2023 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. Collisions. A ball of mass M moves with velocity V_0 on a smooth horizontal surface. It collides elastically with a second ball of mass $m = 2\,\mathrm{kg}$. The second ball is connected to a third ball with a relaxed massless spring $k = 1\,\mathrm{N/m}$. The spring is long enough that the second and the third balls do not collide (Figure 1).

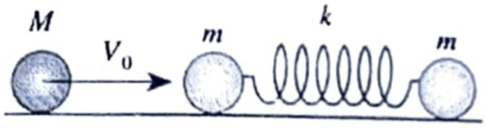

Figure 1

(a) Find the minimum mass of the first ball $M_{\min}$ for which it will collide with the second ball again.

(b) Find the time between the two collisions τ.

First solve the problem approximately. If you have enough time, try to improve on your result using numerical methods.

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 2

Problem. Betatron. The betatron is a compact particle accelerator for electrons which can bring them to relativistic velocities. It consists of two coaxial coils placed symmetrically about a thin cylindrical vacuum chamber (Figure 2). In the chamber there is a small electron source which emits electrons with zero initial velocity.

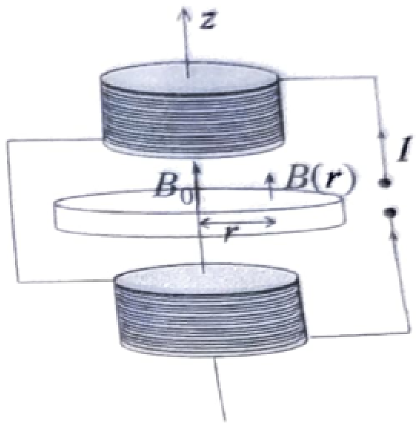

Figure 2

The magnetic field in the plane of the chamber is parallel to the z-axis and varies with the distance r to the coils' axis as follows:

$$B(r) = B_0 \left(1 - \left(\frac{r}{a}\right)^2\right),$$

where B_0 is the field at the centre of the chamber and a is a constant which depends on the size of the coils and the distance between them. After the betatron is turned on, the current in the coils increases, and B_0 grows from zero to some fixed maximum value $B_{\max}$.

(a) The source is placed at a distance r_S from the z-axis so that the electrons it emits are accelerated in the chamber along circular trajectories. Find r_S/a.

(b) Find the angle of the first diffraction maximum Θ_1. You can express your answer as a trigonometric function of Θ_1.

(c) Find the maximum kinetic energy of the electrons $K_{\max}$ for $B_{\max}$ and $a = 0.10\,\mathrm{m}$.

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 3

Problem. Yukawa potential. The interaction energy between a proton and a neutron in the nucleus depends on the distance r between them and is given by

$$U(r) = -U_0 \frac{e^{-r/r_0}}{r/r_0},$$

where $r_0 = 1.3 \times 10^{-15}\,\mathrm{m}$. This dependence was proposed by Hideki Yukawa in 1935. For certain values of U_0 the proton and the neutron are in a bound state, forming a deuteron. The experimental value for the binding energy of the proton and the neutron in a deuteron is $\varepsilon = 2.225\,\mathrm{MeV}$. Using the Heisenberg uncertainty principle, estimate:

(a) The minimum U_0 that allows for a deuteron to form.

(b) The size of the deuteron at the given binding energy.

(c) The value of U_0 at the given binding energy.

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 1. Uniform motion. Two point masses move uniformly with velocities $\mathbf{v_1}$ and $\mathbf{v_2}$ respectively. At time $t = 0$ their positions are $\mathbf{r_{01}}$ and $\mathbf{r_{02}}$.

(a) Find a formula for the time t_{min} when the particles are closest to each other.

(b) Find a formula for this minimum distance d.

(c) Consider an orthogonal basis with unit vectors $\mathbf{e_x}$, $\mathbf{e_y}$, and $\mathbf{e_z}$. The initial position vectors of the masses are $\mathbf{r_{01}} = 0$ and $\mathbf{r_{02}} = (2\,\text{m}) \cdot \mathbf{e_x} + (2\,\text{m}) \cdot \mathbf{e_z}$. Their initial velocities are $\mathbf{v_1} = (1\,\text{m/s}) \cdot \mathbf{e_x} + (1\,\text{m/s}) \cdot \mathbf{e_y}$ and $\mathbf{v_2} = (-1\,\text{m/s}) \cdot \mathbf{e_x} + (1\,\text{m/s}) \cdot \mathbf{e_y}$. Calculate t_{min} and d.

Problem 2. Is the Danube's flow laminar? A river of rectangular cross section has width l and depth h. The river flows laminarly at an angle α to the horizon. Ignore edge effects.

(a) Obtain a formula for the velocity of water $v(z)$ at a distance z from the bottom.

(b) Obtain a formula for the volumetric flow rate of the water Q.

(c) The depth of the Danube is $h = 10\,\text{m}$. The elevation of the water surface is $31\,\text{m}$ at Vidin and $16\,\text{m}$ at Ruse. The distance to the Danube Delta is $790\,\text{km}$ for Vidin and $460\,\text{km}$ for Ruse. Find the velocity of water at the surface of the Danube between Vidin and Ruse. Is the Danube's flow laminar?

The acceleration due to gravity is $g = 10\,\text{m/s}^2$, the density of water is $\rho = 1000\,\text{kg/m}^3$ and the viscosity of water is $\eta = 1.0 \times 10^{-3}\,\text{Pa\,s}$.

Problem 3. Pendulum on an inclined plane. A uniform ball of mass m and radius r is tied to a point on an inclined plane using a string of length l. The plane makes an angle α with the horizon. The acceleration due to gravity is g. The ball can only roll without slipping and the torque due to the string's twist can be neglected. Find the period of small oscillations of the ball about its equilibrium position on the inclined plane. The moment of inertia of the ball with respect to an axis passing through its centre of mass is $I_c = \frac{2}{5}mr^2$.

Problem 4. Voltage rectifier. The circuit on Figure 3 is connected to an ideal AC source of RMS voltage $\mathcal{E}_{\text{eff}} = 12\,\text{V}$ and frequency $\nu = 50\,\text{Hz}$. The resistance used is $R = 100\,\Omega$. The diode is ideal, i.e. zero resistance in one direction and infinite resistance in the other.

(a) Sketch the time dependences for the voltage of the source $\mathcal{E}(t)$ and the voltage across the resistor $U(t)$ on the same graph. The graph must include at least one full period of the AC voltage.

(b) Calculate the minimum capacitance C_{min} for which the voltage fluctuations on the resistor do not exceed $\Delta U = U_{\text{max}} - U_{\text{min}} = 1.0\,\text{V}$.

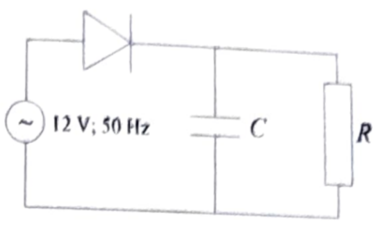

Figure 3

Problem 5. Convoluted grating. A diffraction grating has alternating slits of different length (wide, thin, wide...). The distance between adjacent slits is a, as shown on Figure 4. Monochromatic light of wavelength λ ($\lambda \ll a$) is normally incident on the grating. We observe the diffraction pattern at a large distance L ($L \gg a$) from the grating. If the wide slits are closed and the thin slits are left open, the maxima are of essentially equal intensity I_0. If we close the thin slits and leave the wide ones open, the maxima have intensity $2I_0$.

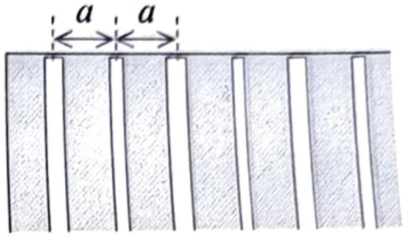

Figure 4

Now we leave all the slits open. Find the distance between the maxima Δx and the intensity of the maxima I_k in terms of their order k.

Problem 6. Optical fiber. A point source is placed at one end of an optical fiber, as shown on Figure 5. It emits a short light pulse of energy $E_0 = 5\,\mu\text{J}$, radiated isotropically within the fiber (uniformly in all directions inside the fiber). The fiber is $L = 10.0\,\text{m}$ long and its refractive index is $n = 1.50$. The fiber is surrounded by air of refractive index 1.

(a) Find the energy E_1 and the length Δt of the light pulse that reaches the other end of the fiber.

(b) Find an expression for the instantaneous power $P(t)$ of the light pulse at the other end of the fiber. The time t is measured starting from the emission of the pulse.

Neglect the dispersion in the medium, as well as any diffraction effects.

Figure 5

Problem 7. Relativistic dynamics.

(a) Express the relativistic momentum p of a particle of mass m in terms of its kinetic energy K.

Consider two particles of rest mass m. The first particle has kinetic energy K_0. It collides elastically with the second particle, which is at rest, and is deflected through an angle θ.

(b) Find the kinetic energy K_1 of the second particle after the collision in terms of K_0 and θ.

Problem 8. Predictions of the Bohr model.

(a) Calculate the magnetic field $\mathbf{B}$ (direction and magnitude) at the centre of a hydrogen atom due to an electron on the first Bohr orbit.

(b) Since the proton possesses a magnetic moment

$$\mu_p = \frac{e\hbar}{2m_p}$$

with possible projections $\pm\mu_p$ along the direction of the field, the ground state energy of the atom will change. Estimate the splitting of the the energy level in that case.

Problem 9. Neutron cooling. A fast neutron experiences elastic central collisons in a medium that acts as a moderator. Find the number of collisions necessary for a neutron of energy $1\,\mathrm{MeV}$ to reach the thermal velocity for temperature $T = 300\,\mathrm{K}$ in graphite.

Problem 10. Heat engine. A heat engine operates on a reversible cycle consisting of several processes. In the process 1-2 the molar heat capacity is proportional to the temperature and increases from $C_1 = 20\,\mathrm{J\,K^{-1}\,mol^{-1}}$ to $C_2 = 50\,\mathrm{J\,K^{-1}\,mol^{-1}}$. The next process 2-3 is adiabatic. The last process 3-1 is isothermal. Find the efficiency of the cycle. Note that the equation of state of the working substance is unknown.

Constants:

Boltzmann constant	k_B	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$
Speed of light in vacuum	c	$3.00 \times 10^{8}\,\mathrm{m/s}$
Electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Reduced Planck constant	$\hbar$	$1.05 \times 10^{-34}\,\mathrm{J\,s}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. All about gravity.

Equipment: Golf boll, table tennis ball, stopwatch, tape measure, tape measure, three-legged stool, wooden blocks, ruler, graph paper

Task 1. Measuring the acceleration due to gravity.
Using the wooden blocks, tilt the table at different angles to the horizon. By measuring the rolling time of the golf ball on the table, find the acceleration due to gravity g. Assume the golf ball is homogeneous. **(6.0 pt)**

Task 2. Measuring the coefficient of restitution for inelastic collisions.
Study the bouncing of the table tennis ball on the three-legged stool. Measure the dependence of total bouncing time of the ball on its initial height. Assume that the ball has stopped when you can no longer hear the sound from the collisions. Using your data, calculate the restitution coefficient of the partially inelastic collisions between the ball and the stool. **(9.0 pt)**

Relevant theory:
1) When a homogeneous ball rolls without slipping on an inclined surface that makes an angle α with the horizon, the time taken for the ball to move a distance l starting from rest is given by

$$t = \sqrt{\frac{14l}{5g\sin\alpha}}.$$

2) The restitution coefficient for a collision between two bodies is defined as $v_{\text{rel, after}}/v_{\text{rel, before}}$, where $v_{\text{rel, before}}$ and $v_{\text{rel, after}}$ are the relative velocities before and after the collision. Assume that this coefficient does not depend on the relative velocity of the bodies.

3) If a body is left to bounce from an initial height h, its total bouncing time (from the instant it is dropped until the instant it comes to rest) is given by

$$T = \frac{1+k}{1-k}\sqrt{\frac{2h}{g}}.$$

Constants and formulae:

Acceleration due to gravity	$g = 9.81\,\text{m/s}^2$
Reynolds number	$\text{Re} = \frac{\rho v L}{\eta}$
Critical Reynolds number for fluid flow around a ball	$\text{Re}_{\text{cr}} \approx 1$
Stokes' law	$F_{\text{drag}} = 6\pi\eta r v$
Newton's law	$F_{\text{drag}} = \frac{1}{2}C\rho S v^2$
Drag coefficient for a sphere	$C \approx \frac{1}{2}$
Density of air at normal temperature and pressure	$\rho = 1.29\,\text{kg/m}^3$
Viscosity of air at normal temperature and pressure	$\eta = 1.86 \times 10^{-5}\,\text{Pa\,s}$

Each problem is worth 15 points.
Time: 5 hours.