BLP Discrete Choice Model With Choice Sets

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1 Model Setup

There are $T = 200$ markets and in each market, there are $J = 10$ products(excluding the outside option), The utility of consumer i consuming product j in market t is given by

$$u_{ijt} = x_{jt}^{'}\beta_i + \alpha_i p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

where

- x_{jt}: Product-market specific characteristics.
- p_{jt}: Product-market specific prices
- ξ_{jt}: Product-market specific mean zero shocks
- ϵ_{ijt}: i.i.d. individual preference shocks ~ T1EV
- β_i: individual $i's$ taste on attributes
- α_i : individual $i's$ taste on price.

The utility of consuming the outside option, u_{i0t} is normalized to ϵ_{i0t}. Indvidual $i's$ taste(β_i &α_i) is affected by i's logged income I_i and V_i . I_i is drawn from $N(\mu_j, \sigma_j^I)$ where μ_j and σ_j^I are the mean and standard deviation of income in market j where individual i resides. On the other hand, V_i is drawn from $N(0, C)$ where C is as follow:

$$\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \end{bmatrix}$$

The equation for $\begin{pmatrix} \beta_i \\ \alpha_i \end{pmatrix}$ can be written as follow:

$$\begin{pmatrix} \beta_i \\ \alpha_i \end{pmatrix} = \begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix} + \Pi I_i + V_i$$

1.1 Exposure to different subsets of products

Due to variation in advertising, some individuals only observe a subset of the products instead of all of the products. Specifically, with probability $p_{j,all}$, every individual in market j observe all the products and with probability $1 - p_{j,all}$, every individual in market j only observes products 1 to 5. The outside option is always observed. The market share of product j in market t, s_{jt}^{all} conditional on every individual in market t observing all the products? (2 points)

$$\sum_{i=1}^{n} \frac{exp(\underbrace{\begin{bmatrix} x_{j,t}^{'} & p_{j,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{j,t})}{1+\sum_{k=1}^{10} exp(\underbrace{\begin{bmatrix} x_{k,t}^{'} & p_{k,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{k,t})}$$

The market share of product j in market t, s_{jt}^{subset} conditional on every individual in market t observing only products 1 to 5:

For the first five products in each market, we have:

$$s_{jt}^{subset} = \sum_{i=1}^{n} \frac{exp(\underbrace{\begin{bmatrix} x_{j,t}^{'} & p_{j,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{j,t})}{1+\sum_{k=1}^{5} exp(\underbrace{\begin{bmatrix} x_{k,t}^{'} & p_{k,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{k,t})}$$

For the next five products in each market, s_{jt}^{subset} equals to zero

$$s_{jt}^{subset} = p_{j,all} \sum_{i=1}^{n} \frac{exp(\underbrace{\begin{bmatrix} x_{j,t}^{'} & p_{j,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{j,t})}{1+\sum_{k=1}^{10} exp(\underbrace{\begin{bmatrix} x_{k,t}^{'} & p_{k,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{k,t})} +$$

$$\mathbf{1}(i \in (1,2,3,4,5))(1-p_{j,all}) \sum_{i=1}^{n} \frac{exp(\underbrace{\begin{bmatrix} x_{j,t}^{'} & p_{j,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{j,t})}{1+\sum_{k=1}^{5} exp(\underbrace{\begin{bmatrix} x_{k,t}^{'} & p_{k,t} \end{bmatrix}}_{1\times 5} \begin{bmatrix} \underbrace{\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}}_{5\times 1} + \underbrace{\begin{pmatrix} \pi \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{5\times 1} \underbrace{I_i}_{1\times 1} + \underbrace{L}_{5\times 5} \times \underbrace{B_i}_{5\times 1} \end{bmatrix} + \xi_{k,t})}$$

2 Optimization Setup

2.1 MPEC

The parameters are estimated via MPEC. The optimization problem is given by:

$$\underset{\theta,\eta,\delta}{min} \quad \eta' W \eta$$

subject to

$$\begin{aligned} g(\delta - x^{'}\begin{pmatrix} \bar{\beta} \\ \bar{\alpha} \end{pmatrix}) &= \eta \\ s(p_t, x_t, \delta; \theta) &= S \\ \pi &\geq 1 \end{aligned}$$

where $\theta = (\bar{\beta}, \bar{\alpha}, \pi)$

3 Test Files and Computation

The data are files are as follow:

- MktData1.csv: Contains the market shares of each product in each market, product-market specific characteristics, and prices
- MktData2.csv: Contains the mean of income, standard deviation of income, probability of every individual observing all products and probability of every individual observing products 1 to 5.

The results of the test files are given by:

Parameters:

Parameters	*Estimates*
β_1	*1.78*
β_2	*1.39*
β_3	*1.33*
β_4	*0.59*
α	*-2.69*
σ_1	*1.16*
σ_2	*0.85*
σ_3	*0.74*
σ_4	*0.74*
σ_5	*0.32*
π	*1.00*

The coviance matrix is as follow:

$$\begin{bmatrix} 28.706 & 1.148 & 0.971 & -0.698 & -0.146 & -0.012 & -0.283 & 0.256 & -0.024 & -0.060 & 21.751 \\ 1.148 & 0.180 & 0.146 & -0.094 & 0.013 & 0.001 & -0.044 & 0.015 & -0.002 & 0.006 & 0.898 \\ 0.971 & 0.146 & 0.121 & -0.079 & 0.007 & 0.000 & -0.032 & 0.014 & -0.002 & 0.003 & 0.754 \\ -0.698 & -0.094 & -0.079 & 0.054 & -0.003 & -0.001 & 0.020 & -0.011 & 0.002 & -0.001 & -0.540 \\ -0.146 & 0.013 & 0.007 & -0.003 & 0.011 & 0.002 & -0.005 & -0.004 & 0.000 & 0.005 & -0.095 \\ -0.012 & 0.001 & 0.000 & -0.001 & 0.002 & 0.003 & -0.001 & 0.001 & -0.002 & 0.001 & -0.007 \\ -0.283 & -0.044 & -0.032 & 0.020 & -0.005 & -0.001 & 0.035 & -0.003 & -0.002 & -0.002 & -0.228 \\ 0.256 & 0.015 & 0.014 & -0.011 & -0.004 & 0.001 & -0.003 & 0.006 & -0.001 & -0.002 & 0.191 \\ -0.024 & -0.002 & -0.002 & 0.002 & 0.000 & -0.002 & -0.002 & -0.001 & 0.002 & 0.000 & -0.018 \\ -0.060 & 0.006 & 0.003 & -0.001 & 0.005 & 0.001 & -0.002 & -0.002 & 0.000 & 0.002 & -0.039 \\ 21.751 & 0.898 & 0.754 & -0.540 & -0.095 & -0.007 & -0.228 & 0.191 & -0.018 & -0.039 & 16.508 \end{bmatrix}$$