2008 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. A solid ball of mass m and radius a (moment of inertia $I = \frac{2}{5}ma^2$) starts rolling without slipping from the top of another fixed ball of radius b. Its initial velocity is negligible. The acceleration due to gravity is g.

(a) Find the angle $\theta = \theta_0$ at which the rolling ball will lose contact with the fixed ball. The angle θ is measured between the upward direction and the segment connecting the centres of the balls.

(b) Find the velocity of the centre of mass v of the rolling ball when it detaches.

(c) What coefficient of friction k would make the upper ball start slipping at an angle $\theta = \alpha < \theta_0$?

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 1. A satellite of mass m moves in a circular orbit of radius r around a planet of mass M. Because of a drag force of the form $F_{\mathrm{dr}} = Av^n$, the orbital radius decreases at a constant rate

$$\frac{\mathrm{d}r}{\mathrm{d}t} = D \ll \frac{r}{T},$$

where T is the orbital period. The gravitational constant is γ.

(a) Find the number n.

(b) Determine $D = f(\gamma, M, m, A)$.

Problem 2. An incompressible fluid of viscosity η flows along a cylindrical pipe of length L and radius R. The pressures at the two ends of the pipe are p_1 and p_2, respectively. The flow is stationary.

(a) Find the flow velocity $v(r)$ in terms of the distance from the axis of the pipe r.

(b) Find the volumetric flow rate through the pipe Q.

Problem 3. A ball of mass M has velocity v_0. It strikes a ball of mass m ($M > m$) at rest. The collision is elastic. The angle between the velocity vectors of M before and after the collision is α.

(a) Find the maximum value of α.

(b) Find the velocities u_M and u_m of the two balls after the collison in the case where maximum α is realised.

Constants:

Acceleration due to gravity	g	$10.0\,\mathrm{m/s^2}$
Elementary charge	e	$1.6 \times 10^{-19}\,\mathrm{C}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J/K}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Diode and paperclip circuit.

Equipment:
Circuit consisting of two identical diodes and a paperclip (the diodes are connected in parallel and the paperclip is in series with one of the diodes), rectifier which can supply either constant voltage or constant current, two multimeters, resistor substitution box (current not to exceed 100 mA), wires, screwdriver, graph paper.

Task 1. Finding the resistance of the paperclip R.
In this part of the problem you will measure the I-V curve of the circuit (without using the substitution box) for both positive and negative (i.e. with reversed polarity) voltages.

Note: Do not exceed a current of 2.5 A.

(a) Sketch the circuit that you have assembled.

(b) Write down the ranges that you use for the multimeters.

(c) Describe how R can be calculated from your measurements.

(d) How will you use the rectifier – to supply a constant voltage or a to supply a constant current?

Note: The characteristics of the diodes have a strong dependence on temperature.

(e) **Quickly** measure the I-V curve of the circuit as the voltage/current is raised. After you have reached the maximum voltage/current, wait until the open diode reaches its equilibrium temperature (be careful not to burn yourself on one of the diodes). Then, **quickly** measure the I-V curve of the circuit as the voltage/current is lowered. Repeat this for voltages of the opposite polarity. Present your results in a table.

(f) Write down whether a diode is open when a positive potential is applied on the terminal with the white band, or vice versa.

(g) Decide on the dataset that you will use for determining R. Choose between the values taken when raising the current/voltage and those taken when lowering the current/voltage.

(h) Plot a graph from which you can find R.

(i) Find R from the graph.

(j) Using the graph, find your error ΔR.

Task 2. Finding the reverse-bias saturation current of the diodes I_S.
The current I_S is the maximum current through a closed diode. The I-V curve of a diode can be modelled by the Shockley diode equation,

$$I = I_S \left(e^{\frac{eU}{nkT}} - 1 \right),$$

where e is the charge of the electron, k is the Boltzmann constant, T is the absolute temperature, and n is a number close to 1.

(a) Find an approximation of the formula above which can be used when measuring the forward I-V curve for voltages on the order of a few hundred mV at room temperature.

(b) Apply a voltage of such polarity that the diode with no paperclip attached to it is open. Measure an appropriate part of the I-V curve for currents under 100 mA. Use the resistor substitution box if necessary. Present your results in a table.

(c) Plot your data in appropriate variables.

(d) Using the plot, find I_S and n.

Call the examiner if you suspect that a multimeter's fuse has blown, or in case of any other technical difficulties.

Each problem is worth 15 points.
Time: 5 hours.