5 A helium balloon is attached to a toy of mass m, and a string of mass per unit length μ hangs under the toy as shown in Fig. 5.1.

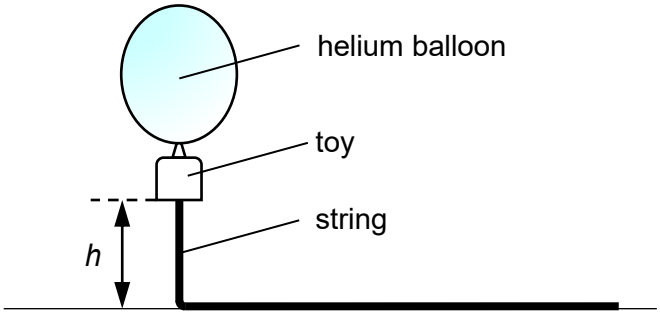

Fig. 5.1

The balloon exerts a constant upward force T on the toy where T is greater than the weight of the toy.

(a) When the toy is at a height h from the ground, it is in equilibrium.

Explain why the relationship between T, m, μ, h and the acceleration of free fall g is given by the expression

$$T = mg + \mu hg .$$

……...

……...

……..... [1]

(b) Subsequently, the toy is displaced downwards by a small distance A_0, and then it is released. Throughout the oscillation, the tail of the string is always in contact with the ground.

Assuming that there are no dissipative forces and taking the mass of the oscillating system to be constant and equal to m, show that the acceleration a of the toy at displacement z from the equilibrium position is given by the expression

$$a = -\left(\frac{\mu g}{m}\right) z .$$

[3]

(c) The mass of the toy is 50 g and the string has a mass per unit length of 5.8 g m^{-1}.

Calculate the period of oscillation of the toy.

period = ... s [3]

(d) Air resistance can cause the oscillation to be lightly damped. Fig. 5.2 shows this variation with time t of the amplitude A of the oscillations of the toy.

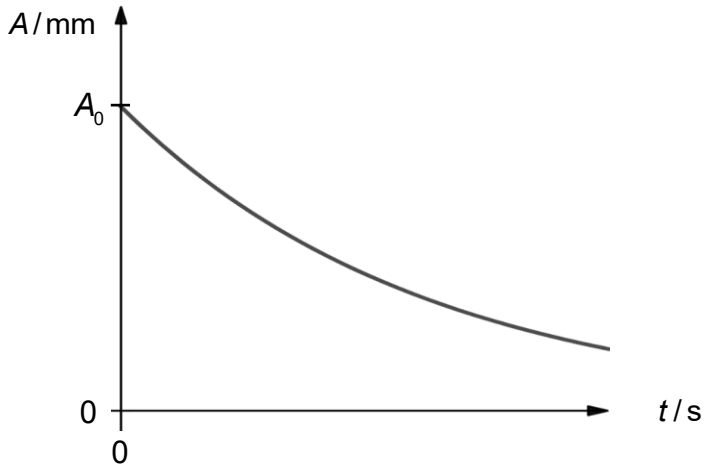

Fig. 5.2

(i) Describe the variation with t of A.

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.....

……..... [1]

(ii) On Fig. 5.2, sketch the variation with t of A when the toy is replaced by another toy with a smaller base surface area. [1]

(iii) Suggest why experimental results for both toys show that the extent of damping is greater than what Fig. 5.2 predicts.

……

……

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……..... [2]

(e) The toy is displaced downwards by a small distance A_0 from the equilibrium position and released at time $t = 0$ s.

(i) On Fig. 5.3, sketch the variation with time t of displacement from the equilibrium position d of the toy when the mass of the string is negligible and there are no dissipative forces.

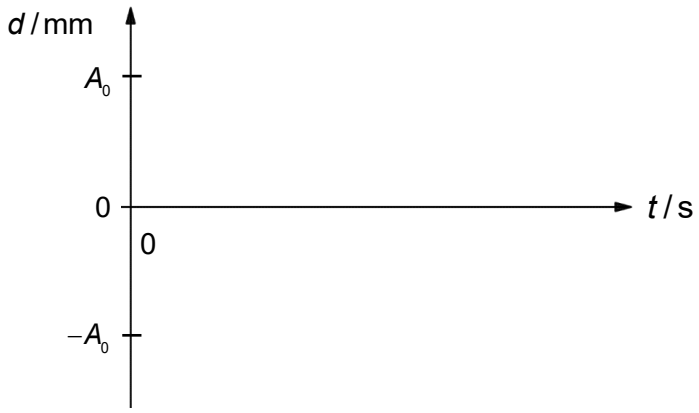

Fig. 5.3

[1]

(ii) Explain the variation with t of d drawn in **(e)(i)**.

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……..... [1]

Section B

Answer **one** question from this Section in the spaces provided.