Homework 1

Requirements:

1. Digital format (can be typeset or photos, ought to write clearly if written by hand), upload to `https://course.pku.edu.cn/`.
2. Submit by next class
3. A problem is not counted if nobody can work it out
4. Each homework 10 points; 1 point deducted for each week's delay

Problems:

1. Compute by hand the inner product of:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

2. Compute the nuclear norm of the following matrix by hand

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

3. Find the dual norm of Mahalanobis norm: $\|\mathbf{x}\|_{\mathbf{M}} = \sqrt{\mathbf{x}^T\mathbf{M}\mathbf{x}}$, where $\mathbf{M}$ is a positive definite matrix.

4. Find the dual norm of $\sqrt{a\|\mathbf{x}\|_A^2 + \|\mathbf{y}\|_B^2}$ for vector $(\mathbf{x}^T, \mathbf{y}^T)^T$, where $\|\cdot\|_A$ and $\|\cdot\|_B$ are two given norms.