Notes on Statistics

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1 Distributions from random samples

1.1 Random samples and statistics

Definition 1.1. The random variables $X_1, \cdots, X_n$ are called a ramdom sample of size n from the population $f(x)$ if they are independent and identically distributed with pdf or pmf $f(x)$, or iid random variables. Let $T(x_1, \cdots, x_n)$ be a real-valued or vector-valued function whose domain includes of sample space of $(X_1, \cdots, X_n)$, then the random variable or random vector $Y = T(X_1, \cdots, X_n)$ is called a statistic, whose distribution is called the sampling distribution of Y.

Theorem 1.2. *This is another theorem.*

Proof. This proves the theorem. $\square$

Example 1.3. This is an example.

2 Point estimation

Example 2.1. This is another example.

Remark 2.2. *This is just a remark.*

3 Hypothesis testing

4 Interval estimation

5 A complete example

6 More examples

7 Analysis of variance

8 Linear regression

A Distribution of transformations of random variables

$$f_{\mathbf{U}}(u_1, \cdots, u_n) = \sum_i f_{\mathbf{X}}(h_{1i}(u_1, \cdots, u_n), \cdots, h_{ni}(u_1, \cdots, u_n)) \left|J_i\right|, \tag{1}$$

B Cochran's theorem

Theorem B.1 (Cochran)**.** *Let X_1, X_2, $\cdots$, X_n be i.i.d. $N(0, \sigma^2)$ distributed random variables, and suppose that*

$$\sum_{i=1}^{n} X_i^2 = \sum_{j=1}^{k} Q_j,$$

where Q_1, Q_2, $\cdots$, Q_k are positive semi-definite quadratic forms in X_1, X_2, $\cdots$, X_n, i.e.,

$$Q_j = \mathbf{X}^\top \mathbf{A}_j \mathbf{X}, \quad j = 1, 2, \cdots, k.$$

Let $r_j = \operatorname{rank}(\mathbf{A}_j)$ be the rank of the matrix $\mathbf{A}_j$. If $\sum_{j=1}^{k} r_j = n$, then

- *Q_1, Q_2, $\cdots$, Q_k are independent;*
- *$Q_j \sim \sigma^2 \chi^2(r_j)$.*

C Two useful relations

References