Notes on Feller Condition

Dec. 32, 2999

1 Fokker-Planck Equation

Consider the following paraboic partial differential equation

$$\frac{\partial u(t,x)}{\partial t} = -\frac{\partial}{\partial x}\left((bx+c)u(t,x)\right) + \frac{\partial^2}{\partial x^2}\left(axu(t,x)\right). \tag{1}$$

This can be view as the corresponding Fokker-Planck equation for the Cox-Ingersoll-Ross (CIR) process

$$dx_t = \kappa(\theta - x_t)dt + \sigma\sqrt{x_t}dW_t, \tag{2}$$

with $a = \sigma^2/2$, $b = -\kappa$, and $c = \kappa\theta$.

References

[1] W. Feller, *Two Singular Diffusion Problems*, Annals of Mathematics **54**, 173 (1951).