Modelling with SAT, a Tutorial

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The purpose of this tutorial is to get familiar with SAT modelling. You will be guided step by step to model a real-life timetabling problem.

1 Introduction

The problem is the following: Let $S = \{s_1, s_2, \ldots s_n\}$ be a set of n nurses. Every nurse $s_i \in S$ has to work some shifts in a horizon of $D = d_1, d_2, \ldots d_m$ of consecutive days. We use a Boolean variable $w_{i,j}$ that is **true** iff the nurse s_i is working during day d_j where $i \in \{1, 2, \ldots n\}$ and $j \in \{1, 2, \ldots m\}$. We study different extensions of this problem.

We use the kissat solver available here https://github.com/siala/kissat. Follow the instructions to install it.

A toy example is provided in instances/tp1/toy.cnf. Have a look at the file to discover the DIMACS CNF format.

Write a simple .cnf file for the following instance:

$$x \vee \neg y \vee z$$
$$y \vee \neg w$$
$$w \vee z$$
$$x \vee \neg z \vee w$$
$$w \vee z \vee x$$

Call the solver on the previous instance. Is it satisfiable? unsatisfiable? Try to run other examples to get used to the format.

2 Modelling the Basic Problem

1. On a paper, write the corresponding SAT model of the basic problem with the constraint that every nurse has to work at least one shift.

2. Write a program takes as input n and m and generates a cnf file (of the form basic$n-$m.cnf) that corresponds to your model.

3. Run the solver on an example and write the solution, i.e., the assignment of the variables, into a separate file of the form basic-$n-$m.sol.

4. Write a program that prints (in a convenient visual way) the schedule corresponding to a basic-$n-$m.sol file.

5. Repeat the previous two steps with many values of n and m.

3 Modelling some extensions

We consider the following extensions:

1. Extension 1:

- Every nurse has to work at least a day
- Every day has to be covered by at least a nurse.

2. Extension 2:

  - Every nurse has to work exactly one day
  - Every day has to be covered by exactly one nurse

3. Extension 3:

  - Every nurse has to work between a and b days
  - Every day has to be covered by c to d nurses.

  Where a, b, c, d are positive integers.

For each extension:

- On a paper, write the corresponding SAT model

- Write a program that takes as input the required parameters and generates a correspondent .cnf file

- Run the solver on the generated cnf files

- Print some solutions (if they exist) visually

Once you reach this point, you can start evaluating the scalability of the different models.