

# Interview CR CNRS for the Competition 06/02

Mohamed Siala

Insight, Centre for Data Analytics, University College Cork, Ireland

Keywords: Constraint Programming, Combinatorial Optimisation, Machine Learning,  
Artificial Intelligence

March 20, 2018

# Plan

- 1 Curriculum Vitae
- 2 Research Paper: Two Clause Learning Approaches for Disjunctive Scheduling, Mohamed Siala, Christian Artigues, and Emmanuel Hebrard, CP'15
- 3 Research Project: Explainable Combinatorial Optimisation via Constraint Reasoning and Machine Learning
- 4 Conclusion

# Academic Career

- **December 2011 to May 2015:**

- PhD in Computer science, LAAS-CNRS, INSA Toulouse, France
- Funding: CNRS, Google, Midi-Pyrénées

- **Since June 2015:**

- Post Doctoral Researcher, Insight, Centre for Data Analytics, University College Cork, Ireland.
- Funding: Science Foundation Ireland 90% and UCC-UTRC 10%.

# Research Areas & Applications

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## Research Areas

- **Constraint programming:** [CP'12, IJCAI'13, CPAIOR'14, Constraints'14, CP'15, EAAI'15, CPAIOR'16, Constraints'16, CP'17, IJCAI'17, CPAIOR'17, CPAIOR'18]

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- **Constraint programming:** [CP'12, IJCAI'13, CPAIOR'14, Constraints'14, CP'15, EAAI'15, CPAIOR'16, Constraints'16, CP'17, IJCAI'17, CPAIOR'17, CPAIOR'18]
- **Boolean SAT:** [CPAIOR'14, CP'15, CP'17, CPAIOR'17]

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- **Boolean SAT:** [CPAIOR'14, CP'15, CP'17, CPAIOR'17]
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- **Scheduling & Sequencing Problems:** [CP'12, IJCAI'13, CPAIOR'14, Constraints'14, CP'15, EAAI'15, Constraints'16]

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## Applications

- **Scheduling & Sequencing Problems:** [CP'12, IJCAI'13, CPAIOR'14, Constraints'14, CP'15, EAAI'15, Constraints'16]
- **Matching under Preferences:** [CPAIOR'16, CP'17, IJCAI'17, ICTAI'17, COCOA'17]

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- Software development: Mistral, under commercialisation with SATALIA (Machine Learning-based Optimisation)
- PC member for: IJCAI, ECAI, CP, etc.
- Reviewing for journals: Artificial Intelligence, JAIR, Constraints, etc

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- Large spectrum of real world applications: space exploration, music, verification, etc
- Excellent trade-off between efficiency, flexibility, and re-usability

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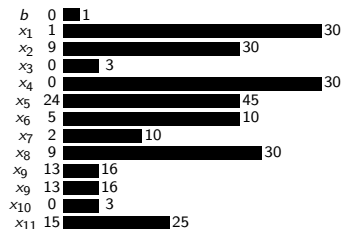
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- Nogood: a conjunction of literals that makes the problem unsatisfiable
- Clause: a disjunction of literals

# Modern Constraint Solvers: Example

$$\begin{aligned}
 &x_1 + x_7 \geq 4 \wedge \\
 &x_2 + x_{10} \geq 11 \wedge \\
 &x_3 + x_9 = 16 \wedge \\
 &x_5 \geq x_8 + x_9 \wedge \\
 &b \leftrightarrow (x_9 - x_4 = 14) \wedge \\
 &b \rightarrow (x_6 \geq 7) \wedge \\
 &b \rightarrow (x_6 + x_7 \leq 9) \wedge \\
 &x_{11} \geq x_9 + x_{10}
 \end{aligned}$$


# Modern Constraint Solvers: Example

$\llbracket x_1 = 1 \rrbracket$

$x_1 + x_7 \geq 4 \wedge$   
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$b$	0	1
$x_1$	1	1
$x_2$	9	30
$x_3$	0	3
$x_4$	0	30
$x_5$	24	45
$x_6$	5	10
$x_7$	2	10
$x_8$	9	30
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# Modern Constraint Solvers: Example

$$\llbracket x_1 = 1 \rrbracket \longrightarrow \llbracket x_7 \geq 3 \rrbracket$$

$$\begin{aligned} x_1 + x_7 &\geq 4 \wedge \\ x_2 + x_{10} &\geq 11 \wedge \\ x_3 + x_9 &= 16 \wedge \\ x_5 &\geq x_8 + x_9 \wedge \\ b &\leftrightarrow (x_9 - x_4 = 14) \wedge \\ b &\rightarrow (x_6 \geq 7) \wedge \\ b &\rightarrow (x_6 + x_7 \leq 9) \wedge \\ x_{11} &\geq x_9 + x_{10} \end{aligned}$$

$b$	0	1
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$x_2$	9	30
$x_3$	0	3
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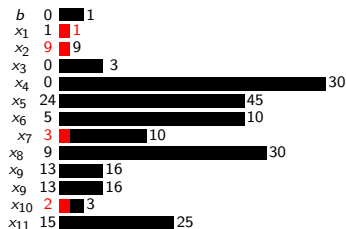
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$$\llbracket x_1 = 1 \rrbracket \longrightarrow \llbracket x_7 \geq 3 \rrbracket$$

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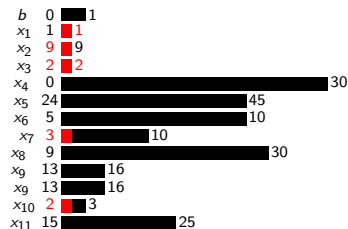
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$$[x_1 = 1] \longrightarrow [x_7 \geq 3]$$

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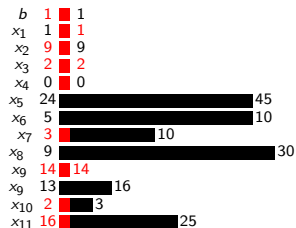
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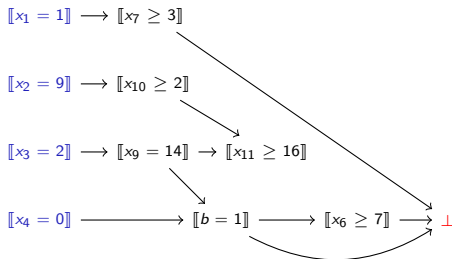
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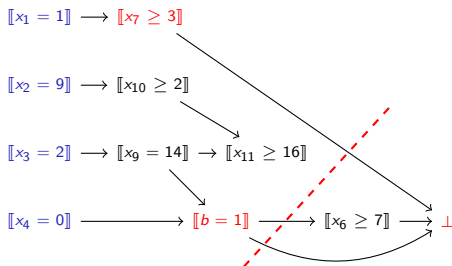
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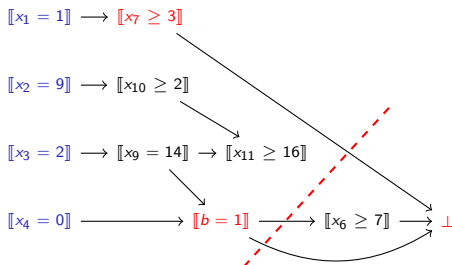


- Conflict analysis:  $\llbracket b = 1 \rrbracket \wedge \llbracket x_7 \geq 3 \rrbracket \Rightarrow \perp$

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- New clause:  $\llbracket b \neq 1 \rrbracket \vee \llbracket x_7 \leq 2 \rrbracket$

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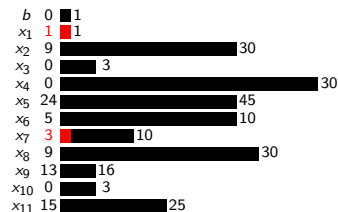
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- Backtrack to level 1

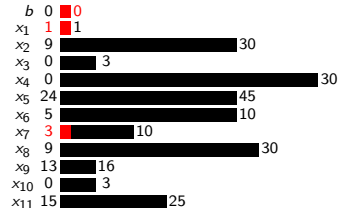
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$$\llbracket x_1 = 1 \rrbracket \longrightarrow \llbracket x_7 \geq 3 \rrbracket \longrightarrow \llbracket b = 0 \rrbracket$$

- Conflict analysis:  $\llbracket b = 1 \rrbracket \wedge \llbracket x_7 \geq 3 \rrbracket \Rightarrow \perp$
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- Backtrack to level 1
- Propagate the learnt clause

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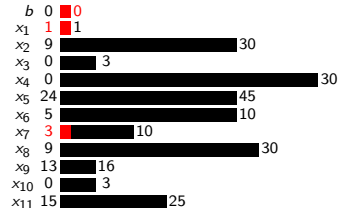


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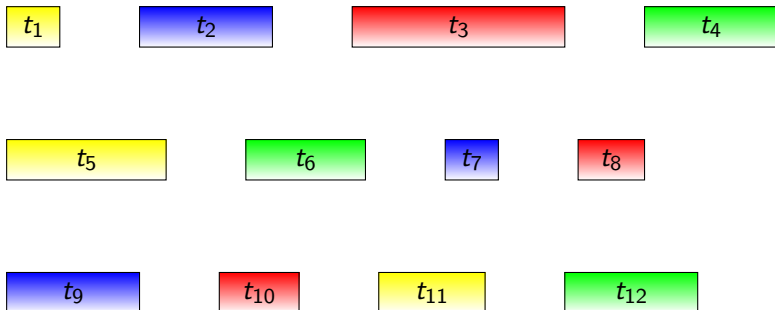
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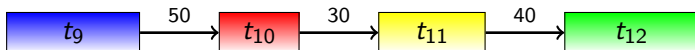
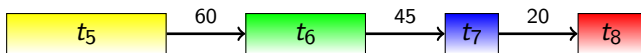


# Context: Disjunctive Scheduling

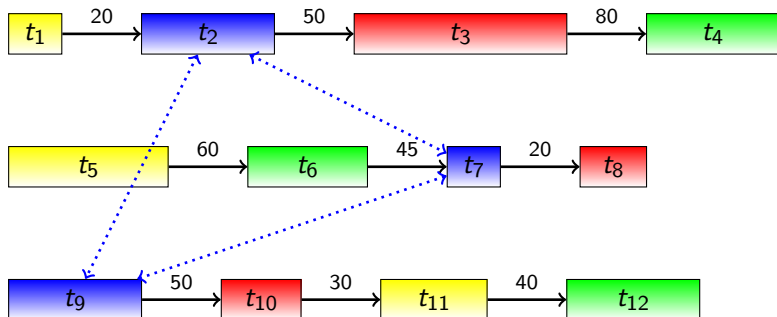
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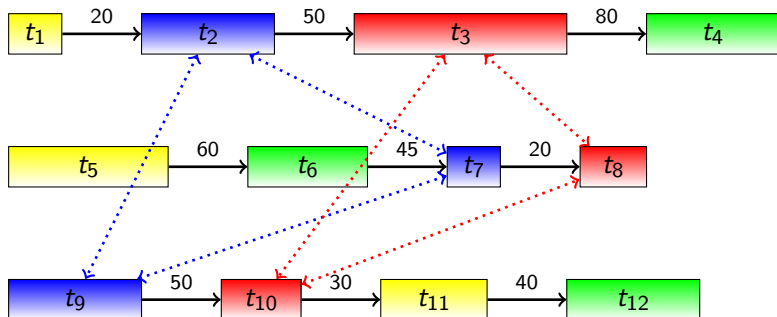
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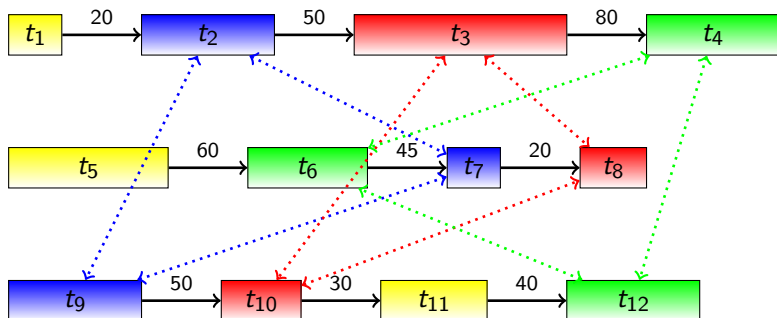


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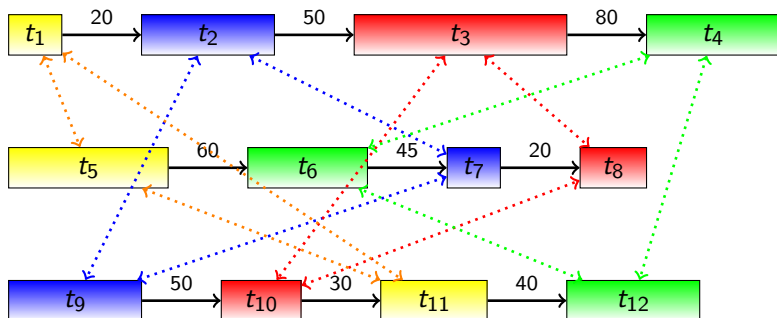




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- Scheduling in CP
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  - Tailored search strategies
- Contribution: Focus on what can be learnt during search!

# Modelling

## Unary Resource Constraint

- $O(n^2)$  Boolean variables  $\delta_{kij}$  ( $i < j \in [1, n]$ ) per machine  $M_k$ .
- Decomposition using the following DISJUNCTIVE constraints:

$$\delta_{kij} = \begin{cases} 0 & \Leftrightarrow t_{ik} + p_{ik} \leq t_{jk} \\ 1 & \Leftrightarrow t_{jk} + p_{jk} \leq t_{ik} \end{cases} \quad (1)$$

# Revisiting Lazy Atom Generation

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## Domain Encoding: standard approach

- 1 Generate domain atoms:  $a \leftrightarrow \llbracket x = d \rrbracket$ ,  $b \leftrightarrow \llbracket x \leq d \rrbracket$
- 2 Generate domain clauses:  $\neg \llbracket x \leq d \rrbracket \vee \llbracket x \leq d + 1 \rrbracket$ ,  $\neg \llbracket x = d \rrbracket \vee \llbracket x \leq d \rrbracket$ , etc
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## Lazy Atom Generation

- ① Atoms and domain clauses are generated during conflict analysis
- ② There is a redundancy issue
- ③ For a domain of size  $k$ ,  $k - 2$  redundant clauses.

# Avoiding redundancy via DOMAINFAITHFULNESS

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## Key Idea

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## Consistency of the domain

Can be enforced in constant amortized time complexity ( $O(1)$ ) down a branch of the search tree

# DISJUNCTIVE-based Learning

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- There exists an explanation for every bound literal  $\llbracket x \leq u \rrbracket$  and  $\llbracket x \geq l \rrbracket$

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- ⊕ Scheduling horizon does not matter in size

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- Well studied benchmarks in OR and CP (more than 2 decades)
- State-of-art results with 7 new lower bounds.

## Conclusions

- Alternative lazy (atom) generation approach avoiding a major redundancy issue
- Novel conflict analysis and learning mechanism
- Efficient in practice, specially for finding proofs

# Plan

- 1 Curriculum Vitae
- 2 Research Paper: Two Clause Learning Approaches for Disjunctive Scheduling, Mohamed Siala, Christian Artigues, and Emmanuel Hebrard, CP'15
- 3 Research Project: Explainable Combinatorial Optimisation via Constraint Reasoning and Machine Learning
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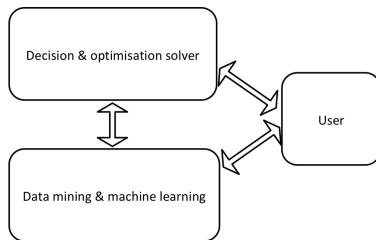
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- Will be mandatory in crucial applications of AI (e.g., medical treatments and legal professions)
- Very little is done regarding explainable combinatorial optimisation!

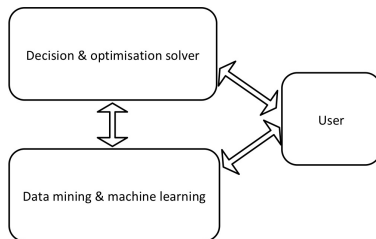
# Research Project

- Title: **Explainable Combinatorial Optimisation via Constraint Reasoning and Machine Learning**
- In a nutshell:
  - How to make decision & optimisation solvers explainable?
  - How can we take into consideration the user's preferences towards explanations?

# Interactive Combinatorial Optimisation



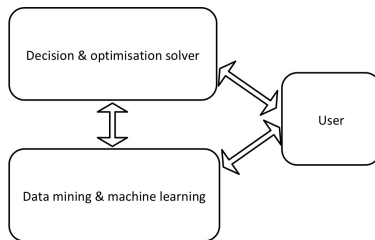
# Interactive Combinatorial Optimisation



- **Decision & optimisation solver:**

- solves combinatorial problems in an interactive way
- is able to explain itself

# Interactive Combinatorial Optimisation

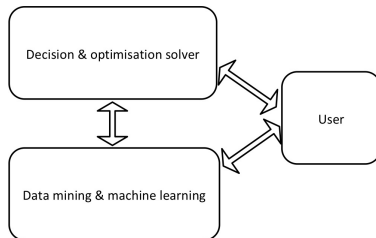


- **User:**

- asks for explanations
- interacts with the “Data mining & machine learning” component in two ways:
  - gives preferences towards the explanations
  - changes data about the problem



# Interactive Combinatorial Optimisation



- **Data mining & machine learning:**

- handles data from the user about the problem
- treats the preferences of the user towards explanations to learn and communicate the preferred explanations to the solver.

# Challenges

- **Challenge 1:** From explaining a filtering algorithm to explaining a backtracking algorithm?
- **Challenge 2:** Making explanations understandable by humans?
- **Challenge 3:** Dealing with model updates: A minor change can have a huge impact on the hardness?

# Roadmap

- ① A uniform language for explanations based on queries
- ② Develop efficient and reusable algorithms for computing explanations.
- ③ Machine learning modules to predict the impact of the interactive updates on
  - the hardness of solving the problem
  - the current solution

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# Conclusion

- Next generation of interactive combinatorial optimisation
- Extremely challenging : both theoretically and practically
- Related to the theme “Data science : data and knowledge engineering, machine learning and reasoning”

# Thank you for your attention

## Thank you for your attention

### Awards

- XCSP3 Solver Competition
- EurlA best thesis award, honourable mention
- CP'12 paper, honourable mention

### Publications

- 3 Journals: Constraints (2), Engineering Applications of Artificial Intelligence (1)
- 11 International Conferences: IJCAI (2), CP (3), CPAIOR (4), ICTAI (1), COCOA (1) (+ since submission **CPAIOR'18**)

### Research Project

- Explainable combinatorial optimisation
- Constraint reasoning meets machine learning for interactive decision making

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