3 A cylinder and piston, used in a car engine, are illustrated in Fig. 3.1.
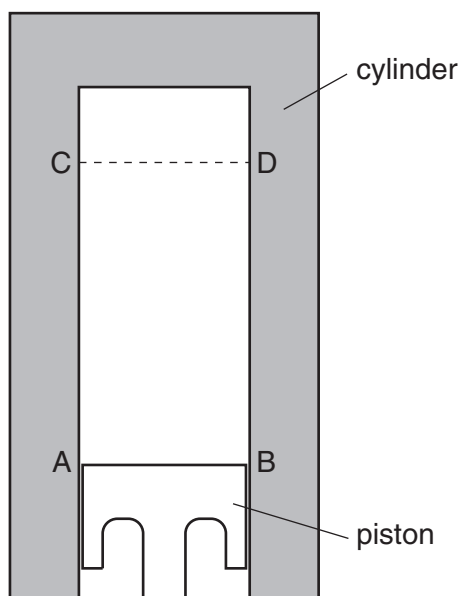
Fig. 3.1

The vertical motion of the piston in the cylinder is assumed to be simple harmonic.
The top surface of the piston is at AB when it is at its lowest position; it is at CD when at its highest position, as marked in Fig. 3.1.

(a) The displacement d of the piston may be represented by the equation

$$d = -4.0 \cos(220t)$$

where d is measured in centimetres.

(i) State the distance between the lowest position AB and the highest position CD of the top surface of the piston.

distance = ... cm [1]

(ii) Determine the number of oscillations made per second by the piston.

number = ... [2]

(iii) On Fig. 3.1, draw a line to represent the top surface of the piston in the position where the speed of the piston is maximum. [1]

(iv) Calculate the maximum speed of the piston.

speed = ... cm s^{-1} [2]

(b) The engine of a car has several cylinders. Three of these cylinders are shown in Fig. 3.2.
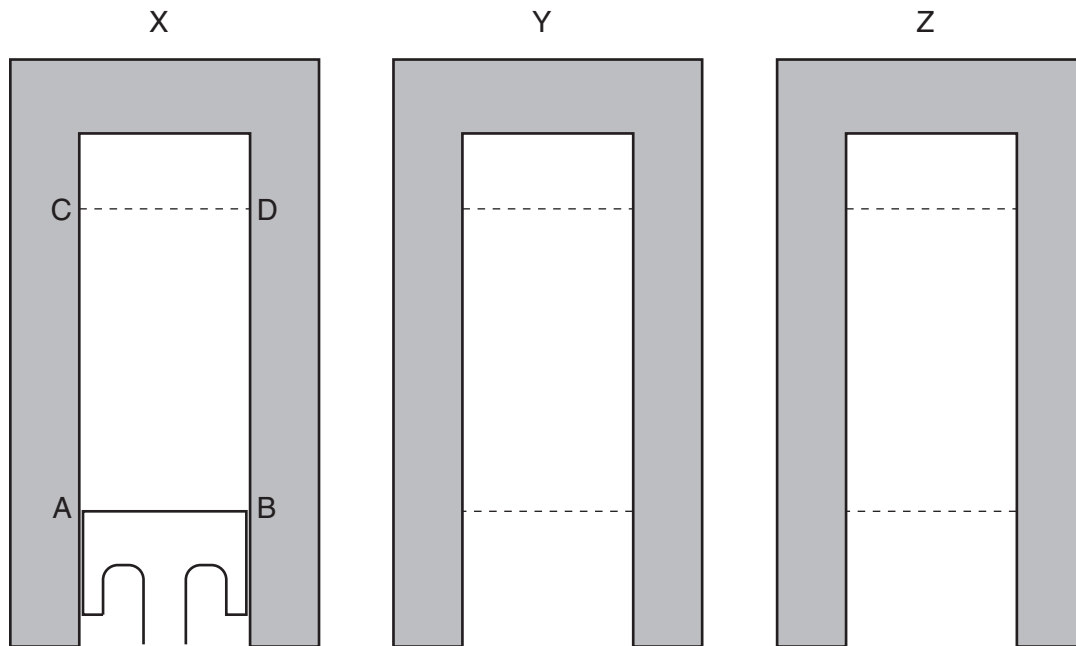
X   Y   Z

Fig. 3.2

X is the same cylinder and piston as in Fig. 3.1.

Y and Z are two further cylinders, with the lowest and the highest positions of the top surface of each piston indicated.

The pistons in the cylinders each have the same frequency of oscillation, but they are not in phase.

At a particular instant in time, the position of the top of the piston in cylinder X is as shown.

(i) In cylinder Y, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 120° ($\frac{2}{3}\pi$ rad).

Complete the diagram of cylinder Y, for this instant, by drawing

    1. a line to show the top surface of the piston,   [1]

    2. an arrow to show the direction of movement of the piston.   [1]

(ii) In cylinder Z, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 240° ($\frac{4}{3}\pi$ rad).

Complete the diagram of cylinder Z, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

(iii) For the piston in cylinder Y, calculate its speed for this instant.

speed = ... cm s^{-1} [2]