

# *Mathematical Drawing as Poetry*

Rahman Mohammadpour

In the green sky I saw the new moon reaping,

And minded was I of my own life's field:

What harvest wilt thou to the sickle yield

When through thy fields the moon-shaped knife goes sweeping?

Hafez



- Various reasons for the existence (!) and use of drawings in mathematics:
  - ▶ improving mathematical write-ups
  - ▶ pedagogical purposes
  - ▶ facilitation in the process of perception
  - ▶ a different kind of representation
  - ▶ etc.



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  - ▶ a different kind of representation
  - ▶ etc.
- Mathematicians use the language of mathematics to talk about abstract objects in the realm of mathematics - if we can distinguish between the two at all!





Detail of Hall of Bulls, Lascaux II

- ▶ Learning and the desire to pass on knowledge were crucial to the development of human civilisations and led in particular to the invention of writing systems.
- ▶ There is ample (pre-)historical evidence that drawings of figures and bodies played a crucial role in the transmission and documentation of existing knowledge from one generation to the next.



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In later civilisations, too, we could observe how figures and lines facilitated communication between individuals, of which the hieroglyphics in ancient Egypt are a notable example; mathematics was no exception.



Hieroglyphs at Amada, at temple founded by Tuthmosis III

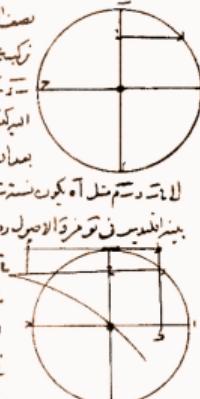
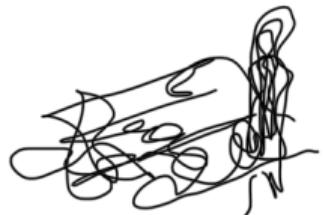
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- We can readily see how notation plays a role different from characters in mathematical scripts.
  - The invention of mathematical symbols (notation) has been a decisive factor in the spread of mathematics, and subsequently science, among various nations worldwide.



## Cubic equation and intersection of conic sections, Khayyam (1048-1131)

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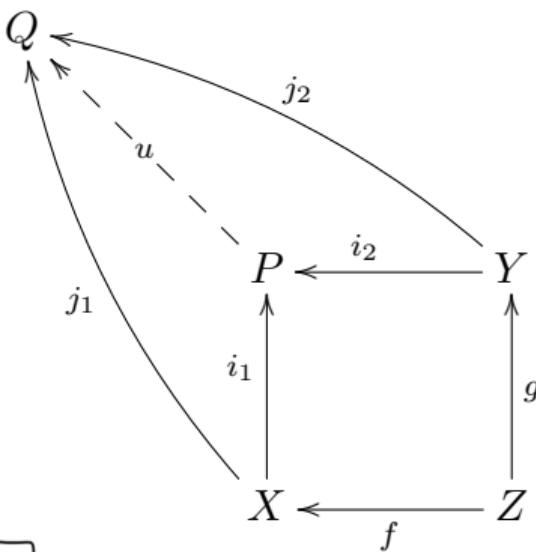
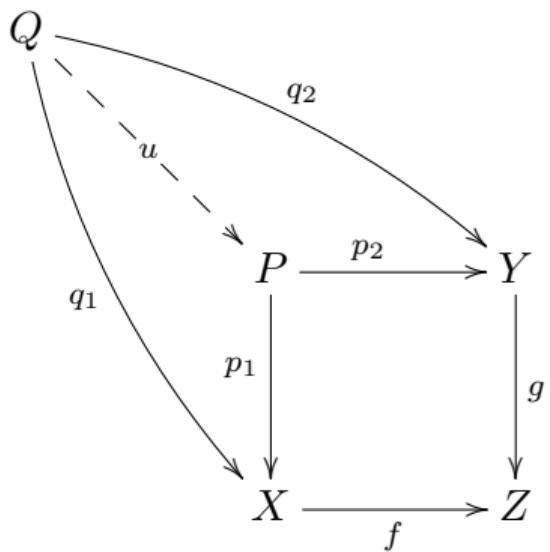
Mathematical symbols are not equivalent to the characters; they appear as figurative representations of phrases, which emerge in a miniature form of a drawing and speak to us substantially different from our words and phrases. E.g., consider the integral!

### *The Stokes Theorem*

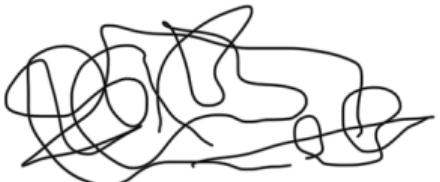
$$\oint_{\Gamma} \mathbf{F} \cdot d\mathbf{T} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$



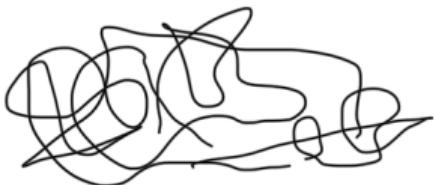
A more modern mathematical symbol that appears as a drawing uses arrows!



- ▶ In a mathematical manuscript, various forms of communication are usually used, including mathematical language, personal and common simplifications, human languages and, of course, notation.



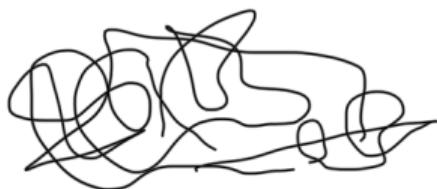
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- ▶ The problem is that one would find no sensation and creativity in a soulless sequence of simple and limited symbols!

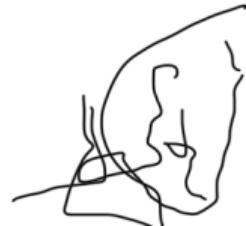


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- ▶ To elaborate further, while we could formulate any mathematical argument precisely using a legitimate formal language, we do not always use a purely formal language to write an article.
- ▶ The problem is that one would find no sensation and creativity in a soulless sequence of simple and limited symbols! We are not machines!



## *My thoughts*

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## *My thoughts*

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- ▶ I discovered, on many occasions, in my mathematical practice, that my drawings form a significant step towards understanding and deploying a mathematical idea. However, they are strangers to me at first. The moment I am about to figure them out is when I start communicating with my “self” through drawings.
- ▶ I have found the process of picturing a mathematical intuition on complex objects very similar to an artistic experience, where I, as a mathematician, would see (perceive) abstract entities and ideas that are incredibly fragile that may not be immediately realisable on a paper.



## *My thoughts*

- ▶ This poetic form of a mathematical idea that can convey lots of information through a personal interpretation of a mathematical idea is a must in my mathematical development.
- ▶ Furthermore, I use the same feature to transfer my knowledge to a colleague without involving complicated formulas; otherwise, we cannot keep the conversation alive; we must have common sense in the form of a picture.



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Mathematical drawings, like artistic drawings, are performances that are neither questionable nor disputable nor open to one-sided interpretation, but can be practical, personal and poetic;



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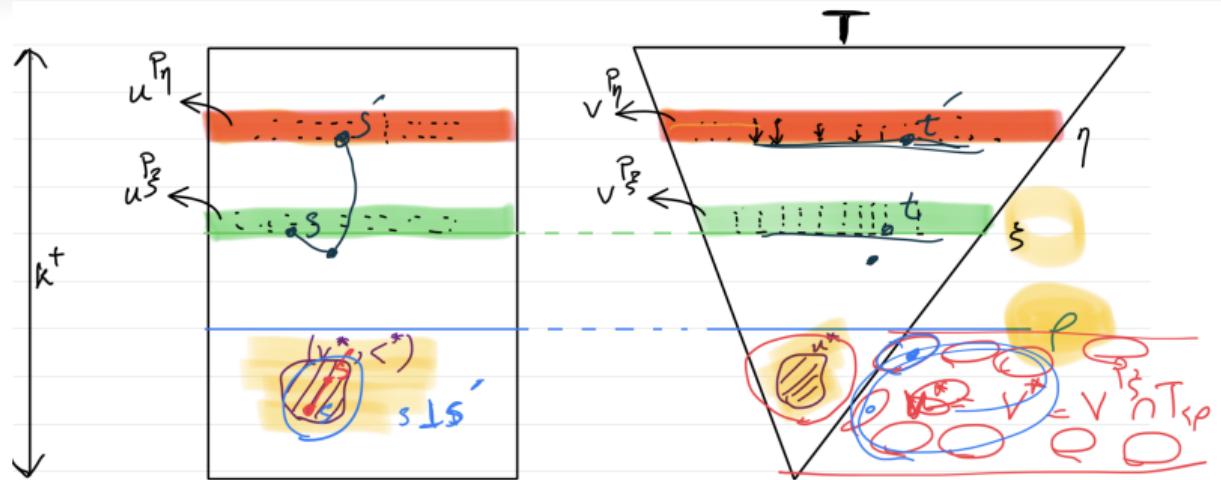
Mathematical drawings, like artistic drawings, are performances that are neither questionable nor disputable nor open to one-sided interpretation, but can be practical, personal and poetic; they have much to say and yet remain simple in form; they are the poetry of our mathematical language as they convey vague ideas, latent notions or any undeveloped concept through figures and sketches and between people.



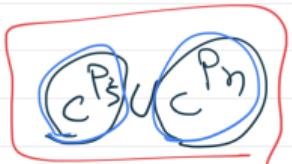
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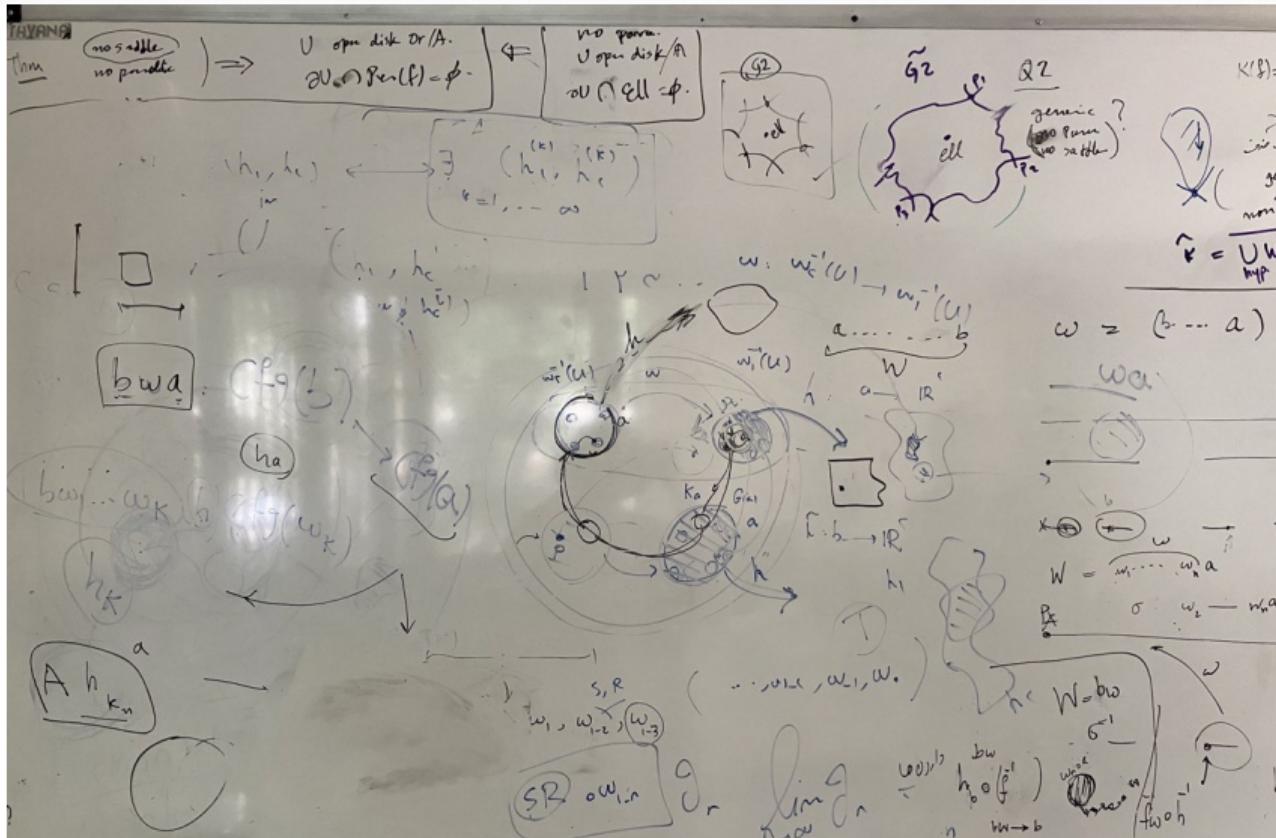


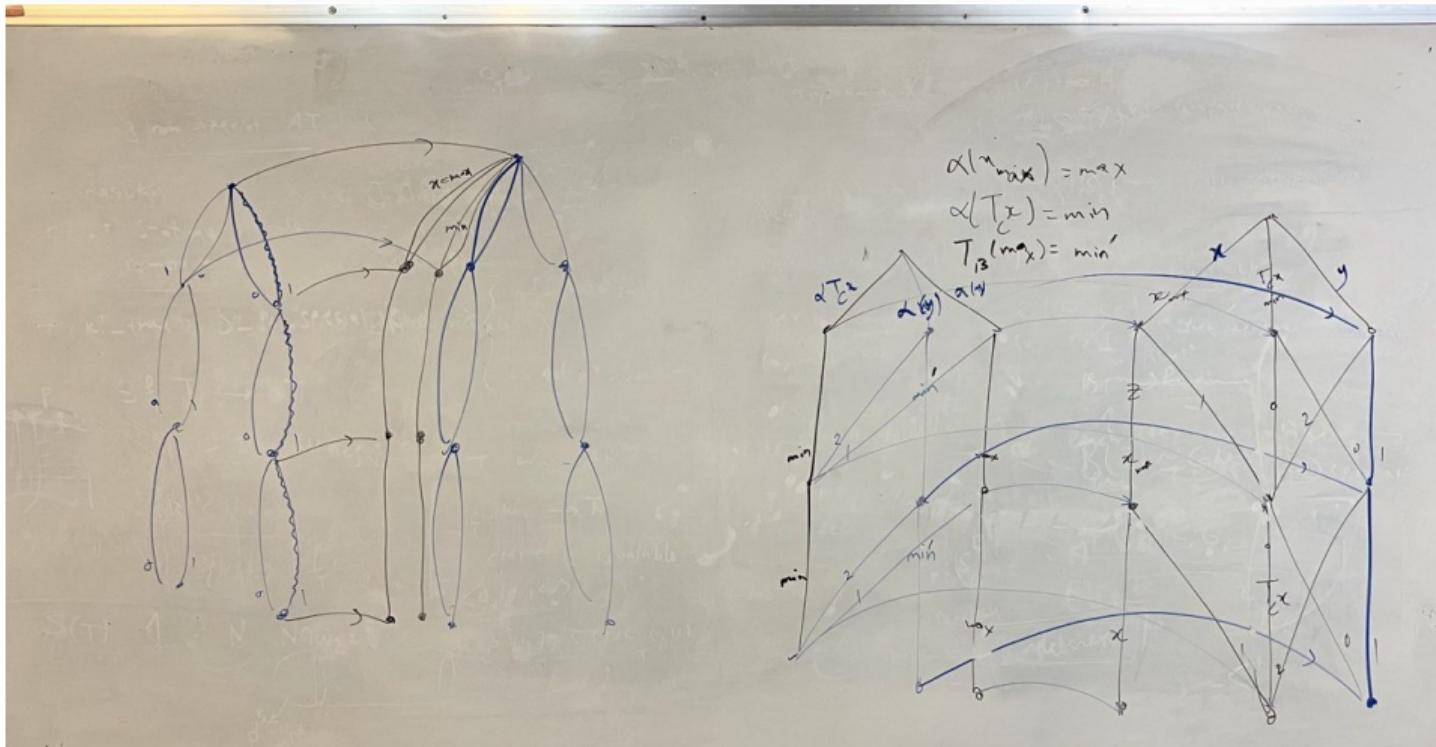
$\exists \xi < \eta \quad (*) \quad \forall t \in \cup P_\xi \setminus V^{P_\eta} \quad \forall t' \in V^{P_\eta} \setminus V^{P_\xi}$



$(\cup P_\xi \cup \cup P_\eta \cup \Xi) \setminus (\cup P_\xi \cup V^{P_\eta}) = -$

$(s, t) \quad (s', t')$





Prop. Let  $B$  and  $C$  be decisive ord. Gr. diagrams and let  $f: B \rightarrow C$  be an ordered premapism as in Df...  
 Consider Verchik maps  $\overline{T}_B: X_B \rightarrow X_B$  and  $\overline{T}_C: X_C \rightarrow X_C$ .

Let  $\alpha = v(f): X_C \rightarrow X_B$  be the induced continuous surjective map. Then  $\alpha$  is a topological factoring iff

for any  $y \in X_C^{\max}$  and any  $x \in \alpha^{-1}(y)$

$$\alpha(x) \in X_B^{\max}$$

One of the following occurs:

(1)  $x \in X_C^{\max}$  and for any  $n \in \mathbb{N}$

there is a minimal edge  $p_n$  in  $F_n$

with  $s(p) = r(e_n)$  and  $r(p) = r(s'_n)$

where  $e_n$  is the  $n$ -th edge of  $\overline{T}_B(y)$

and  $s'_n$  is the  $n$ -th edge of  $\overline{T}_C(y)$

(2)  $x \notin X_C^{\max}$  and it  $x = s_{p_k}$  and  $\overline{T}_C x = s'_k$

and  $k$  is the smallest number with  $r(s) = r(s'_k)$ , then

$$\alpha(\overline{T}_C x) = \overline{T}_B(y)$$

there are  $p_1, p_2, \dots, p_k$  in  $F_n$  s.t. for any  $n \in \mathbb{N}$ ,

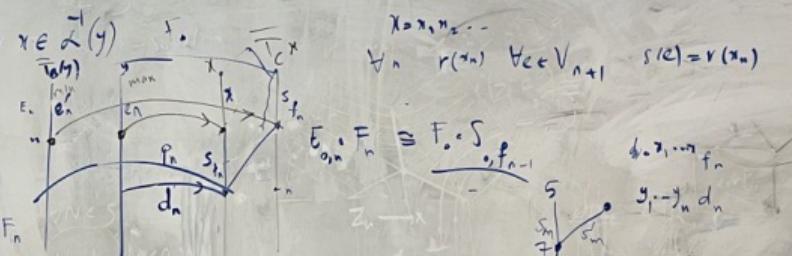
$p_n$  is a minimal edge in  $F_n$

with  $s(p_n) = r(e_n)$  and  $r(p_n) = r(s'_n)$

and for any  $i > k$ ,  $p_n$  is the

successor of  $p_n$  where

$d_n$  is the distance between  $s(p_n)$  and  $r(p_n)$  and for  $n < k$  the minimal is  $d_n$ . There are edges in  $F_n$  realising  $d_n$  and for  $n > k$  there are edges in  $F_n$  realising  $d_n$ .



$$\forall n \quad r(e_n) \quad \forall i \in V_{n+1} \quad s(i) = r(e_n)$$

$$\overline{T}_B: F_n \cong F_{n+1} \xrightarrow{S_{p_n}, f_{n+1}}$$

$$r(s'_n) = r(s'_k)$$

$$f_k \quad f_k$$



S. Sherlah in his office, Jerusalem

*Intro*  
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*Contextualisation*  
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*Mathematical Language*  
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*Mathematical Drawing as an independent matter*  
ooo

*Gallery*  
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# *A technique*

## *A technique*



*Diagonalization*, mixed media on paper, A3

*Intro*  
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*Contextualisation*  
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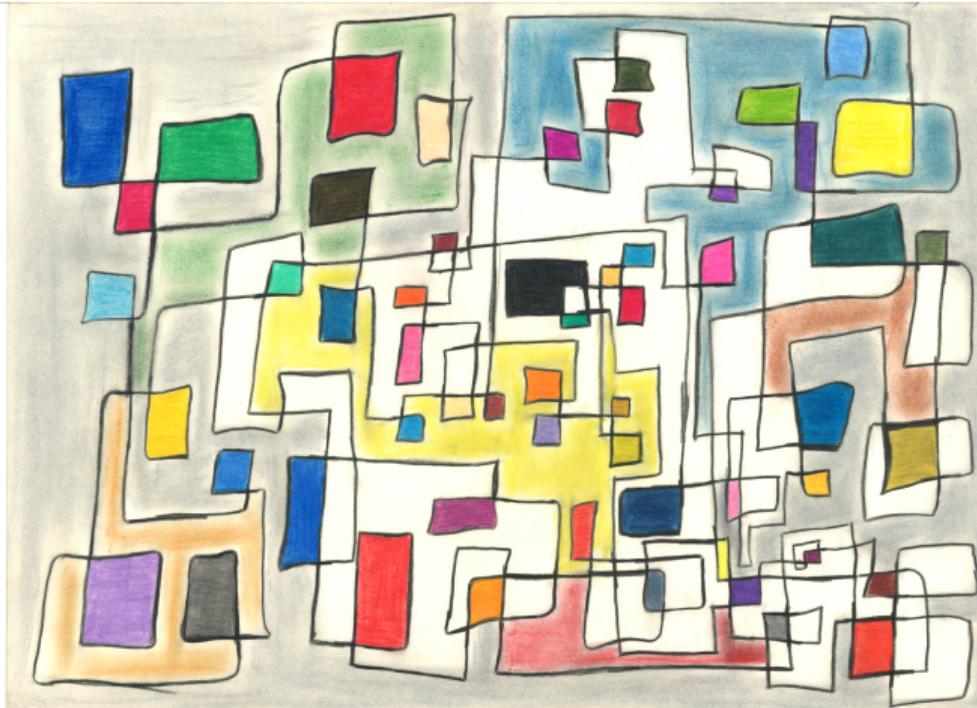
*Mathematical Language*  
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# *An axiom*

## *An axiom*



*Regularity*, mixed media on paper, A3

*Intro*  
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*Contextualisation*  
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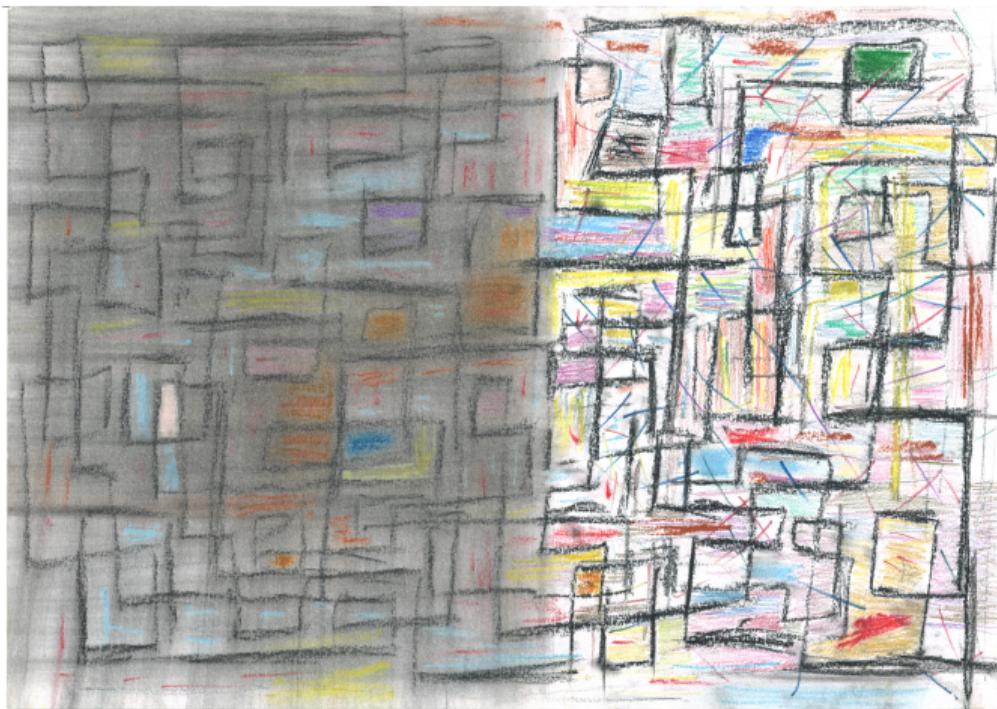
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# *A concept*

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*Complexity*, mixed media on paper, A3

*Thank you!*

