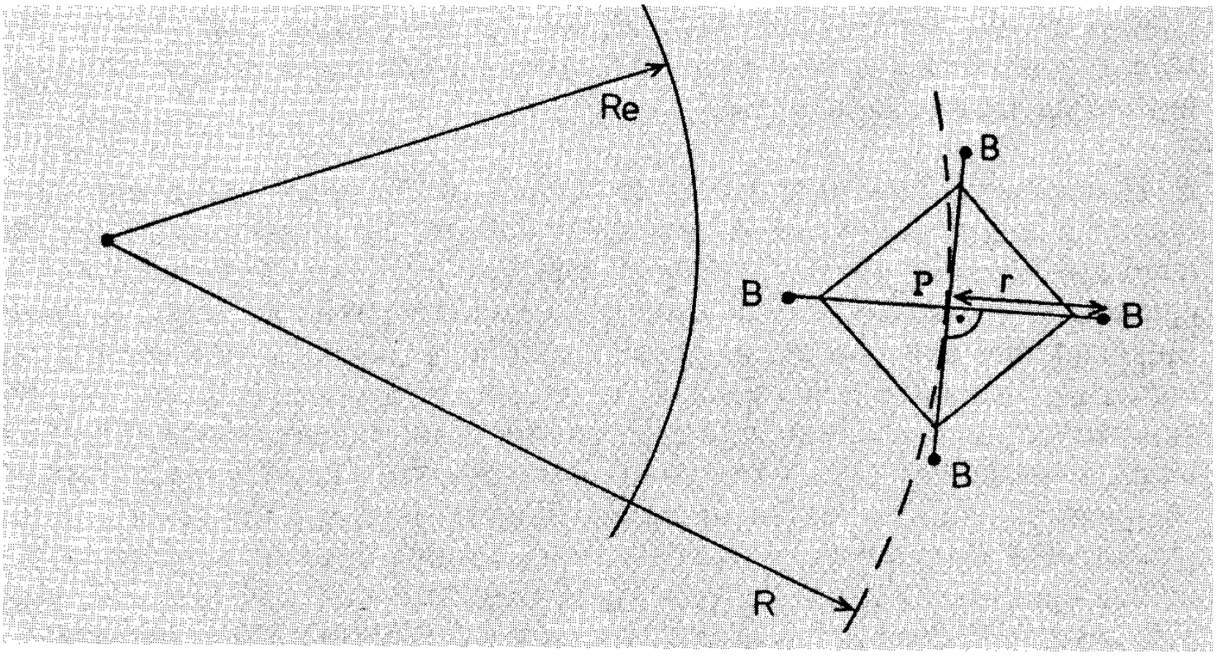
# IPhO 1992 Theory Problem 1 (modified)

This is a modified version of IPhO 1992 T1. While I like the problem, it is plagued by some issues that make it annoying to do. This includes unusual and unclear wording. I have improved the wording, and shortened the table (which was quite annoying to complete).

The figure shows a satellite orbiting Earth in a circular orbit of radius $R$ in the Earth's equatorial plane. The satellite consists of a massless central body $P$ and four small peripheral bodies $B$. The four bodies B each have mass $m \ll M_E$: they are fastened to $P$ with thin, light, inelastic wires of length $r \ll R$. $P$ and the four bodies $B$ are coplanar with the equatorial plane and they rotate within this plane with angular velocity of magnitude $\omega$. The four radial wires are linked to each other by additional link wires which keep the angles between the radial wires at a constant 90°, making the satellite act like a rigid body. You may assume that the link wires do not provide any radial force (along the line joining $B$ and $P$).




Analyze the following questions for general $\omega$, considering all possible situations. This includes both directions of rotation of the bodies $B$, where $\boldsymbol{\omega}$ can be **parallel** or **antiparallel** to the orbital angular velocity $\boldsymbol{\Omega}$. Also obtain numerical values for (a) and (b) using the data provided at the end of the problem. You may still assume that $r \ll R$ and $m \ll M_E$.

a) [3.0] The drawing shows the satellite in the position where for the various wires: $\boldsymbol{r}$ is parallel, anti-parallel or perpendicular to $\boldsymbol{R}$. (The vector $\boldsymbol{r}$ runs from body $P$ to a body $B$ and has length $r$; the vector $\boldsymbol{R}$ runs from the centre of mass of the Earth to the body P.)

   Find the tension in the radial wires when the bodies $B$ are in the 4 positions indicated. These positions correspond to the maximum and minimum forces.

b) [2.0] Inside the four bodies $B$ there are four identical machines, powered by solar energy, connected to the radial wires. Each machine expends energy to pull the wire in, towards $P$, for a short time whenever the tension in the wire (as indicated in the previous question) is near maximum. The machine then restores energy by letting the same length of wire out again when the tension is near minimum. The length of wire that is pulled in and let out is 1% of the mean length of the radial wire. The mean length does not change with time.

   What is the average net power converted by one machine?

c) [5.0] Discuss the changes in the motion of the satellite that are caused by the action of the machines. In particular, analyze any changes that may occur in each of the situations listed in the table overleaf. Take into account both directions of the rotation $\omega$, where $\omega$ can take any value (i.e. do not use the quantities provided).

## Data:

- Radius of orbit $R = R_E + 500\,\text{km}$
- Mean length of radial wires $r = 100\,\text{km}$
- Each body $B$ has a mass of $m = 1\,000\,\text{kg}$
- The satellite rotates with respect to the fixed stars at 10 revolutions/hour

## Constants:

- Mass of Earth $M_E = 5.97 \times 10^{24}\,\text{kg}$
- Gravitational constant $G = 6.673 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$
- Radius of Earth $R_E = 6\,378\,\text{km}$
- Define $K \equiv GM_E = 3.983 \times 10^{14}\,\text{m}^3\,\text{s}^{-2}$

# Answer Table

Fill the following table as part of your answer. Write down equalities or inequalities and/or short explanations where necessary.

<table>
<tr><td>Quantity</td><td>increases if...</td><td>decreases if...</td><td>is unchanged if...</td></tr>
<tr><td>Radius<br>of orbit<br>$R$</td><td></td><td></td><td></td></tr>
<tr><td>Magnitude<br>of angular<br>velocity<br>$\omega$</td><td></td><td></td><td></td></tr>
<tr><td colspan="4">Could the satellite reach a higher orbit as a result of work done by the machine?<br><br>Yes / No</td></tr>
<tr><td colspan="4">Could the satellite reach an arbitrarily high orbit? Why?<br><br>Answer:</td></tr>
</table>