

Homework 9

1. Digital format (can be typeset or photos, ought to write clearly if written by hand), upload to <https://course.pku.edu.cn/>.
2. Submit by next class
3. A problem is not counted if nobody can work it out
4. Each homework 10 points; 1 point deducted for each week's delay

Problems:

1. Consider the function

$$f(\mathbf{x}) = \frac{x_1^4}{4} + \frac{x_2^2}{2} - x_1x_2 + x_1 - x_2.$$

Apply the DFP algorithm to minimize it with the following starting initial conditions: (i) $[0, 0]^T$; (ii) $[1.5, 1]^T$. Use $\mathbf{H}_0 = \mathbf{I}_2$. Does the algorithm converge to the same point for the two initial conditions? If not, explain.

2. Consider the problem to minimize $(3 - x_1)^2 + 7(x_2 - x_1^2)^2 + 9(x_3 - x_1 - x_2^2)^2$. Starting from the point $(1, 1, 1)^T$, write codes to solve the problem by the following procedures:

- a. The method of Davidon-Fletcher-Powell (DFP).
- b. The method of Broyden-Fletcher-Goldfarb-Shanno (BFGS).
- c. Compare the positive definiteness of \mathbf{H} 's of the two methods.

3. Use L-BFGS to solve extended Rosenbrock function

$$f(\mathbf{x}) = \sum_{i=1}^{n/2} [\alpha(x_{2i} - x_{2i-1}^2)^2 + (1 - x_{2i-1})^2],$$

where α is a parameter that you can vary (for example, 1 or 100) and $n = 2000$. The solution is $\mathbf{x}^* = (1, 1, \dots, 1)^T$, $f^* = 0$. Choose the starting point as $(-1, -1, \dots, -1)^T$. Observe the behavior of your program for various values of the memory parameter m .

4. Solve $\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{Ax} - \mathbf{b}\|^2 + \lambda \|\mathbf{x}\|_1$ using the following majorant minimization methods:

- a. Use the Lipschitz gradient majorant function of $\frac{1}{2} \|\mathbf{Ax} - \mathbf{b}\|^2$;
- b. Use the variational majorant function $\min_{\mathbf{d} > \mathbf{0}} \frac{1}{2} (\mathbf{x}^T \mathbf{D} \mathbf{x} + \mathbf{1}^T \mathbf{D}^{-1} \mathbf{1})$ of $\|\mathbf{x}\|_1$, where $\mathbf{1}$ is an all-one vector and \mathbf{D} is a diagonal matrix with \mathbf{d} on the diagonal.

Compare their performances.

Note: For coding problems, please write reports and hand in both codes and reports. Remember to restart your computer before running your codes.