2016 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. The planets E and M are in circular orbits around the star S. Their orbital radii are respectively $r_E = 150 \times 10^6\,\text{km}$ and $r_M = 230 \times 10^6\,\text{km}$. The rotational period of planet E around the star S is $T_E = 365\,\text{d}$.

(a) Find the rotational period T_M of planet M around the star S, in days.

The inhabitants of planet E wish to get to planet M. The spaceship is to travel with the engines turned off on an orbit tangent to both the orbits of planet E and planet M. The gravitational forces between the spaceship and the planets can be neglected.

(b) Find the duration T_{EM} of the flight from planet E to planet M, in days.

(c) Find the angle $\angle ESM$ at the instant when the spaceship takes off from planet E.

(d) Find the time T_2 after which the angle between the planets (i.e. their relative position) is again suitable for launching an identical spaceship from planet E to planet M.

(e) Find the velocity v of the spaceship at launch with respect to the star (in km/s).

The problem is worth 5 points.
Time: 60 minutes.

Theoretical Exam

Problem 1. A half-cylinder of radius r lies on the ground with its flat surface down. A uniform rod of rectangular cross section is placed symmetrically on top of the half-cylinder perpendicularly to its axis. The rod has mass m, length l, and height h. The acceleration due to gravity is g.

(a) How should the parameters be related if the rod's equilibrium is stable?

(b) If the equilibrium is stable, find a formula for the oscillation period of the rod T when it is displaced from its equilibrium position.The rod does not slip on the half-cylinder's surface.

Problem 2. A disc of radius r and mass m is placed on a horizontal surface. Initially the disc rotates with angular velocity ω about its axis of symmetry. The initial velocity of its centre of mass is v (where $\omega r \gg v$). Find the initial friction force F acting on the disc. The coefficient of friction between the disc and the surface is k. The acceleration due to gravity is g.

Problem 3. Two point masses, $m = 1\,\mathrm{kg}$ each, lie on a smooth horizontal surface. The masses are connected by a stiff massless spring with relaxed length $d = 1\,\mathrm{m}$ and spring constant $k = 1\,\mathrm{N/m}$. Initially the spring is relaxed, one of the masses is at rest, and the other is given a horizontal velocity $v = 1\,\mathrm{m/s}$ perpendicular to the spring. Find the maximum elongation of the spring x, accurate to $1\,\mathrm{mm}$.

Constants:

Acceleration due to gravity	g	$9.81\,\mathrm{m/s^2}$

Each problem is worth 3 points.
Time: 5 hours.

Experimental Exam

Problem 1. Bifilar torsional pendulum.

Equipment:
2 rulers (each of unknown mass m), 2 coins (each of mass $M = 7.00\,\mathrm{g}$), tape measure, stop-watch, string, scissors, tape.

A bifilar torsional pendulum consists of a homogeneous rod of mass m and length L attached to two strings of length h at points equidistant from the centre of mass. The distance between the strings is d. The pendulum oscillates with period T about a vertical axis passing through its centre of mass. The moment of inertia I of a rod of mass m and length l about an axis passing through its centre of mass perpendicularly to the rod is $I = \frac{1}{12}ml^2$. Record your results in the answer sheet.

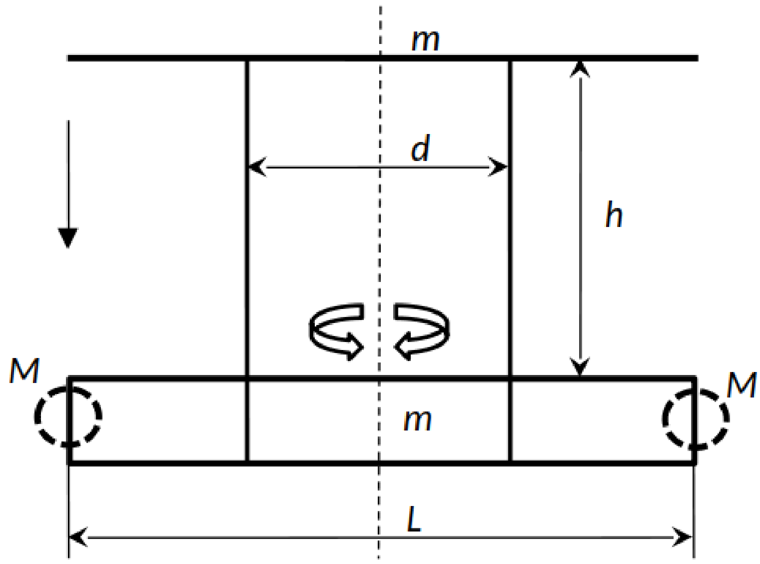

Figure 1

(a) Find the centre of mass of the ruler. Write down the division of the ruler where it is located. What is the value of L that you will be working with? **(0.5 pt)**

(b) Suspend the ruler as described above (see Figure 1, the plane of the ruler should be vertical). If the period T depends on the length of the strings h as $T \propto h^n$, find the number n experimentally. Round your value to one of the numbers $\left(\pm\frac{1}{3}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 3\right)$. **(3.5 pt)**

(c) If the period T depends on the distance between the strings d as $T \propto d^k$, find the number k experimentally. Round your value to one of the numbers $\left(\pm\frac{1}{3}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 3\right)$. **(3.5 pt)**

(d) All in all, the period of the pendulum is given by

$$T = 2\pi C \frac{L}{\sqrt{g}} h^n d^k,$$

where C is some number. Find the value of C experimentally. **(3.5 pt)**

(e) Let the period of the pendulum be $T(0)$ for some constant d and h. If we attach a coin to each end of the ruler (so that the centre of each coin is exactly on the rim of the ruler), the period becomes $T(M)$. The period of the pendulum $T(\mu)$ depends on its mass μ and its moment of inertia $I(\mu)$ as $T(\mu) \propto \sqrt{\frac{I(\mu)}{\mu}}$. Find a formula $m = f(M, T(0), T(M))$ from which the mass of the ruler m can be determined. Take the necessary measurements and find m. **(4.0 pt)**

Each problem is worth 15 points.
Time: 5 hours.