SAT vs. CP

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January 18, 2022

Back to Constraint Programming

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- The constraint satisfaction problem (CSP) is the problem of deciding if a constraint network has a solution
- Mostly solvable by backtracking algorithms (Search and Filtering)

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"To succeed, try first where you are most likely to fail"

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Value Ordering

'Succeed-first' [Geelen, 1992]:
"Follow the best chances leading to a solution"

Filtering

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Arc Consistency

Let C be a constraint and D be a list of domains for the variables in the scope of C.
C is Arc Consistent (AC) iff for every variable x in the scope of C, for every value $v \in D(x)$, there exists an assignment w in D satisfying C in which v is assigned to x

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- If all the domains are singleton, the propagator must be able to check if the assignment corresponds to a solution or not.

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- CP vs. SAT: a fundamental difference is the presence of global reasoning win CP.

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- **Can we find something that takes advantages from both worlds?** $\rightarrow$ **Clause learning in CP**

Modern Constraint Solvers: Hybrid CP/SAT

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Modern Constraint Solvers: Hybrid CP/SAT

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Learning in CP

$x_1 + x_7 \geq 4 \wedge$
$x_2 + x_{10} \geq 11 \wedge$
$x_3 + x_9 = 16 \wedge$
$x_5 \geq x_8 + x_9 \wedge$
$b \leftrightarrow (x_9 - x_4 = 14) \wedge$
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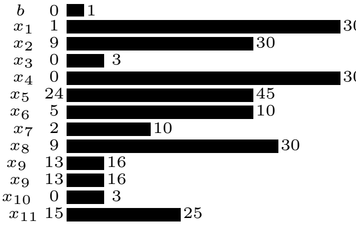

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$[\![x_1 = 1]\!]$

b	0	1
x_1	1	1
x_2	9	30
x_3	0	3
x_4	0	[illegible]
x_5	24	45
x_6	5	10
x_7	2	10
x_8	9	30
x_9	13	16
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x_{10}	0	3
x_{11}	15	25

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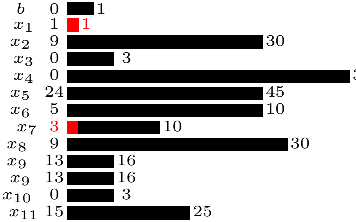

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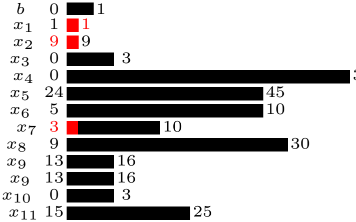

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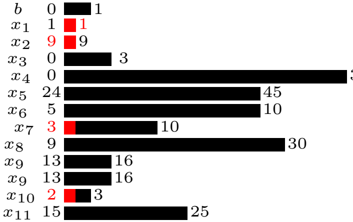

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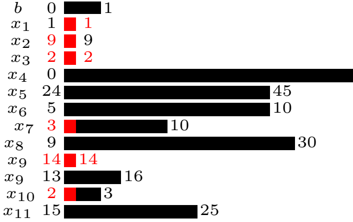

Learning in CP

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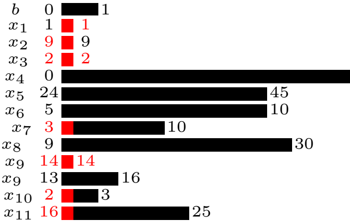

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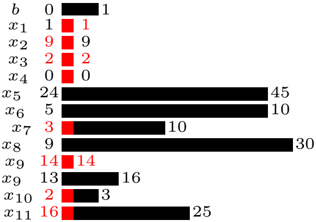

Learning in CP

$$\begin{tikzcd}
\llbracket x_1 = 1 \rrbracket \arrow[r] & \llbracket x_7 \geq 3 \rrbracket & \\
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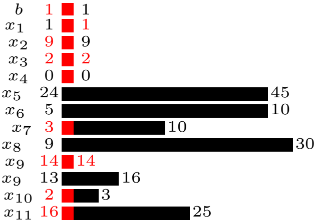

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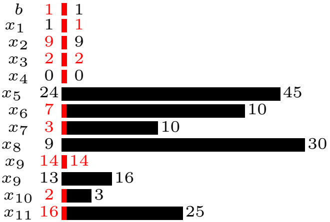

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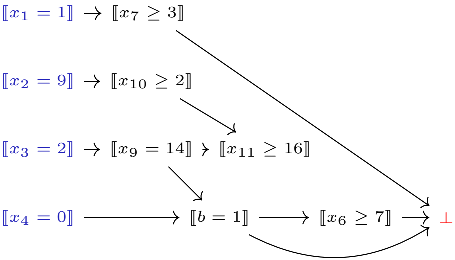

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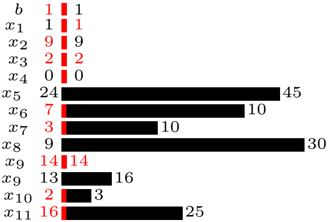

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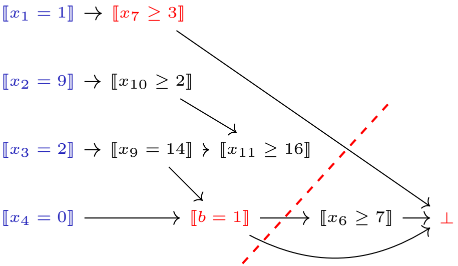

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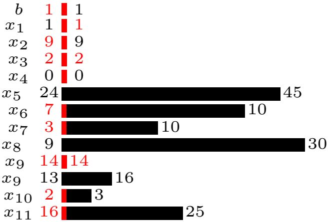

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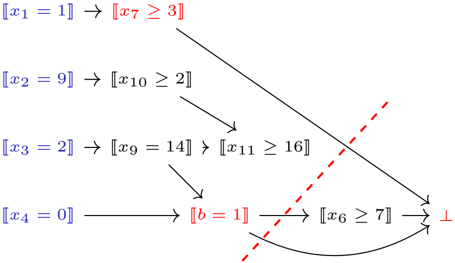

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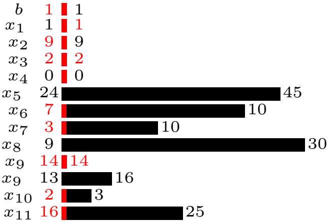

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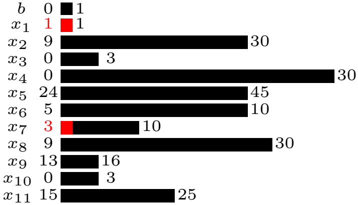

Learning in CP

$[\![x_1 = 1]\!] \rightarrow [\![x_7 \geq 3]\!] \longrightarrow [\![b = 0]\!]$

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- Propagate the learnt clause

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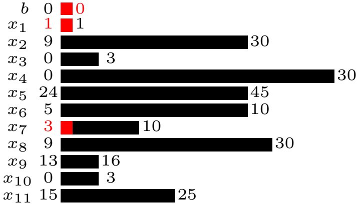
\end{aligned}
$$

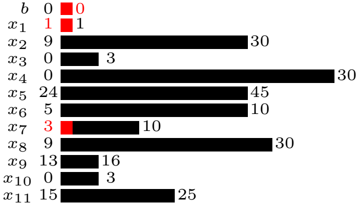


Learning in CP

$[\![x_1 = 1]\!] \rightarrow \; [\![x_7 \geq 3]\!] \longrightarrow [\![b = 0]\!]$

- Conflict analysis: $[\![b = 1]\!] \wedge [\![x_7 \geq 3]\!] \Rightarrow \perp$
- New clause: $[\![b \neq 1]\!] \vee [\![x_7 \leq 2]\!]$
- Backtrack to level 1
- Propagate the learnt clause
- Continue exploration

$$
\begin{aligned}
&x_1 + x_7 \geq 4 \wedge \\
&x_2 + x_{10} \geq 11 \wedge \\
&x_3 + x_9 = 16 \wedge \\
&x_5 \geq x_8 + x_9 \wedge \\
&b \leftrightarrow (x_9 - x_4 = 14) \wedge \\
&b \rightarrow (x_6 \geq 7) \wedge \\
&b \rightarrow (x_6 + x_7 \leq 9) \wedge \\
&x_{11} \geq x_9 + x_{10}
\end{aligned}
$$



Conflict analysis

Algorithm 1: 1-UIP-with-Propagators

$\mathit{\Psi} \leftarrow explain(\perp)$;
while $|\{q \in \mathit{\Psi} \mid level(q) = current\ level\}| > 1$ **do**
  $p \leftarrow \arg\max_q(\{rank(q) \mid level(q) = current\ level\ \wedge\ q \in \mathit{\Psi}\})$;
  $\mathit{\Psi} \leftarrow \mathit{\Psi} \cup \{q \mid q \in explain(p) \wedge level(q) > 0\} \setminus \{p\}$;
return $\mathit{\Psi}$;

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- To enable clause learning in CP, every propagator must be able to explain their filtering in the form of clauses ("Lazy Clause Generation").
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- Example: Explain the constraint $X \leq Y$ with two scenarios (failure and propagation).

Exercise

- Let $(x_1, \ldots, x_n)$ be a sequence of Boolean variables, and let d be a positive integer.
- The CARDINALITY$(x_1, \ldots, x_n, d)$ constraint holds iff exactly d variables from the sequence $(x_1, \ldots, x_n)$ are true.
- Write a filtering algorithm for CARDINALITY.
- What is the time complexity?
- Does it enforce arc consistency?
- Explain the CARDINALITY filtering.

Correction

Algorithm 4: CARDINALITY($[x_1, \ldots, x_n], d$)

if $\left|\{x_j \mid \mathcal{D}(x_j) = \{1\}\}\right| > d$ **then**
    $\mathcal{D} \leftarrow \perp$;
if $\left|\{x_j \mid \mathcal{D}(x_j) = \{0\}\}\right| > n - d$ **then**
    $\mathcal{D} \leftarrow \perp$;
if $\left|\{x_j \mid \mathcal{D}(x_j) = \{1\}\}\right| = d$ **then**
    foreach $i \in \{1..n\}$ **do**
        if $\mathcal{D}(x_i) = \{0, 1\}$ **then**
            $\mathcal{D}(x_i) \leftarrow \{0\}$;
else
    if $\left|\{x_j \mid \mathcal{D}(x_j) = \{0\}\}\right| = n - d$ **then**
        foreach $i \in \{1..n\}$ **do**
            if $\mathcal{D}(x_i) = \{0, 1\}$ **then**
                $\mathcal{D}(x_i) \leftarrow \{1\}$;
return $\mathcal{D}$;

Explaining The Cardinality Constraint

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$$x^1 \wedge x^2 \wedge x^{d+1} \rightarrow \perp$$

Where $D(x^i) = \{1\}$

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- Explaining the propagating the value 1: the conjunction of all the assigned variables

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Where $D(x^i) = \{0\}$

- Explaining the propagating the value 1: the conjunction of all the assigned variables
- Explaining the propagating the value 0: the conjunction of all the assigned variables

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