

Notes on Feller Condition

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1 Fokker-Planck Equation

Consider the following parabolic partial differential equation [1]

$$\frac{\partial u(t, x)}{\partial t} = -\frac{\partial}{\partial x} ((bx + c)u(t, x)) + \frac{\partial^2}{\partial x^2} (axu(t, x)). \quad (1)$$

This can be view as the corresponding Fokker-Planck equation for the Cox-Ingersoll-Ross (CIR) process

$$dx_t = \kappa(\theta - x_t)dt + \sigma\sqrt{x_t}dW_t, \quad (2)$$

with $a = \sigma^2/2$, $b = -\kappa$, and $c = \kappa\theta$.

References

- [1] W. Feller, *Two Singular Diffusion Problems*, Annals of Mathematics **54**, 173 (1951).