2024 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. Ballistic projectile. A rocket is launched from the surface of the Earth with velocity v_0 equal to the surface orbital velocity. The launch angle with respect to the horizon is α (Figure 1).

(a) What is the maximum height of the rocket above the Earth's surface H?

(b) What is the maximum range of the rocket L (measured along the surface of the Earth)?

Express your answers in terms of α and R. Neglect air drag.
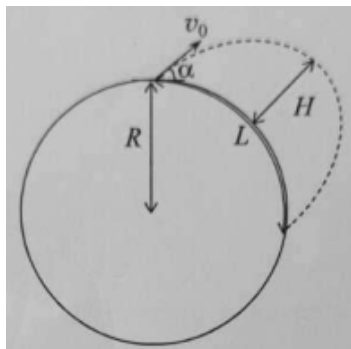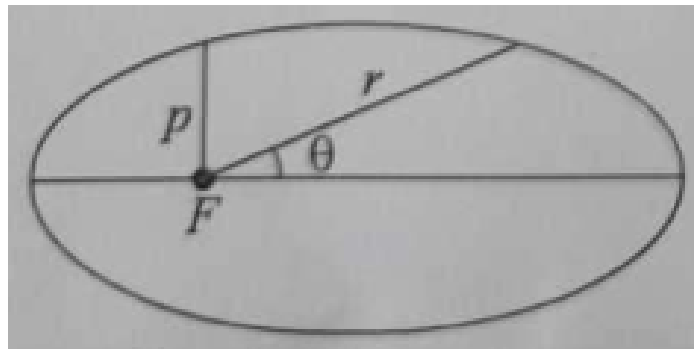

Figure 1

Figure 2

Hint: The equation of an ellipse in polar coordinates is

$$r = \frac{p}{1 - e\cos\theta},$$

where p is the semi-latus rectum (one half of the chord through one focus, perpendicular to the major axis), e is the eccentricity, and r is the distance to one of the foci F. The angle θ is measured at this focus F starting from the direction towards the most distant point on the ellipse (Figure 2).

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 2

Problem. Falling rod in a magnetic field. A capacitance C is connected to the ends of two long parallel vertical conducting wires of negligible resistance. The wires are a distance l apart. A rod of mass m and resistance R starts sliding along the wires. The system is placed in a homogeneous horizontal magnetic field B. The acceleration due to gravity is g.

(a) Find the current through the rod $I(\infty)$ after a long time.

(b) Find the acceleration of the rod $a(\infty)$ after a long time.

(c) Find the time dependence $I(t)$ for the current through the rod.

(d) Find the acceleration of the rod a_C for $R = 0$.

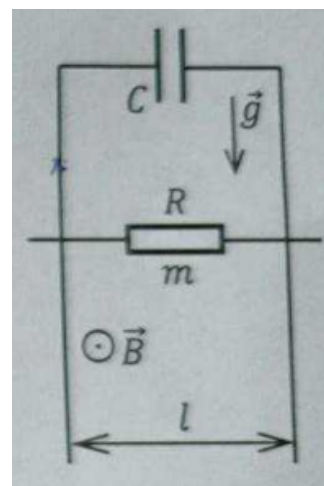Figure 3

(e) Find the velocity of the rod $v_R(\infty)$ after a long time if there was a wire in place of the capacitor.

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The problem is worth 5 points.
Time: 60 minutes.

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Short Exam 3

Problem. Phonon gas. At low temperatures (about $30\,\mathrm{K}$) the thermal properties of a dielectric crystal are governed by the atoms' harmonic oscillations. These oscillations are of a quantum nature, and correspond to waves that propagate through the crystal with velocity $v = 5 \times 10^3\,\mathrm{m/s}$. The ensemble of oscillating atoms can be modelled as a phonon gas. The phonons occupy the volume of the crystal V at temperature T. The energy of a phonon ε is related to its momentum p by $\varepsilon = vp$.

(a) Find a relation between the pressure of the phonon gas P and its internal energy density $u(T) = U/V$ at thermal equilibrium. Treat the crystal as a cube of edge l where the phonons collide elastically with the surfaces.

(b) Find the internal energy density u in terms of T.

(c) Using the uncertainty relation $\Delta E \Delta t \sim \hbar$, find the constant coefficient in your expression for u in terms of v and any relevant fundamental constants. Estimate the value of the coefficient.

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The problem is worth 5 points.
Time: 60 minutes.

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Theoretical Exam

Problem 1. Deflection. A ball of mass M has velocity v_0. It strikes a ball of mass m $(m < M)$ which is initially at rest. The collision is elastic and two-dimensional. Determine the maximum deflection angle $\theta_{\max}$ of the heavier particle following the collision.

Problem 2. Falling ladder. A ladder of length l (to be treated as a uniform rod) is leaning on a wall vertically. The lower end of the ladder is given a tiny push and the ladder starts falling down while keeping in contact with the wall and the ground. Find the height h at which the upper end of the ladder loses contact with the wall (Figure 4). Neglect friction.
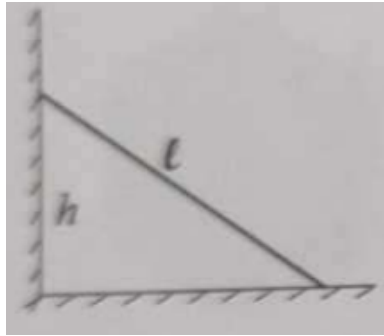

Figure 4

Problem 3. Rolling. A cylinder of mass m and radius R is placed on a horizontal surface. A small part of the cylinder is thinned out to a radius r. A light tether is wound around that part, as shown on Figure 5. A force F is applied to the free end of the tether at an angle α to the horizon. The cylinder starts to roll without slipping. Let the acceleration of the centre of the cylinder C be $\mathbf{a_C}$. Find the magnitude and the direction of $\mathbf{a_C}$ (left or right on the figure) in terms of α.
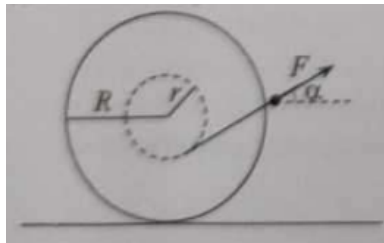

Figure 5

Problem 4. Resistances. A regular dodecahedron is a polyhedron composed of 12 regular pentagonal faces. Three faces meet at each of its 20 vertices. Likewise, a regular icosahedron is a polyhedron composed of 20 regular triangular faces (Figure 6). Five faces meet at each of its 12 vertices. Consider such polyhedra made out of identical rods of resistance R each.

(a) Find the resistances R_1 and R_2 between two adjacent and two opposite vertices of the dodecahedron.

(b) Find the resistances R_3 and R_4 between two adjacent and two opposite vertices of the icosahedron.

Problem 5. Soap bubble. Find the minimum thickness $d_{\min}$ of a thin soap film of refractive index $n = 1.33$ which best reflects light of wavelength $640\,\text{nm}$ and does not reflect light of wavelength $400\,\text{nm}$. The light is incident at an angle of $30°$. The soap film is surrounded by

air (of refractive index unity). At thickness d_{min}, what are the other wavelengths in the visible spectrum that correspond to minima or maxima?
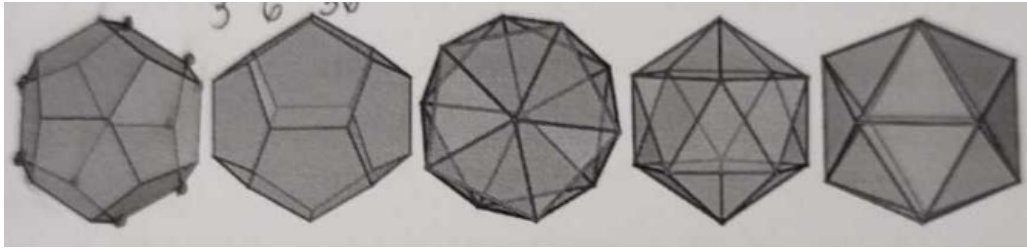

Figure 6

Problem 6. Solar halo, lunar halo. In cold winter days or nights, when the weather is clear, high up in the mountains one can see a bright circle around the Sun or the Moon. This is called a halo, and it is caused by the refraction of light in ice crystals. Assume that the ice crystals are in the shape of regular hexagonal prisms and that the light rays travel in planes parallel to their bases. The refractive index of ice in the middle of the visible spectrum is $n = 1.31$. Find the radius of the halo, that is, the angle between a point on the halo and the light source. What is the colour of the halo near its inner edge and its outer edge?

Problem 7. Heat engine. A heat engine operates on the cycle shown on Figure 7. The minimum temperature is T_1 and the maximum temperature is T_2.

(a) State a geometric interpretation for the work done by the heat engine.

(b) Find the efficiency of the cycle η in terms of T_1 and T_2. What is the minimum data necessary for calculating η?

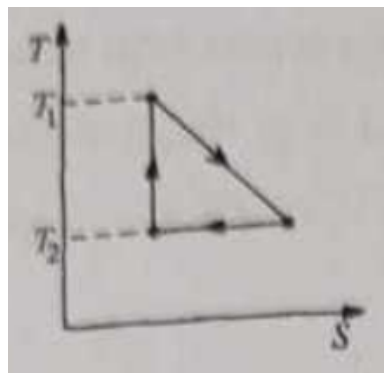(c) Suggest particular values for this data and calculate η.

Figure 7

Problem 8. Ventilated room. The air in a room of volume $V = 27\,\mathrm{m}^3$ is slowly heated. A small window is left ajar so that air can leave through the opening. The outside pressure is $P = 1\,\mathrm{atm}$.

(a) What is the amount of heat Q necessary to heat up the air in the room from $0\,°\mathrm{C}$ to $20\,°\mathrm{C}$?

(b) Find the change in the internal energy of the air ΔU.

(c) Find the entropy change of the air in the room ΔS during the heating.

Problem 9. Pion mass and transuranium elements.

(a) Modern physics explains the attractive nuclear forces with the exchange of particles called pions. Use the uncertainty relations to find the mass of a pion m.

(b) Classical physics allows for the electrostatic field of an atomic nucleus (of atomic number $Z > 92$) to attract an electron close to itself. On the other hand, the Heisenberg uncertainty principle implies that the electron's localisation in the nucleus leads to a sharp increase in its kinetic energy. Find the atomic number of the transuranium element which can keep an electron close to its nucleus. Assume that this element is stable.

Problem 10. Square. Two protons and two positrons are held at rest at the vertices of a square of side length $a = 1\,\mathrm{cm}$ such that like particles lie on opposite vertices. The particles are simultaneously released. Estimate their velocities after a long time. Treat protons and positrons as classical point masses and neglect their gravitational interactions.

Constants:

Boltzmann constant	k_B	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Vacuum permittivity	ε_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Speed of light in vacuum	c	$2.998 \times 10^8\,\mathrm{m/s}$
Electron/Positron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$\approx 2000 m_e$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
Reduced Planck constant	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Incandescent light bulb.

Equipment:
DC power supply (supplies voltages $[0, 30\,\text{V}]$ or currents $[0, 5\,\text{A}]$), light bulb with tungsten filament, 2 multimeters, 6 wires, ruler, tables, graph paper (Figure 8).
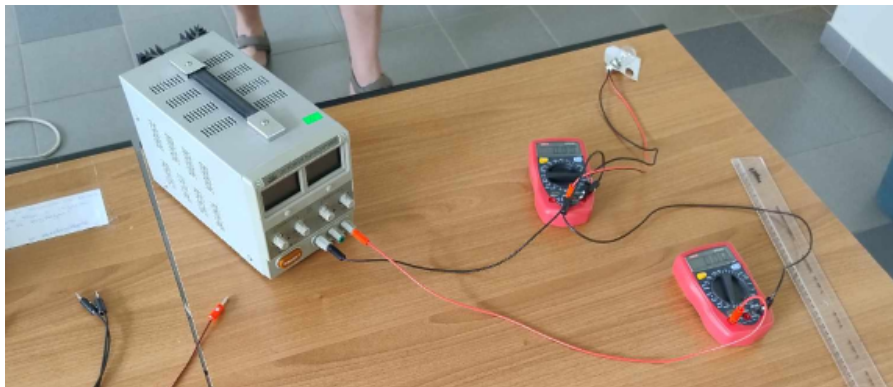

Figure 8

The nominal parameters of the lamp are $12\,\text{V}$ and $21\,\text{W}$. In the first four tasks it is good enough for our purposes to assume that the resistance of tungsten is proportinal to its absolute temperature. Record all measurements in tables. Write down your results in the answer sheet.

Note: When using the multimeters you can only work with two of the ranges: $20\,\text{V}$ and $10\,\text{A}$.
Note: Do not supply voltages larger than $13\,\text{V}$ or the lamp may blow.
Note: The lamp is hot when operating. Do not touch it and do not leave it in contact with other objects.

(a) Assemble a circuit that will allow you to measure the I-V curve of the lamp. Sketch the circuit. **(1.0 pt)**

At high temperatures you can assume that the power emitted by the filament is proportinal to T^n, where T is the absolute temperature of the filament and n is a real number.

(b) Study the I-V curve of the lamp at high temperatures and find the number n using a graph. **(7.0 pt)**

At low temperatures (i.e. close to room temperature) the power emitted by the filament is proportional to $(T - T_0)$, where T is the absolute temperature of the filament and T_0 is the temperature of the surroundings.

(c) Study the I-V curve of the lamp at low temperatures and find the resistance of the filament at room temperature R_0 using a graph. **(5.0 pt)**

(d) Using your data, your intermediate results, as well as the appropriate assumptions, find the working temperature of the filament T_1 at the nominal voltage $12\,\text{V}$. **(1.0 pt)**

Figure 9 shows experimental values for the resistivity of tungsten with temperature. You are also given a second-degree polynomial fit for the data.

(e) Using your data together with the resistivity plot (or formula), calculate the working temperature of the filament T_2 at the nominal voltage $12\,\text{V}$ again. **(1.0 pt)**

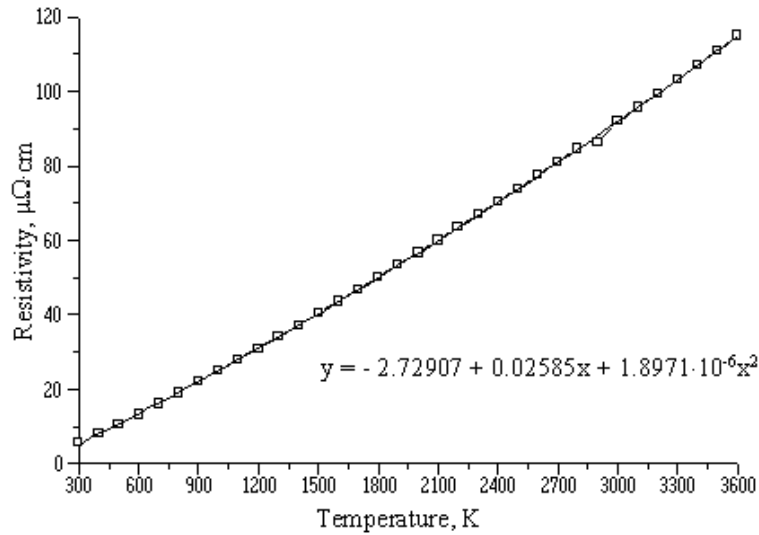Call the examiner in case of any technical difficulties.

Figure 9

Problem 2. Laws of hydrodynamics.

Introduction:
Water and glycerine have very different properties. In most everyday situations the viscosity of water can be neglected and it can be treated as an ideal fluid. In contrast, glycerine is one of the most viscous fluids and its motion is governed by internal friction. In this problem you will study the physical laws which describe the fluid flow out of a syringe with a narrow opening. You will work with an ideal fluid and a highly viscous fluid. You will also determine the viscosity of a glycerine solution.

Figure 10

Consider a syringe consisting of a wide cylindrical chamber of cross section S and a cylindrical nozzle of length l and cross section $\sigma \ll S$ (Figure 10). The syringe is placed vertically and is filled to a level h above the lower end of the nozzle.
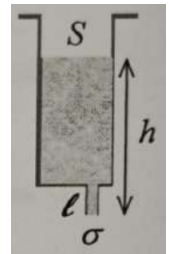
In the case of an ideal fluid the viscosity of the liquid is negligible and the speed of efflux at the lower end of the nozzle is given by Torricelli's law:

$$(1) \qquad\qquad v = \sqrt{2gh}.$$

In the case of a highly viscous fluid the volumetric flow rate Φ out of the nozzle is given by Poiseuille's law:

$$(2) \qquad\qquad \Phi = \frac{\pi r^4 \rho g h}{8\eta l},$$

where r is the radius of the nozzle, ρ is the density of the liquid, and g is the acceleration due to gravity.

Use the following values, which can be considered exact:

- Acceleration due to gravity: $g = 9.81\,\mathrm{m/s^2}$

- Density of the glycerine solution: $\rho = 1200\,\mathrm{kg/m^3}$

Equipment:

1. Syringe (without a piston) with a scale graduated to $20\,\mathrm{ml}$ (Figure 11). The length of the nozzle is $l = 1.20\,\mathrm{cm}$. This value can be considered exact.

2. Plastic cup and bottle of glycerine solution.

3. Empty cups for pouring the water and the glycerine.

4. Disposable plate, to be placed below the syringe so as to avoid spilling water on the workstation.

5. Ruler.

6. Stopwatch.

7. Two sheets of graph paper, sheet with two tables, two blank sheets. If you need additional tables, make your own.

8. Kitchen roll to absorb water with.

Figure 11

(a) The syringe is placed vertically and is filled with fluid up to a level h_1 above the lower end of the nozzle. The fluid starts to flow out of the syringe. Prove that the time t taken for the fluid to drain to a level h_2 above the lower end of the nozzle ($h_1 > h_2 > l$) is given by

$$\text{(3a)} \qquad t = \frac{S}{\sigma} \left(\sqrt{\frac{2h_1}{g}} - \sqrt{\frac{2h_2}{g}} \right) \qquad \text{for an ideal fluid,} \qquad \textbf{(1.0 pt)}$$

$$\text{(3b)} \qquad t = \frac{8\pi \eta l S}{\sigma^2 \rho g} \ln\left(\frac{h_1}{h_2}\right) \qquad \text{for a viscous fluid.} \qquad \textbf{(1.0 pt)}$$

(b) Find the inner cross section S of the chamber of the syringe. Describe your methods and calculate your measurement error ΔS. **(2.0 pt)**

(c) Using water as your ideal fluid, take appropriate measurements in order to confirm Equation (3a). Present your results in tabular and graphical form using appropriate variables. **(3.5 pt)**

(d) Using your data from (c), find the cross section of the nozzle σ. **(2.0 pt)**

If you cannot find the value of σ, you may use the approximate value $\sigma = 3 \, \text{mm}^2$ in what follows. This will cost you 2 points.

    (e) Using the glycerine solution as your viscous fluid, take appropriate measurements in order to confirm Equation (3b). Present your results in tabular and graphical form using appropriate variables. **(3.5 pt)**

    (f) Using your data from (e), find the viscocity of the glycerine solution η. **(2.0 pt)**

Each problem is worth 15 points.
Time: 5 hours.