

- 7 Read the passage below and answer the questions that follow.

### Invincible Class Submarine

The Republic of Singapore Navy (RSN) recently introduced the Invincible-class conventionally-powered attack submarines, replacing the ageing Challenger-class. Fig. 7.1 shows a submarine of similar configuration.

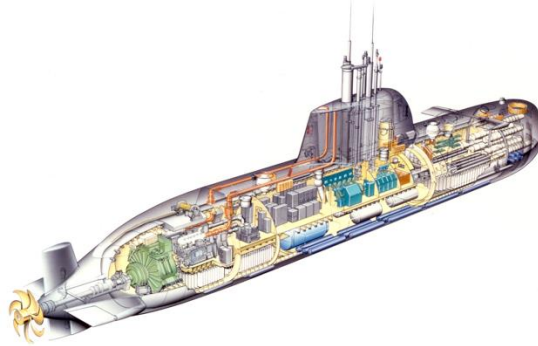


Fig. 7.1

Table 7.1 shows the basic information of the submarine.

Table 7.1

feature	quantity
length $L$ / m	70.0
diameter $d_0$ / m	6.30
maximum speed (surface) / km h <sup>-1</sup>	27.8
maximum speed (submerged) / km h <sup>-1</sup>	37.0

The most efficient geometry for a submarine's pressure hull are circular cross-sections that transfer normal pressure loading to in-plane compressive forces i.e. hydrostatic pressure. Failures in the structure of a submarine could lead to an implosion. For simplicity, the hull can be analysed as a cylinder.

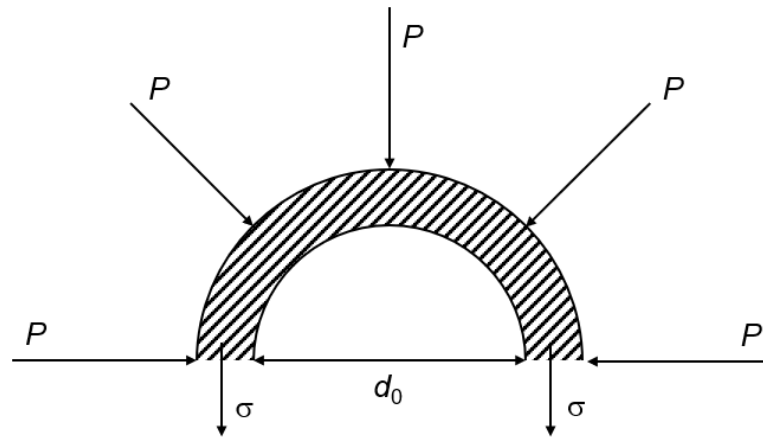
The hoop stress  $\sigma$  on a thin walled cylinder is given by

$$\sigma = \frac{d_0 P}{2t}$$

where  $P$  is the hydrostatic pressure acting normally at the hull,  $d_0$  is the diameter and  $t$  is the wall thickness of the hull.

The compressive stress limit defines the maximum stress a metal can endure before experiencing permanent deformation and failure. Usually, the hull of a submarine is made of high-strength steel (HY-80) that has a compressive stress limit of 550 MPa.

The cross-sectional view of the hull is shown on Fig. 7.2, illustrating the main stresses acting along the circumference of the hull.



**Fig. 7.2**

Singapore is an international shipping hub where more than 1000 ships visit daily. To navigate safely underwater amidst the heavy traffic, submarines rely heavily on sonar. SONAR is the acronym for SOund Navigation And Ranging. Sonar uses sound waves to create a picture of the surroundings. This allows submarines to discreetly navigate, detect potential threats, and even communicate with other underwater vessels.

An active sonar transmits an acoustic signal, which propagates to a reflector and subsequently reflects the signal back to the sonar receiver. Phased array sonar is a type of active sonar that is typically used by military submarines. The equipment is made up of an array of transducers to transmit and receive sound waves. Phased array sonars can be used to create multiple beams at the same time, which can be useful for tasks such as tracking multiple targets or creating a three-dimensional image of an area.

Drag forces are a constant challenge for submarines navigating underwater. These arise from the friction between the submarine's hull and the surrounding water. This force is given by

$$F = \frac{1}{2} C_d \rho v^2 A$$

where  $\rho$  is the water density,  $v$  is the velocity of the submarine,  $A$  is the submarine's projected frontal area, and  $C_d$  is the drag coefficient of a dimensionless value representing the shape-dependent resistance of the object.

**[Turn over**

- (a) The new Invincible-class submarine is required to patrol the South China Sea periodically where it is deeper. The density,  $\rho$ , of seawater in this region is  $1030 \text{ kg m}^{-3}$ .

- (i) Determine the pressure exerted by the seawater on the submarine's hull when it is 400 m in depth.

pressure = ..... MPa [2]

- (ii) Hence, show that the required minimum mean thickness of the hull is 2.4 cm at this depth.

[2]

- (iii) Explain why the submarine hull can be considered as a thin wall.

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..... [1]

- (iv) The Young's modulus is a mechanical property of solid materials that is defined as the ratio of the stress applied to the object and the resulting deformation (compression) within Hooke's limit of proportionality.

Young's modulus  $E$  is given by

$$E = \frac{\sigma}{\left(\frac{d_x}{d_0}\right)}$$

where  $d_0$  is the original diameter and  $d_x$  is the net compression of the diameter.

Determine the new diameter of the submarine when it is at 400 m depth and HY-80 has a Young's modulus,  $E$  of 205 GPa.

new diameter = ..... m [2]

- (v) According to the hoop stress equation, when a submarine is required to travel in deeper waters, the design of its hull thickness must be increased.

State two limitations of this method.

1. ....

.....

2. ....

..... [2]

[Turn over

- (b) (i) Sonar uses seawater as a medium for the propagation of sound.

Describe how sound travels in the deep sea.

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..... [2]

- (ii) Medwin's formula is a useful approximation for speed of sound in seawater:

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.010T)(S - 35) + 0.016D$$

where  $c$  is the speed of sound,  $S$  is the salinity in parts per thousand (ppt),  $T$  is the temperature ( $^{\circ}\text{C}$ ), and  $D$  is the depth (m).

Table 7.2 shows the data about the seawater in the region.

**Table 7.2**

average salinity	33 parts per million (ppm)
average temperature	28 $^{\circ}\text{C}$

Determine the speed of sound in the seawater at a depth of 400 m.

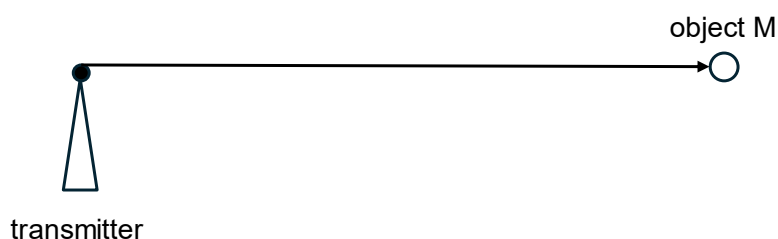
speed .....  $\text{m s}^{-1}$  [2]

- (iii) The speed of sound in air is about  $330 \text{ m s}^{-1}$ .

Suggest why the speed of sound in the seawater is faster.

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 .....  
 ..... [1]

- (iv) Fig. 7.3 shows a transmitter of the sonar transducer emitting a short sound pulse “ping” to detect a spherical object M, with cross-sectional area  $S$  underwater. The mean power of the transmitter is  $P$ . The transmitter can be taken to be a point source.



**Fig. 7.3**

The echo “ping” is received by M. Assume that there are no losses in the transmission and M reflects all the sound energy it receives over a hemispherical surface. The intensity of sound received at the transmitter from M is  $I_M$ .

Show that the distance,  $d$  of the object in terms of  $P$ ,  $S$  and  $I_M$  is

$$d = \left[ \frac{PS}{(I_M)8\pi^2} \right]^{\frac{1}{4}}.$$

[3]

**[Turn over**

- (c) (i) The Invincible-class submarine is designed to be hydrodynamically efficient and the drag coefficient  $C_d$  can be assumed to be 0.1.

Determine the minimum engine power of the submarine so that it can move at its maximum constant speed underwater.

power = ..... W [2]

- (ii) Suggest why in reality, the movement of the submarine will require more engine power than (c)(i) when traveling at the same constant speed.

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.....  
..... [1]

[Total: 20]

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