Conflict Driven Clause Learning

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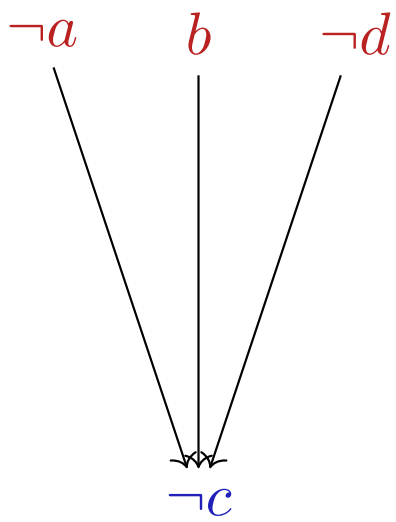
- **Can be seen as a CP Solver (Search, propagation) augmented by clause learning**
- But also :
  - Activity-based branching
  - Lazy data structures (2-Watched Literals)
  - Clause Database Reduction
  - Simplifications
  - Restarts
  - ...

Exercise: Propose a filtering algorithm for clauses. The algorithm takes as input a clause and has access (read and write) for the variables domains.

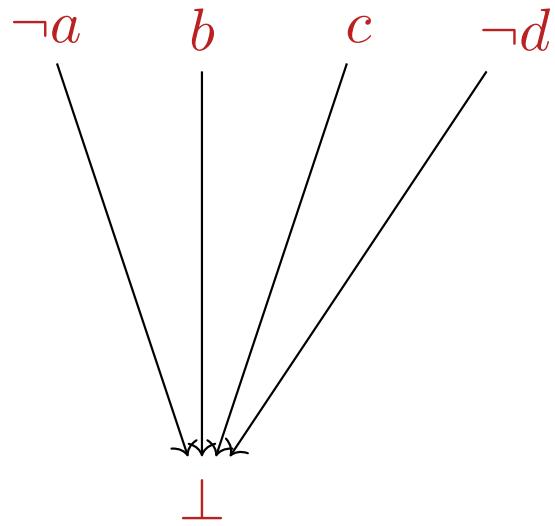
Unit Propagation

Given a clause C of arity n. If $n-1$ literals are false then set the last one to be true.

Example: $(a \vee \neg b \vee \neg c \vee d)$



$\neg a \wedge b \wedge \neg d \Rightarrow \neg c$



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Two Watched Literals

- Unit propagation is implemented with an "intelligent" data structure called Two-watched literals
- Observe first that propagation happens only in two cases:
  - The clause becomes unit (i.e., all variables except one is instantiated): Propagate the only uninstantiated literal to satisfy the clause
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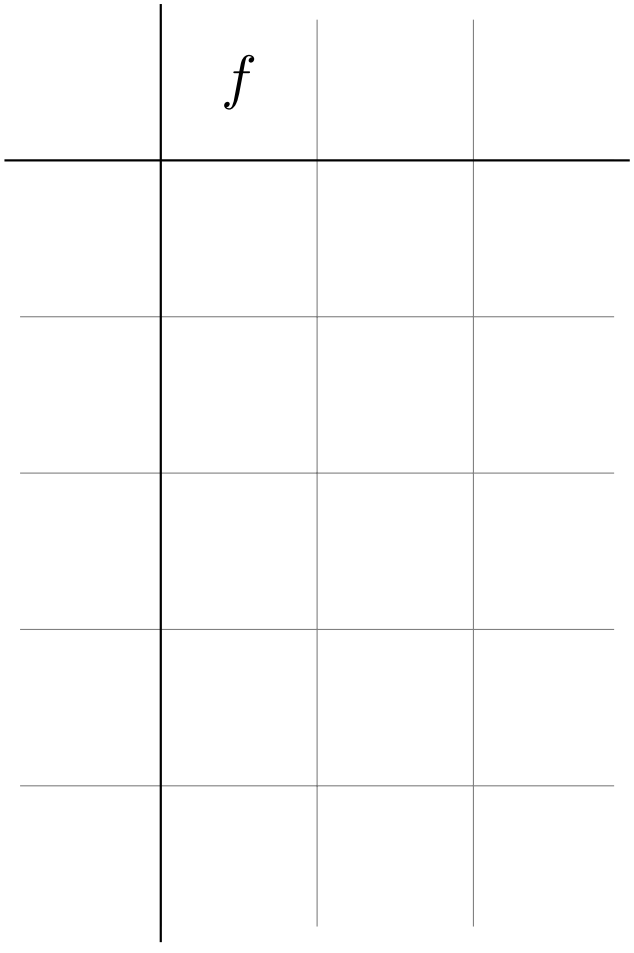
- If a literal watching a clause C becomes $false$, look for replacement. If no replacement found, then perform propagation

Implication Graph



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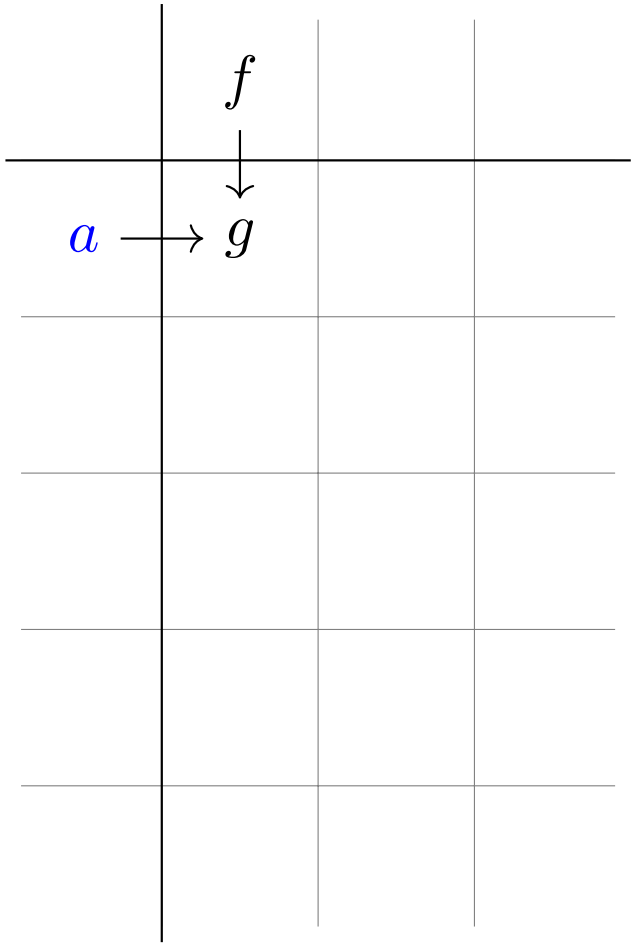

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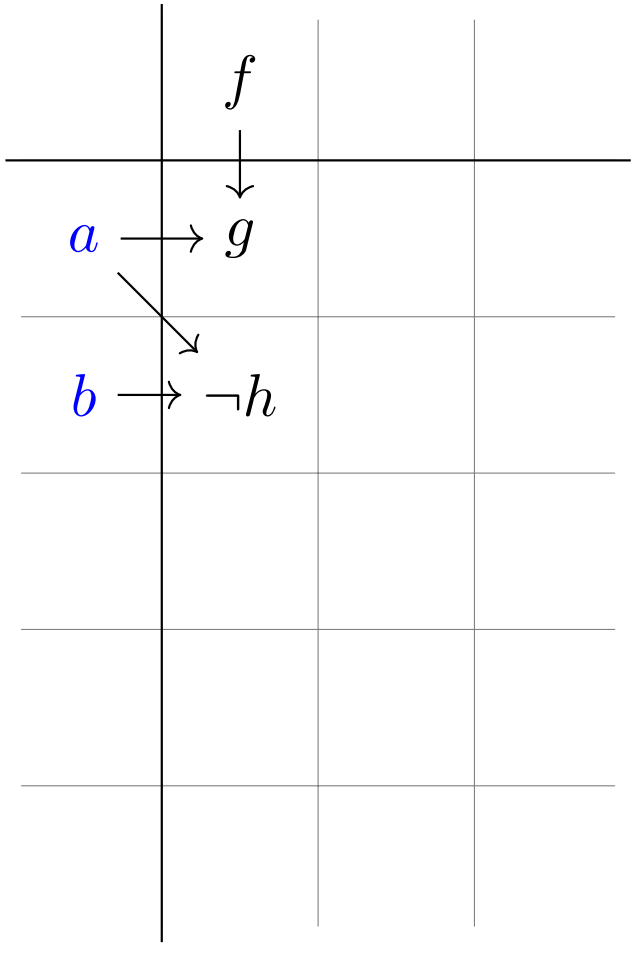

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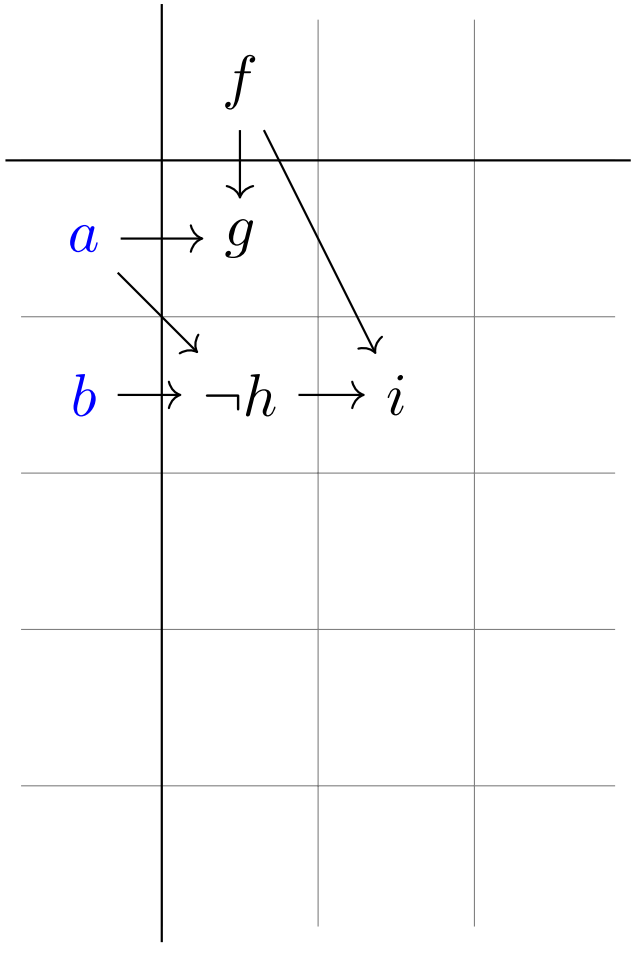

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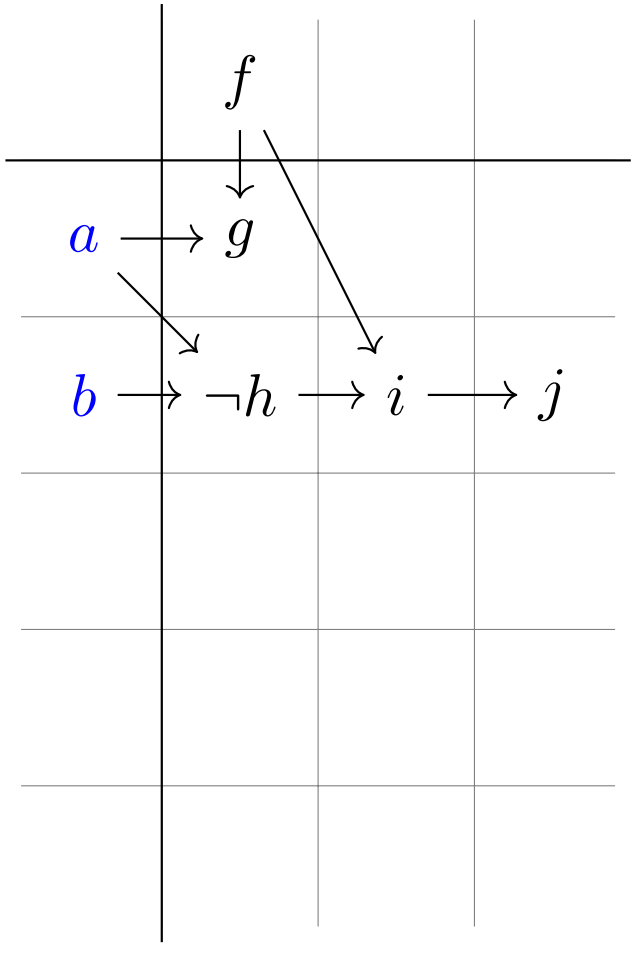

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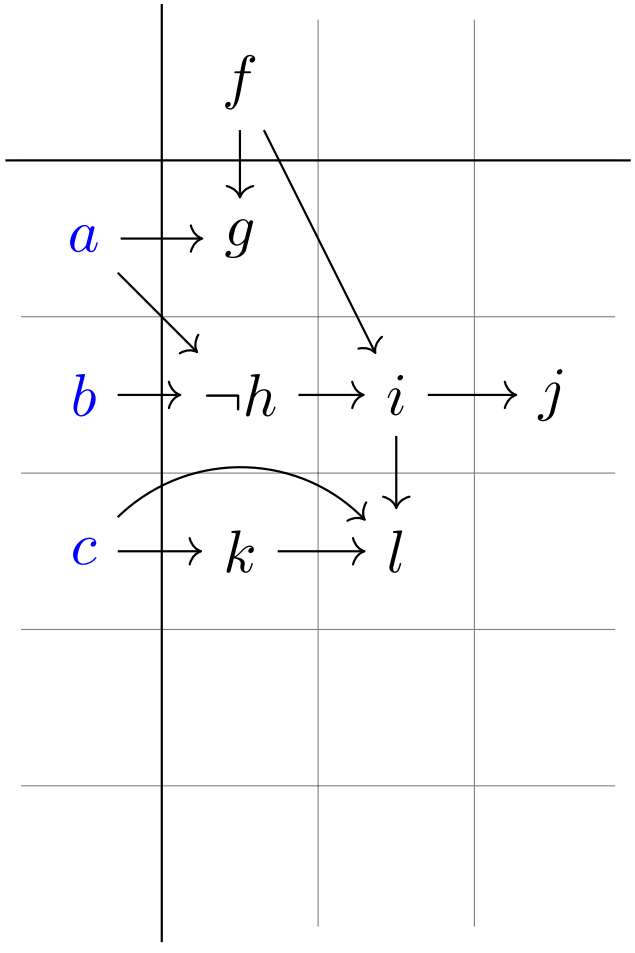

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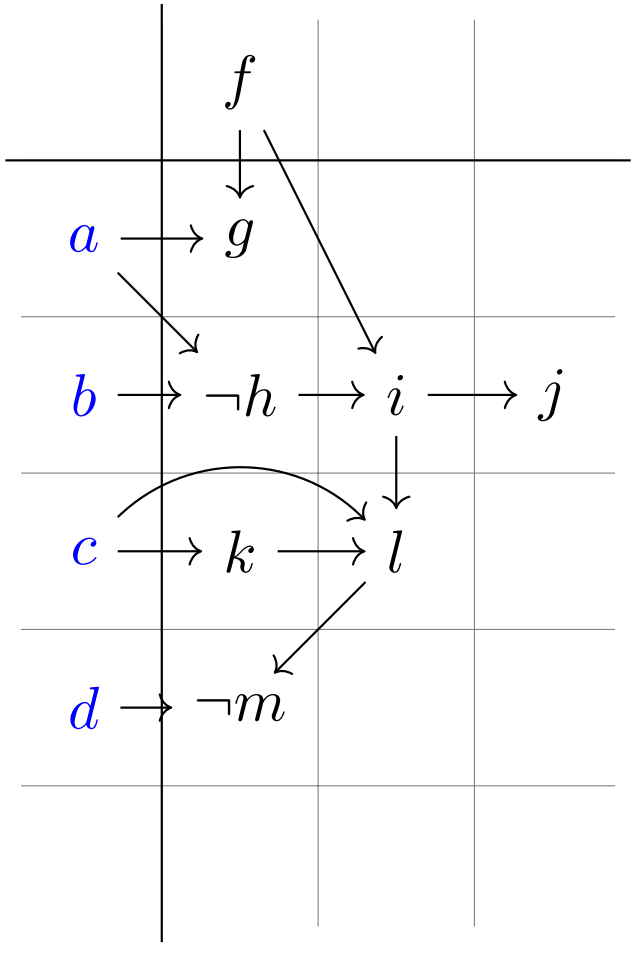

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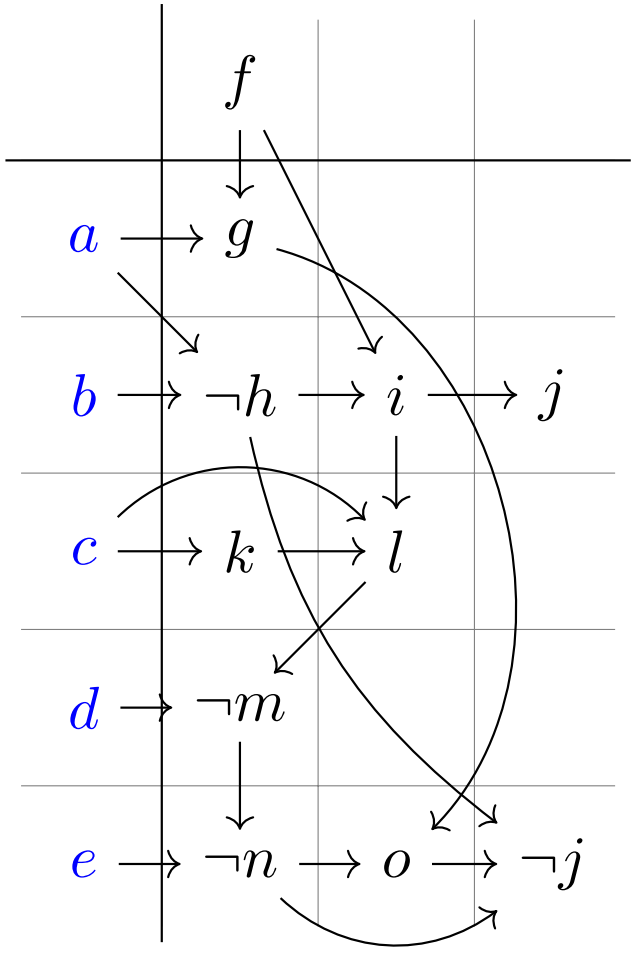

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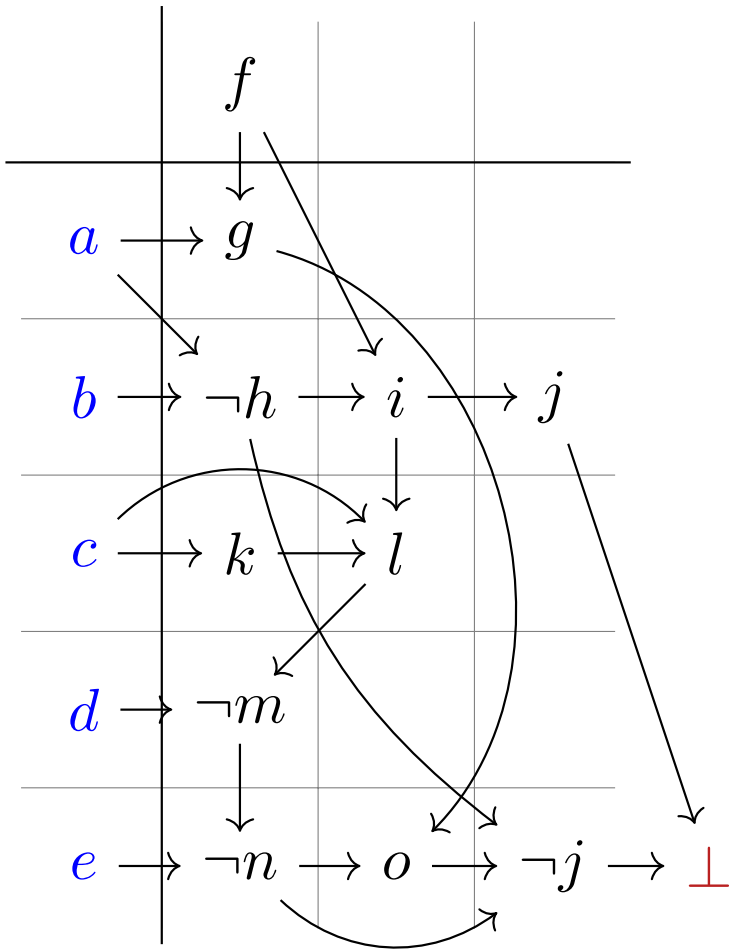

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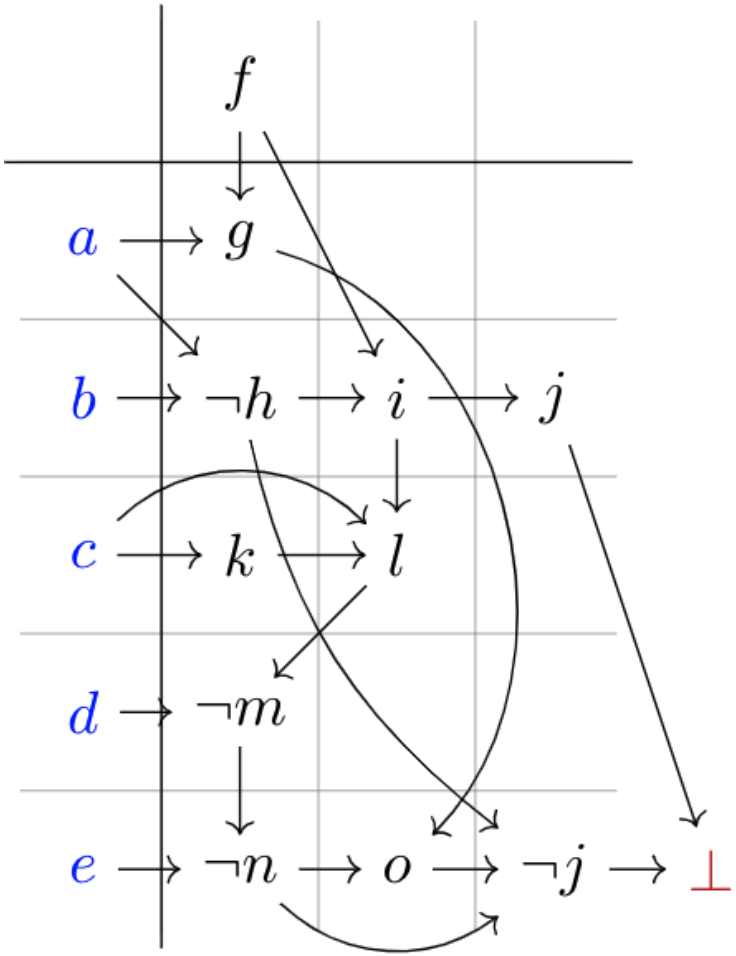

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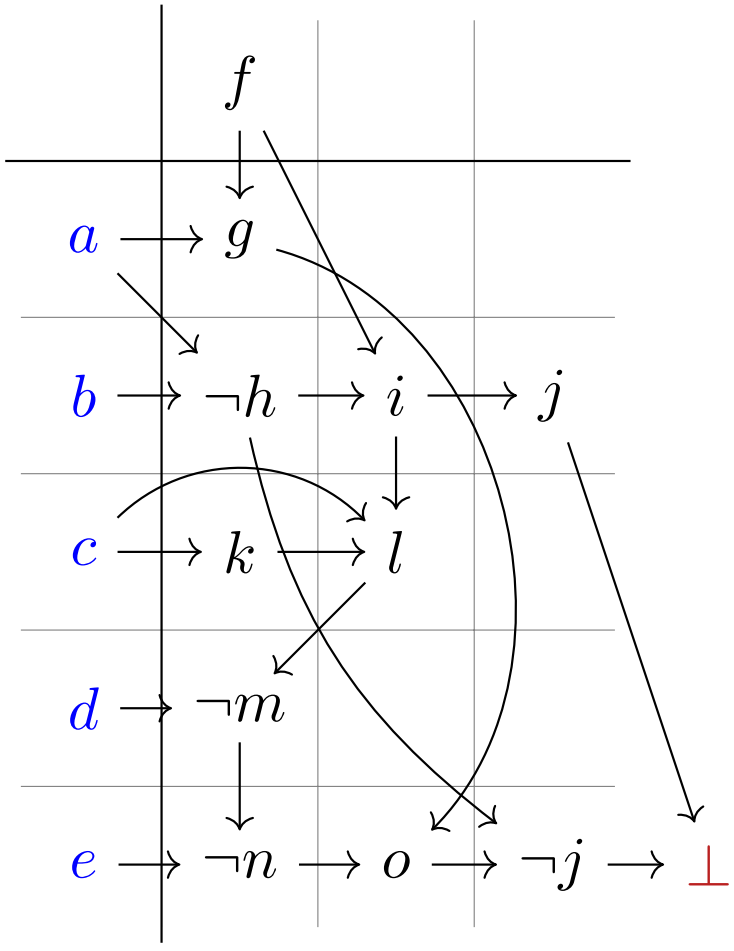
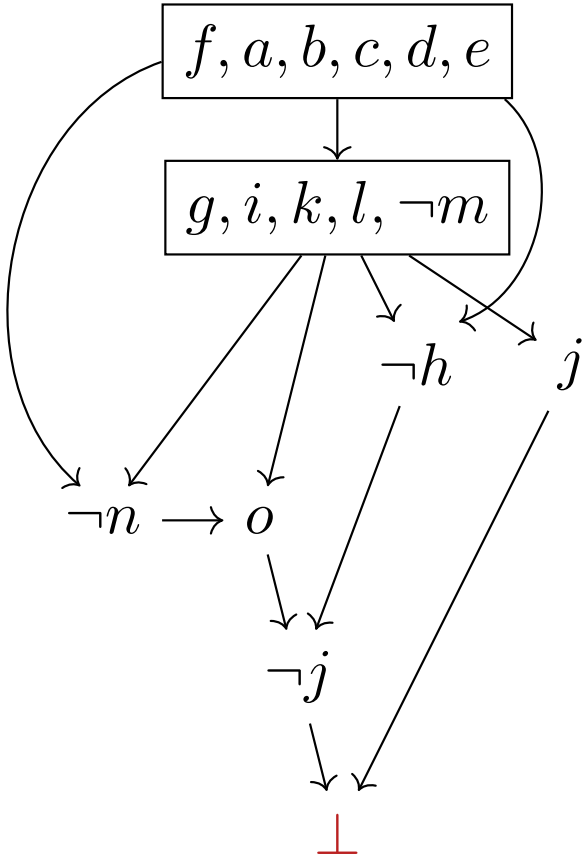
Conflict Analysis





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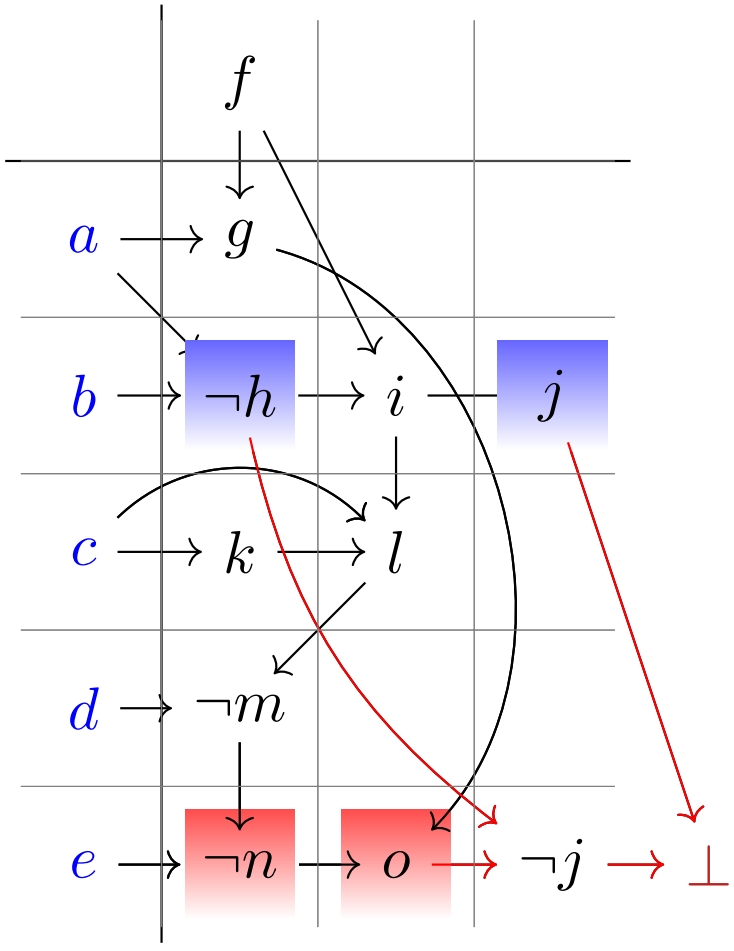

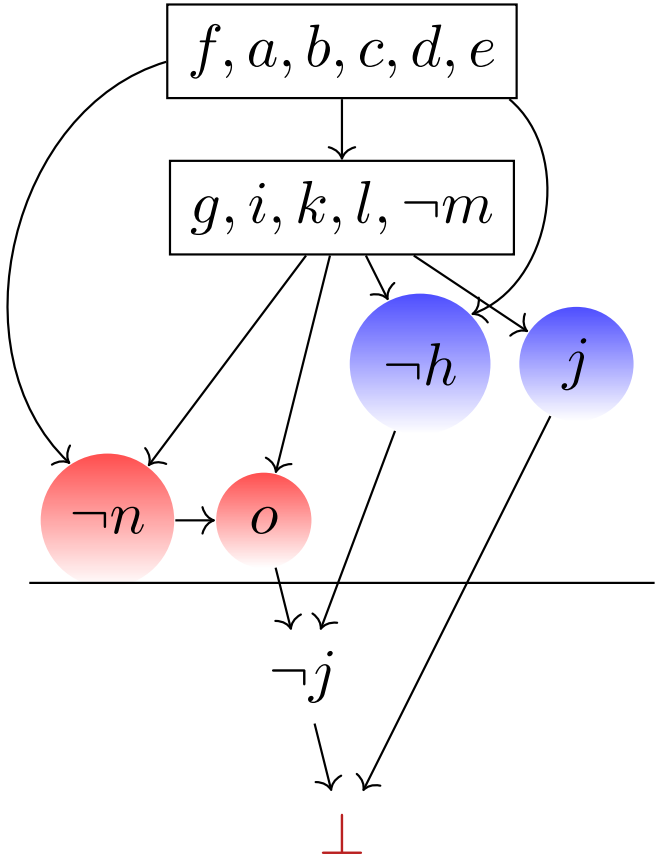

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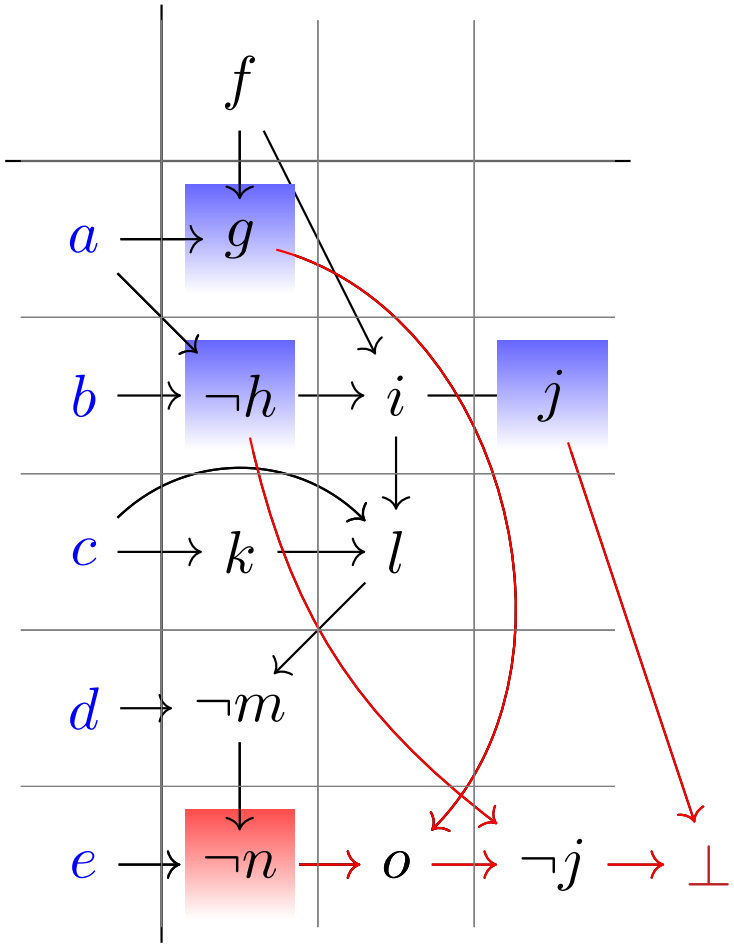

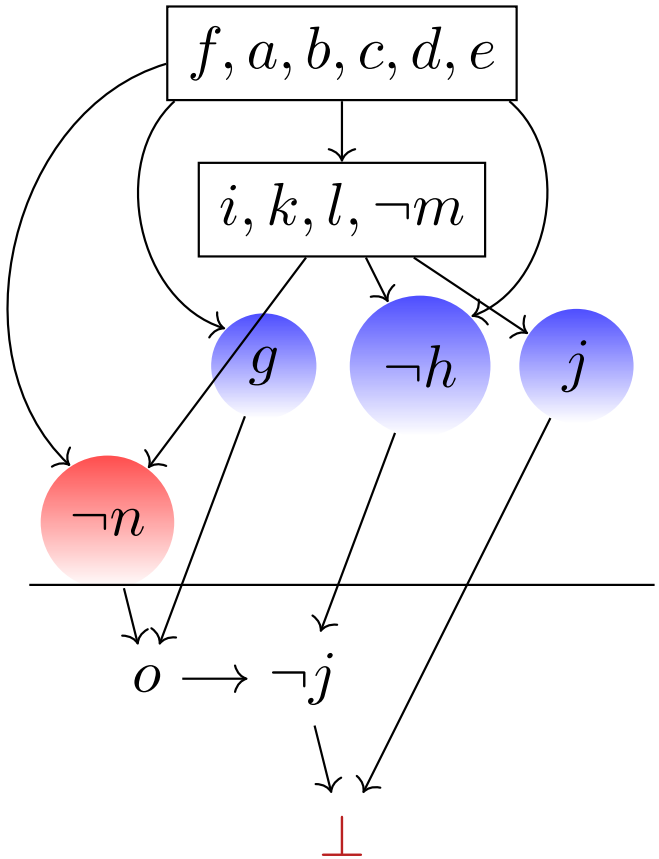

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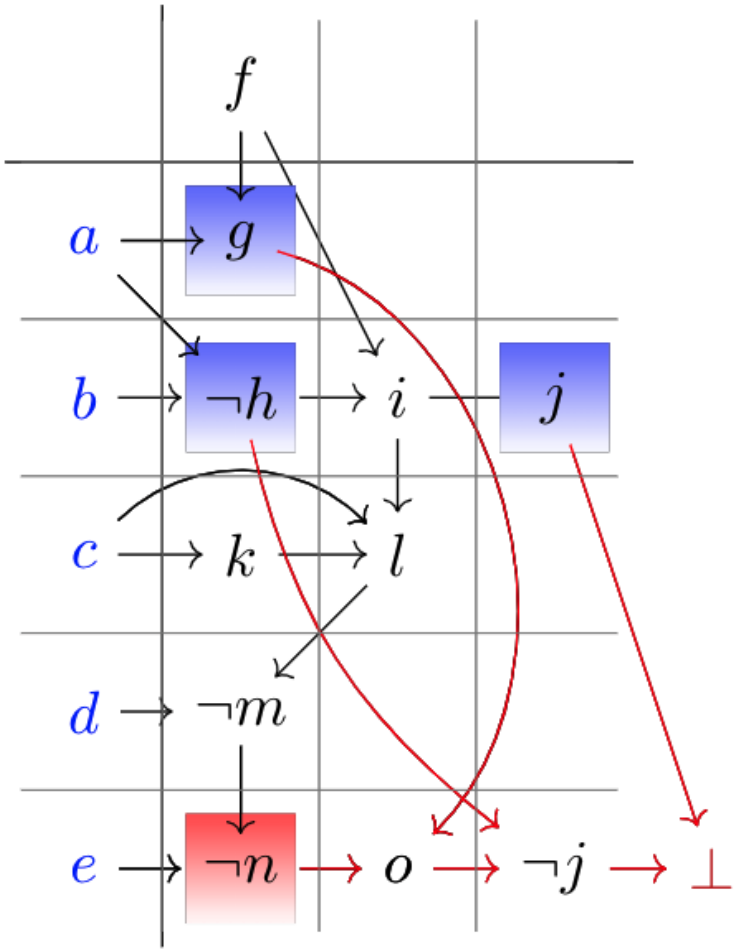

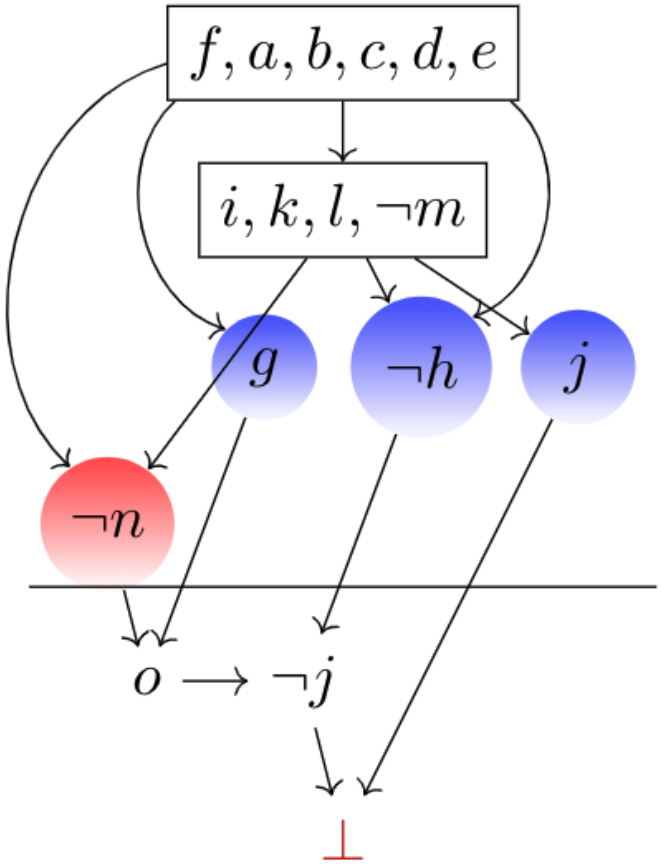

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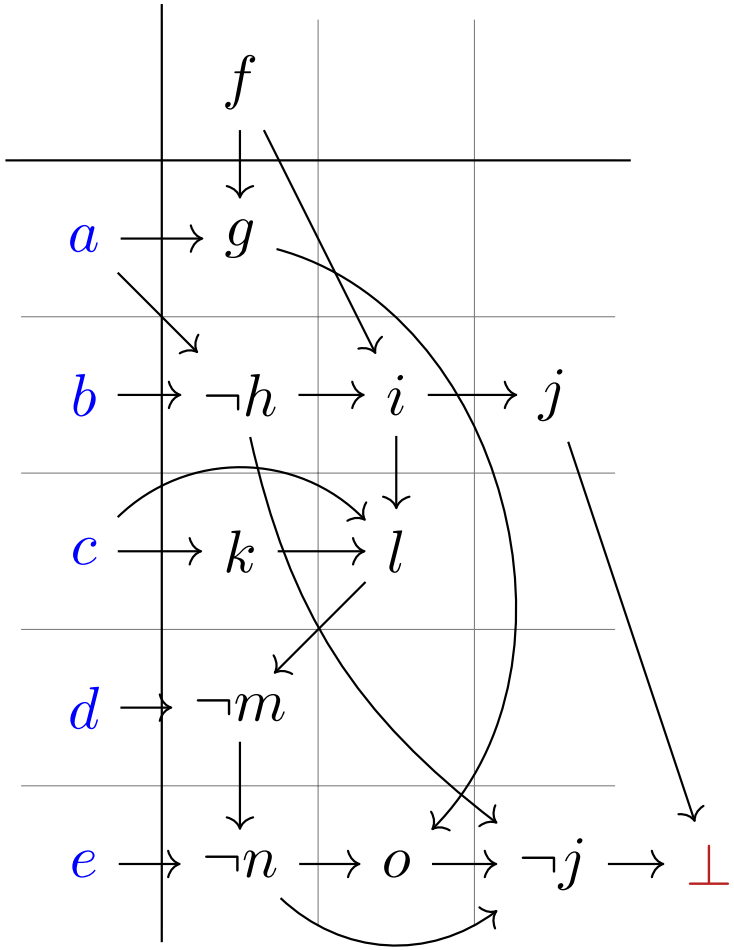

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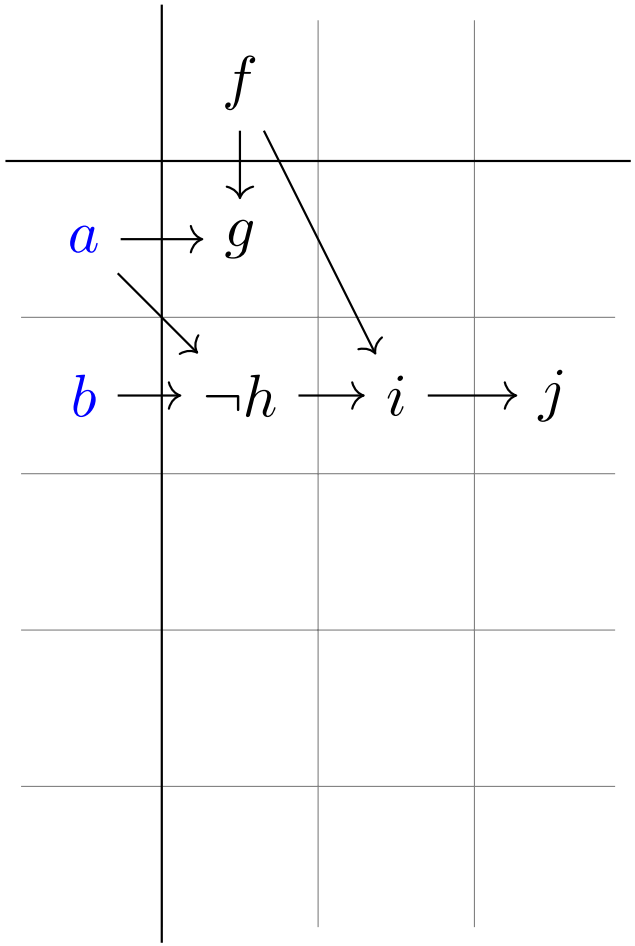

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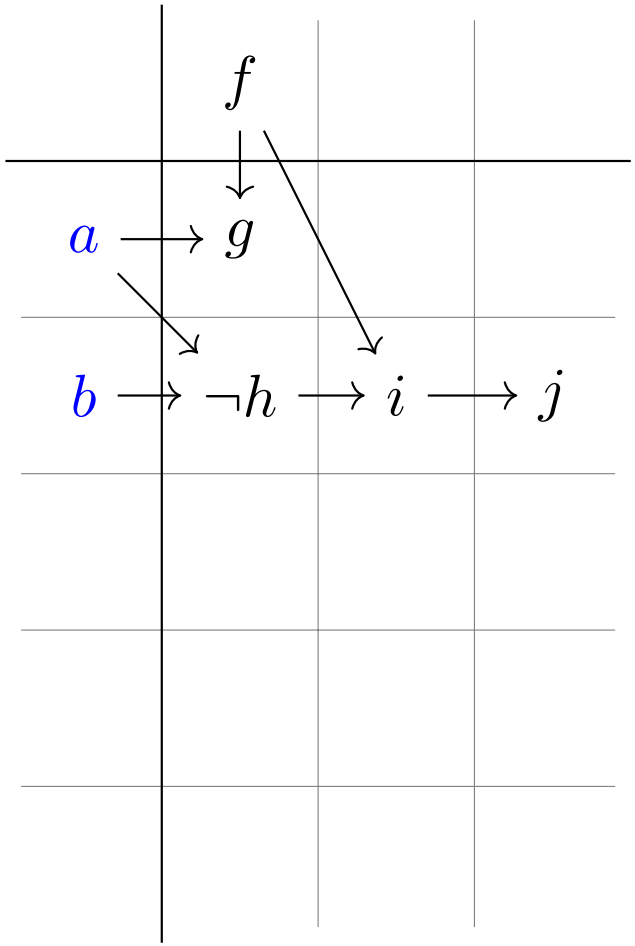

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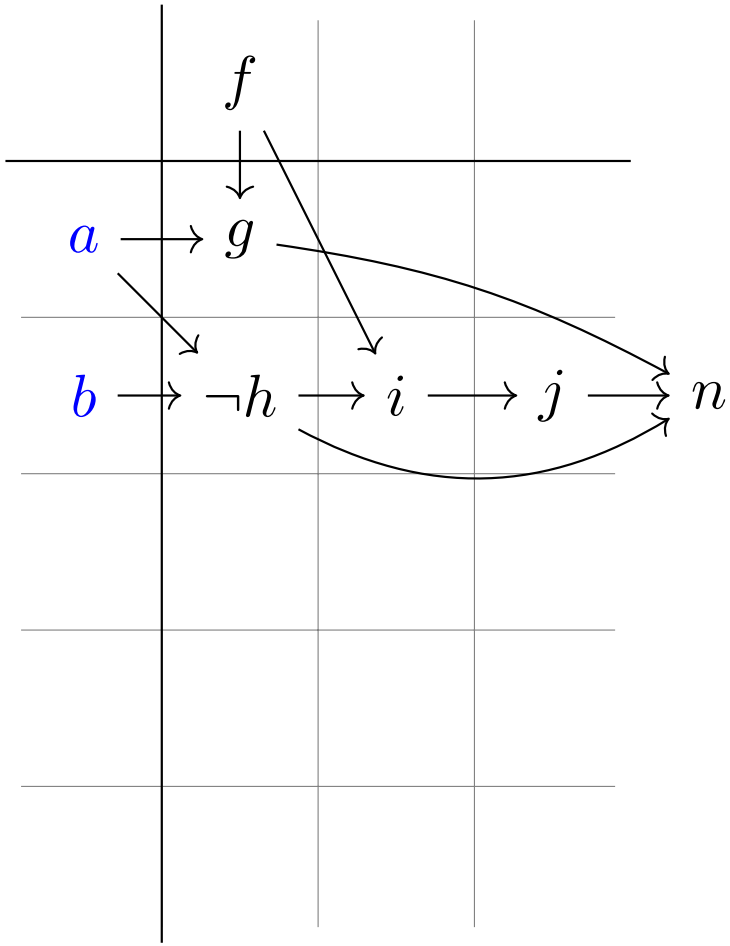
Conflict analysis



$\neg a \vee \neg f \vee g$
$\neg a \vee \neg b \vee \neg h$
$a \vee c$
$a \vee \neg i \vee \neg l$
$a \vee \neg k \vee \neg j$
$b \vee d$
$b \vee g \vee \neg n$
$b \vee \neg f \vee n \vee k$
$\neg c \vee k$
$\neg c \vee \neg k \vee \neg i \vee l$

$c \vee h \vee n \vee \neg m$
$c \vee l$
$d \vee \neg k \vee l$
$d \vee \neg g \vee l$
$\neg g \vee n \vee o$
$h \vee \neg o \vee \neg j \vee n$
$\neg i \vee j$
$\neg d \vee \neg l \vee \neg m$
$\neg e \vee m \vee \neg n$
$\neg f \vee h \vee i$
$\boxed{\neg g \vee h \vee \neg j \vee n}$

Learning and Backjumping

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- When there is only one literal uip propagated in the last level in the current explanation, learn the associated new clause C, backjump (to the last level of propagated literals in C), propagate the new clause $\neg uip$, and continue the exploration

Boosting Search through Randomization and Restarts [Gomes et al., 1998]

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Heavy-tail phenomena (SAT and CP)

At any time during the experiment there is a non-negligible probability of hitting a problem that requires exponentially more time to solve than any that has been encountered before.

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- Randomization: breaking ties, random decision between k best choices, ...
- Restarts: Geometric/Luby

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SAT Solvers (Few examples)

- MiniSat: http://minisat.se/
- Glucose: `http://www.labri.fr/perso/lsimon/glucose/`
- LingeLing `http://fmv.jku.at/lingeling`
- Any Solver by Armin Biere
  `http://fmv.jku.at/software/index.html`
- Any winner from past and future SAT competitions:
  https://www.satcompetition.org/

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