Notes on Piterbarg's Formulation of Heston Model

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1 Shifted Lognormal Model

Consider the following shifted lognormal process for underlying S,

$$dS_t = \lambda\Big(bS_t + (1-b)\,L\Big)dW_t, \tag{1}$$

the price of a call option written on S is given by

$$C(t, S_t, T, K) = \frac{1}{b}\Big[\Big(bS_t + (1-b)\,L\Big)N\left(d_+\right) - \Big(bK + (1-b)\,L\Big)N\left(d_-\right)\Big], \tag{2}$$

where

$$d_\pm = \frac{1}{|\lambda b|\sqrt{T-t}}\left(\log\left(\frac{bS_t + (1-b)\,L}{bK + (1-b)\,L}\right) \pm \frac{1}{2}\lambda^2 b^2 (T-t)\right), \tag{3}$$

S_t is the spot price of the underlying at time t, and the strike and the expiry of the call option are K and T, respectively.

References

[1] S. Heston, *A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options*, Review of Financial Studies **6**, 327 (1993).

[2] P. Carr and D. Madan, *Option valuation using the fast Fourier transform*, The Journal of Computational Finance **2**, 61 (1999).

[3] A. Lewis, *A Simple Option Formula for General Jump-Diffusion and Other Exponential Levy Processes*, `https://ssrn.com/abstract=282110`.