookrightarrow |\psi\rangle^\dagger = \begin{bmatrix} a^* & b^* \end{bmatrix}$

dagger of a matrix:

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix}$

Hadmard: The Operator

$|0\rangle \rightarrow \boxed{H} \rightarrow ?$

$\downarrow \qquad \downarrow$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{\text{back to Dirac}} \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}|1\rangle + |0\rangle$

Tensor Product ($\otimes$)

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|00\rangle = |0\rangle \otimes |0\rangle$

$\Downarrow$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}$

BINARY
0 = 0
1 = 01
2 = 10
3 = 11
4 = 100
5 = 101
6 = 110
7 = 111

number of qubits = number of terms

binary value = position

ex.

$|101\rangle \hookrightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

operators

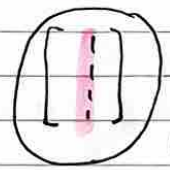
Hadmard & Tensor

$$H \otimes H \, |00\rangle \Rightarrow H|0\rangle \otimes H|0\rangle$$

$$\Downarrow$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \text{again}$$

$$\Downarrow$$



$$\frac{1}{2}((|00\rangle) + (|01\rangle + (|10\rangle) + (|11\rangle))$$

$$|0\rangle \longrightarrow \boxed{H} \longrightarrow ?$$

$$|1\rangle \longrightarrow$$

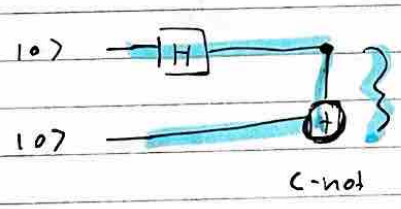
$$H^{\otimes 2} |01\rangle \Rightarrow H \otimes H \, |01\rangle$$

$$\Downarrow$$

$$H|0\rangle \otimes H|1\rangle \qquad \otimes = \cdot$$

Circuits:

$$|0\rangle \longrightarrow \boxed{H} \longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



$$(H|0\rangle \otimes |0\rangle) \text{ C-NOT}$$

$$\hookrightarrow \frac{1}{\sqrt{2}}|0\rangle|0\rangle + |1\rangle H)$$

Examples:

$$H^{\otimes 2}|00\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$H^{\otimes 2}|01\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$H^{\otimes 2}|10\rangle = \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$$

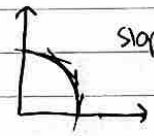
$$H^{\otimes 2}|11\rangle = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$$

Differentiation

when given functions

↳ apply → $\frac{d}{dx}[x^n] = n x^{n-1}$

derivative is notated as $\dot{x}$



slope of lines

the bottom variable is "what its respect to"

Lagrange

$L = KE - PE$ ⇒ $L = T - V$

↳ (T) ↳ (V)

lagrange equation

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx}$$

x = distance

$\dot{x}$ = velocity

euler - lagrange ⇒ equation of motion

---

scenario (1)

free falling ball

$PE\ (V) = mg \cdot h$

$KE\ (T) = \frac{1}{2} m v^2$

$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx}$

(positive or negative)

$L = \left(\frac{1}{2} m v^2\right) - (mg \cdot h)$

↳ $\boxed{L = \frac{1}{2} m \dot{x}^2 - mgx}$

$\frac{dL}{dx} \Rightarrow \frac{d}{dx}\left(\frac{1}{2} m \dot{x}^2\right) - \frac{d}{dx}(mgx)$

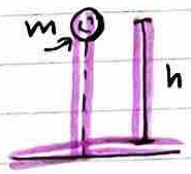
$\Rightarrow 0 - mg \Rightarrow \boxed{-mg}$

$\frac{dL}{d\dot{x}} \Rightarrow \frac{d}{d\dot{x}}\left(\frac{1}{2} m \dot{x}^2\right) - \frac{d}{d\dot{x}}(mgx)$

$\Rightarrow m\dot{x} - 0 \Rightarrow \boxed{m\dot{x}}$

$\frac{d}{dt}(m\dot{x}) \Rightarrow \boxed{m\ddot{x}}$ → $-mg = m\ddot{x}$

$\boxed{-g = \ddot{x}}$ ← EQUATION OF MOTION



$-g = -9.8\ m/s^2$

$\ddot{x}$ = acceleration

lagrange

$$x = A\cos(\omega t) + B\sin(\omega t)$$

A = amplitude

B = -amplitude

$$\boxed{\omega^2 = k/m}$$

$$\ddot{x} + \omega^2 x = 0$$

ω = natural frequency

scenario ②

MASS on a spring

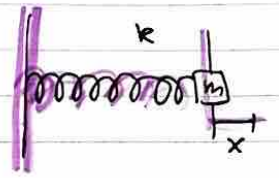
$PE\ (V): \frac{1}{2}kx^2$

$KE\ (T): \frac{1}{2}m\dot{x}^2$

k is the spring constant

$$L = \left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right)$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx} \qquad L = KE - PE$$



$$\frac{d}{d\dot{x}}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right) \Downarrow \boxed{m\dot{x}}$$

$$\hookrightarrow \frac{d}{dt}(m\dot{x}) \Rightarrow m\ddot{x}$$

$$\frac{d}{dx}\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2\right) \Downarrow \boxed{-kx}$$

$$\boxed{m\ddot{x} = -kx}$$ EQUATION OF MOTION

scenario ③

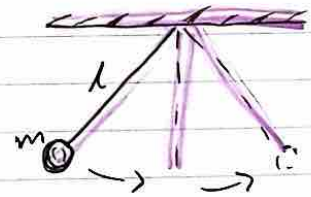
pendulum

$PE\ (V): gm(l - l\cos\theta)$

$KE\ (T): \frac{1}{2}I\omega^2 \rightarrow \frac{1}{2}ml^2\dot{\theta}^2$

$$L = \left(\frac{1}{2}ml^2\dot{\theta}^2 - gm(l - l\cos\theta)\right)$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = \frac{dL}{d\theta} \qquad L = KE - PE$$



$$\frac{d}{d\dot{\theta}}\left(\frac{1}{2}ml^2\dot{\theta}^2\right) - gm(l - l\cos\theta) \Downarrow \boxed{ml^2\dot{\theta}}$$

$$\hookrightarrow \frac{d}{dt}(ml^2\dot{\theta}) \Rightarrow \boxed{ml^2\ddot{\theta}}$$

$$\frac{d}{d\theta}\left(\frac{1}{2}ml^2\dot{\theta}^2\right) - gm(l - l\cos\theta) \Downarrow \boxed{-mgl\sin\theta}$$

$$ml^2\ddot{\theta} = -mgl\sin\theta$$

EQUATION OF MOTION $\boxed{l\ddot{\theta} = -g\sin\theta}$

disk vs wheel

Disk - $I = \frac{1}{2}mr^2$

wheel - $I = mr^2$

or

"ring"

scenario ④

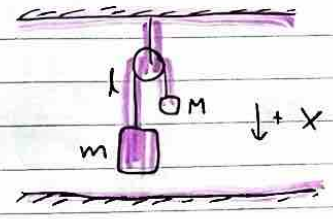
pulley

$PE: -mgx - Mg(l - x)$

$KE: \frac{1}{2}m\dot{x}^2 + \frac{1}{2}M_2\dot{x}^2$

down is the positive x direction

$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}M\dot{x}^2 + mgx + Mgl - Mgx$



$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx} \qquad L = KE - PE$$

$$\frac{d}{dt}(m\dot{x} + M\dot{x})$$

$$\Downarrow$$

$$\boxed{\ddot{x}(m + M)} = \boxed{g(m - M)}$$ ← equation of motion

scenario ⑤

spring

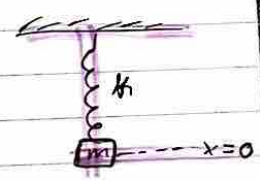
(vertical)

$PE: \frac{1}{2}kx^2 - mgx$

$KE: \frac{1}{2}m\dot{x}^2$

down is the positive x direction

$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + mgx$



$$\frac{d}{dt}\left(\frac{dKE}{d\dot{x}}\right) - \frac{dKE}{dx} + \frac{dPE}{dx} = 0 \qquad L = KE - PE$$

$$\frac{dKE}{dx} = 0 \qquad \frac{dPE}{dx} = kx - mg \qquad \frac{dKE}{d\dot{x}} = m\dot{x} \qquad \frac{d}{dt}\left(\frac{dKE}{d\dot{x}}\right) = m\ddot{x}$$

$$\ddot{x} + \frac{k}{m}x - g = 0 \Rightarrow \boxed{\ddot{x} = -\frac{k}{m}x + g}$$ equation of motion

scenario ⑥

spring

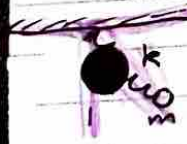
(swinging)

$PE: -mg(l + x)\cos\theta + \frac{1}{2}kx^2$

$KE: \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m(l + x)^2\dot{\theta}^2$

two degrees of freedom (spring, angle)

$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m(l + x)^2\dot{\theta}^2 + mg(l + x)\cos\theta + \frac{1}{2}kx^2$



(using x)

$$\frac{dL}{dx} = m(l + x)\dot{\theta}^2 + mg\cos\theta - kx$$

$$\frac{dL}{d\dot{x}} = m\dot{x} \qquad \frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = m\ddot{x}$$

$$m\ddot{x} = m(l + x)\dot{\theta}^2 + mg\cos\theta - kx$$

$$\boxed{\ddot{x} = (l + x)\dot{\theta}^2 + g\cos\theta - kx}$$

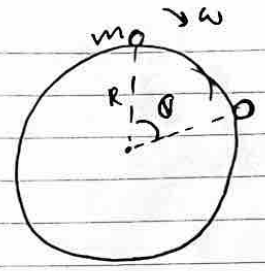
(using θ)

$$\frac{dL}{d\theta} = -mg(l + x)\sin\theta \qquad \frac{dL}{d\dot{\theta}} = m(l + x)^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = m\left(2(l + x)\dot{x}\dot{\theta} + (l + x)^2\ddot{\theta}\right)$$

$$\boxed{-g\sin\theta = 2\dot{x}\dot{\theta} + (l + x)\ddot{\theta}}$$

Particle sliding down a dome



KE: $\frac{1}{2}mr^2\dot{\theta}^2$ PE: $mgr\cos\theta$

$\cancel{L = \frac{1}{2}mr^2\dot{\theta}^2 - mgr\cos\theta}$

$\frac{dL}{d\dot{\theta}} = mr^2\dot{\theta}$ $\frac{dL}{d\theta} = mgr\sin\theta$

$mr^2\ddot{\theta} = mgr\sin\theta$

equation of motion → $\boxed{r\ddot{\theta} = g\sin\theta}$

$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = mr^2\ddot{\theta}$

$g\cos\theta = r\dot{\theta}^2$

equation of motion → $\boxed{g\cos\theta = r\omega^2}$

where the particle leaves the dome:

$$\theta = \cos^{-1}\left(\frac{u^2 + 2gR}{3gR}\right)$$

$\boxed{u^2 = gR \text{ then } \theta = 0}$

⇓

$$\theta = \cos^{-1}(2/3) \approx 48.1896851^\circ$$

$\boxed{\omega = u/R \text{ "omega" :)}}$

newtonian way (help):

$$mg\cos\theta - F_n = F_c$$

$$\left\{\begin{array}{l} F_c = a_c m \\ a_c = \omega^2 r \end{array}\right\}$$

$$mg\cos\theta - F_n = \omega^2 r m$$

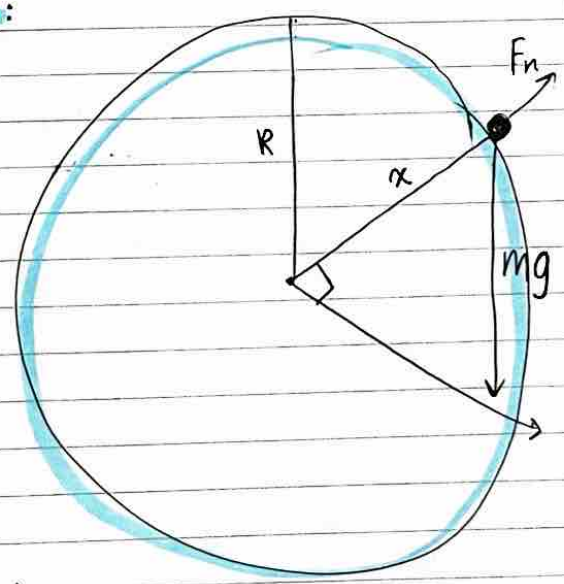
when $F_n = 0$, or no contact → $\boxed{g\cos\theta = \omega^2 r}$

⇓

$$\theta = \cos^{-1}\left(\frac{\omega^2 r}{g}\right)$$

↳ $\cos\theta = \left(\frac{\omega^2 r}{g}\right)$

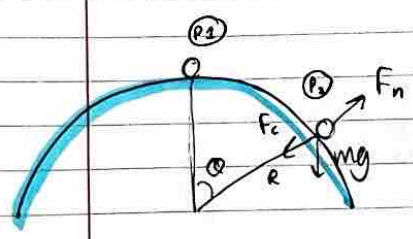
(which should match the previous equation for theta...)



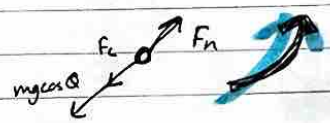
when $\cos\theta = 1$, it will fly off

Particle sliding (cont.)

newtonian way (better)



Forces: $mg\cos\theta - F_n = \frac{mv^2}{R}$



Energy: $mgR + \frac{1}{2}mu^2 = mgR\cos\theta + \frac{1}{2}mv^2$

P1 PE: mgR
KE: $\frac{1}{2}mu^2$

P2 PE: $mgR\cos\theta$
KE: $\frac{1}{2}mv^2$

1. to find when (what θ) the particle lifts, assume $F_n = 0$

2. to find when (what u) the particle will fly, $\cos\theta = 1$

Particle in a box

$$\mathcal{E} = \frac{n^2\hbar^2\pi^2}{2mL^2} \quad n = 1..2..3..$$

$$\mathcal{E} = \frac{h^2n^2}{8mL^2} \quad \text{bc.. } \boxed{\hbar = \frac{h}{2\pi}}$$

equation:

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x n}{L}\right)$$

to make it time dependant:

multiply the state by $e^{\frac{-i\mathcal{E}_n t}{\hbar}}$

More Differentiation

$$\frac{d(\sin\theta)}{d\theta} = \cos\theta$$

$$\frac{d(\sin\theta)}{dt} = \frac{d(\sin\theta)}{d\theta} \cdot \frac{d\theta}{dt} = \cos\theta \cdot \dot{\theta}$$

$$\frac{d(\cos\theta)}{dt} = \frac{d(\cos\theta)}{d\theta} \cdot \frac{d\theta}{dt} = -\sin\theta \cdot \dot{\theta}$$

$$\frac{d(\cos\theta)}{d\theta} = -\sin\theta$$

$$\boxed{\frac{dx^n}{dx} = n \cdot x^{n-1}}$$

$$\boxed{\frac{d(\sin x)}{dx} = \cos x}$$

$$\frac{de^x}{dx} = e^x \qquad \frac{de^{kx}}{dx} = ke^{kx} \qquad \frac{d(\sin kx)}{dx} = k\cos kx \qquad \frac{d(\sin x^2)}{dx} = 2x\cos x^2$$

Hamiltonian

$$H = KE + PE$$

$$m\frac{d}{dt}\left(\frac{dH}{dp}\right) + \frac{dH}{dx} = 0$$

$$p_r = m\dot{r}$$

$$p_\theta = mr^2\omega$$

$$\omega = \dot{\theta}$$

$$\frac{dH}{dp_\theta} = \omega \qquad \frac{dH}{dp_r} = \dot{r}$$

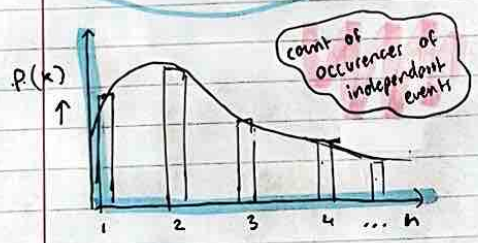
$$\frac{dH}{dr} = \dot{p}_r \qquad \frac{dH}{d\theta} = \dot{p}_\theta$$

Stats

Stats

Poisson Distribution → "a frequency with which an event occurs"

$$P(k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$$



λ = mean "average" → "varience"
↳ "expectation"

k = the frequency we are looking for

e = "euler's number" ≈ 2.718281828 46

Expectation for Poisson:

$$E(n) = \sum_{n} n \cdot p(n) = \lambda$$ → also varience

Entropy for Poisson:

$$H = \sum_{k} \log(k!) \frac{m^{k} e^{-m}}{k!}$$

if:
$m > 10$
then:
$H = \frac{1}{2}\log(2\pi e m)$

Variance for Poisson:

$$Var(x) = \mathcal{E}(X^2) - (\mathcal{E}(x))^2$$

Boltzmann Distribution

$$P_i = \frac{e^{\frac{-\mathcal{E}_i}{(k_B T)}}}{Z}$$

$\mathcal{E}_i$ ⇒ energy of particle i
k_B ⇒ Boltzmann Constant
T ⇒ Temprature (k)
Z ⇒ Summation of all Energy

Canonical Ensemble:
Identical, but distinguishable

Thermodynamics

3 laws of thermodynamics

thermo-dynamics

① energy can NOT be created nor destroyed, only tranformed

$U \leftarrow$ internal energy
$Q \leftarrow$ Heat added
$W \leftarrow$ work done

$\Delta U = Q - W$ } doing work
$\Delta U = Q + W$ } work done to me

② Entropy: the disorder of the world will always increase

as entropy ↑, energy to do useful work ↓

"heat will always move from a hotter to colder body"
if a process generates entropy, it will happen spontaneously and will be irreversible unless you put more energy into it.

$$S = k_b \ln \Omega$$

S = entropy
Ω = number of microscopic configurations
k_b = boltzmann constant

$$k_b = \frac{PV}{TN}$$

P = pressure
V = volume
T = absolute temp
N = number of molecules

③ perfect zero: the entropy of a perfect crystal reaches absolute zero as temperture of absolute zero

a crystal is because all the atoms of it are perfectly aligned (any imperfections of a crystal carry energy)

this is a complete theory, as we cannot make "zero temp"

0th Law: if two thermodynamic systems are both in thermal equilibrium with a third system, then the two systems are in thermal equilibrium with each other
(transitive law)

Ising Model

a string, layer, or more of particles, charged + or −

the hamiltonian:

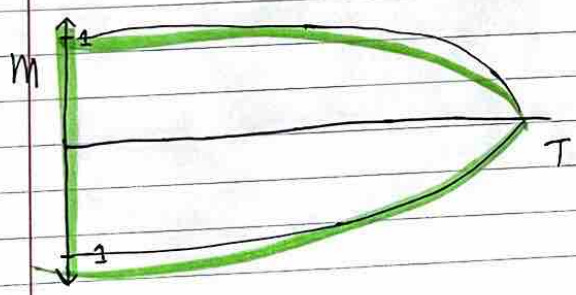
$$H(\sigma) = -J\sum_{i=1}^{L} \sigma_i \sigma_{i+1} - h\sum_{i=1}^{L} \sigma_i$$

when $J=0$
↳ micro $= 2^L$
↳ macro $= L+1$

groundstate → all ↑
energy → $-hL$
multiplicity → 1

when $h=0$
↳ micro $= 2^L$
↳ macro $= L+1$

ground state → all ↑ or all ↓
energy → $-JL$
multiplicity → 2



as temp > 0, the system becomes more disordered

(ferromagnetic materials, bc spin becomes unaligned)

The energy is calculated by the spins → ↑↑ = −1 ↑↓ = 1
↳ you take all pairs and add the pairs

Min possible energy:
for an $L \times L$ system, there are $2L(L-1)$ possible connections
↳ so $\mathcal{E} = -2JL(L-1)$ and $M = \pm h^2$

Max possible energy:
$\mathcal{E} = +2JL(L-1)$ and $M = 0$ is L is even

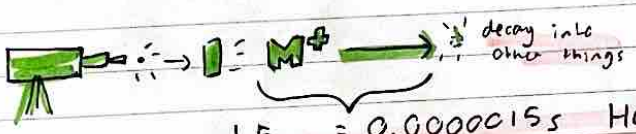
Ising Model

Muons (muon decay experiment)

cosmic rays shower down particles all the time

↳ pions (π^-) electrons (e^-) gammas (γ)
kaons (K) neutrons (n) x-rays (γ)
positrons (e^+) neutrinos (ν) muons (μ^+)

Muons (μ^+) in a lab:



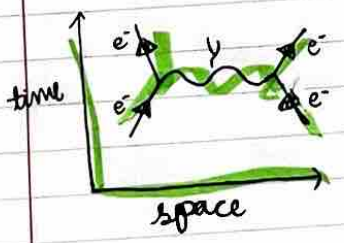
$1.5\,ms = 0.0000015\,s$ Half-life

life expectancy: $2.2\,\mu s$

muon decay

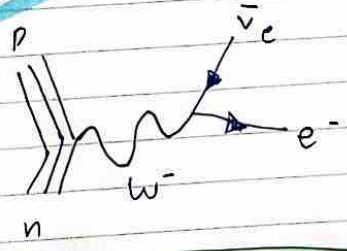
Because of time dialation, these muons can travel to earths surface before decaying. From the muons perspective, the distance to earth is shorter because of length dialation

Feynman Diagrams



a particle (electron) emits a photon to another electron

*see print for more



nuclear decay: $n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$

fermion
photon
gluon
boson (W, Z)

W^+ is when $p \rightarrow n$
W^- is when $n \rightarrow p$

Particle	Antiparticle
electron ⊖	positron ⊕
proton ⊕	antiproton ⊖
neutrino	anti neutrino

Feynman Diagrams

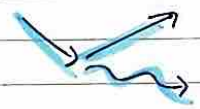
RULES:

x-axis - left to right

y-axis - down or up

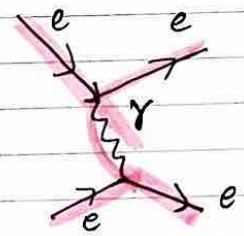
RULES:

- each vertex has an arrow in, and arrow out
- quarks or leptons are solid lines
- exchange particles (W^+, W^-, Z^0, γ) are curly
- arrows from right to left are anti-particles
- Junctions linked by a line representing exchange particles

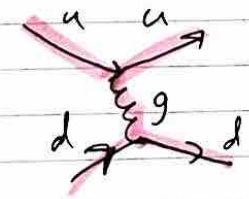


Some diagrams

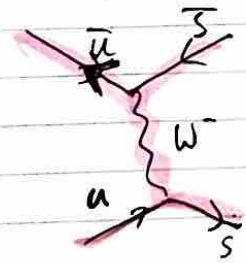
① electron repulsion



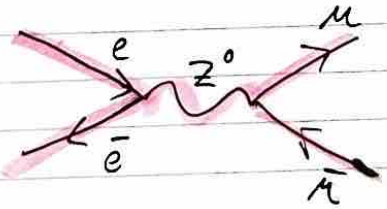
② quark-quark attraction



③ $\pi^- + p \rightarrow K^0 + \Lambda^0$



④ $e + \bar{e} \rightarrow \mu + \bar{\mu}$

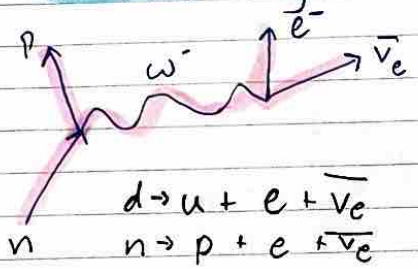


Feynman Diagrams

Weak Nuclear Feynman diagrams

(these diagrams assume positive time ↑)

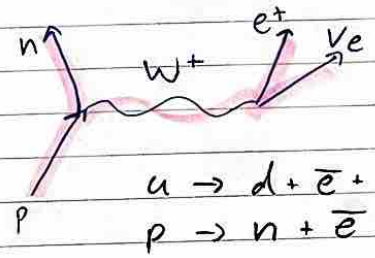
① beta minus decay



$d \to u + e + \bar{\nu}_e$

$n \to p + e + \bar{\nu}_e$

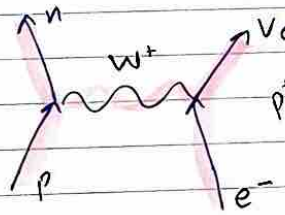
② beta plus decay



$u \to d + \bar{e} + \nu$

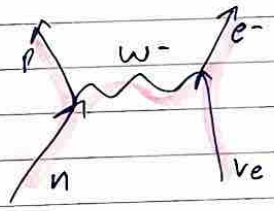
$p \to n + \bar{e} + \nu$

③ electron capture



$p^+ + e^- \to n + \nu_e$

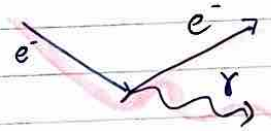
④ neutron-neutrino interaction



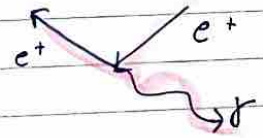
$n + \nu_e \to p + e^-$

Others:

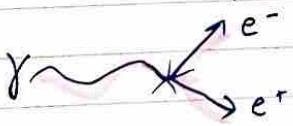
① electron emitting a photon



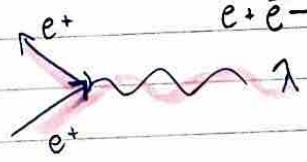
② positron emits a photon



③ photon produces electron and positron
↳ (pair production

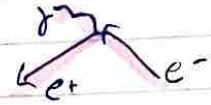


④ annihilation



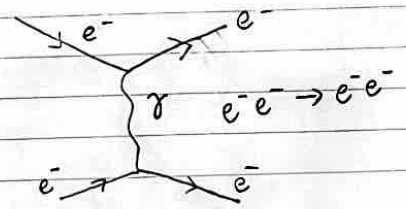
$e + \bar{e} \to 2\gamma$

⑤ positron scattering



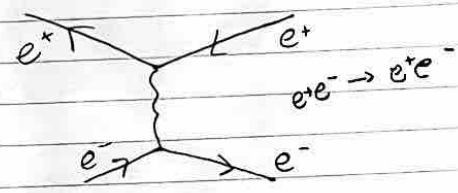
More:

⑥ electron - electron scattering:



$e^-e^- \to e^-e^-$

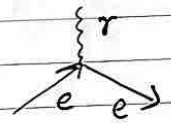
⑦ electron - positron scattering:



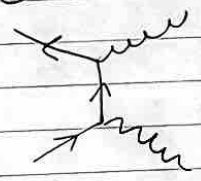
$e^+e^- \to e^+e^-$

QED: (quantum electrodynamics)

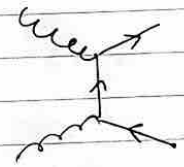
⑧ primitive vertex:



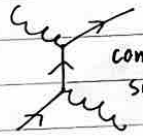
⑨ pair annihilation



⑩ pair production

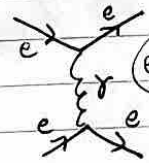


⑪



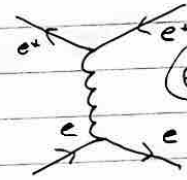
compton scattering

⑫ møller scattering



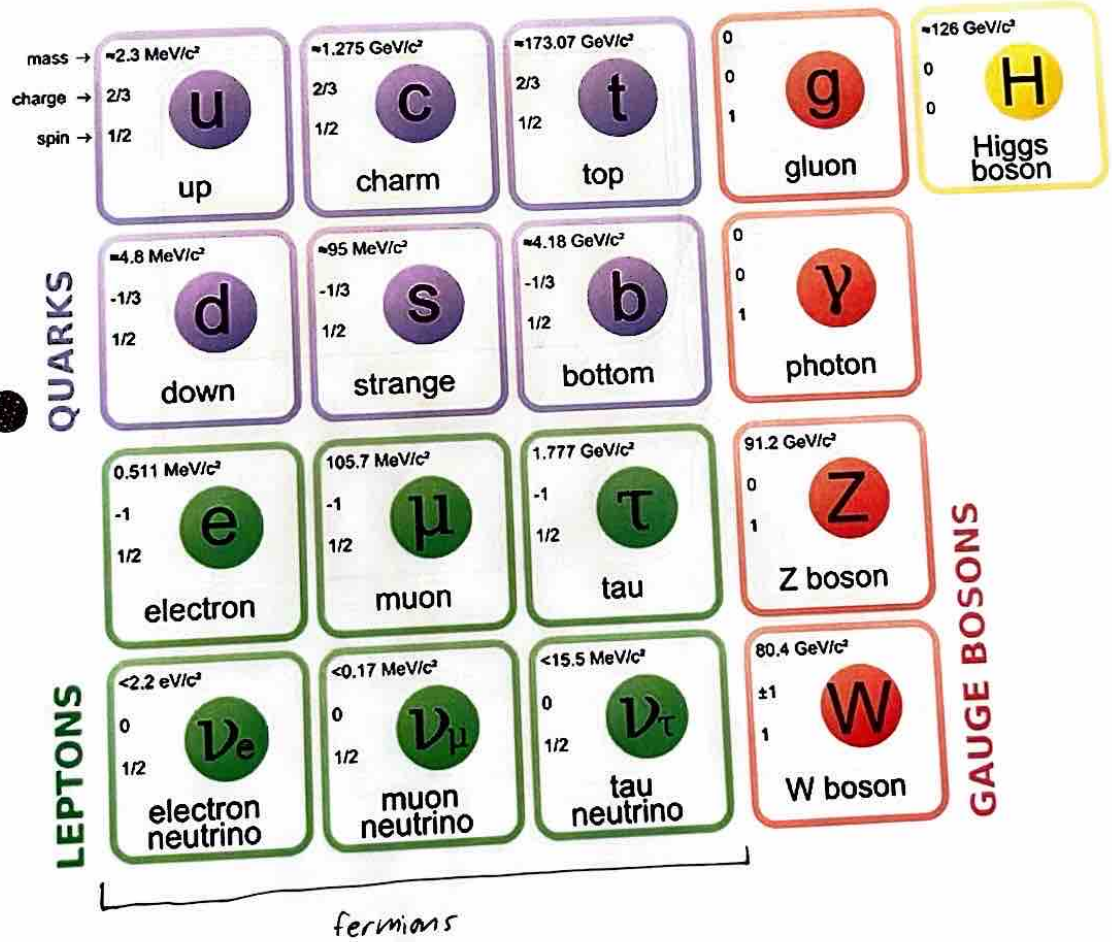
ex. 6

⑬ bhabha scattering



ex. 7

The Standard Model



standard model

ADVANCED PLACEMENT PHYSICS 1 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \mathrm{m^3/(kg \cdot s^2)} = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ 1 atmosphere of pressure, $1\ \mathrm{atm} = 1.0 \times 10^5\ \mathrm{N/m^2} = 1.0 \times 10^5\ \mathrm{Pa}$	Magnitude of the acceleration due to gravity at Earth's surface, $g = 9.8\ \mathrm{m/s^2}$ Magnitude of the gravitational field strength at Earth's surface, $g = 9.8\ \mathrm{N/kg}$

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

UNIT SYMBOLS				
	hertz,	Hz	newton,	N
	joule,	J	pascal,	Pa
	kilogram,	kg	second,	s
	meter,	m	watt,	W

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam**:**

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.
- Fluids are assumed to be ideal, and pipes are assumed to be completely filled by fluid, unless otherwise stated.

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

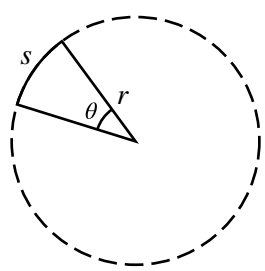
Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$
$s = r\theta$

Rectangular Solid
$V = \ell wh$

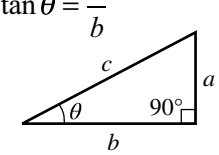
Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$



A = area
b = base
C = circumference
h = height
ℓ = length
r = radius
s = arc length
S = surface area
V = volume
w = width
θ = angle

Right Triangle
$a^2 + b^2 = c^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



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MECHANICS AND FLUIDS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x\left(x - x_0\right)$

$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$

$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\left|\vec{F}_f\right| \le \left|\mu \vec{F}_n\right|$

$\vec{F}_s = -k\Delta\vec{x}$

$a_c = \frac{v^2}{r}$

$K = \frac{1}{2}mv^2$

$W = F_{\parallel}d = Fd\cos\theta$

$\Delta K = \sum W_i = \sum F_{\parallel,i}d_i$

$U_s = \frac{1}{2}k(\Delta x)^2$

$U_G = -\frac{Gm_1m_2}{r}$

$\Delta U_g = mg\Delta y$

$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$

$P_{\text{inst}} = F_{\parallel}v = Fv\cos\theta$

$\vec{p} = m\vec{v}$

$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$

$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$

$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i\vec{v}_i}{\sum m_i}$

a = acceleration
d = distance
E = energy
F = force
J = impulse
k = spring constant
K = kinetic energy
m = mass
p = momentum
P = power
r = radius, distance, or position
t = time
U = potential energy
v = velocity or speed
W = work
x = position
y = height
θ = angle
μ = coefficient of friction

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$

$v = r\omega$

$a_T = r\alpha$

$\tau = r_{\perp}F = rF\sin\theta$

$I = \sum m_i r_i^2$

$I' = I_{\text{cm}} + Md^2$

$\alpha_{\text{sys}} = \frac{\Sigma\tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$

$K = \frac{1}{2}I\omega^2$

$W = \tau\Delta\theta$

$L = I\omega$

$L = rmv\sin\theta$

$\Delta L = \tau\Delta t$

$\Delta x_{\text{cm}} = r\Delta\theta$

$T = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$x = A\cos(2\pi ft)$

$x = A\sin(2\pi ft)$

$\rho = \frac{m}{V}$

$P = \frac{F_{\perp}}{A}$

$P = P_0 + \rho gh$

$P_{\text{gauge}} = \rho gh$

$F_b = \rho Vg$

$A_1v_1 = A_2v_2$

$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$

a = acceleration
A = amplitude or area
d = distance
f = frequency
F = force
h = height
I = rotational inertia
k = spring constant
K = kinetic energy
ℓ = length
L = angular momentum
m = mass
M = mass
P = pressure
r = radius, distance, or position
t = time
T = period
v = velocity or speed
V = volume
W = work
x = position
y = vertical position
α = angular acceleration
θ = angle
ρ = density
τ = torque
ω = angular speed

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ADVANCED PLACEMENT PHYSICS 2 TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Universal gas constant,	$R = 8.31\ \text{J/(mol} \cdot \text{K)}$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}\ \text{J/K}$
1 atmosphere of pressure,	$1\ \text{atm} = 1.0 \times 10^5\ \text{N/m}^2 = 1.0 \times 10^5\ \text{Pa}$
Coulomb constant,	$k = \dfrac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ \dfrac{\text{N} \cdot \text{m}^2}{\text{C}^2}$
Proton mass,	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Neutron mass,	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Electron mass,	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Elementary charge,	$e = 1.60 \times 10^{-19}\ \text{C}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\ (\text{T} \cdot \text{m})/\text{A}$
1 electron volt,	$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s} = 4.14 \times 10^{-15}\ \text{eV} \cdot \text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J} \cdot \text{m} = 1240\ \text{eV} \cdot \text{nm}$
Speed of light,	$c = 3.00 \times 10^8\ \text{m/s}$
Wien's constant,	$b = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8}\ \text{W}/(\text{m}^2 \cdot \text{K}^4)$
1 unified atomic mass unit,	$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg} = 931\ \text{MeV}/c^2$

Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \text{m}^3/(\text{kg} \cdot \text{s}^2) = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$
Magnitude of the acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Magnitude of the gravitational field strength at Earth's surface, $g = 9.8\ \text{N/kg}$

UNIT SYMBOLS	
ampere,	A
coulomb,	C
degree Celsius,	°C
electron volt,	eV
farad,	F
hertz,	Hz
joule,	J
kelvin,	K
kilogram,	kg
meter,	m
mole,	mol
newton,	N
ohm,	Ω
pascal,	Pa
second,	s
tesla,	T
volt,	V
watt,	W

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam:

- The frame of reference of any problem is assumed to be inertial unless otherwise stated.
- Air resistance is assumed to be negligible unless otherwise stated.
- Springs and strings are assumed to be ideal unless otherwise stated.
- The electric potential is zero at an infinite distance from an isolated point charge.
- The direction of current is the direction in which positive charges would drift.
- All batteries, wires, and meters are assumed to be ideal unless otherwise stated.

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ELECTRICITY

$$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} = k\frac{|q_1 q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} = k\frac{|q|}{r^2}$$

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}$$

$$\Delta V = \frac{\Delta U_E}{q}$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$\left|\vec{E}\right| = \left|\frac{\Delta V}{\Delta r}\right|$$

$$C = \frac{Q}{\Delta V}$$

$$C = \kappa\varepsilon_0\frac{A}{d}$$

$$E_C = \frac{Q}{\kappa\varepsilon_0 A}$$

$$U_C = \frac{1}{2}Q\Delta V$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$P = I\Delta V$$

$$I = \frac{\Delta V}{R}$$

$$R_{\text{eq},s} = \sum_i R_i$$

$$\frac{1}{R_{\text{eq},p}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{C_{\text{eq},s}} = \sum_i \frac{1}{C_i}$$

$$C_{\text{eq},p} = \sum_i C_i$$

$$\tau = R_{\text{eq}}C_{\text{eq}}$$

A = area
C = capacitance
d = distance
E = electric field
F = force
I = current
ℓ = length
P = power
q = charge
Q = charge
r = distance, radius, or position
R = resistance
t = time
U = potential energy
V = electric potential
κ = dielectric constant
ρ = resistivity
τ = time constant

MAGNETISM

$$F_B = qvB\sin\theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$F_B = I\ell B\sin\theta$$

$$\Phi_B = \vec{B}\cdot\vec{A}$$

$$\Phi_B = \left|\vec{B}\right|\cos\theta\left|\vec{A}\right|$$

$$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right|$$

$$\mathcal{E} = B\ell v$$

A = area
B = magnetic field
F = force
I = current
ℓ = length
q = charge
r = distance, radius, or position
t = time
v = velocity or speed
$\mathcal{E}$ = emf
θ = angle
Φ = flux

THERMAL PHYSICS

$$P = \frac{F_\perp}{A}$$

$$K_{\text{avg}} = \frac{3}{2}k_B T = \frac{1}{2}mv_{\text{rms}}^2$$

$$\frac{Q}{\Delta t} = \frac{kA\Delta T}{L}$$

$$PV = nRT = Nk_B T$$

$$U = \frac{3}{2}nRT = \frac{3}{2}Nk_B T$$

$$W = -P\Delta V$$

$$\Delta U = Q + W$$

$$Q = mc\Delta T$$

A = area
c = specific heat
F = force
k = thermal conductivity
K = kinetic energy
L = length
m = mass
n = number of moles
N = number of atoms
P = pressure
Q = energy transferred to a system by heating
t = time
T = temperature
U = internal energy
v = velocity or speed
V = volume
W = work done on a system

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WAVES, SOUND, AND OPTICS

$\lambda = \frac{v}{f}$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$

$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$

$\Delta D = m\lambda$

$\Delta D = a\sin\theta$

$a\left(\frac{y_{\min}}{L}\right) \approx m\lambda$

$\Delta D = d\sin\theta$

$d\left(\frac{y_{\max}}{L}\right) \approx m\lambda$

$v_{\text{string}} = \sqrt{\frac{F_T}{m/\ell}}$

$T = \frac{1}{f}$

$x(t) = A\cos(\omega t) = A\cos(2\pi ft)$

$y(x) = A\cos\left(2\pi\frac{x}{\lambda}\right)$

$|f_{\text{beat}}| = |f_1 - f_2|$

a = width
A = amplitude
d = separation
D = path length
f = frequency or focal length
F = force
h = height
ℓ = length
L = distance
m = order or mass
M = magnification
n = index of refraction
s = position
t = time
T = period
v = speed
x = position
y = position
λ = wavelength
θ = angle
ω = angular frequency

MODERN PHYSICS

$E = hf$

$\lambda = \frac{h}{p}$

$\lambda = \frac{c}{f}$

$\lambda_{\max} = \frac{b}{T}$

$P = A\sigma T^4$

$K_{\max} = hf - \phi$

$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$

$E = mc^2$

$N = N_0 e^{-\lambda t}$

$\lambda = \frac{\ln 2}{t_{1/2}}$

A = area
b = constant
E = energy
f = frequency
K = kinetic energy
m = mass
N = number of particles
p = momentum
P = power
t = time
T = absolute temperature
θ = angle
λ = wavelength or decay constant
σ = constant
ϕ = work function

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

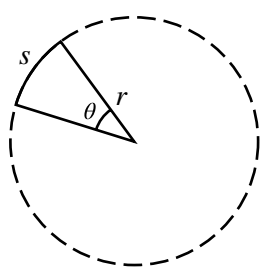
Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$
$s = r\theta$

Rectangular Solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$



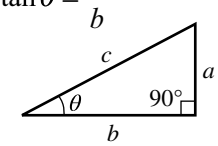
A = area
b = base
C = circumference
h = height
ℓ = length
r = radius
s = arc length
S = surface area
V = volume
w = width
θ = angle

Right Triangle
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



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MECHANICS AND FLUIDS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x\left(x - x_0\right)$

$\vec{x}_{\text{cm}} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$

$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$

$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$

$\left|\vec{F}_f\right| \le \left|\mu \vec{F}_n\right|$

$\vec{F}_s = -k\Delta\vec{x}$

$a_c = \frac{v^2}{r}$

$K = \frac{1}{2}mv^2$

$W = F_{\parallel}d = Fd\cos\theta$

$\Delta K = \sum W_i = \sum F_{\parallel,i}d_i$

$U_s = \frac{1}{2}k(\Delta x)^2$

$U_G = -\frac{Gm_1m_2}{r}$

$\Delta U_g = mg\Delta y$

$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$

$P_{\text{inst}} = F_{\parallel}v = Fv\cos\theta$

$\vec{p} = m\vec{v}$

$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$

$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$

$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i\vec{v}_i}{\sum m_i}$

a = acceleration
d = distance
E = energy
F = force
J = impulse
k = spring constant
K = kinetic energy
m = mass
p = momentum
P = power
r = radius, distance, or position
t = time
U = potential energy
v = velocity or speed
W = work
x = position
y = height
θ = angle
μ = coefficient of friction

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$

$v = r\omega$

$a_T = r\alpha$

$\tau = r_{\perp}F = rF\sin\theta$

$I = \sum m_i r_i^2$

$I' = I_{\text{cm}} + Md^2$

$\alpha_{\text{sys}} = \frac{\Sigma\tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$

$K = \frac{1}{2}I\omega^2$

$W = \tau\Delta\theta$

$L = I\omega$

$L = rmv\sin\theta$

$\Delta L = \tau\Delta t$

$\Delta x_{\text{cm}} = r\Delta\theta$

$T = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$x = A\cos(2\pi ft)$

$x = A\sin(2\pi ft)$

$\rho = \frac{m}{V}$

$P = \frac{F_{\perp}}{A}$

$P = P_0 + \rho gh$

$P_{\text{gauge}} = \rho gh$

$F_b = \rho Vg$

$A_1v_1 = A_2v_2$

$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$

a = acceleration
A = amplitude or area
d = distance
f = frequency
F = force
h = height
I = rotational inertia
k = spring constant
K = kinetic energy
ℓ = length
L = angular momentum
m = mass
M = mass
P = pressure
r = radius, distance, or position
t = time
T = period
v = velocity or speed
V = volume
W = work
x = position
y = vertical position
α = angular acceleration
θ = angle
ρ = density
τ = torque
ω = angular speed

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AP STATISTICS

Formula Sheet and Tables

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Formulas for AP Statistics

I. Descriptive Statistics

$$\bar{x} = \frac{1}{n}\sum x_i = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2} = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}}$$

$$\hat{y} = a + bx$$

$$\bar{y} = a + b\bar{x}$$

$$r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b = r\frac{s_y}{s_x}$$

II. Probability and Distributions

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Probability Distribution	Mean	Standard Deviation
Discrete random variable, X	$\mu_X = E\left(X\right) = \sum x_i \cdot P\left(x_i\right)$	$\sigma_X = \sqrt{\sum\left(x_i - \mu_X\right)^2 \cdot P\left(x_i\right)}$
If X has a **binomial** distribution with parameters n and p, then: $P\left(X = x\right) = \binom{n}{x} p^x \left(1-p\right)^{n-x}$ where $x = 0, 1, 2, 3, \ldots, n$	$\mu_X = np$	$\sigma_X = \sqrt{np\left(1-p\right)}$
If X has a **geometric** distribution with parameter p, then: $P\left(X = x\right) = \left(1-p\right)^{x-1} p$ where $x = 1, 2, 3, \ldots$	$\mu_X = \frac{1}{p}$	$\sigma_X = \frac{\sqrt{1-p}}{p}$

III. Sampling Distributions and Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard error of the statistic}}$

Confidence interval: $\text{statistic} \pm \left(\text{critical value}\right)\left(\text{standard error of statistic}\right)$

Chi-square statistic: $\chi^2 = \sum \dfrac{\left(\text{observed} - \text{expected}\right)^2}{\text{expected}}$

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III. Sampling Distributions and Inferential Statistics *(continued)*

Sampling distributions for proportions:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\hat{p}$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	$s_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
For two populations: $\hat{p}_1 - \hat{p}_2$	$\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2$	$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$s_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ When $p_1 = p_2$ is assumed: $s_{\hat{p}_1-\hat{p}_2} = \sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $\hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$

Sampling distributions for means:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For one population: $\bar{X}$	$\mu_{\bar{X}} = \mu$	$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$	$s_{\bar{X}} = \frac{s}{\sqrt{n}}$
For two populations: $\bar{X}_1 - \bar{X}_2$	$\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2$	$\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$s_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

Sampling distributions for simple linear regression:

Random Variable	Parameters of Sampling Distribution		Standard Error* of Sample Statistic
For slope: b	$\mu_b = \beta$	$\sigma_b = \frac{\sigma}{\sigma_x\sqrt{n}}$, where $\sigma_x = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$	$s_b = \frac{s}{s_x\sqrt{n-1}}$, where $s = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-2}}$ and $s_x = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$

*Standard deviation is a measurement of variability from the theoretical population. Standard error is the estimate of the standard deviation. If the standard deviation of the statistic is assumed to be known, then the standard deviation should be used instead of the standard error.

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Tables for AP Statistics

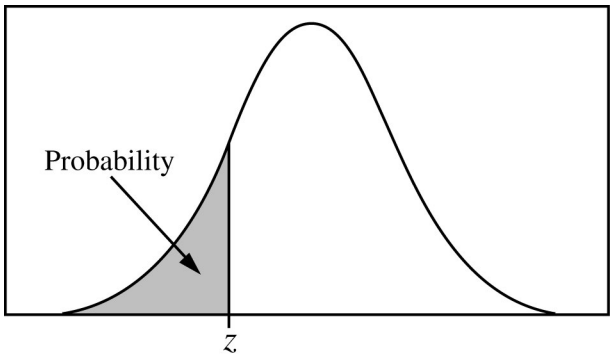

Table entry for z is the probability lying below z.

Table A Standard Normal Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

continued on next page

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Table A Standard Normal Probabilities *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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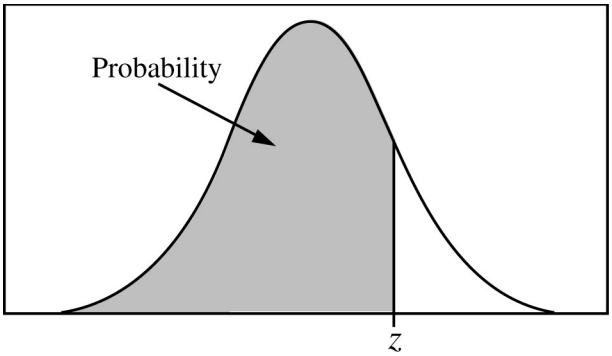

Table entry for z is the probability lying below z.

Table A Standard Normal Probabilities *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936

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Table A Standard Normal Probabilities *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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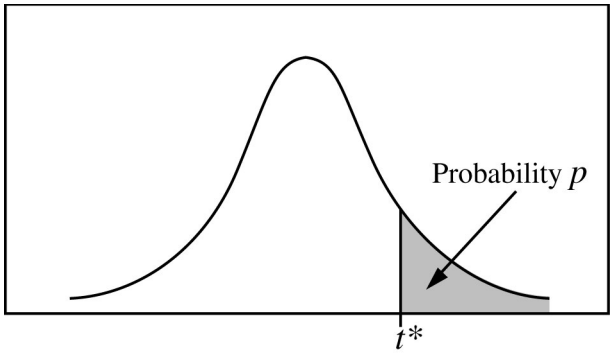

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

Table B t Distribution Critical Values

	Tail Probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850

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Table B t Distribution Critical Values *(continued)*

	Tail Probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

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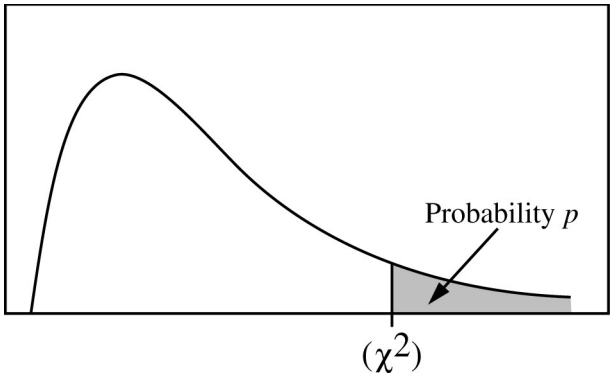

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 Critical Values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50

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Table C χ^2 Critical Values *(continued)*

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

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AP® CHEMISTRY EQUATIONS AND CONSTANTS

UNIT SYMBOLS	
gram,	g
mole,	mol
liter,	L
meter,	m
second,	s
hertz,	Hz
atmosphere,	atm
millimeter of mercury,	mm Hg
degree Celsius,	°C
kelvin,	K
joule,	J
volt,	V
coulomb,	C
ampere,	A

UNIT CONVERSIONS
1 hertz = 1 s^{-1}
1 atm = 760 mm Hg = 760 torr
K = °C + 273.15
1 volt = $\frac{1 \text{ joule}}{1 \text{ coulomb}}$
1 ampere = $\frac{1 \text{ coulomb}}{1 \text{ second}}$

METRIC PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

ATOMIC STRUCTURE

$$E = h\nu$$

$$c = \lambda\nu$$

$$F_{coulombic} \propto \frac{q_1 q_2}{r^2}$$

E = energy
ν = frequency
λ = wavelength
F = force
q = charge
r = separation
Planck's constant, $h = 6.626 \times 10^{-34}$ J s
Speed of light, $c = 2.998 \times 10^{8}$ m s^{-1}
Avogadro's number $= 6.022 \times 10^{23}$ mol^{-1}

GASES, LIQUIDS, AND SOLUTIONS

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$PV = nRT$$

$$P_A = P_{total} \times X_A, \text{ where } X_A = \frac{\text{moles A}}{\text{total moles}}$$

$$P_{total} = P_A + P_B + P_C + \ldots$$

$$n = \frac{m}{\boldsymbol{M}}$$

$$D = \frac{m}{V}$$

$$KE = \frac{1}{2}mv^2$$

$$M = \frac{n_{solute}}{L_{solution}}$$

$$A = \varepsilon bc$$

P = pressure
V = volume
T = temperature
n = number of moles
X = mole fraction
m = mass
$\boldsymbol{M}$ = molar mass
D = density
KE = kinetic energy
v = velocity
M = molarity
A = absorbance
ε = molar absorptivity
b = path length
c = concentration
Gas constant, $R = 8.314$ J mol^{-1} K^{-1}
$= 0.08206$ L atm K^{-1} mol^{-1}
STP = 273.15 K and 1.0 atm
Ideal gas at STP = 22.4 L mol^{-1}

KINETICS

$$[A]_t - [A]_0 = -kt$$

$$\ln[A]_t - \ln[A]_0 = -kt$$

$$\frac{1}{[A]_t} - \frac{1}{[A]_0} = kt$$

$$t_{1/2} = \frac{0.693}{k}$$

k = rate constant
t = time
$t_{1/2}$ = half-life

EQUILIBRIUM

$$K_c = \frac{[C]^c[D]^d}{[A]^a[B]^b}, \text{ where } a\,A + b\,B \rightleftarrows c\,C + d\,D$$

$$K_p = \frac{(P_C)^c(P_D)^d}{(P_A)^a(P_B)^b}$$

$$K_w = [H_3O^+][OH^-] = 1.0 \times 10^{-14} \text{ at } 25°C$$

$$pK_w = 14 = pH + pOH \text{ at } 25°C$$

$$pH = -\log[H_3O^+], \qquad pOH = -\log[OH^-]$$

$$K_a = \frac{[H_3O^+][A^-]}{[HA]}, \qquad K_b = \frac{[OH^-][HB^+]}{[B]}$$

$$pK_a = -\log K_a, \qquad pK_b = -\log K_b$$

$$K_w = K_a \times K_b, \qquad pK_w = pK_a + pK_b$$

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

Equilibrium Constants

K_c (molar concentrations)
K_p (gas pressures)
K_w (water)
K_a (acid)
K_b (base)

THERMODYNAMICS/ELECTROCHEMISTRY

$$q = mc\Delta T$$

$$\Delta H^\circ_{reaction} = \sum \Delta H^\circ_{f\ products} - \sum \Delta H^\circ_{f\ reactants}$$

$$\Delta S^\circ_{reaction} = \sum S^\circ_{products} - \sum S^\circ_{reactants}$$

$$\Delta G^\circ_{reaction} = \sum \Delta G^\circ_{f\ products} - \sum \Delta G^\circ_{f\ reactants}$$

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$
$$= -RT \ln K$$
$$= -nFE^\circ$$

$$I = \frac{q}{t}$$

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln Q$$

q = heat
m = mass
c = specific heat capacity
T = temperature
S° = standard entropy
H° = standard enthalpy
G° = standard Gibbs free energy
R = gas constant
K = equilibrium constant
n = number of moles of electrons
E° = standard potential
I = current (amperes)
q = charge (coulombs)
t = time (seconds)
Q = reaction quotient
Faraday's constant, F = 96,485 coulombs / 1 mol e^-

AP® CHEMISTRY EQUATIONS AND CONSTANTS

UNIT SYMBOLS	
gram,	g
mole,	mol
liter,	L
meter,	m
second,	s
hertz,	Hz
atmosphere,	atm
millimeter of mercury,	mm Hg
degree Celsius,	°C
kelvin,	K
joule,	J
volt,	V
coulomb,	C
ampere,	A

UNIT CONVERSIONS
1 hertz = 1 s^{-1}
1 atm = 760 mm Hg = 760 torr
K = °C + 273.15
1 volt = $\frac{1 \text{ joule}}{1 \text{ coulomb}}$
1 ampere = $\frac{1 \text{ coulomb}}{1 \text{ second}}$

METRIC PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

ATOMIC STRUCTURE

$$E = h\nu$$

$$c = \lambda\nu$$

$$F_{coulombic} \propto \frac{q_1 q_2}{r^2}$$

E = energy
ν = frequency
λ = wavelength
F = force
q = charge
r = separation
Planck's constant, $h = 6.626 \times 10^{-34}$ J s
Speed of light, $c = 2.998 \times 10^{8}$ m s^{-1}
Avogadro's number $= 6.022 \times 10^{23}$ mol^{-1}

GASES, LIQUIDS, AND SOLUTIONS

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$PV = nRT$$

$$P_A = P_{total} \times X_A, \text{ where } X_A = \frac{\text{moles A}}{\text{total moles}}$$

$$P_{total} = P_A + P_B + P_C + \ldots$$

$$n = \frac{m}{\boldsymbol{M}}$$

$$D = \frac{m}{V}$$

$$KE = \frac{1}{2}mv^2$$

$$M = \frac{n_{solute}}{L_{solution}}$$

$$A = \varepsilon bc$$

P = pressure
V = volume
T = temperature
n = number of moles
X = mole fraction
m = mass
$\boldsymbol{M}$ = molar mass
D = density
KE = kinetic energy
v = velocity
M = molarity
A = absorbance
ε = molar absorptivity
b = path length
c = concentration
Gas constant, $R = 8.314$ J mol^{-1} K^{-1}
$= 0.08206$ L atm K^{-1} mol^{-1}
STP = 273.15 K and 1.0 atm
Ideal gas at STP = 22.4 L mol^{-1}

KINETICS

$$[A]_t - [A]_0 = -kt$$

$$\ln[A]_t - \ln[A]_0 = -kt$$

$$\frac{1}{[A]_t} - \frac{1}{[A]_0} = kt$$

$$t_{1/2} = \frac{0.693}{k}$$

k = rate constant
t = time
$t_{1/2}$ = half-life

EQUILIBRIUM

$$K_c = \frac{[C]^c[D]^d}{[A]^a[B]^b}, \text{ where } a\,A + b\,B \rightleftarrows c\,C + d\,D$$

$$K_p = \frac{(P_C)^c(P_D)^d}{(P_A)^a(P_B)^b}$$

$$K_w = [H_3O^+][OH^-] = 1.0 \times 10^{-14} \text{ at } 25°C$$

$$pK_w = 14 = pH + pOH \text{ at } 25°C$$

$$pH = -\log[H_3O^+], \qquad pOH = -\log[OH^-]$$

$$K_a = \frac{[H_3O^+][A^-]}{[HA]}, \qquad K_b = \frac{[OH^-][HB^+]}{[B]}$$

$$pK_a = -\log K_a, \qquad pK_b = -\log K_b$$

$$K_w = K_a \times K_b, \qquad pK_w = pK_a + pK_b$$

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

Equilibrium Constants

K_c (molar concentrations)
K_p (gas pressures)
K_w (water)
K_a (acid)
K_b (base)

THERMODYNAMICS/ELECTROCHEMISTRY

$$q = mc\Delta T$$

$$\Delta H^\circ_{reaction} = \sum \Delta H^\circ_{f\ products} - \sum \Delta H^\circ_{f\ reactants}$$

$$\Delta S^\circ_{reaction} = \sum S^\circ_{products} - \sum S^\circ_{reactants}$$

$$\Delta G^\circ_{reaction} = \sum \Delta G^\circ_{f\ products} - \sum \Delta G^\circ_{f\ reactants}$$

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$
$$= -RT \ln K$$
$$= -nFE^\circ$$

$$I = \frac{q}{t}$$

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\ln Q$$

q = heat
m = mass
c = specific heat capacity
T = temperature
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