2005 Bulgarian IPhO Team Selection Test

Theoretical Exam

Problem 1. A channel with a rectangular cross-section is confined by a concrete slab of height $a = 2.0\,\mathrm{m}$ and width $b = 0.2\,\mathrm{m}$, as shown on Figure 1. The coefficient of friction between the slab and the bottom of the channel is $k = 1$. The density of concrete is $\rho_1 = 2.3 \times 10^3\,\mathrm{kg/m^3}$ and the density of water is $\rho_0 = 1.0 \times 10^3\,\mathrm{kg/m^3}$. Find the water level H that the slab can hold.

Problem 2. A frame consists of four identical uniform hinged rods, each of mass m, as shown on Figure 2. The end A is fixed to a wall, while the opposite end C moves away from the wall with a velocity v. Find the total kinetic energy of the frame at the instant when the rods form a square.

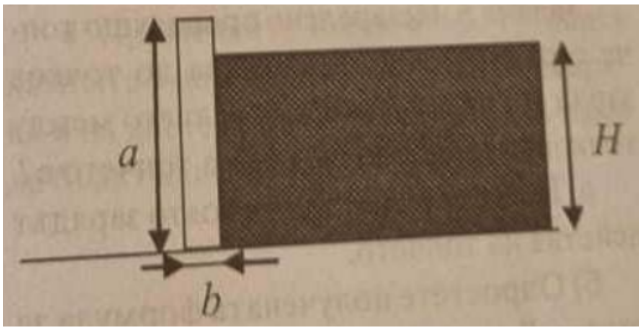

Figure 1

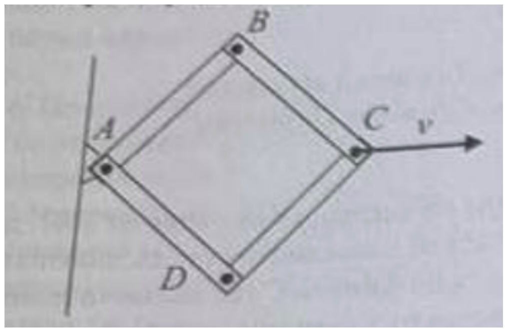

Figure 2

Problem 3. What is the maximum velocity $v_{\max}$ at which a car of mass $m = 1000\,\mathrm{kg}$ can make a turn of radius $r = 500\,\mathrm{m}$? The coefficient of friction between the tyres and the road is $k = 0.5$. The car experiences air drag of the form $F_{\text{drag}} = cv^2$, where v is the velocity of the car and $c = 2\,\mathrm{kg/m}$ is a constant.

Problem 4. A uniform ball of mass m and radius r is put near the bottom of a fixed hemispherical bowl of radius R. The ball is then let go, and it rolls without slipping. The acceleration due to gravity is g. Find the period of oscillations T.

Problem 5. A neutral conducting ball of radius R is put near a point charge Q. The distance between the point charge and the centre of the ball is l.

(a) Find the force F with which the charge acts on the ball.

(b) Simplify the formula for F in the case $l \gg R$.

Problem 6. Consider the interference of N waves at a point, their equations being

$$x_k = a\cos\left(\omega t + (k-1)\alpha\right), \qquad k = 1, 2, \ldots, N.$$

Find the amplitude A of the resulting wave at that point.

Problem 7. A thermodynamic system's internal energy depends only on its temperature. The system is used in a quasistatic cyclic process which consists of an isothermal expansion, an isobaric compression, and an adiabatic compression. The ratio between the maximum and the minimum temperature throughout the cyclic process is $\alpha = 2$.

(a) Draw the cycle on a pV diagram.

(b) Find the efficiency η of a heat engine that runs this cycle.

Problem 8. An electron near the surface of a liquid helium layer will interact with its image. The interaction energy is described by

$$U(x) = \begin{cases} +\infty, & x < 0, \\ -\Lambda/x, & x > 0, \end{cases}$$

where x is the distance to the helium's surface, and

$$\Lambda = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{\varepsilon - 1}{\varepsilon + 1}, \qquad \text{with } \varepsilon = 1.057.$$

(a) Using the Heisenberg uncertainty principle, find the minimum energy of the electron near the helium's surface.

(b) Using a quantisation condition similar to that of the Bohr model for the hydrogen atom, find the energy levels of the electron.

Problem 9. Let m_e be the mass of the electron. A photon of energy $E_\gamma = 2m_ec^2$ is scattered by an electron at rest, losing half of its energy. Find the angle α between the directions of motion of the particles afterwards.

Problem 10. Show that the electron gas in metals (consisting of the valence electrons) can be taken as ideal. To that end, compare the average kinetic energy of the electrons due to localisation with the average interaction potential energy per electron. Provide your numerical estimates for sodium, which has a molar mass $\mu = 23\,\mathrm{g/mol}$ and density $\rho = 1\,\mathrm{g/cm^3}$.

Constants:

Acceleration due to gravity	g	$10\,\mathrm{m/s^2}$
Vacuum permittivity	ε_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Elementary charge	e	$1.6 \times 10^{-19}\,\mathrm{C}$
Avogadro constant	N_A	$6.0 \times 10^{23}\,\mathrm{mol^{-1}}$
Electron mass	m_e	$9.1 \times 10^{-31}\,\mathrm{kg}$
Planck constant	h	$6.6 \times 10^{-34}\,\mathrm{J\,s}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1.
Equipment:

1. Golf ball (a uniform ball; ignore the uneven surface)
2. Table
3. Books which can be put under the table legs
4. Tape measure
5. Stopwatch
6. Graph paper

Tasks:

(a) Describe a method for determining the acceleration due to gravity g using the equipment given, including the relevant theory.

(b) Describe how you proceed with performing the experiment.

(c) Take the necessary measurements and present them in a table. Plot your results.

(d) Calculate g and estimate your error.

Problem 2.
Equipment:

1. Lightbulb from a flashlight, with wires soldered to it ($\times 2$, the second one is a spare)
2. Rectifier
3. Multimeter ($\times 2$)
4. Wires ($\times 6$)
5. Graph paper

Tasks:

(a) Assume that the lightbulb emits as a black body, and that the tungsten filament's resistance is proportional to its (thermodynamic) temperature. Propose a method for determining the exponent n in Stefan-Boltzmann's law.

(b) Assemble the circuit that you will use for your measurements. Make a sketch of the circuit.

(c) Take the necessary measurements and present them in a table. Plot your results.

(d) Calculate n and estimate your error.

(e) Discuss the reasons why your value may deviate from theory.

Note:

- Use the ammeter's 20 A range for current measurements. Breaking the multimeters will incur a penalty.
- The voltage across the lightbulb should stay in the range [4 V, 6 V]. If both your lightbulbs blow, you will not get a spare.

Each problem is worth 10 points.
Time: 5 hours.