2022 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. Radial oscillations. Two identical point masses A and B, each of mass m, are connected with a light inextensible string. The string passes through an opening in a horizontal table O. Mass A hangs below the opening while mass B is placed on top the table at a distance l_0 from the opening. All friction is neglected. The acceleration due to gravity is g.

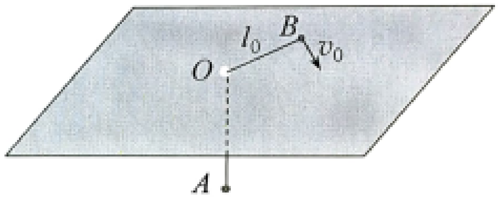

Figure 1

(a) What initial velocity v_0 should mass B be given so that it rotates in a circle around the opening O? **(1.0 pt)**

(b) After mass B has been given this velocity v_0, we attach an additional mass Δm to mass A. As a result, ball A will move downwards, pulling along the string. Find the length of string l left on top of the table when A reaches its lowest point. **(2.5 pt)**

(c) If $\Delta m \ll m$, mass A will oscillate harmonically between its lowest and highest positions. Find the oscillation period T. **(1.5 pt)**

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 2

Problem. Binary phase diffraction grating. Consider a slab of thickness h made up of alternating strips of width b and refractive indices n_1 and n_2, respectively. The width of the whole grating is $2Nb$, where $N \gg 1$ is an integer. A parallel monochromatic beam of wavelength λ is normally incident on the plate. We observe the diffracted parallel beams at different angles Θ to the normal of the plate. Neglect reflections at the surfaces of the plate.

(a) What is the minimum thickness $h_{\min}$ so that the intensity at the centre of the diffraction pattern is zero, $I(\Theta = 0) = 0$? In what follows, work with a diffraction grating of thickness $h_{\min}$.

(b) Find the angle of the first diffraction maximum Θ_1. You can express your answer as a trigonometric function of Θ_1.

(c) Find the angle of the second diffraction maximum Θ_2. You can express your answer as a trigonometric function of Θ_2.

(d) Find the general intensity pattern of the diffraction grating $I(\Theta)$.

The problem is worth 5 points.
Time: 60 minutes.

Short Exam 3

Problem. Throttling. Throttling is a process in which gas is forced through a porous medium that eliminates its macroscopic motion. This is usually carried out in thermally insulated systems. The most notable example of throttling is called a Joule-Thomson process, as shown on Figure 2.

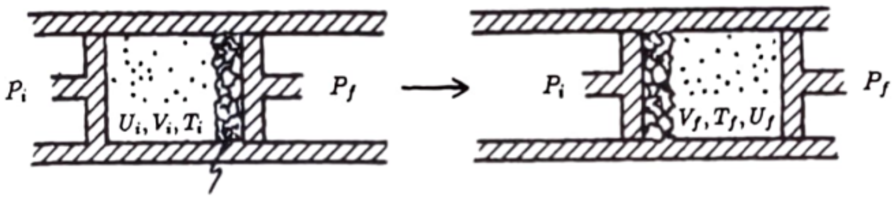

Figure 2

A fixed porous plug divides a thermally insulated cylinder into two parts. Initially the gas is to the left of the plug, and it has volume V_i, temperature T_i, and pressure P_i. After the gas passes to the right of the plug, its pressure is $P_f < P_i$. The pressures are kept constant throughout the whole process. As a result, the gas experiences a temperature change $\Delta T = T_f - T_i$ and a pressure change $\Delta P = P_f - P_i$. This is called the Joule-Thomson effect.

(a) Using the first law of thermodynamics, find an equation between the thermodynamic variables of the gas in the initial (i) and the final (f) state. Neglect the energy of macroscopic motion. **(2.0 pt)**

(b) Consider a gas described by the van der Waals equation of state,

$$\left(P + \frac{\nu^2 a}{V^2}\right)(V - \nu b) = \nu R T.$$

Show that a gas with $a = 0$ is always heated up by the Joule-Thomson process. Find the temperature increase of the gas following its expansion. Assume that $\frac{\nu b}{V} \ll 1$. **(1.5 pt)**

(c) Show that a gas with $b = 0$ is always cooled down by the Joule-Thomson process when expanding. Find the temperature decrease of the gas. **(1.5 pt)**

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 1. Car. A car of mass $m = 1000\,\text{kg}$ moves with speed $v_0 = 30\,\text{m/s}$ on a straight horizontal road. The driver disengages the transmission and the car starts to slow down, as given by

$$v(t) = \frac{v_0}{1 + \frac{t}{\tau}},$$

where $\tau = 60\,\text{s}$ is a constant.

(a) Find the distance s_1 the car has moved until its velocity decreases to $v_1 = 20\,\text{m/s}$.

(b) After the car has reached speed v_1, the driver engages the transmission again. Find the useful power of the engine P_1 that is necessary to maintain uniform motion at this speed.

Problem 2. Accelerating rocket. A rocket has initial mass m_0. It accelerates from rest in outer space without any gravitational forces acting on it. The payload of the rocket is $0.1m_0$, and the remaining $0.9m_0$ is fuel. The fuel is ejected with a velocity $u = 3000\,\text{m/s}$ relative to the rocket in a direction opposite that of its motion. The fuel supply is automatically adjusted so that the acceleration of the rocket remains constant at $g = 9.8\,\text{m/s}^2$ (for the convenience of the passengers). Find the distance L that the rocket covers before all its fuel is exhausted.

Problem 3. Electron beam. An electron beam enters a circular region of radius $a = 1.0\,\text{cm}$. In this region there is a magnetic field $B = 0.11\,\text{T}$ parallel to its axis. The initial velocity of the electrons is perpendicular to the field lines and is directed towards the centre of the region, as shown on Figure 3. After exiting the region, the electrons have deviated at an angle $\theta = 64°$ with respect to their initial direction. Find an expression for the velocity of the electrons v. Calculate v.

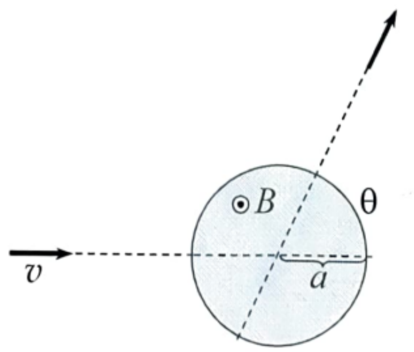

Figure 3

Problem 4. Five charges on a sphere. Five identical like charges q are constrained to the surface of a sphere of radius r. We search for the configuration which minimises their electrostatic potential energy.

(a) One option is for 3 of the charges to form the largest possible equilateral triangle, while the other 2 charges complement the triangle to a regular triangular bipyramid (that is, two regular triangular pyramids with a common base. Find the potential energy E_1 of this arrangement.

(b) A second option is for 4 of the charges to form a square, with the fifth charge complementing the square to a regular square pyramid. Find the potential energy E_2 of this arrangement in terms of some angle that specifies the pyramid.

(c) Find this angle with an accuracy that allows for a good estimate of the minimum potential energy of such an arrangement. Which arrangement has a lower potential energy: the first one or the second one?

Problem 5. RC filter. An AC circuit is shown on Figure 4. The input voltage is $U_{\text{in}}(t) = U_{\text{in}0}\cos(\omega t)$. At the output we measure a voltage $U_{\text{out}}(t) = U_{\text{out}0}\cos(\omega t + \varphi)$. Work in the special case $R_1 = R_2$, $C_1 = C_2$.

(a) Find a formula for the ratio $K(\omega) = U_{\text{out}0}/U_{\text{in}0}$.

(b) What is the value of ω which maximises $K(\omega)$?

(c) Find the maximum K.

Problem 6. Thermal screens. Two infinite parallel planes are maintained at constant temperatures T_0 and T_{N+1} in vacuum. We place another N infinite parallel planes between the first two. The bodies reach equilibrium temperatures T_1, T_2, ... , T_N. Treat all the planes as blackbodies.

(a) Let the heat exchange rate between the bodies at the two ends be P_0 when there are no bodies between them, and P_N when there are N bodies between them. Find the ratio P_0/P_N.

(b) Find a formula for the temperature of the k-th body T_k (for all $k \in [1, N]$).

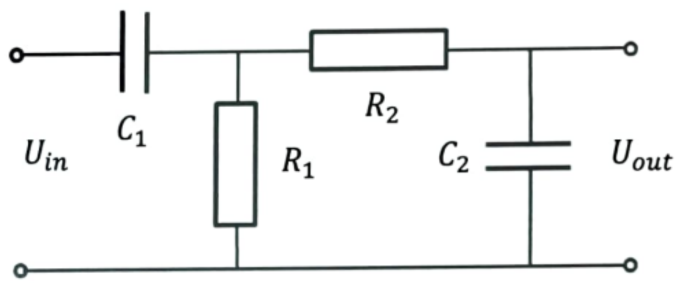

Figure 4

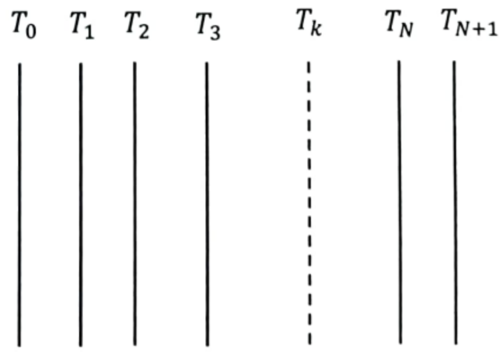

Figure 5

Problem 7. Thermodynamic process. A gas is compressed in such a way that the heat released to the surroundings is equal in magnitude to the change of the internal energy.

(a) Find the heat capacity of such a process.

(b) Find this heat capacity for a van der Waals gas in terms of its pressure P and volume V.

Problem 8. Liquefaction of helium. A thermally insulated vessel is filled with helium gas at temperature $T_0 = 10\,\text{K}$. The gas is slowly expelled through a nozzle until the pressure in the vessel is $P_1 = 1\,\text{atm}$ and the temperature is $T_1 = 4.2\,\text{K}$. In the end there is only liquid helium in the vessel. The enthalpy of boiling for helium at $4.2\,\text{K}$ is $r = 84\,\text{J/mol}$. Find the initial pressure P_0 of the gas in the vessel. Treat the helium gas as ideal. Assume that the gas expulsion is quasistatic.

Problem 9. Quantum gas. Estimate the characteristic temperature T_0 at which the quantum properties of helium gas become significant. The density of the gas is $\rho = 0.18\,\text{g/cm}^3$. The molar mass of helium is $\mu = 4\,\mathrm{g/mol}$.

Problem 10. Quantum vortices. When a cylindical vessel full of superfluid liquid helium is rotating about its axis, vortices will form in its volume, as shown on Figure 6. The velocity of the atoms in a vortex is given by

$$v = \frac{K}{r},$$

where r is the distance to the axis and the constant K is called the circulation quantum of the vortex. Calculate the minimum K.

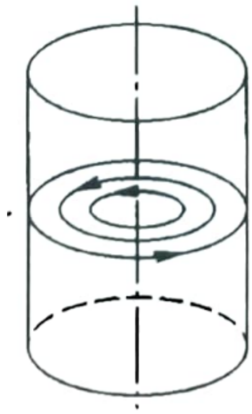

Figure 6

Constants:

Boltzmann constant	k_B	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Coulomb's constant	k_C	$8.99 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$
Vacuum permittivity	ε_0	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\mathrm{N/A^2}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m/s}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Wien's constant	b	$2.90 \times 10^{-3}\,\mathrm{m\,K}$
Electron mass	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Proton mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
Reduced Planck constant	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Geiger-Müller counter.

The Geiger-Müller counter is the simplest detector of ionising radiation (Figure 7). It consists of a thick cylindrical insulated metal tube (cathode C) and a thin co-axial wire (anode A). A high voltage is applied between the electrodes. The tube is filled with rarefied gas. At one of its ends there is a thin metal window through which the high energy particles (alpha or beta radiation) can enter the tube. Each particle that enters the tube ionises the gas, which leads to a short current pulse across the circuit. This gives rise to a voltage at the load. This voltage is then registered by a digital pulse counter.

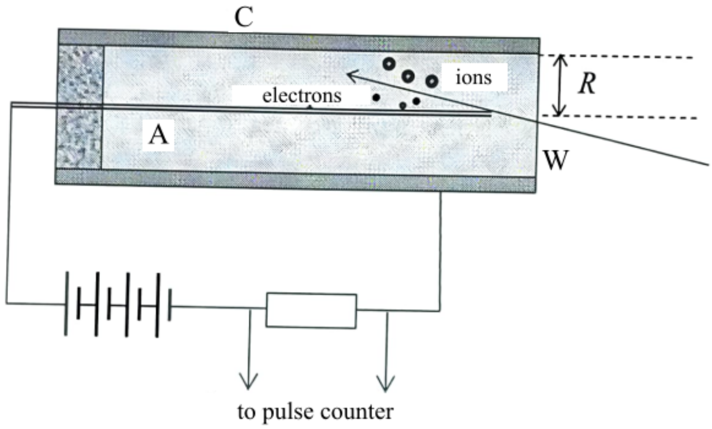

Figure 7

The time interval in which a current flows in the circuit due to a particle is called the **dead time** τ of the counter. If another particle enters the tube during this period, its pulse will 'merge' with the previous one and this particle will not be counted. This is why the number of detected particles N in a time interval t is less than the actual number of particles that have entered the tube. The number N is given by

$$N = \frac{\Phi t}{1 + \Phi \tau}, \tag{1}$$

where Φ is the particle flux, i.e. the average number of particles entering through the window per unit time. The flux Φ is measured in s^{-1}.

In this problem we will study a Geiger-Müller counter with a window radius $R = 1.0\,\mathrm{cm}$. When using the simulation software `gmcounter.exe`[1] you can set a measurement time $t \in [10\,\mathrm{s}, 100\,\mathrm{s}]$ in steps of 1 s. If you enter a decimal, it will be rounded to the nearest integer.

Task 1. Determining the dead time of the counter.
You are given five sources of beta radiation with activities[2] $A = 2.1, 5.0, 10.3, 15.2, 17.9\,\mathrm{Bq}$. Assume that the sources are pointlike and isotropic (i.e. radiating equally in all directions). The sources can only be studied separately. They are placed right next to the window of the tube. The half-lives of the sources are much longer than the duration of the experiment, so the activities of the sources may be assumed constant.

[1] You can find the program in my archives, at `bg/2022/2022-IV/exp`.

[2] The activity of a source is the average number of radioactive decays in the source, $A = \frac{\mathrm{d}N}{\mathrm{d}t}$. The SI unit for activity is the **becquerel** (Bq), which is equivalent to s^{-1}, i.e. one decay per second.

Select **Part 1** in the main menu of the program. A list of sources from 1 to 5 will appear on the screen. If you press 0, you will return to the main menu. After selecting a source, you need to enter the measurement time t. When the measurement is over, the program outputs the number of detected pulses N. Then the program returns to the source selection menu.

(a) For each source measure the number of detected pulses at various time intervals.

(b) Present your data in tabular and graphical form. Use auxiliary variables for which a linear dependence is expected.

(c) Find the dead time of the counter and estimate your error.

Note: Bear in mind that the number of registered particles N is subject to unavoidable random fluctuations about some mean value $\bar{N}$. The expected relative deviation from the mean value is given by

$$\frac{|N - \bar{N}|}{\bar{N}} \propto \frac{1}{\sqrt{\bar{N}}}.$$

This could help you select the measurement time so that the measurement error is within certain bounds.

Task 2. Determining the half-life of uranium.
You are given a radioactive source containing $m = 5\,\text{mg}$ of the uranium isotope $^{238}_{92}\text{U}$. Uranium-238 is subject to alpha decays with a very long half-life, and the amount of uranium remains constant throughout the experiment. The source can be assumed pointlike and isotropic. The source is located on the axis of the counter. It can be placed at distances $d \in [0.5\,\text{cm}, 5.0\,\text{cm}]$ from the window in steps of $0.1\,\text{cm}$ ($1\,\text{mm}$).

Select **Part 2** in the main menu of the program. Here you can set the measurement time t (in seconds) and the distance d between the window and the source (in centimetres). To return to the main menu, enter a negative distance or a negative measurement time.

(a) Using the law of radioactive decay, derive a formula which relates the activity of the source A to the total mass of the radioactive isotope m. The formula may include the molar mass of the isotope μ, its halflife $T_{1/2}$, and the Avogadro constant $N_A = 6.02 \times 10^{23}\,\text{mol}^{-1}$.

(b) Using measurements (simulations) at different distances d, find the range L of alpha particles in air. Find the kinetic energy E of the alpha particles released by the decay of uranium. Use the formula

$$L/\text{cm} = 0.3(E/\text{MeV})^{3/2}.$$

(c) Using measurements (simulations) at different distances $d < L$, find the activity of the source A.

(d) Using your data, find the half-life $T_{1/2}$ of $^{238}_{92}\text{U}$ in years, and estimate your error.

Hint: The solid angle Ω subtended by a cone with an angle θ between its axis and its generatrix (Figure 8) is

$$\Omega = 2\pi(1 - \cos\theta).$$

The solid angle that corresponds to an entire sphere (i.e. all space) is $4\pi\,\text{sr}$ (steradians).

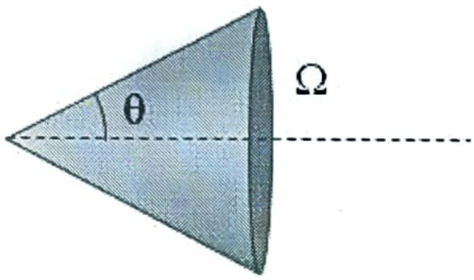

Figure 8

Problem 2. External photoemission.

Photoemission from a metal is studied using the circuit on Figure 9. The light source is a mercury lamp. A monochromator filters its light to produce monochromatic light of different wavelengths. This light is incident on the cathode of an evacuated tube. You are given measurements of the photocurrent at various potential differences U across the electrodes. The measurements are taken at 5 different wavelengths λ. Two values for the power of the light P have been used for each wavelength.

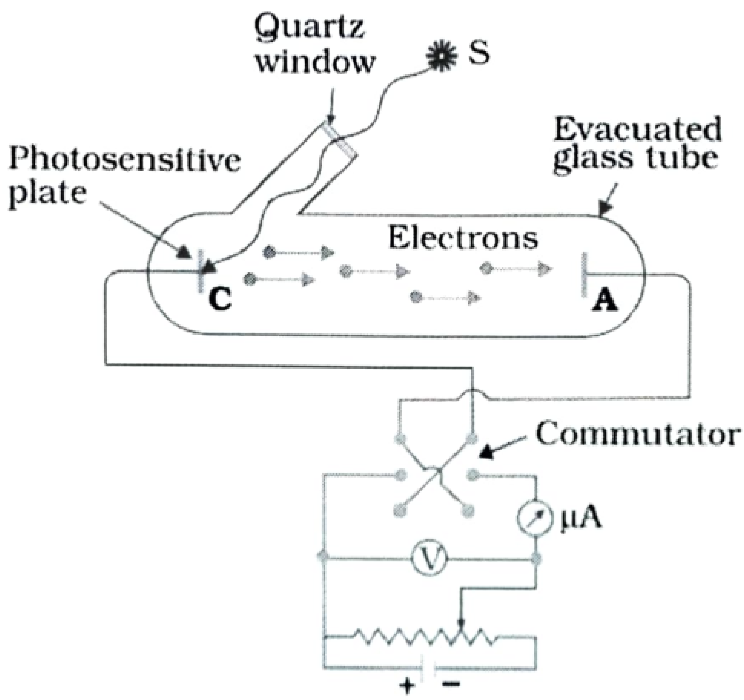

Figure 9

(a) Find the work function A of the cathode used in the experiment (in eV). Use the data in the table and plot the relevant graph. **(7.0 pt)**

(b) The quantum efficiency QE of the photoelectric effect is defined as the ratio between the number of emitted electrons and the number of photons incident on the cathode. Using the data in the table, plot a graph of the quantum efficiency QE against the wavelength of the monochromatic light λ (in nm). At what wavelength $\lambda_{QE=0}$ does the quantum efficiency become zero? **(7.0 pt)**

(c) Using the data for the work functions A (in eV) of different chemical elements, determine the metal at the cathode. **(1.0 pt)**

λ, nm →	365,5		404,7		435,8		546,1		578,0	
P, W →	0,961	1,73	2,44	4,40	5,56	10,00	5,45	9,80	1,23	2,30
U, V ↓	I, μA	I, μA	I, μA	I, μA	I, μA	I, μA	I, μA	I, μA	I, μA	I, μA
-1,60	0	0	0	0	0	0	0	0	0	0
-1,50	0	0	0	0	0	0	0	0	0	0
-1,40	2	4	0	0	0	0	0	0	0	0
-1,30	7	12	0	0	0	0	0	0	0	0
-1,20	12	22	0	0	0	0	0	0	0	0
-1,10	17	29	2	3	0	0	0	0	0	0
-1,00	22	39	10	19	0	0	0	0	0	0
-0,90	27	47	18	34	0	0	0	0	0	0
-0,80	32	56	27	50	25	45	0	0	0	0
-0,70	40	71	36	66	50	90	0	0	0	0
-0,60	50	89	46	84	75	135	0	0	0	0
-0,50	62	111	58	106	105	185	0	0	0	0
-0,40	73	132	70	128	140	250	0	0	0	0
-0,30	88	160	83	152	210	370	3	5	0	0
-0,20	105	190	109	192	295	535	15	27	0	0
-0,10	125	222	139	247	425	760	30	54	3	5
0,00	156	281	185	330	700	1270	50	90	6	11
0,10	192	347	240	430	980	1750	90	160	10	18
0,20	233	417	300	535	1200	2160	140	250	15	27
0,30	270	487	360	640	1370	2460	210	380	23	40
0,40	290	522	422	740	1520	2745	300	540	33	60
0,50	303	541	505	900	1620	2910	420	760	55	100
0,60	313	562	620	1110	1680	3030	760	1370	85	150
0,70	321	578	700	1270	1730	3115	1080	1940	132	240
0,80	325	584	750	1345	1760	3155	1180	2120	230	410
0,90	328	590	775	1390	1785	3220	1250	2240	245	440
1,00	330	595	800	1440	1810	3250	1280	2300	250	450
2,00	335	600	825	1488	1830	3280	1335	2400	256	460
3,00	337	605	835	1500	1840	3310	1360	2450	261	465
4,00	340	610	840	1510	1850	3300	1380	2480	265	470
5,00	340	610	840	1510	1850	3300	1380	2480	265	470

Ag	4.5	Al	4.16	As	3.75	Au	5.25	B	4.45	Ba	2.6
Be	4.98	Bi	4.31	C	5	Ca	2.87	Cd	4.08	Ce	2.9
Co	5	Cr	4.5	Cs	1.95	Cu	4.7	Eu	2.5	Fe:	4.75
Ga	4.32	Gd	2.90	Hf	3.90	Hg	4.475	In	4.09	Ir	5.3
K	2.29	La	3.5	Li	2.9	Lu	3.3	Mg	3.66	Mn	4.1
Mo	4.6	Na	2.36	Nb	4.4	Nd	3.2	Ni	5.2	Os	5.93
Pb	4.25	Pd	5.4	Pt	5.5	Rb	2.261	Re	4.72	Rh	4.98
Ru	4.71	Sb	4.6	Sc	3.5	Se	5.9	Si	4.7	Sm	2.7
Sn	4.42	Sr	2.59	Ta	4.4	Tb	3.00	Te	4.95	Th	3.4
Ti	4.33	Tl	3.84	U	3.75	V	4.3	W	4.8	Y	3.1
Yb	2.60	Zn	4.2	Zr	4.05						

Each problem is worth 15 points.
Time: 5 hours.