Multi-model Analysis of Hemodynamics in VSD

Group 7

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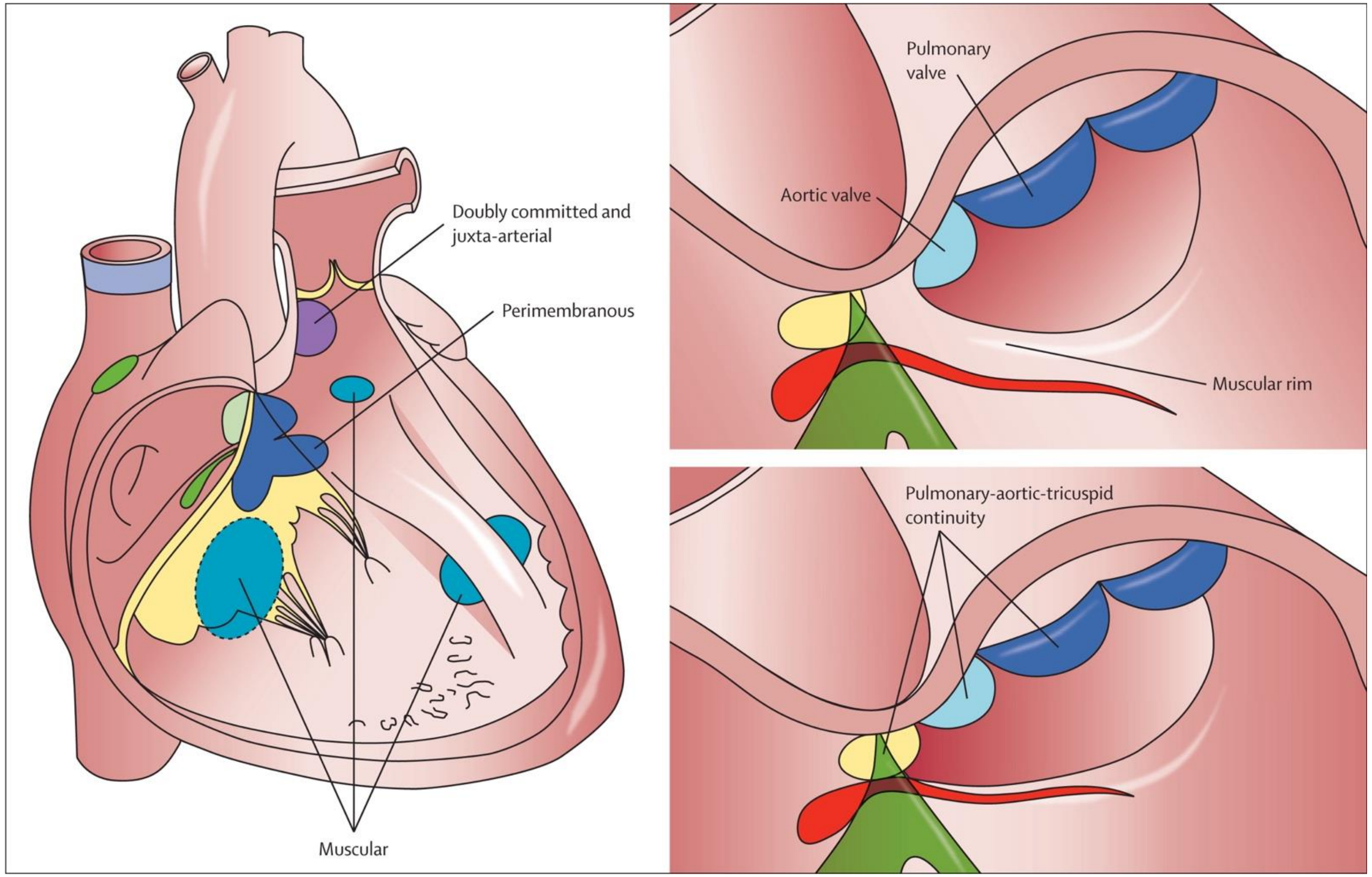
BACKGROUND

VSD

Xu Shen

Introduction of VSD



Penny, D. J., & Vick, G. W. (2011)

Ventricular septal defect (VSD) is one of the most common congenital heart diseases, accounting for approximately **20-30%** of all congenital cardiac defects.

Larger defects can lead to symptoms such as failure to thrive, recurrent pulmonary infections, and heart failure

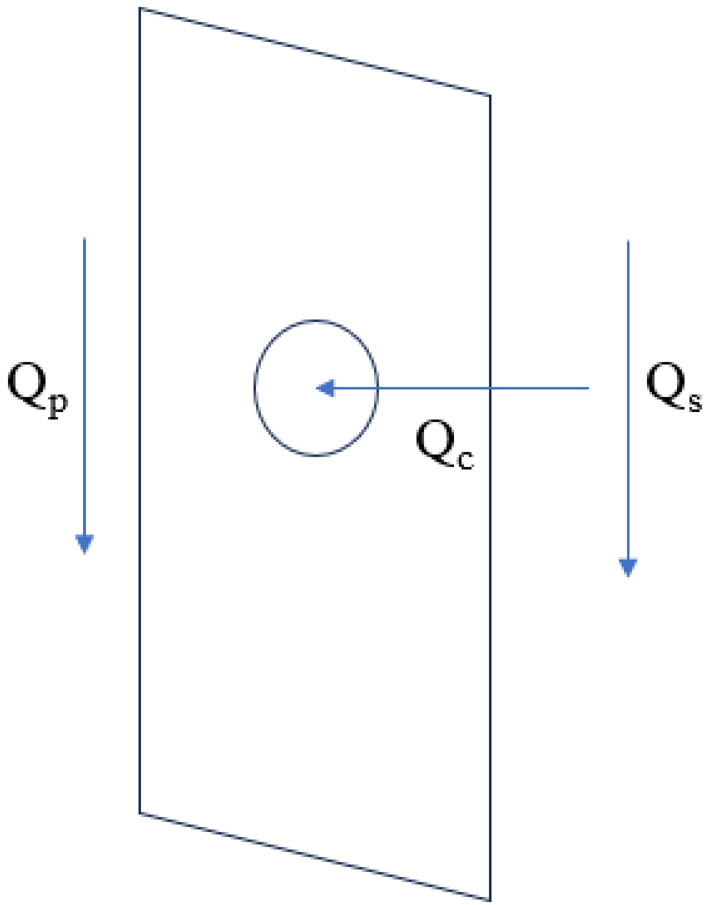
VSD leads to left-to-right shunting. [1]**Qp/Qs≥1.5** indicates hemodynamic significance, requiring intervention.

1 Qp: Pulmonary Blood Flow; Qs: Systemic Blood Flow

ORIFICE PLATE MODEL

Xu shen

Orifice Plate Model

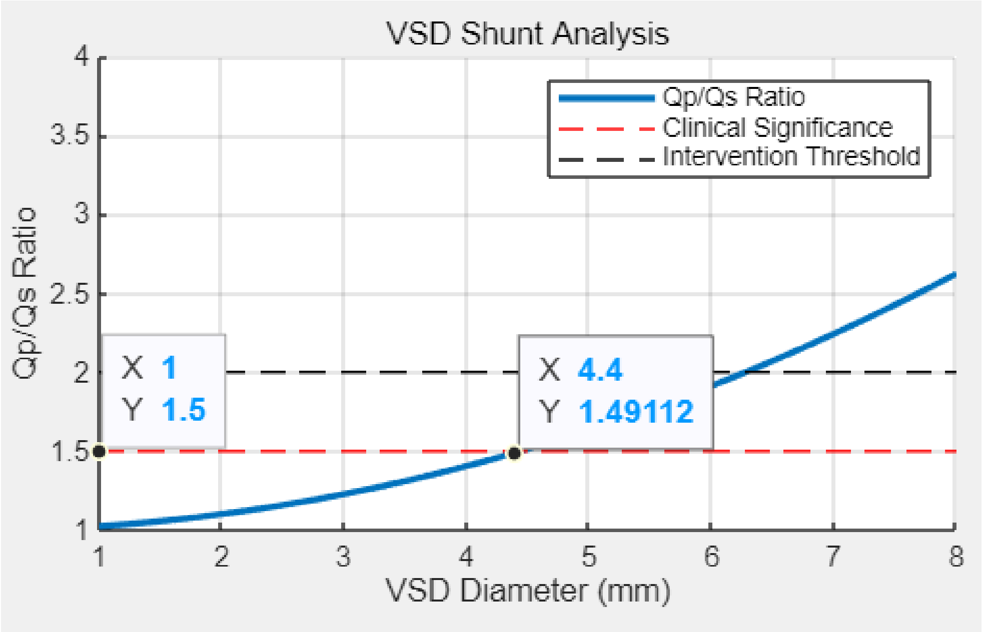


The classic formula for the orifice flow model

$$Q = c_d A \sqrt{\frac{2\Delta p}{\rho}}$$

Q: the **flow rate** through the orifice
c_d: is the **flow coefficient**
A: the **effective cross-sectional area** of the orifice
Δp: the **pressure difference** across the orifice
ρ: the **density** of the fluid

Orifice Plate Model

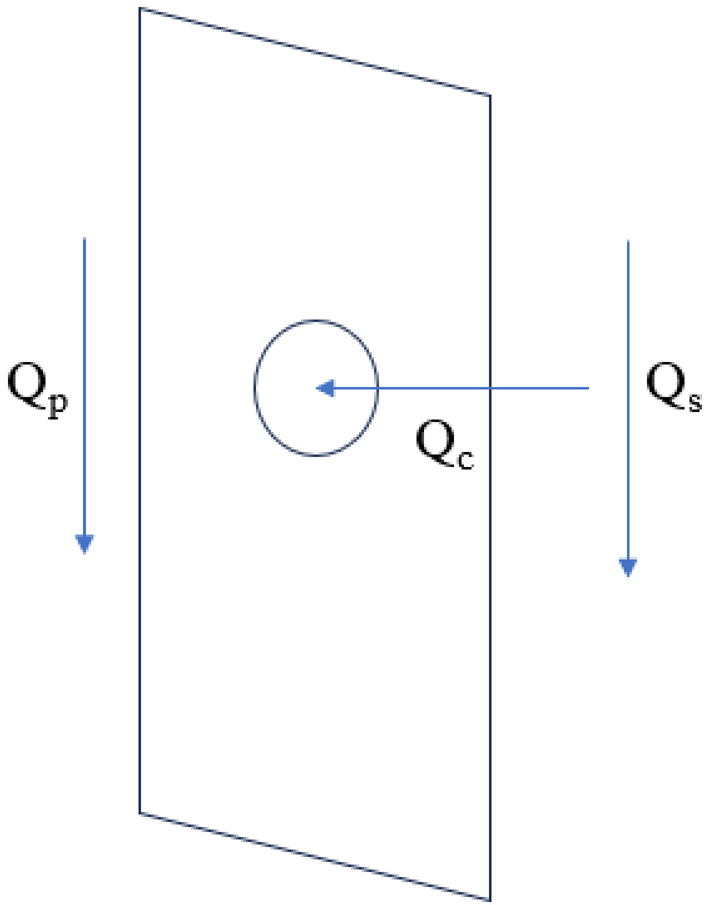


$$Q_c = c_d A \sqrt{\frac{2\Delta p}{\rho}}$$

$$\frac{Q_p}{Q_s} = \frac{Q_s + Q_c}{Q_s} = 1 + \frac{Q_c}{Q_s}$$

c_d = 0.6
Δp = (60, 110) mmHg
ρ = 1.06 g/ml
Q_c = shunt flow in the cardiac septal defect
Q_p = blood ejected from the right ventricle
Q_s = blood returning from the systemic circulation

Orifice Plate Model



Limitations

- Neglect **pressure variations** caused by cardiac cycles
- Disregard **vascular elasticity**
- Idealize the defect as a **perfect circle**

Future studies

- Enhance the model by incorporating **time-varying pressure terms**
- Considering vascular wall **elasticity**

TUBE MODEL

Model

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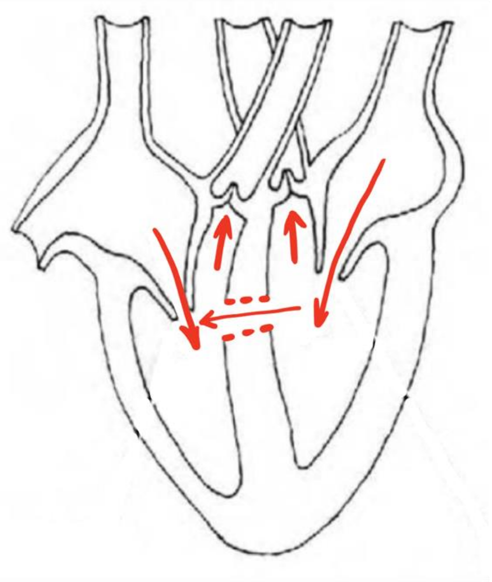
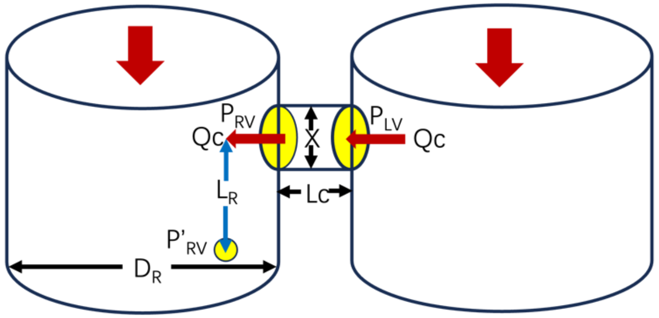
Simplification





Wall

cylindrical tubes

- Rigid
- Uniform
- Straight

Blood

Newtonian

- Steady
- Laminar
- Incompressible

Simplification

$$P'_{RV} = f(X, L_C)$$

INPUT

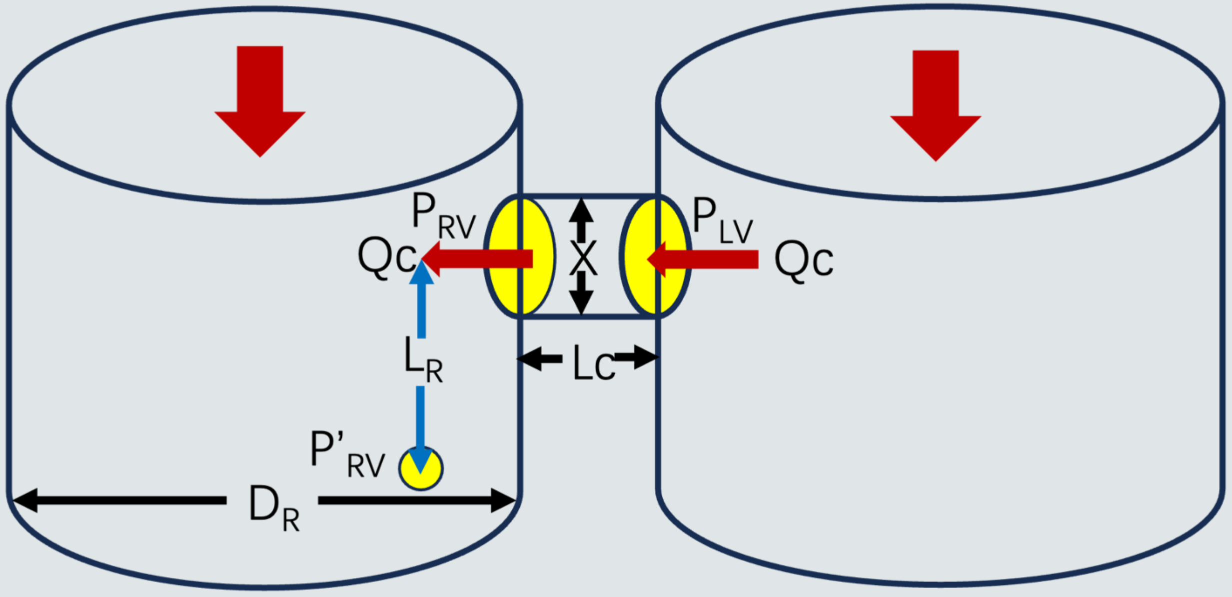
Constants

D_R P_{RV}

L_C P_{LV}

Variables

L_R X



OUTPUT

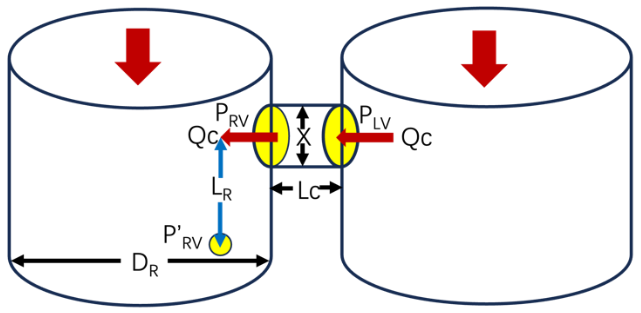
P'_{RV}

Description

Poiseuille's law

$$R = \frac{128\eta L}{\pi D^4}$$

$$Q = \frac{\Delta P}{R} = \frac{\Delta P \pi D^4}{128\eta L}$$



$$\Delta Q_R = Q_c$$

$$Q_c = \frac{(P_L - P_R)\pi x^4}{128\eta L_c}$$

$$\Delta Q_R = \frac{P_R - P'_{R0}}{R_R} - \frac{P_R - P'_R}{R_R}$$

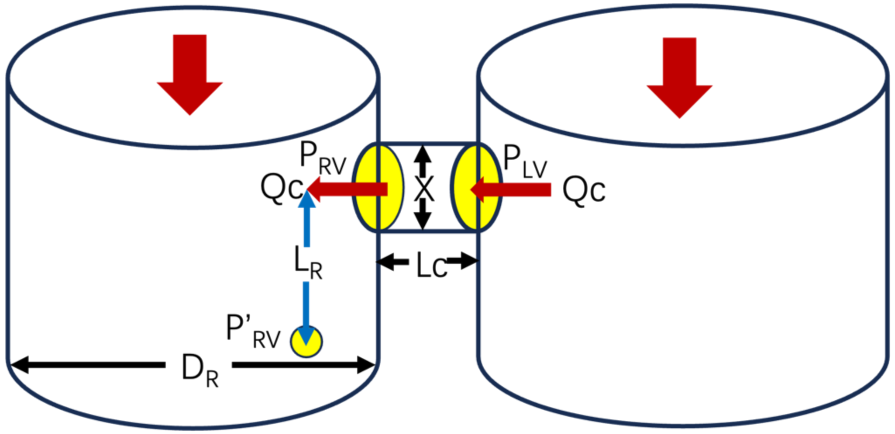
$$= \frac{P'_{R0} - P'_R}{128\eta L_R}\pi D_R^4$$

Description

Poiseuille's law

$$R = \frac{128\eta L}{\pi D^4}$$

$$Q = \frac{\Delta P}{R} = \frac{\Delta P \pi D^4}{128\eta L}$$



$$\Delta Q_R = Q_c$$

$$Q_c = \frac{(P_L - P_R)\pi x^4}{128\eta L_c}$$

$$\Delta Q_R = \frac{P_R - P'_{R0}}{R_R} - \frac{P_R - P'_R}{R_R}$$
$$= \frac{P'_{R0} - P'_R}{128\eta L_R}\pi D_R^4$$

$$\Delta P_R = P'_R - P'_{R0}$$
$$= \frac{(P_L - P_R)x^4 \cdot L_R}{L_c \cdot D_R^4}$$

TUBE MODEL

Result

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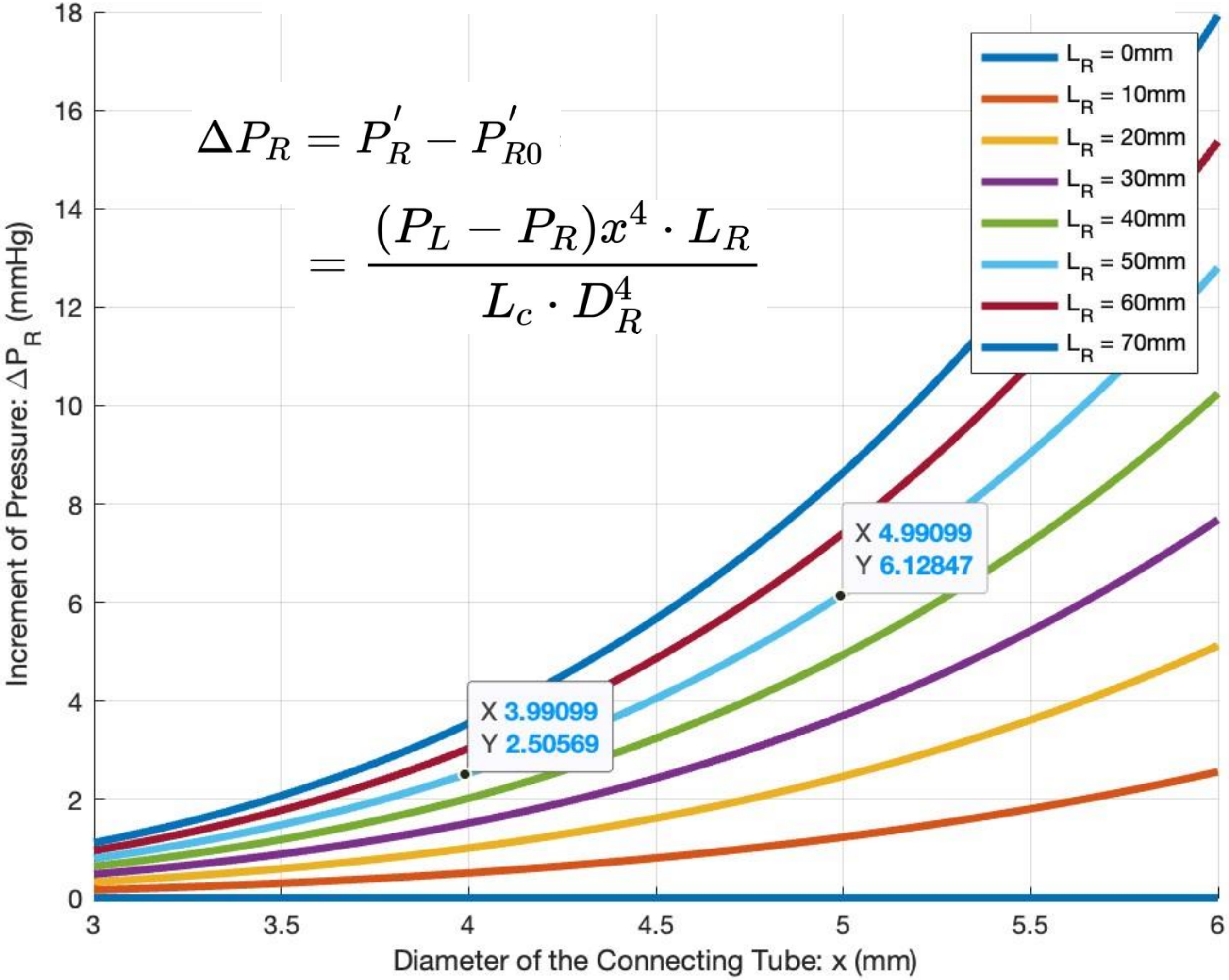
Tendency

Pressure Increment vs Defect Diameter



$$P_L = 110\,\text{mmHg} \quad D_R = 15\,\text{mm}$$

$$P_R = 25\,\text{mmHg} \quad L_c = 8.5\text{mm}$$

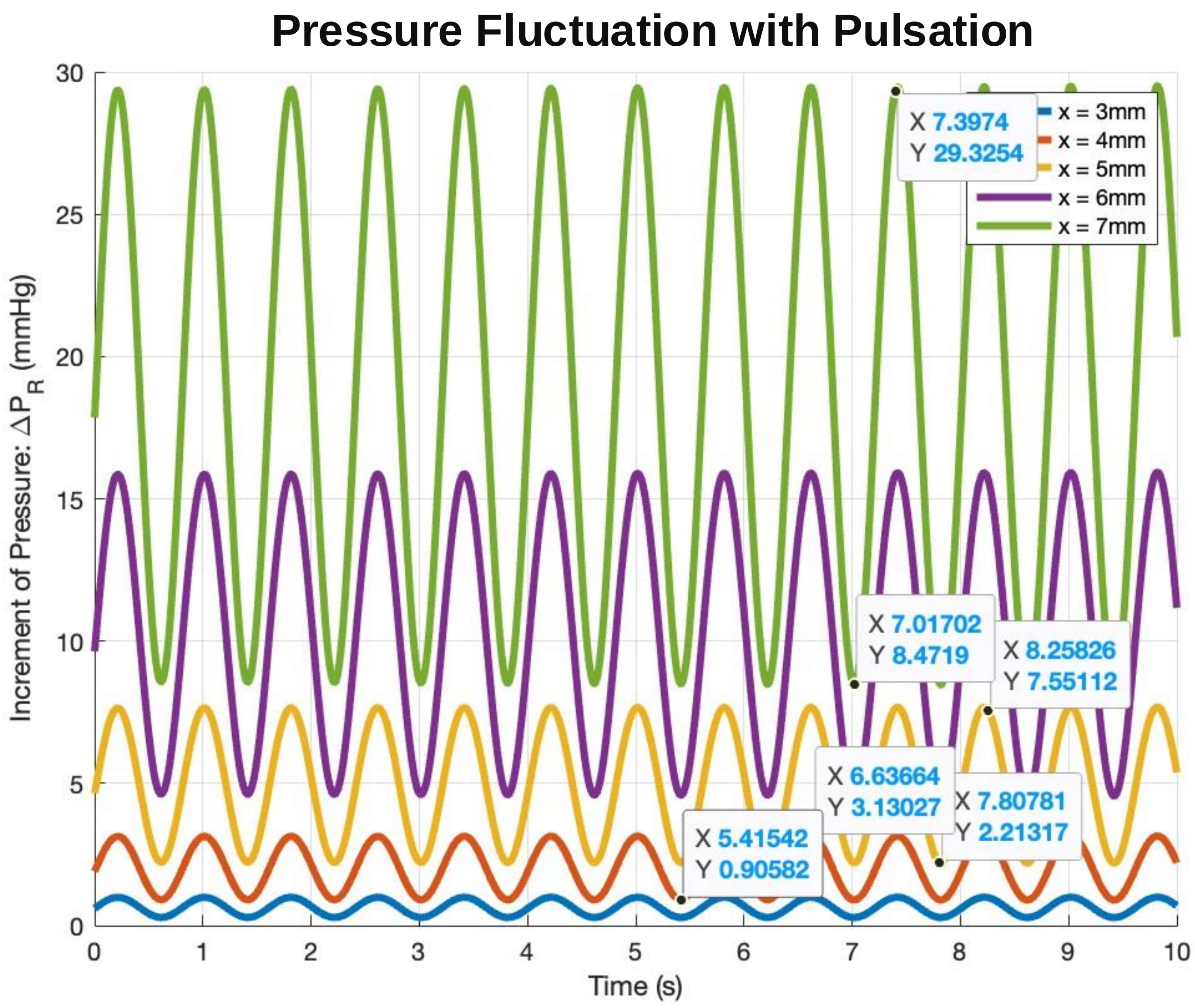
More Hallmarks for Clinical determination

increases sharply

---- call for precise clinical threshold

Increases with L_R

Dynamic



$$L_R = 40\,\text{mm}$$

$$p_{LV}(t) = 65 + 55\sin(7.85t)$$

$$p_{RV}(t) = 15 + 10\sin\left(7.85t + \frac{\pi}{6}\right)$$

+ Time Dimension

enables a more accurate assessment

X ↑

- **mean pressure**
- **oscillation amplitude**

↑

TUBE MODEL

Discussion

Sijie Li

Discussion

Model Limitations

- **Assumptions Matter!**
- **Highly Specific**

Not suitable for evaluating other metrics like $\frac{Q_p}{Q_s}$

BUT, may not necessary!

Idealized Simulation

- **Sinusoidal approximates pulsation**
- **captures general trends!**

Future Work

- **identify intervention thresholds**

Data correlation. eg. meta-analysis to establish pressure-damage relationships

- **Incorporate realistic conditions**
  - Non-Newtonian blood
  - flow dynamics

Pressure-Volume Model

Yushan Wei

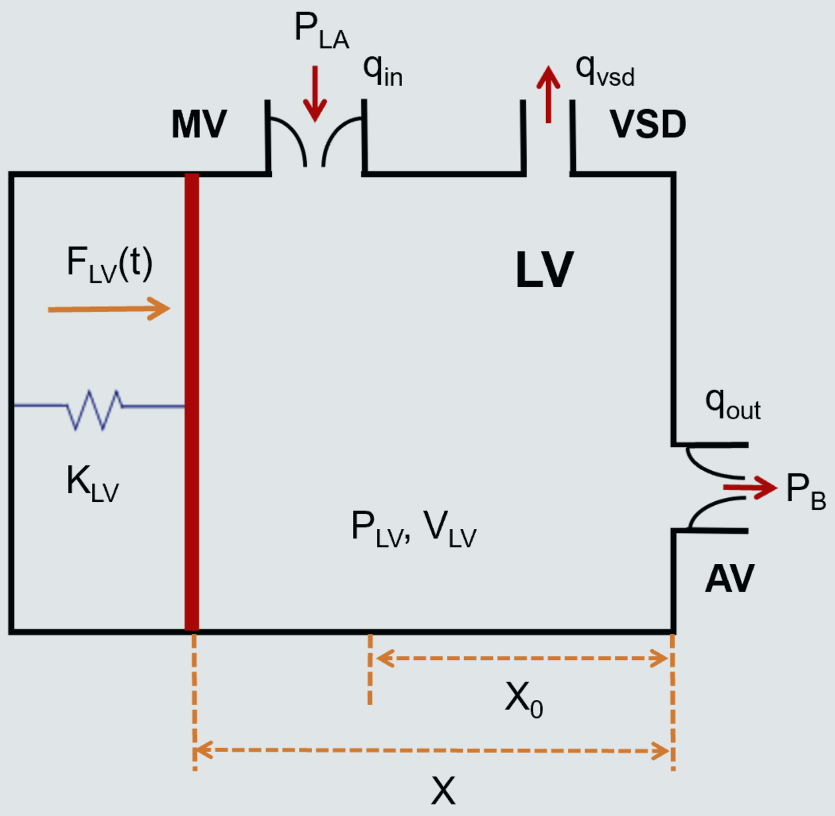
Simplification

Ventricular Parameter

V_{LV} P_{LV}

K_{LV} $F_{LV}(t)$

X X_0



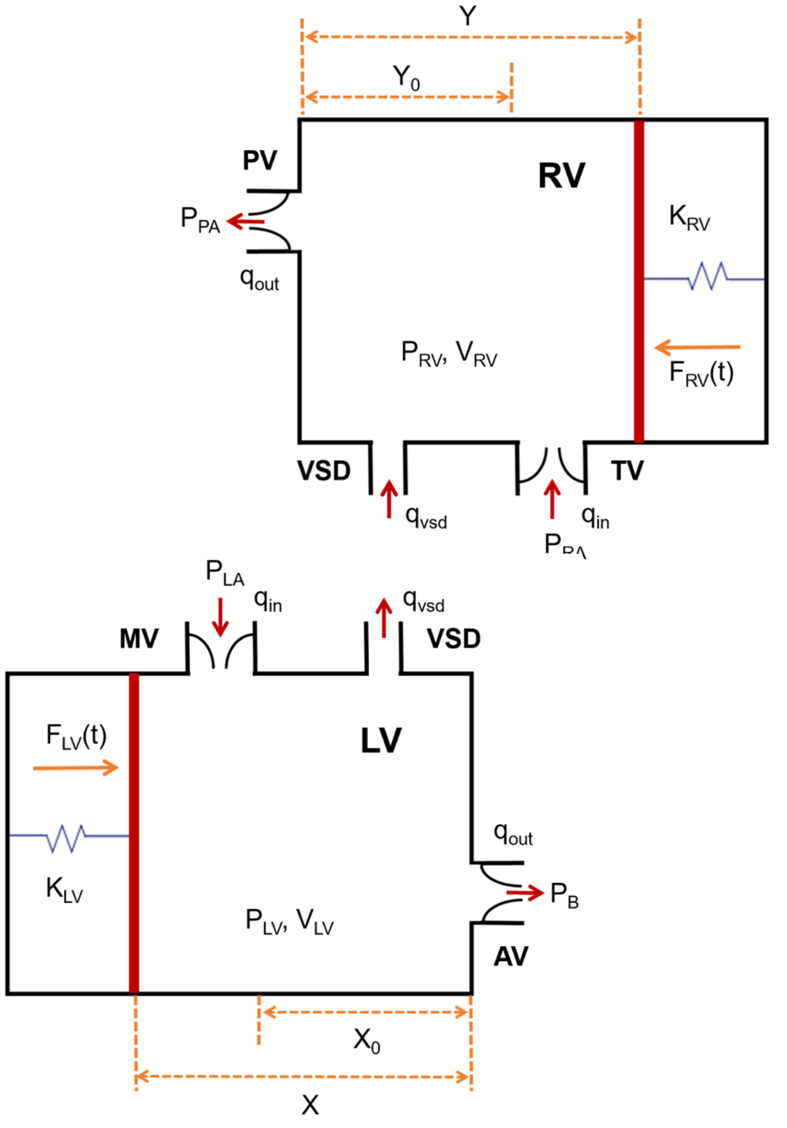
Blood Parameter

P_{LA} P_B

q_{in} q_{out}

q_{vsd}

Simplification



Model

Rigid

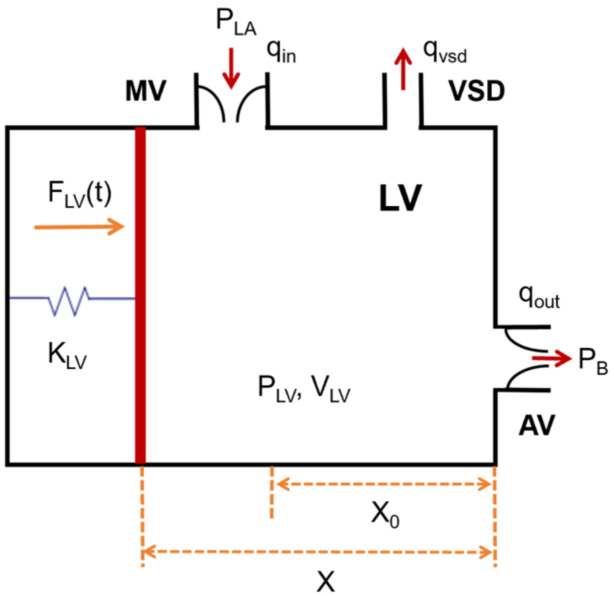
Massless

Blood

Steady

Incompressible

Simplification



$$\frac{dV_{LV}}{dt} = q_{in} - q_{out} - q_{vsd}$$

$$\frac{dV_{LV}}{dt} = \frac{P_{LA} - P_{LV}}{R} - \frac{P_{LV} - P_B}{R} - C_d A \sqrt{\frac{2\Delta P}{\rho}}$$

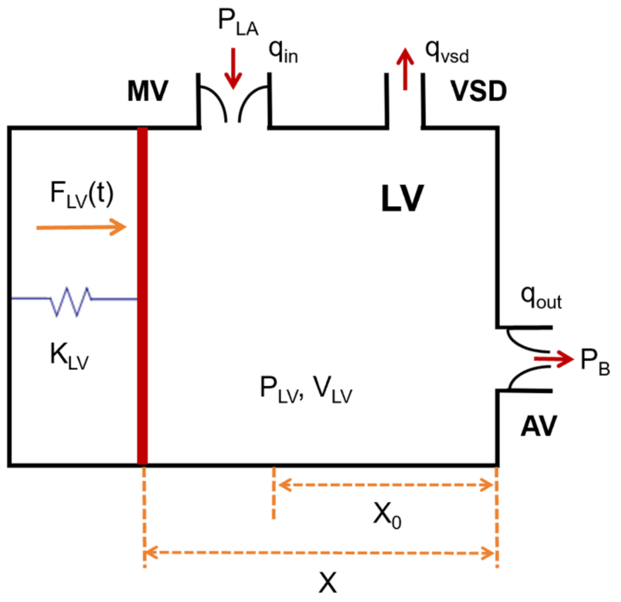
Flow rate in valve

$$q = \frac{P_u - P_d}{R}$$

Orifice Plate Model

$$q_{vsd} = C_d A \sqrt{\frac{2\Delta P}{\rho}}$$

Simplification



$$\frac{dV_{LV}}{dt} = \frac{P_{LA} - P_{LV}}{R} - \frac{P_{LV} - P_B}{R} - C_d A\sqrt{\frac{2\Delta P}{\rho}}$$

$$P_{LV}A = K_{LV}(X - X_0) + F_{LV}(t)$$

$$P_{LV} = E_{LV}(V_{LV} - V_{unlv}) + G_{LV}(t)$$

$$\frac{dP_{LV}}{dt} = E_{LV}\frac{dV_{LV}}{dt} + \frac{dG_{LV}(t)}{dt}$$

Replace A with V

$$E_{LV} = \frac{K_{LV}}{A^2}$$

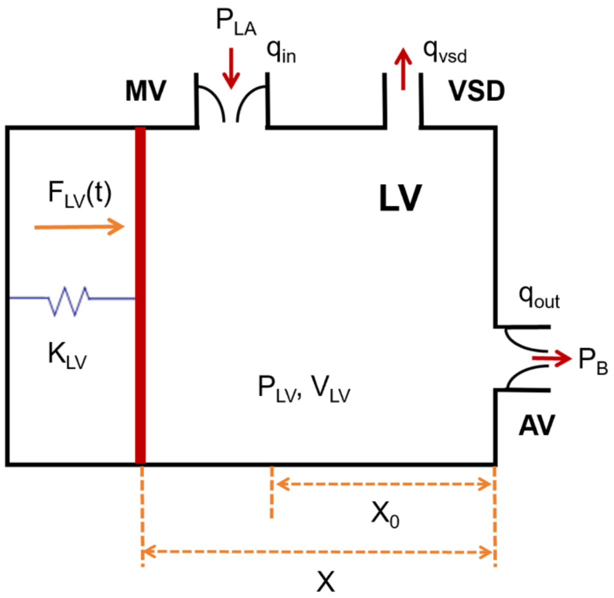
$$V_{unlv} = X_0 A$$

$$G_{LV}(t) = \frac{F_{LV}(t)}{A}$$

$G_{LV}(t)$ can be approximated as a sin function

$$\frac{dG_{LV}(t)}{dt} = 113.75\pi sin\frac{\pi t}{0.4}$$

Simplification

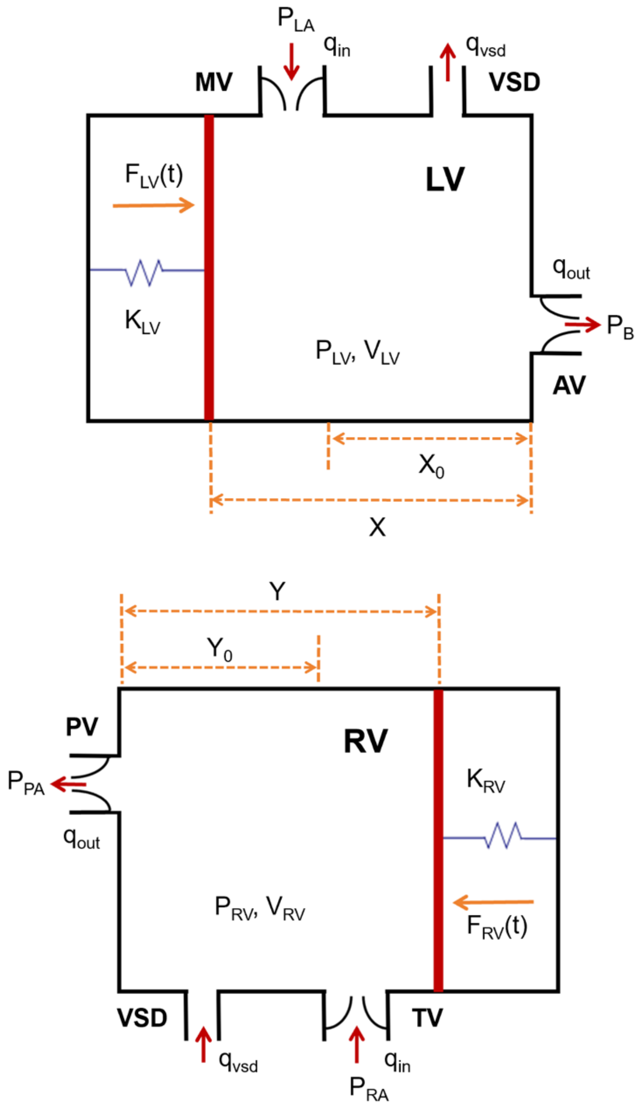


$$\frac{dV_{LV}}{dt} = \frac{P_{LA} - P_{LV}}{R} - \frac{P_{LV} - P_B}{R} - C_d A\sqrt{\frac{2\Delta P}{\rho}}$$

$$\frac{dP_{LV}}{dt} = E_{LV}\frac{dV_{LV}}{dt} + \frac{dG_{LV}(t)}{dt}$$

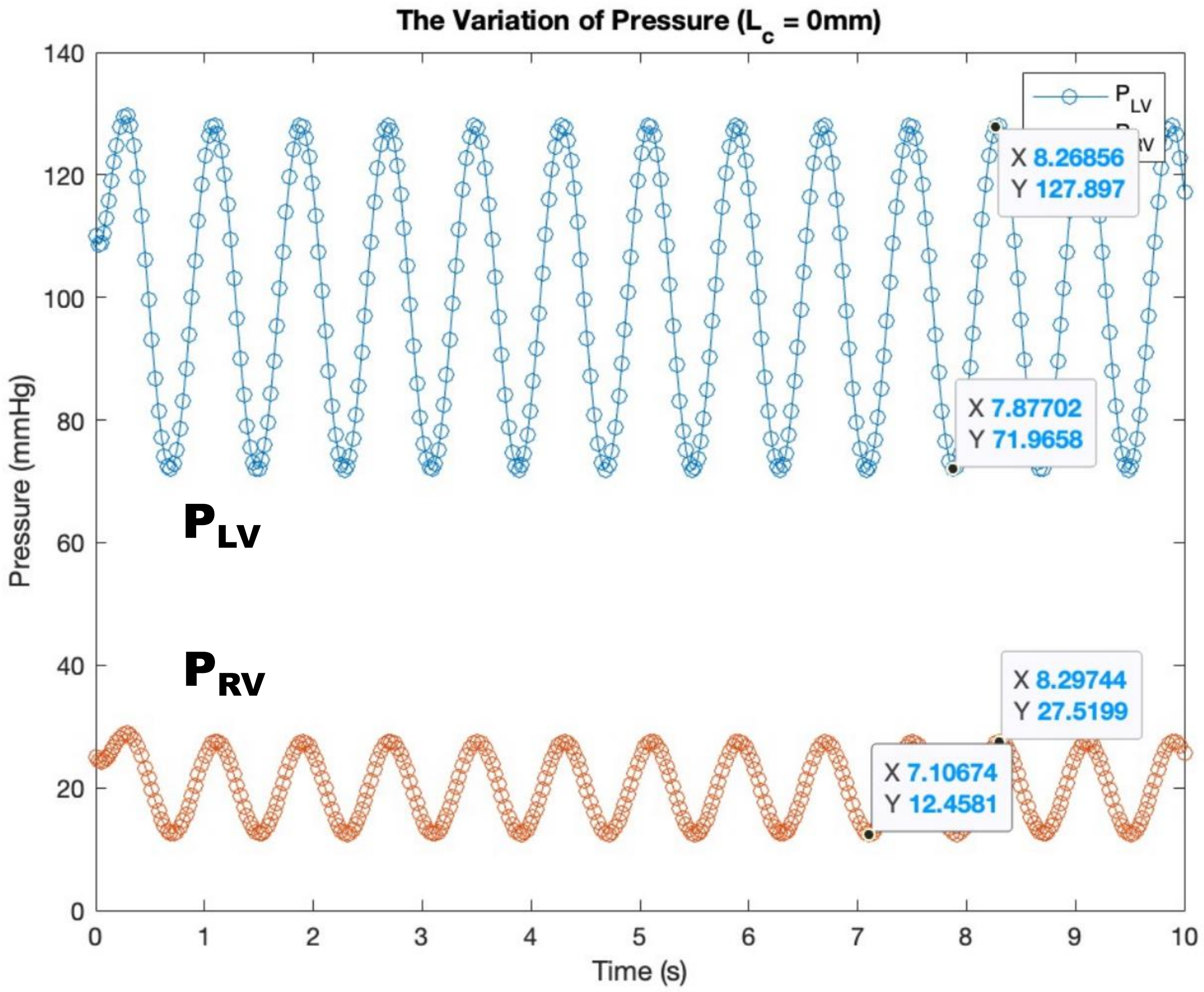
$$\frac{dP_{LV}}{E_{LV}dt} + \frac{2}{R}P_{LV} + C_d A\sqrt{\frac{2(P_{LV} - P_{RV})}{\rho}} = \frac{dG_{LV}(t)}{E_{LV}dt} + \frac{P_{LA}}{R} + \frac{P_B}{R}$$

Simplification



$$\begin{cases} \dfrac{dP_{LV}}{E_{LV}dt} + \dfrac{2}{R}P_{LV} + C_d A\sqrt{\dfrac{2(P_{LV} - P_{RV})}{\rho}} = \dfrac{dG_{LV}(t)}{E_{LV}dt} + \dfrac{P_{LA}}{R} + \dfrac{P_B}{R} \\ \\ \dfrac{dP_{RV}}{E_{RV}dt} + \dfrac{2}{R}P_{RV} - C_d A\sqrt{\dfrac{2(P_{LV} - P_{RV})}{\rho}} = \dfrac{dG_{RV}(t)}{E_{RV}dt} + \dfrac{P_{RA}}{R} + \dfrac{P_{PA}}{R} \end{cases}$$

Results



Initial Value:

P_{LV0} = 110 mmHg

P_{RV0} = 25 mmHg

Special parameter

$$E_{LV} = 0.1\ mmHg/ml$$

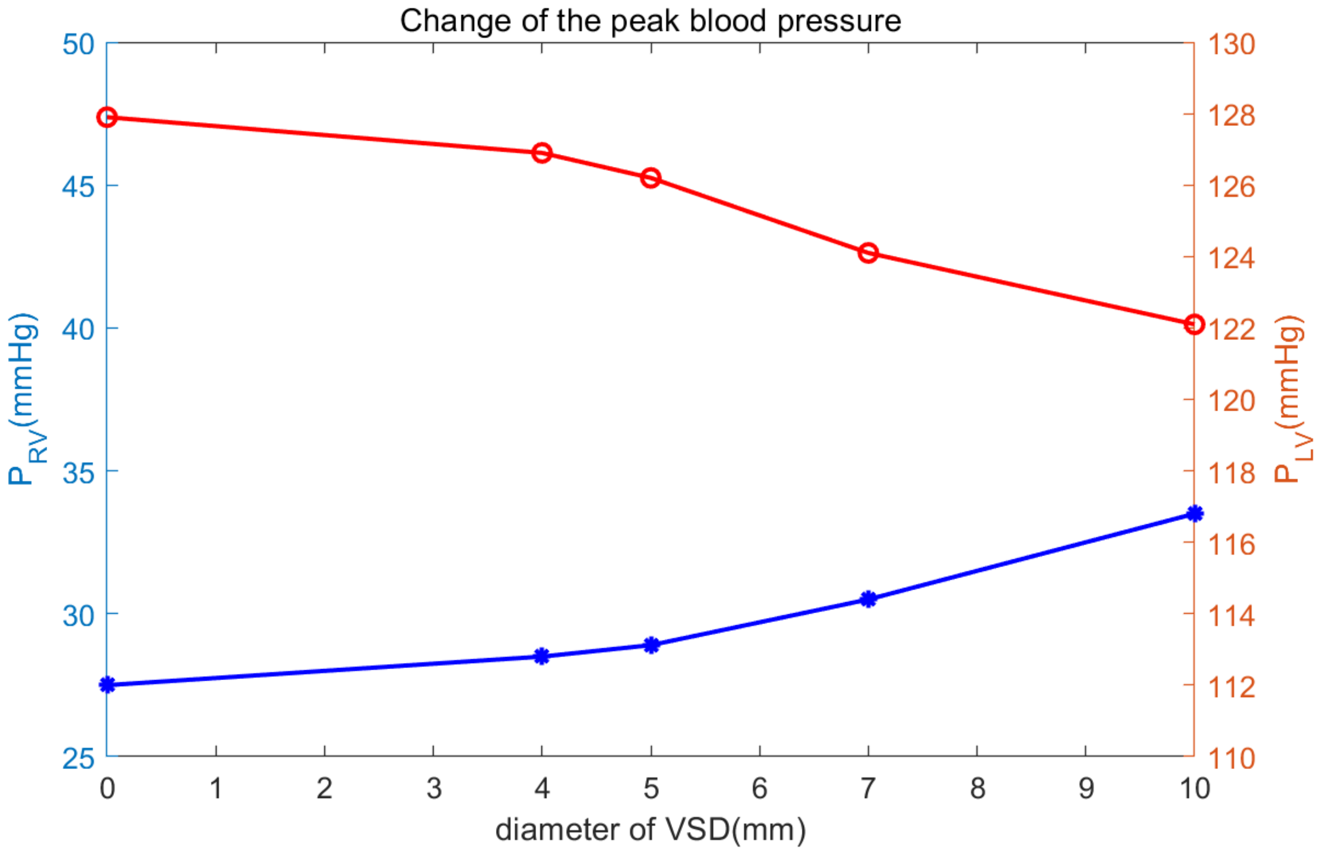
$$E_{RV} = 0.07\ mmHg/ml$$

$$\frac{dG_{LV}(t)}{dt} = 113.75\pi \sin\frac{\pi t}{0.4}$$

$$\frac{dG_{RV}(t)}{dt} = 25.25\pi \sin\frac{\pi t}{0.4}$$

$$R = 0.04$$

Results



Left ventricular blood pressure decreases with VSD

Right ventricular blood pressure increases with VSD

Discussion

Limitation

Many average values are used for simplicity

Advantage

It considers the interaction between ventricular pressure and volume

simulate the key components of the ventricle

Future Work

The model can be improved to study the effect of blood pressure on cardiac deformation

Reference List

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