

Notes on Statistics

Dec. 32, 2999

1 Distributions from random samples

1.1 Random samples and statistics

Definition 1.1. The random variables X_1, \dots, X_n are called a random sample of size n from the population $f(x)$ if they are independent and identically distributed with pdf or pmf $f(x)$, or iid random variables. Let $T(x_1, \dots, x_n)$ be a real-valued or vector-valued function whose domain includes of sample space of (X_1, \dots, X_n) , then the random variable or random vector $Y = T(X_1, \dots, X_n)$ is called a statistic, whose distribution is called the sampling distribution of Y .

Theorem 1.2. *This is another theorem.*

Proof. This proves the theorem. □

Example 1.3. This is an example.

2 Point estimation

Example 2.1. This is another example.

Remark 2.2. *This is just a remark.*

3 Hypothesis testing

4 Interval estimation

5 A complete example

6 More examples

7 Analysis of variance

8 Linear regression

A Distribution of transformations of random variables

$$f_{\mathbf{U}}(u_1, \dots, u_n) = \sum_i f_{\mathbf{X}}(h_{1i}(u_1, \dots, u_n), \dots, h_{ni}(u_1, \dots, u_n)) |J_i|, \quad (1)$$

B Cochran's theorem

Theorem B.1 (Cochran). *Let X_1, X_2, \dots, X_n be i.i.d. $N(0, \sigma^2)$ distributed random variables, and suppose that*

$$\sum_{i=1}^n X_i^2 = \sum_{j=1}^k Q_j,$$

where Q_1, Q_2, \dots, Q_k are positive semi-definite quadratic forms in X_1, X_2, \dots, X_n , i.e.,

$$Q_j = \mathbf{X}^\top \mathbf{A}_j \mathbf{X}, \quad j = 1, 2, \dots, k.$$

Let $r_j = \text{rank}(\mathbf{A}_j)$ be the rank of the matrix \mathbf{A}_j . If $\sum_{j=1}^k r_j = n$, then

- Q_1, Q_2, \dots, Q_k are independent;
- $Q_j \sim \sigma^2 \chi^2(r_j)$.

From linear algebra, we know that if a matrix \mathbf{A} is symmetric and idempotent, then $\text{rank}(\mathbf{A}) = \text{tr}(\mathbf{A})$. Since \mathbf{A} is idempotent, i.e., $\mathbf{A}^2 = \mathbf{A}$, the eigenvalues of the matrix \mathbf{A} are either 0 or 1. Therefore, the rank of the matrix is equal to the sum of its eigenvalues. We will use this fact to determine the degrees of freedom for the χ^2 distributions corresponding to the quadratic forms. In this case, we will only need to verify that the matrices are idempotent.

C Two useful relations

References