2018 Bulgarian IPhO Team Selection Test – Solutions

Short Exam 1

Problem. Falling ladder. The two ends of a rod of mass m and length l sit on a horizontal floor and on a vertical wall, respectively. The rod lies in a plane which is perpendicular both to the floor and the wall. The acceleration due to gravity is g. Initially the rod is at rest and it makes an angle α_0 with the floor. The rod is let go and starts falling. Ignore friction. The moment of inertia of a rod about an axis passing through its centre of mass perpendicularly to the rod is $I = \frac{1}{12}ml^2$.

(a) Find the velocity of the centre of mass of the rod v, as well as its angular velocity ω, as a function of the angle α which the rod makes with the floor during its descent.

(b) At what angle α_1 does the rod lose contact with the wall?

(c) Find the velocity v_∞ with which the rod will slide on the floor after it has fallen down.

Solution.

Short Exam 2

Problem. Induction motor. Figure 1 shows an asynchronous motor. The rotor is made up of two metal rings attached to the axis, and a large number of rods N which connect the rings. A system of inductors (not shown on the figure) creates a homogeneous magnetic field B perpendicular to the axis of the motor. The inductors are powered by three-phase power, and as a result the magnetic field vector rotates around the axis of the motor with an angular velocity ω_0.

The radius of the rings is a and the length of the rods is l. Each rod has a resistance R and the resistance of the rings is negligible.

(a) Obtain an expression for the torque M acting on the rotor when it stays fixed.

(b) Assume the rotor powers some mechanical device, as a result of which it rotates with an angular velocity ω ($0 < \omega < \omega_0$). Find an expression for the torque in terms of ω. What is the maximum possible mechanical power of the motor $P_{\max}$?

Hint: You may want to work in another frame of reference.

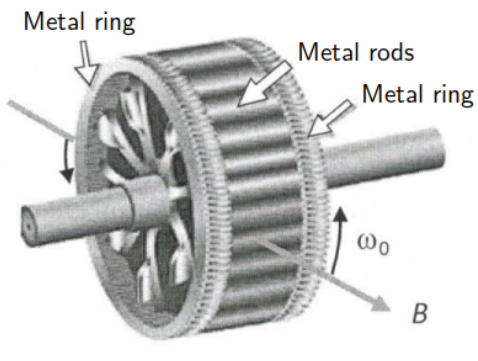

Figure 1

Solution.

Short Exam 3

Problem. Bose-Einstein condensate. An ideal monatomic gas ($^4_2$He) composed of bosons is cooled down at constant volume V and constant particle number N. As its temperature decreases, we reach a temperature T_0 below which the properties of the gas arise from the quantum properties of the bosons – i.e. their wavelike nature and their indistinguishability.

(a) Find T_0. The wavelike properties become significant when the de Broglie wavelength at the average thermal energy is approximately equal to the mean distance between the particles. Provide a numerical estimate for the number density $n = N/V$ if $T_0 = 4\,\mathrm{K}$.

At temperatures $T < T_0$ the particles of the gas can be separated into two groups, each encompassing a nonnegligible number of particles. The first group consists of N_0 particles at the lowest energy level ($\varepsilon = 0$), which do not take part in the thermal motion. The second group consists of N^* particles distributed across various energy levels (with $\varepsilon > 0$). These do take part in the thermal motion, and their number is given by

$$N^* = N\left(\frac{T}{T_0}\right)^{3/2}.$$

This is called a degenerate Bose gas.

(b) Find the heat capacity of the gas C_V when $T < T_0$. For this temperature range, find the equation of a reversible adiabatic process in the variables T and V.

(c) Find the pressure of the gas P when $T < T_0$. What is interesting about this result?

Apart from the thermodynamic variables T, V, and N, your results must include the Planck constant h, the Boltzmann constant k, and the mass of the Helium atom, $m_{\mathrm{He}} = 6.7 \times 10^{-27}\,\mathrm{kg}$.

Solution.

Theoretical Exam

Problem 1. Oscillations and dissipation. Two bodies, each of mass $m = 100\,\mathrm{g}$, are attached to the two ends of a spring with relaxed length $l_0 = 5.00\,\mathrm{cm}$ and spring constant $k = 100\,\mathrm{N/m}$. The bodies are placed on a horizontal surface, where their coefficient of friction with the surface is $\mu = 1.00$. Initially Body 2 is at rest and the spring is relaxed. Body 1 is imparted a velocity $v_0 = 1.00\,\mathrm{m/s}$ directed towards Body 2.

(a) Find the maximum extension of the spring Δl during the subsequent motion of the system.

(b) Find the displacement x_2 of Body 2 between its initial position and its position at the instant when the spring is most extended.

Solution.

Problem 2. Spray. A long horizontal cylindrical pipe of length l and diameter $d \ll l$ rotates with an angular velocity ω about a vertical axis passing through one of its ends. The pipe contains some ideal incompressible fluid of density ρ which forms a column of length l due to the rotation. There are small holes at both ends of the pipe.

(a) Find a formula for the velocity with which the fluid exits the pipe. Do not account for gravity in this part of the problem.

(b) Calculate the distance from the axis at which the stream strikes the floor. The pipe is $2\,\mathrm{m}$ long and the fluid column is $1\,\mathrm{m}$ long. The pipe rotates with a period $T = 0.5\,\mathrm{s}$ at a height $H = 2.0\,\mathrm{m}$ above the floor. Neglect air drag.

Solution.

Problem 3. Car suspension. The front axle and the rear axle of a car are at a distance $l = 2.0\,\mathrm{m}$ from each other. The centre of mass of the car is at a distance $h = 0.4\,\mathrm{m}$ above the ground, and it is located midway between the front and the rear wheels. When the car is at rest, the suspension spring of each of the wheels is compressed by $\Delta = 10\,\mathrm{cm}$ with respect to its relaxed length. As the car moves, the driver steps on the brakes and the wheels of the car start slipping along the road. The coefficient of friction between the tyres and the road is $\mu = 1.0$. Calculate the angle at which the car's body would tilt with respect to the horizon. The mass of the wheels is negligible.

Solution. This is Problem 2.25 from MIPT, Volume 1.

Problem 4. Field strength. An infinite sheet is given a uniform charge density σ. A hole of radius a is cut out from the sheet (Figure 2). Find expressions for the electric field at:

(a) A point A lying on the axis of the hole, at a distance z away from the sheet.

(b) A point B lying in the plane of the sheet, at a small distance r ($r \ll a$) away from the centre of the hole.

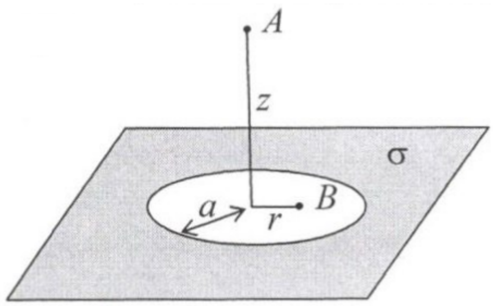

Figure 2

Solution.

Problem 5. Accelerating ring. A copper ring of cross-section $S = 1\,\mathrm{mm}^2$ and radius $r = 5\,\mathrm{cm}$ starts rotating around its axis with a constant angular acceleration $\alpha = 1000\,\mathrm{rad/s^2}$. Find the magnetic field B at the centre of the ring. The resistivity of copper is $\rho = 1.68 \times 10^{-8}\,\Omega\mathrm{m}$. *Hint:* You may want to work in the reference frame of the rotating ring. What is the force that gives rise to an EMF in the ring?

Solution. This problem covers the Stewart-Tolman effect, which is also described in Problem 18 from Kevin Zhou's Handout ERev (i.e. MPPP 173). The reason why a current appears in the ring is that the electrons will lag behind the lattice ions by a little, despite the mutual interactions that try to keep them together. This will become clearer if we work in the reference frame of the lattice. So, let's look at the ring at some instant when its angular velocity is ω, and switch to the frame that rotates with ω and accelerates with $\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$.

In this case, we'll get some inertial forces. The full expression for the force experienced by an object of mass m is

$$\mathbf{F}' = m\mathbf{a}' = \mathbf{F} - m\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} \times \mathbf{r}' - 2m\boldsymbol{\omega} \times \mathbf{v}' - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}'),$$

where $\mathbf{F}$ covers the actual physical forces, the second term is the Euler force, the third term is the Coriolis force, and the fourth term is the centrifugal force. Here $\mathbf{r}'$ is measured from the origin, which is somewhere on the rotation axis. This formula is difficult to remember, but the terms can be simplified. Forget about the origin, and denote by ρ the distance from the rotation axis to our object. We'll write the respective unit vector as $\hat{\boldsymbol{\rho}}$, and we'll use $\hat{\boldsymbol{\phi}}$ for the tangential unit vector. After applying the triple product rule, we'll find for the centrifugal term

$$\mathbf{F}_{\mathrm{R}} = -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') = (m\omega^2\rho)\hat{\boldsymbol{\rho}}.$$

The Coriolis term is somewhat nicer without the minus:

$$\mathbf{F}_{\mathrm{C}} = 2m\mathbf{v}' \times \boldsymbol{\omega}.$$

For some reason I always forget whether the $\mathbf{v}'$ or the $\boldsymbol{\omega}$ comes first. In such cases I find it useful to work with mnemonics. For example, I associate the above formula with the phrase "**to view**" (and the m is obvious, because it's a force).

The Euler term reduces to

$$\mathbf{F}_{\mathrm{E}} = -(m\rho\alpha)\hat{\boldsymbol{\phi}}.$$

Considering that $\rho\alpha$ is just the linear acceleration of the object, this force is actually rather familiar. It's the same thing as the inertial force $-ma$ that pushes you against your seat when you're accelerating in a car. You should just imagine that the rectilinear motion of the car is part of a turn with a very large radius ρ and a very small angular acceleration α.

Back to our problem. Here, the centrifugal and the Coriolis forces are both radial, while the Euler force is tangential. This means that if an electron does a lap around the ring, the Euler force will do work on it equal to

$$A = \oint \mathbf{F}_{\mathrm{E}} \cdot \mathrm{d}\mathbf{r} = (-m\alpha r)(2\pi r).$$

But this can be interpreted as an electromotive force $\varepsilon = A/(-e)$. So, we have a current

$$I = \frac{\varepsilon}{R} = \frac{AS}{(-e)2\pi r\rho} = \frac{m\alpha r S}{e\rho}.$$

The magnetic field in the centre of the ring is then

$$B = \frac{\mu_0 I}{2r} = \boxed{\frac{\mu_0 m\alpha S}{2e\rho} = 2.1 \times 10^{-13}\,\mathrm{T}.}$$

This remains the same in the lab frame, because the centre of the ring is motionless in both frames. The reason is that the Lorentz transformations for the EM field are local, meaning that they depend only on the field values *right here*. And in this problem we're talking about a point where the velocity is zero, so we don't expect any changes.

Problem 6. Lens and plate. A source of monochromatic light (of wavelength $\lambda = 532\,\mathrm{nm}$) is placed at a distance $a = 4\,\mathrm{cm}$ from a lens of radius $R = 2\,\mathrm{cm}$ and focal length $f = 3\,\mathrm{cm}$ (Figure 3). A screen is placed in the focal plane on the other side of the lens.

(a) Find the radius r of the illuminated spot on the screen. You can neglect the diffraction from the rim of the lens.

(b) A thin plate of thickness $d = 20\,\mathrm{\mu m}$ and refractive index $n = 1.5$ is put between the source and the lens, perpendicularly to the optical axis of the lens (Figure 4). The illuminated spot on the screen turns into alternating concentric bright and dark rings. Find the number of bright rings N.

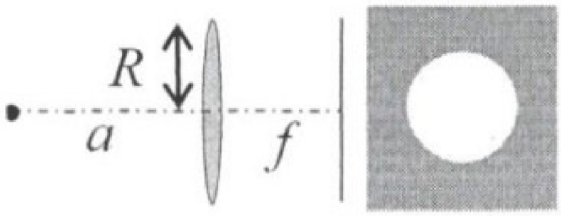

Figure 3

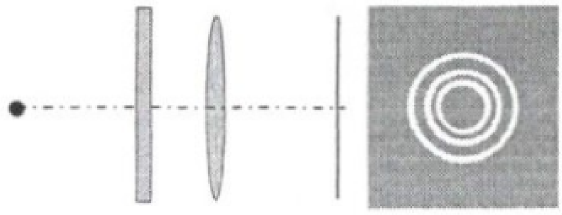

Figure 4

Solution.

Problem 7. Relativistic force. A particle of rest mass m starts moving under a constant force F.

(a) Find the total distance covered by the particle in the lab frame until it reaches a velocity $v = 0.8c$, where c is the speed of light in vacuum.

(b) Find the proper time taken for the particle to reach a velocity $v = 0.8c$.

Hint: $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{1+x^2}} = \ln\left(x+\sqrt{1+x^2}\right) + C$

Solution. (a) We will set $c = 1$ to reduce the writing load, and we'll restore the c factors at the end via dimensional analysis. We start with $F = \frac{\mathrm{d}p}{\mathrm{d}t}$. We multiply both sides of $\mathrm{d}p = F\mathrm{d}t$ by v, and we see that the distance covered is

$$x = \int \mathrm{d}x = \frac{1}{F}\int v\,\mathrm{d}p = \frac{1}{F}\left((vp)\Big|_0^{0.8} - \int_0^{0.8} p\,\mathrm{d}v\right) = \frac{m}{F}\left(\frac{v^2}{\sqrt{1-v^2}}\Big|_0^{0.8} - \int_0^{0.8}\frac{v}{\sqrt{1-v^2}}\,\mathrm{d}v\right).$$

After evaluating the integral using a substitution, we reach

$$x = \frac{m}{F}\left(\frac{v^2}{\sqrt{1-v^2}} - \sqrt{1-v^2}\right)\Big|_0^{0.8} = \frac{m}{F}\left(\frac{1}{\sqrt{1-v^2}} - 1\right) = \frac{2m}{3F} \Leftrightarrow \boxed{\frac{2mc^2}{3F}.}$$

(b) To count the total proper time, we need to sum all the little proper time intervals $\mathrm{d}\tau = \mathrm{d}t/\gamma$. This means we need to compute

$$\tau = \int \sqrt{1-v^2}\,\mathrm{d}t.$$

This isn't as bad as it looks, because we can find the dependence $v(t)$ from $\mathrm{d}p = F\mathrm{d}t$. After integrating, we have $v/\sqrt{1-v^2} = Ft/m$, which is the same as $1 - v^2 = \frac{1}{1+(Ft/m)^2}$. So:

$$\tau = \frac{m}{F}\int_0^{Ft_0/m}\frac{1}{\sqrt{1+(Ft/m)^2}}\,\mathrm{d}(Ft/m),$$

where t_0 is the time taken to reach $v = 0.8$. Using the dependence above, we find $t_0 = 4m/3F$, and therefore

$$\tau = \frac{m}{F}\int_0^{4/3}\frac{1}{\sqrt{1+u^2}}\,\mathrm{d}u = \frac{m}{F}\ln\left(u+\sqrt{1+u^2}\right)\Big|_0^{4/3} = \frac{m}{F}(\ln 3 - \ln 1) \Leftrightarrow \boxed{\frac{mc}{F}\ln 3.}$$

Problem 8. Fast and slow. A mole of ideal gas is put in a vertical cylinder under a light freely moving piston. The pressure of the gas is p_0 and its temperature is T_0. Compare the final temperatures T_1 and T_2 of the gas at the end of the following processes:

1. The external pressure increases (or decreases) from p_0 to p instantaneously.
2. The external pressure increases (or decreases) from p_0 to p slowly.

The gas is thermally insulated from the surroundings and it has an adiabatic index of γ.

Solution. This is a repeat of Problem 2B from the 2003 Bulgarian National Round (which has a wrong solution). You can also find this one in many Russian books. It's Problem 1.71 in MIPT, Volume 1 – but the solution there is also wrong. We'll write $p/p_0 \equiv \kappa$, and we'll denote $\nu = 1\,\text{mol}$.

Let's start with T_1. After the outside pressure is changed from p_0 to p, the gas will expand violently. Eventually it has to reach an equilibrium with the surroundings, and its pressure then must be p. This is a nonquasistatic process, and we know nothing about it except for the parameters in the initial and the final state. Thus, we're only allowed to use the first law of thermodynamics. There's no external heat input ($Q_{\text{in}} = 0$), or in other words, the process is adiabatic:

$$A_{\text{gas}} + \Delta U = -A_{\text{surroundings}} + \nu C_v(T_1 - T_0) = p(V_{\text{final}} - V_{\text{initial}}) + \frac{1}{\gamma - 1}\nu R(T_1 - T_0) = 0.$$

Next, we have $p_0 V_{\text{initial}} = \nu R T_0$ and $pV_{\text{final}} = \nu R T_1$. We reach

$$(T_1 - \kappa T_0) + \frac{1}{\gamma - 1}(T_1 - T_0) = 0 \quad \Rightarrow \quad T_1 = \frac{\kappa(\gamma - 1) + 1}{\gamma} T_0 = \left(1 + \frac{\gamma - 1}{\gamma}\frac{(p - p_0)}{p_0}\right) T_0.$$

Now for T_2. This process is quasistatic and adiabatic, so the expansion of the gas will obey $pV^{\gamma} = \text{const}$. Using the ideal gas equation, we see that this is the same as $p^{\frac{1-\gamma}{\gamma}} T = \text{const}$. It follows that

$$T_2 = \kappa^{\frac{\gamma-1}{\gamma}} T_0 = \left(1 + \frac{p - p_0}{p_0}\right)^{\frac{\gamma-1}{\gamma}} T_0.$$

We'll introduce $x \equiv \frac{p-p_0}{p_0}$ and $a \equiv \frac{\gamma-1}{\gamma}$, noting that $x > -1$ and $a \in (0,1)$. The expressions for the temperature are then $T_1 = (1 + ax)$ and $T_2 = (1 + x)^a$. They are equal when $x = 0$. To compare them for all x, we'll need to look at their derivatives:

$$\frac{\mathrm{d}T_1}{\mathrm{d}x} = a, \qquad \frac{\mathrm{d}T_2}{\mathrm{d}x} = \frac{a}{(1 + x)^{1-a}}.$$

Since $1 - a$ is a positive number, the derivative of T_2 will be smaller than that of T_1 for $x > 0$, and larger than that of T_1 for $x < 0$. This means that the graph of $T_1(x)$ always lies above that of $T_2(x)$, even though they touch at $x = 0$. Thus $\boxed{T_1 \geq T_2}$ (always). In case you're curious, we just proved a special case of the so-called Bernoulli inequality.

Problem 9. Click-clack. A monatomic ideal gas is subjected to a process where the number of collisons Z between the atoms per unit volume per unit time remains constant.

(a) Find how Z depends on the pressure of the gas p and the temperature of the gas T.

(b) Find the equation of the process in terms of p and V.

(c) Find the molar heat capacity of the process C.

Solution. (a) Each atom has a characterisic cross-section σ. If the centre of any other atom is incident on this cross-section, then this counts as a collision. We'll look at the rate of collisions experienced by a single atom, working in its rest frame. Imagine that the quadratic mean velocity of the atoms is v in the lab frame. In the rest frame, however, we have

$$\left\langle \mathbf{v}_{\text{relative}}^2 \right\rangle = \left\langle (\mathbf{v}_1 - \mathbf{v}_2)^2 \right\rangle = \left\langle \mathbf{v}_1^2 + \mathbf{v}_2^2 - 2\mathbf{v}_1 \cdot \mathbf{v}_2 \right\rangle = 2\left\langle \mathbf{v}_1^2 \right\rangle,$$

because $\mathbf{v}_1$ and $\mathbf{v}_2$ are uncorrelated. We see that in the rest frame, the RMS speed is $\sqrt{2}v$ instead of just v. This multiplier $\sqrt{2}$ will also carry over when considering the change of the arithmetic mean velocity $\bar{v}$.

Our atom is bombarded by other atoms with arithmetic mean velocity $\sqrt{2}\bar{v}$ from all directions. However, since it offers the same cross-section σ no matter what the angle of incidence is, nothing will change in terms of the collision rate if we imagine that all the atoms are flying in from one fixed direction. And so the number of collisions in time $\mathrm{d}t$ can be found from counting the atoms contained within a cylinder of base area σ:

$$\mathrm{d}N = \sqrt{2}n\bar{v}\sigma\,\mathrm{d}t.$$

Here n is the number density of the atoms in the gas. Note that by using the arithmetic mean velocity, we've applied appropriate averaging for the fact that atoms of different speeds will fly in at different rates.

Now we know that one atom experiences $\sqrt{2}n\bar{v}\sigma$ collisions per unit time. But there are n atoms per unit volume, and collisions happen in pairs (divide by two!). Hence $Z = n^2\bar{v}\sigma/\sqrt{2}$. Next we use that $n = p/k_BT$ and $\bar{v} = \sqrt{8k_BT/\pi m}$. This yields

$$Z = \left(\frac{2\sigma}{\sqrt{\pi k_B^3 m}}\right)\left(\frac{p^2}{\sqrt{T^3}}\right) \sim \boxed{p^2T^{-3/2}}.$$

The examiner doesn't care about the prefactors, just the p and T dependence, so we could've been less fussy.

(b) Using $p^2T^{-3/2} = \text{const}$ and $pV/T = \text{const}$, we find $\boxed{pV^{-3} = \text{const.}}$

(c) We have $C = \mathrm{d}Q/\nu\mathrm{d}T$, where ν is the number of moles. From $\mathrm{d}Q = \mathrm{d}U + p\mathrm{d}V$ and $\mathrm{d}U = \nu C_v\mathrm{d}T$ we get

$$C = C_v + \frac{p\mathrm{d}V}{\nu\mathrm{d}T} = \frac{3}{2}R + \frac{p\mathrm{d}V}{\nu\mathrm{d}T}.$$

To obtain $p\mathrm{d}V$, we need to look at the equation of the process. We take its logarithm and differentiate, yielding

$$\frac{\mathrm{d}p}{p} - 3\frac{\mathrm{d}V}{V} = 0 \quad \Rightarrow \quad 3p\mathrm{d}V = V\mathrm{d}p.$$

This enables us to find $\mathrm{d}(pV) = p\mathrm{d}V + V\mathrm{d}p = 4p\mathrm{d}V$. But $\mathrm{d}(pV) = \nu R\mathrm{d}T$, and so

$$C = \frac{3}{2}R + \frac{1}{4}\frac{\nu R\mathrm{d}T}{\nu\mathrm{d}T} = \frac{3}{2}R + \frac{1}{4}R = \boxed{\frac{7}{4}R.}$$

Problem 10. Electron beam. Using Heisenberg's uncertainty principle, estimate the minimum diameter d of the spot which an electron beam makes on a screen, given that the electrons take $\tau = 10^{-8}\,\mathrm{s}$ to get from the collimator (a circular opening) to the screen.

Solution. This is Problem 2.29 from MIPT, Volume 3. We'll assume that the collimator has a diameter D. In a world with no diffraction, we'd have $d = D$. Alas, things are not so simple. We'll denote the distance to the screen by l. The electrons will leave the collimator with some fixed momentum p_y along its axis, and also a small orthogonal component p_x that can be found from the uncertainty principle, $p_x D \sim \hbar$. The angular spread θ of the beam is then given by

$$\tan\theta = \frac{p_x}{p} = \frac{\hbar}{mvD} = \frac{\hbar\tau}{mlD}.$$

The diameter of the spot is

$$d = D + 2l\tan\theta = D + \frac{2\hbar\tau}{mD}.$$

We want to pick the D which minimises this expression. Using either derivatives or the AM-GM inequality, we find that the best possible choice is $D = \sqrt{2\hbar\tau/m}$. Substituting this back into our result for d, we get

$$d = 2\sqrt{\frac{2\hbar\tau}{m}} \approx \boxed{3\,\mu\text{m}.}$$

Experimental Exam

Problem 1. Measuring the density of irregular-shaped bodies.

Equipment:
Kitchen scale ($m < 500\,\mathrm{g}$!), stand, binder clip, glass cylinder, plastic cup, stopwatch, ruler, tape measure, bottle with 1.5 l of tap water, scissors, funnel, string, graph paper, and the following five bodies:

1. Fishing sinker (grey ball with a channel through the diameter)
2. Hinge from a cupboard (grey, rectangular, with 4 holes)
3. White piece of metal
4. Reddish piece of metal
5. Bouncy ball with a smiley face

Some of the equipment is shown on Figures 5 and 6. Record all measurements in tables. Write down your results in the answer sheet.

Figure 5

Figure 6

Task 1. Measurements with a scale.

1. Devise a method for measuring the density ρ_b of irregular-shaped bodies (without having to calculate their volume in advance) using the readings of the scale in the following cases:
   1. The body is placed directly on the scale (m_s).
   2. The body is left at the bottom of a plastic cup filled with water (m_b).
   3. The body is tied on a string and is fully submerged in water, but it does not touch the bottom of the cup (m_s).

   The density of water is ρ_w. Find the formula $\rho_\mathrm{b} = f(\rho_\mathrm{w}, m_\mathrm{s}, m_\mathrm{b}, m_\mathrm{s})$. **(1.0 pt)**
2. Take enough good measurements and calculate the density of each body. **(4.0 pt)**

Task 2. Measurements with a stopwatch.

Make a simple pendulum of length $l \approx 0.7\,\mathrm{m}$ using the stand, the clip, the sinker, and some string.

1. Measure the periods of oscillation T_a and T_w when the sinker is in the air and when the sinker is underwater (inside the glass cylinder). **(1.5 pt)**

2. Assume that the drag forces in both media have no effect on the oscillation periods. Find a formula for the density of the sinker $\rho_\mathrm{b} = f(\rho_\mathrm{w}, T_\mathrm{a}, T_\mathrm{w})$. Calculate this density and compare it to your result in **1(b)**. **(1.0 pt)**

Task 3. Measurements with a stopwatch and a ruler.

1. Measure the dependence of the amplitude of the pendulum $A(t)$ on time when it oscillates in air and in water, $A_\mathrm{a}(t)$ and $A_\mathrm{w}(t)$ respectively. Assume that this dependence is of the form $A(t) = A(0)e^{-\gamma t}$. Using appropriate plots, find the damping coefficients γ_a and γ_w. **(5.0 pt)**

2. For a given pendulum, let the period of damped oscillations with coefficient γ be T, and the period of undamped oscillations be T_0. These are related by

$$T_0 = \frac{1}{\sqrt{\frac{1}{T^2} + \left(\frac{\gamma}{2\pi}\right)^2}}.$$

   Use the data from **2(a)** and **3(a)** to find $T_{\mathrm{a}0}$ and $T_{\mathrm{w}0}$, the oscillation periods of the pendulum in air and in water if there were no damping. Calculate the density of the body $\rho_{\mathrm{b}0} = f(\rho_\mathrm{w}, T_{\mathrm{a}0}, T_{\mathrm{w}0})$ again. Has your result improved compared to the one you obtained in **2(b)**? **(1.0 pt)**

Task 4. Working with a different model.

1. The oscillations are exponentially damped only if the drag force is proportional to the velocity, which is typical of low Reynolds numbers, $\mathrm{Re} < 1$. At large Reynolds numbers ($\mathrm{Re} \gg 1$) the drag force is proportional to the square of the velocity instead. Then, assuming weak damping, the angular amplitude of a simple pendulum depends on time as follows:

$$\alpha(t) = \frac{\alpha(0)}{1 + \alpha(0)\delta t}.$$

   Using your measurements for oscillations in water from **3(a)**, make an appropriate plot and use it to find δ. **(1.0 pt)**

2. For quadratic drag $F_\mathrm{dr} = bv^2$ acting on a ball of cross-section S in a medium of density ρ, the coefficient b is given by $b = \frac{1}{2}C\rho S$. The constant C is called a drag coefficient. If b and δ from **4(a)** are related by $\delta = \frac{8}{3}\frac{bl}{mT}$ (l is the length of the string, m is the mass of the bob, T is the oscillation period), find C for your experimental setup. **(0.5 pt)**

Problem 2. Black box.

Equipment:

1. Black box with three numbered terminals (Figure 7)
2. Multimeter ($\times 1$)
3. Stopwatch ($\times 1$)
4. Graph paper ($\times 2$)
5. Ruler, blank paper

The black box contains three components in a Y-connection – a battery, a resistor, and a capacitor, as shown on Figure 8. Each terminal of the black box leads to the free end of a component. You do not have access to the centre of the Y-connection. Initially the capacitor is either completely discharged or left with a voltage less than 0.5 V.

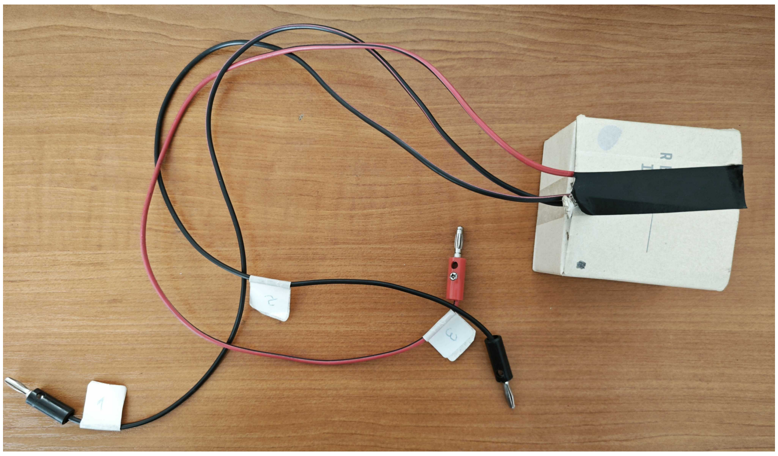

Figure 7

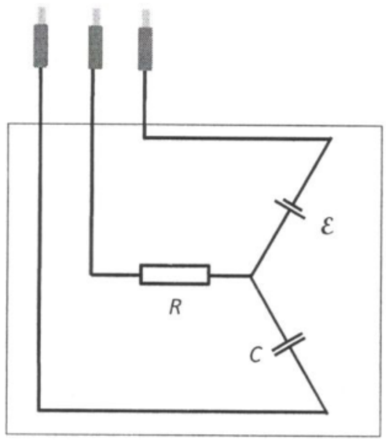

Figure 8

Tasks:

1. Describe the measurements that you will have to make in order to find out which terminal is connected to which component.
2. Take the appropriate measurements and sketch the circuit inside the black box, indicating the numbers of the terminals corresponding to each component. Also indicate which end of the battery connects to the terminal.
3. Write down the relevant theory and take the appropriate measurements so as to find values for:
   - the EMF of the battery $\mathcal{E}$
   - the capacitance C
   - the resistance R.
4. Estimate the errors in $\mathcal{E}$, C, and R.

You can neglect the internal resistance of the battery and the internal resistance of the multimeter in ammeter mode. The internal resistance of the multimeter in voltmeter mode is not infinitely large and you must treat it as a separate load in the circuit.