# Conflict Driven Clause Learning

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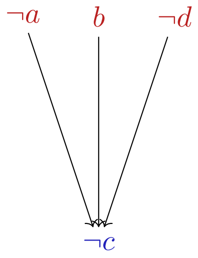
- **Can be seen as a CP Solver (Search, propagation) augmented by clause learning**
- But also :
  - Activity-based branching
  - Lazy data structures (2-Watched Literals)
  - Clause Database Reduction
  - Simplifications
  - Restarts
  - ...

**Exercise:** Propose a filtering algorithm for clauses. The algorithm takes as input a clause and has access (read and write) for the variables domains.

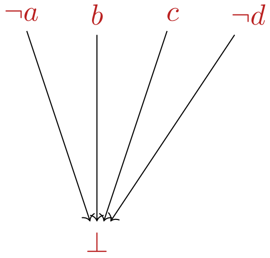
# Unit Propagation

Given a clause $C$ of arity $n$. If $n-1$ literals are false then set the last one to be true.

Example: $(a \vee \neg b \vee \neg c \vee d)$




$\neg a \wedge b \wedge \neg d \Rightarrow \neg c$




$\neg a \wedge b \wedge c \wedge \neg d \Rightarrow \perp$

# Two Watched Literals

- Unit propagation is implemented with an “intelligent” data structure called Two-watched literals
- Observe first that propagation happens only in two cases:
  - The clause becomes unit (i.e., all variables except one is instantiated): Propagate the only uninstantiated literal to satisfy the clause
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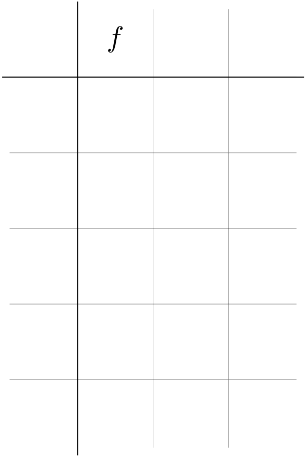
- If a literal watching a clause $C$ becomes $false$, look for replacement. If no replacement found, then perform propagation

# Implication Graph




$$\neg a \vee \neg f \vee g$$
$$\neg a \vee \neg b \vee \neg h$$
$$a \vee c$$
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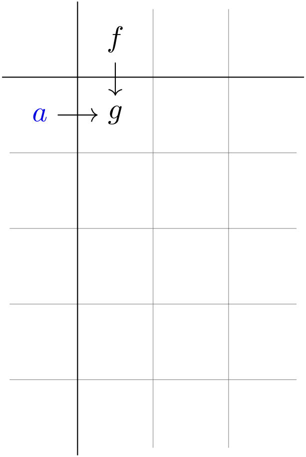


$\neg a \lor \neg f \lor g$

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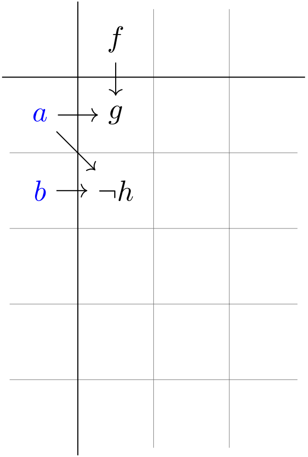


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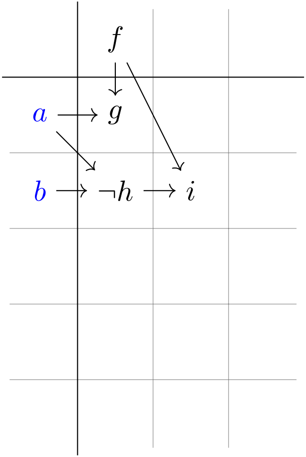


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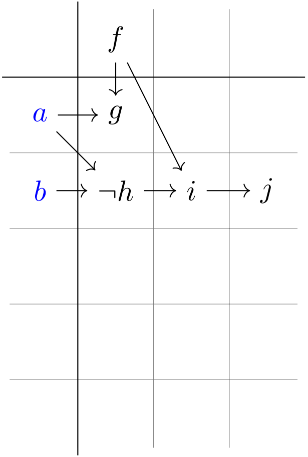


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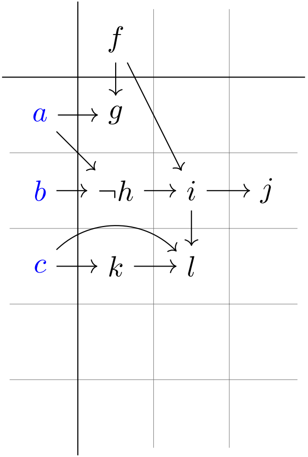


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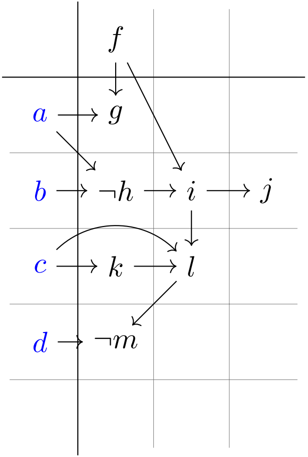

$$
\begin{array}{ll}
\neg a \vee \neg f \vee g & c \vee h \vee n \vee \neg m \\
\neg a \vee \neg b \vee \neg h & c \vee l \\
a \vee c & d \vee \neg k \vee l \\
a \vee \neg i \vee \neg l & d \vee \neg g \vee l \\
a \vee \neg k \vee \neg j & \neg g \vee n \vee o \\
b \vee d & h \vee \neg o \vee \neg j \vee n \\
b \vee g \vee \neg n & \neg i \vee j \\
b \vee \neg f \vee n \vee k & \neg d \vee \neg l \vee \neg m \\
\neg c \vee k & \neg e \vee m \vee \neg n \\
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\end{array}
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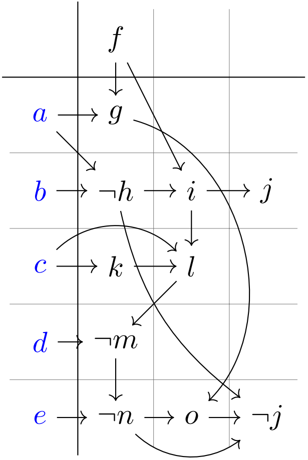


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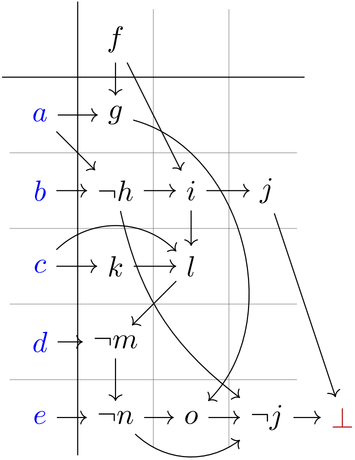


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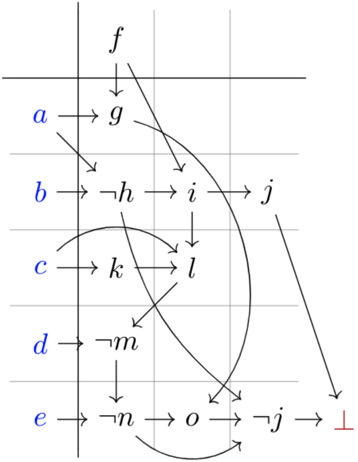


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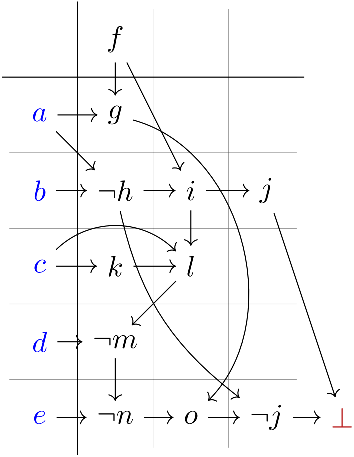


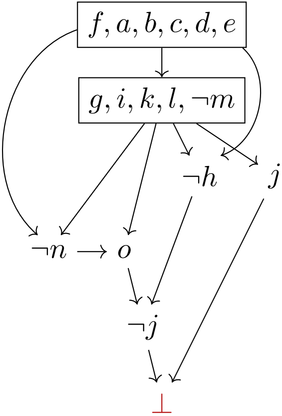

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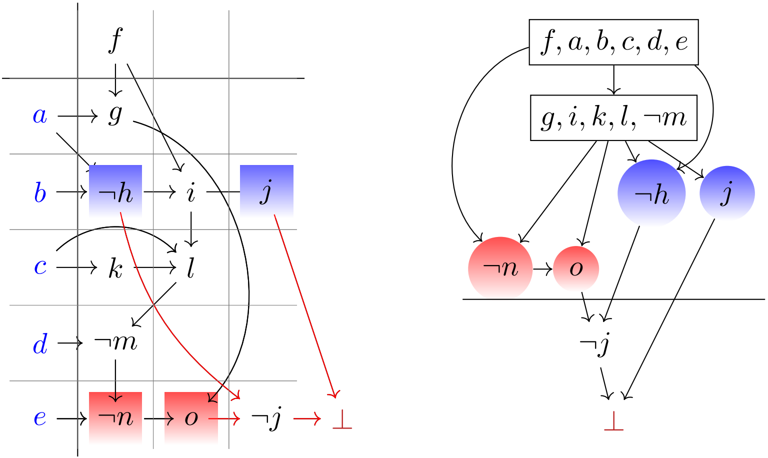

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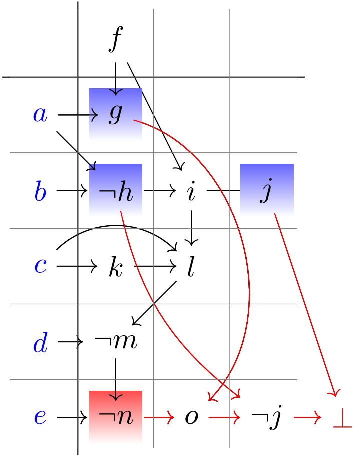


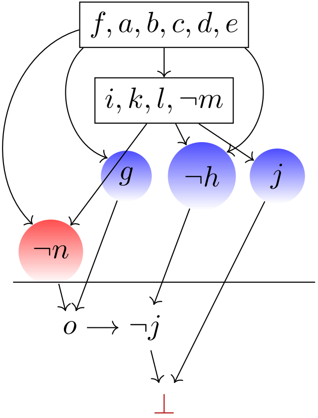

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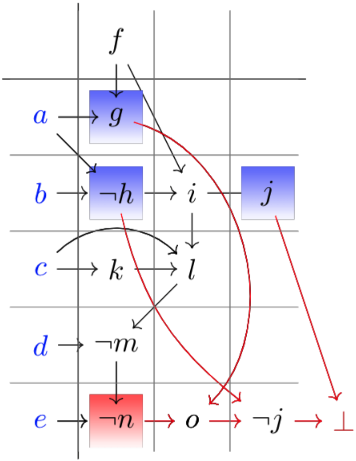


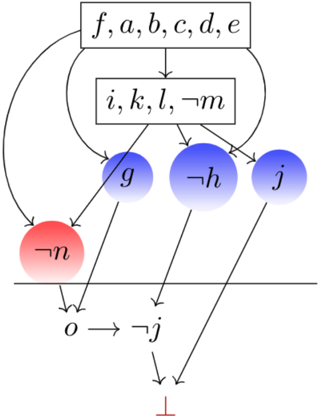

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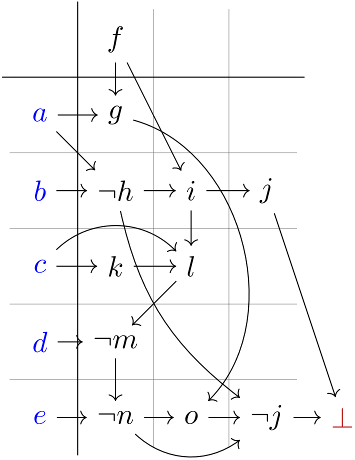


| | |
|---|---|
| $\neg a \vee \neg f \vee g$ | $c \vee h \vee n \vee \neg m$ |
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| $b \vee g \vee \neg n$ | $\neg i \vee j$ |
| $b \vee \neg f \vee n \vee k$ | $\neg d \vee \neg l \vee \neg m$ |
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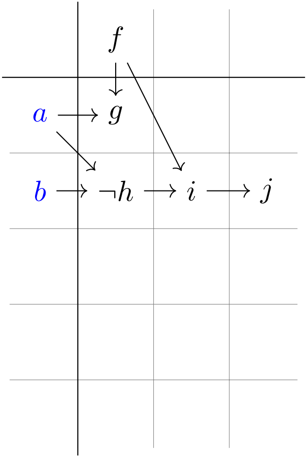


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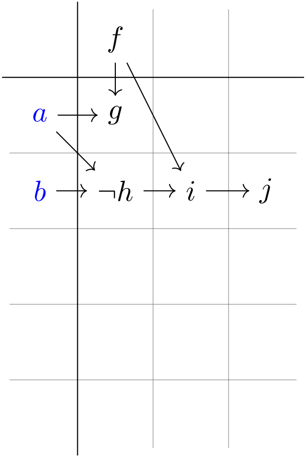


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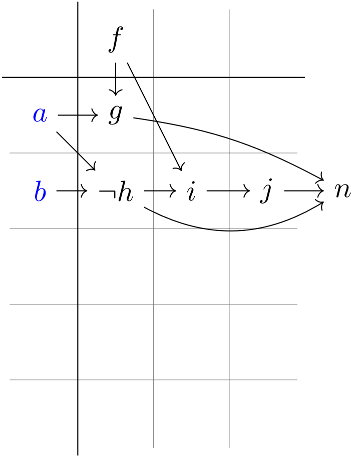


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- At each conflict learn a new clause as following:
- Start with the explanation from the clause triggering failure in the form of $l_1 \wedge \ldots \wedge l_n \rightarrow \perp$ and let it be the initial explanation

# Learning and Backjumping

- Definition: Explaining a failure: $l_1 \wedge \ldots \wedge l_n \rightarrow \perp$ where $\neg l_1 \vee \ldots \vee \neg l_n$ is the clause triggering failute
- Definition: Explaining a propagation of $l$: $l_1 \wedge \ldots \wedge l_n \rightarrow l$ where $\neg l_1 \vee \ldots \vee \neg l_n \vee \neg l$ is the triggering clause
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- When there is only one literal $uip$ propagated in the last level in the current explanation, learn the associated new clause $C$, backjump (to the last level of propagated literals in $C$), propagate the new clause $\neg uip$, and continue the exploration

# Boosting Search through Randomization and Restarts [Gomes et al., 1998]

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### Heavy-tail phenomena (SAT and CP)

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- Randomization: breaking ties, random decision between $k$ best choices, ...
- Restarts: Geometric/Luby

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# SAT Solvers (Few examples)

- MiniSat: http://minisat.se/
- Glucose: `http://www.labri.fr/perso/lsimon/glucose/`
- LingeLing `http://fmv.jku.at/lingeling`
- Any Solver by Armin Biere `http://fmv.jku.at/software/index.html`
- Any winner from past and future SAT competitions: https://www.satcompetition.org/

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