2009 Bulgarian IPhO Team Selection Test

Short Exam 2

Problem. A point charge q of mass m is initially at rest. The charge is placed in a homogeneous electric field E and a homogeneous magnetic field B at right angles to E. It turns out that the charge's trajectory is a periodic curve, i.e. the trajectory will consist of many identical segments. Note that the charge's motion can be represented as a superposition of two simpler sorts of motion.

(a) Find the time T in which the charge covers one segment of its trajectory. In other words, find the time interval between two consecutive instants at which the charge is at rest.

(b) Find the spatial periodicity of the curve L. In other words, find the distance between two neighbouring positions where the charge is at rest.

(c) Find the maximum velocity of the charge v_{max}.

(d) Find the 'width' of the trajectory H.

(e) What are the two simpler sorts of motion that give rise to the resultant motion of the charge? What are their parameters?

(f) Find the radius of curvature r_{curv} of the trajectory at the points of maximum velocity v_{max}.

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 4.

(a) Find the electric field E at a point A on the axis of an ideal electric dipole p. The distance between point A and the dipole is r.

(b) Find the force F on another dipole of the same magnitude and orientation, located at point A.

(c) Find the energy of this system of dipoles W.

(d) Find the torque M acting on the dipole at point A if we rotate it at an angle θ with respect to its initial orientation.

Problem 5. Find the equivalent resistance of the circuit on Figure 1. Each segment represents a resistance R. Draw the equivalent circuits that you have used to reach the answer.

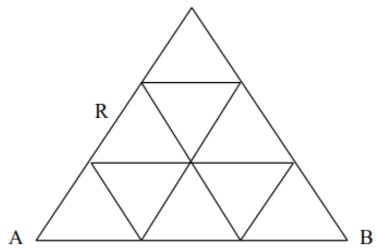

Figure 1

Problem 6. A convex lens of diameter $D = 10\,\text{cm}$ and focal length $f = 1\,\text{m}$ is cut into two identical halves, removing a glass layer of thickness $a = 0.5\,\text{mm}$. The two halves are then stuck back together. A point source of monochromatic light of wavelength $\lambda = 500\,\text{nm}$ is placed in the focal plane of the system. A screen is placed perpendicularly to the optical axis at a distance of $L = 10\,\text{m}$ on the other side of the lens.

(a) Find the distance Δx between the maxima of the resulting interference pattern.

(b) Find the number of maxima on the screen N.

(c) Approximately at what size of the point source δ does the interference pattern vanish?

(d) Find the distance L_{max} at which the screen should be placed so that the number of maxima is largest. What is this number N_{max}?

Constants:

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Seiche.

Equipment:
Rectangular box, 3 l of water, measuring vessel (in ml), stopwatch, ruler, graph paper.

Consider a harmonic wave of wavelength λ propagating along the surface of an infinite layer of liquid of depth h. Neglecting the viscosity and the surface tension of the liquid, and assuming a wave amplitude $A \ll h$, the propagation speed is given by

$$v = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi h}{\lambda}\right)}, \tag{1}$$

where $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ and g is the acceleration due to gravity. This formula yields approximations for the propagation speed v in the cases of 'shallow' and 'deep' water, respectively:

$$v_{\text{shallow}} = \sqrt{gh}, \tag{2}$$

$$v_{\text{deep}} = \sqrt{\frac{g\lambda}{2\pi}}. \tag{3}$$

A seiche is a standing wave formed on the surface of an enclosed body of water. The surface of the water remains nearly flat, however the water level at the two ends of the vessel oscillates in antiphase with a period T. The seiche can be considered as a standing wave formed by the superposition of two harmonic travelling waves of equal amplitudes that propagate in opposite directions.

In shallow enough water, instead a seiche one could create a travelling nonlinear wave with a single peak, called a soliton. The velocity of this type of wave can also be assumed to obey (1).

The aim of this problem is to study the period T of a seiche and of a soliton for different values of the depth h, i.e. both for 'shallow' water and 'deep' water.

Before you start taking measurements, play around with the setup at different depths. Induce waves along both sides of the box and see when you can reliably measure the timescales. Hence decide on your experimental procedure and the ranges in which you will measure.

(a) Induce a seiche or a soliton at different depths h and measure the period of the wave T. Explain how you induce the waves. **(0.5 pt)**

Present your results in a table. **(3.5 pt)**

(b) Write down the depths h at which you can create a seiche and those at which you can create a soliton. **(0.5 pt)**

(c) Plot a graph of the period T against the depth h. **(3.0 pt)**

(d) For what depths is the period T constant (or nearly constant)? **(0.5 pt)**

(e) Assuming that in 'deep' water the seiche has some effective wavelength λ_{eff}, find the ratio $n = \frac{\lambda_{\text{eff}}}{2L}$, where L is the length of the box in the direction of propagation of the seiche. **(0.5 pt)**

(f) Assuming that in 'shallow' water the wave velocity is given by

$$v \propto h^k, \tag{4}$$

plot this dependence in appropriate coordinates so that k can be determined from the graph. **(3.0 pt)**

Also present the data in the graph in tabular form. **(0.5 pt)**

(g) Calculate k. **(1.0 pt)**

(h) Find the values of the ratio h/L for which (4) holds. **(1 pt)**

(i) Explain qualitatively why (1) also holds for a soliton. **(0.5 pt)**

(j) At what depths does viscosity become significant for your experiment? **(0.5 pt)**

Call the examiner in case of any technical difficulties.

Each problem is worth 15 points.
Time: 5 hours.