7 Read the passage and answer the questions that follow.

In World War II, Nazi Germany fought a bombing campaign against the British. The Germans developed a radio navigation system called *X-Gerät* ("X-Apparatus") to help their bombers find their targets more accurately. In turn, the British developed countermeasures, aiming to jam the German signals or lead the bombers off-course. This period of the war came to be known as the "Battle of the Beams."

X-Gerät consists of radio arrays called *Knickebein* ("crooked leg"). Each *Knickebein* array had eight radio antennae, which behave like point sources of radio waves, suspended 6.0 m apart, transmitting radio waves at a frequency of 30.0 MHz. The waves from these antennae superpose to produce a single beam of radio waves, which spreads out with an angle α as shown in Fig. 7.1. This guides an aircraft equipped with a radio receiver, to fly in a straight line towards their target.

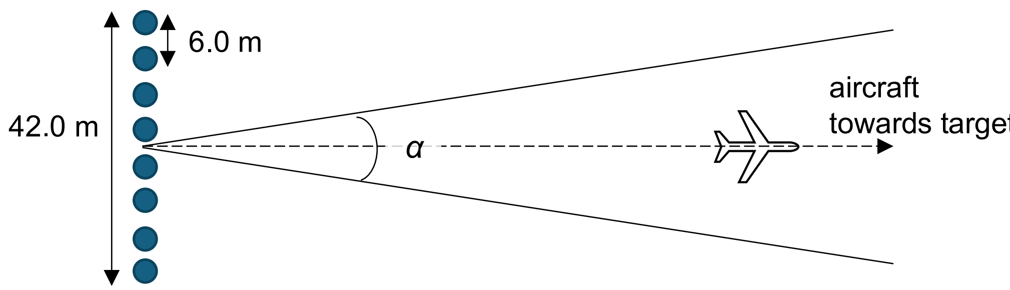

Fig. 7.1

The accuracy of this method of navigation is limited by the width of the beam, especially at long ranges. The narrower the beam, the more accurate the guidance system – but it is difficult to make the angle α small enough to be practical at the long ranges required. However, the German engineers managed to come up with a solution that significantly reduced the *Knickebein* beam's actual angular width.

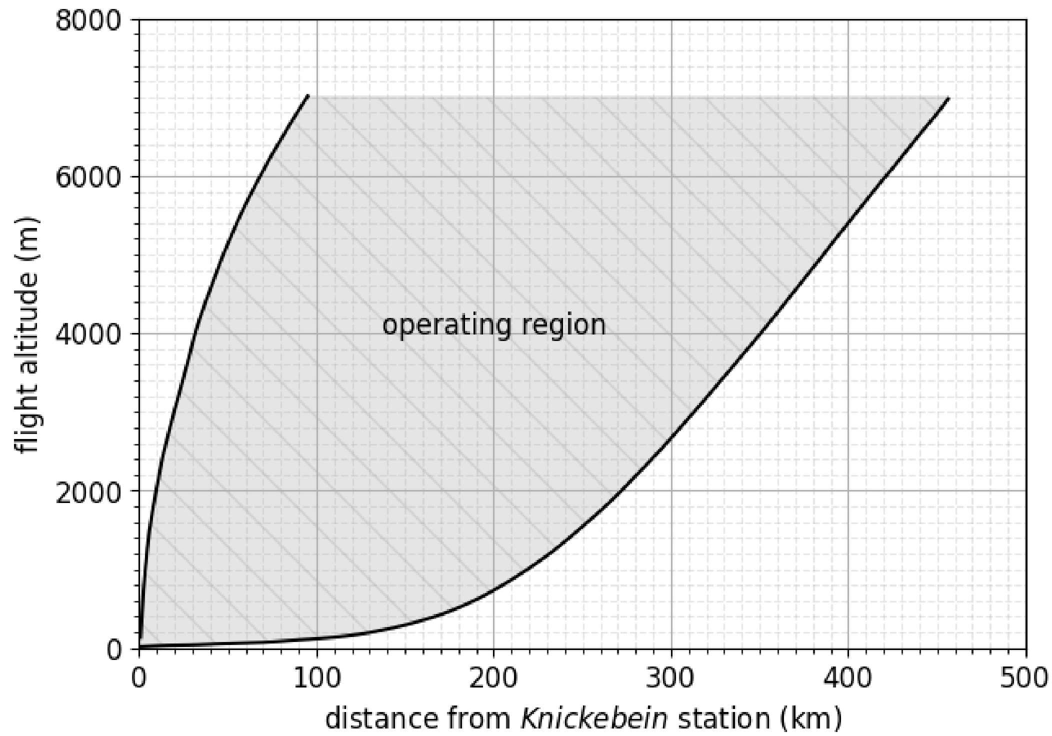

Fig. 7.2

Whether the signal can be detected by the aircraft depends on its altitude and distance from the *Knickebein* station. Fig. 7.2 is a graph of flight altitude against distance from the *Knickebein* station. The shaded "operating region" between the two curves represents the region where the intensity of the radio waves is large enough for the aircraft to detect it.

In the *X-Gerät* system, the bomber flies along one *Knickebein* beam (the "guide beam") in a straight line towards its target at a constant speed and altitude. As the bomber approaches the target, it flies through two more *Knickebein* beams (*Oder* and *Elbe*), which are perpendicular to its path, as shown in Fig. 7.3. These beams are used to control when the bomb should be released.

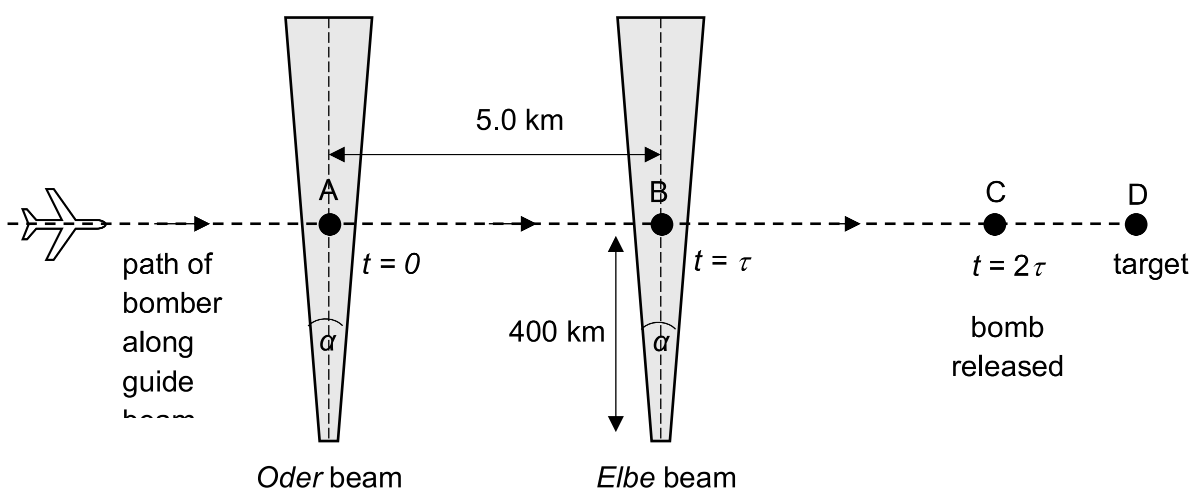

Fig. 7.3

When the bomber detects the *Oder* beam (at point A), a stopwatch is started. After travelling 5.0 km, the bomber detects the *Elbe* beam (at point B), and the stopwatch reading τ is noted. At point C, the stopwatch reading is 2τ, and the bomb is released, which falls and lands on the target at D. The *Oder* and *Elbe* beams originate from *Knickebein* stations in occupied Europe, about 400 km away, and thus have non-negligible widths. This contributes some uncertainty that may cause the bomb to miss.

(a) **(i)** The *Knickebein* array of radio antennae results in interference of radio wave, similar to visible light that has been passed through a diffraction grating.

Using the data from Fig. 7.1, show that the *Knickebein* array produces only a single beam.

[2]

(ii) This beam of radio wave spreads out as it travels, like a wave passing through a single slit of the same width as the entire array of radio antennae.

For the arrangement of sources in Fig. 7.1, calculate α, the angular width of the beam.

α = ...° [3]

(b) **(i)** The bomber is flying at the minimum altitude for it to have a detectable signal 400 km from the *Knickebein* station.

State this altitude.

altitude = …………………………… m [1]

(ii) The time shown on the stopwatch when the bomber crosses the Elbe beam is τ = 51.4 s.

Show that the speed of the bomber is 97.3 m s^{-1}.

[1]

(iii) Determine CD in Fig. 7.3, the horizontal distance from the release point to the target.

CD = …………………………… m [3]

(iv) Now, the bomber is flying at a greater speed than your answer to **(b)(ii)**, but the same altitude as in **(b)(i)**; the positions of the *Elbe* and *Oder* beams (A and B respectively) are unchanged.

Explain whether the bomb released at point C will still hit the target.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [2]

(v) The German engineers managed to reduce the angular width α of the beam to 0.3° instead of the value you found in **(a)(ii)**.

Determine the uncertainty in the position of point A due to the angular width of the *Oder* beam.

uncertainty = ... m [2]

(c) The German bomber used with the *Knickebein* was the Heinkel He 111H, which had the technical specifications in Table 7.1.

Crew	5
Propulsion for propellers	2 piston engines
Power of one piston engine	990 kW
Maximum speed	435 km/h
Service ceiling	6700 m
Range	1950 km
Empty weight	8680 kg
Maximum take-off weight	14000 kg

Table 7.1

Forces acting on a Heinkel He 111H flying at constant speed and altitude are shown in Fig. 7.4.

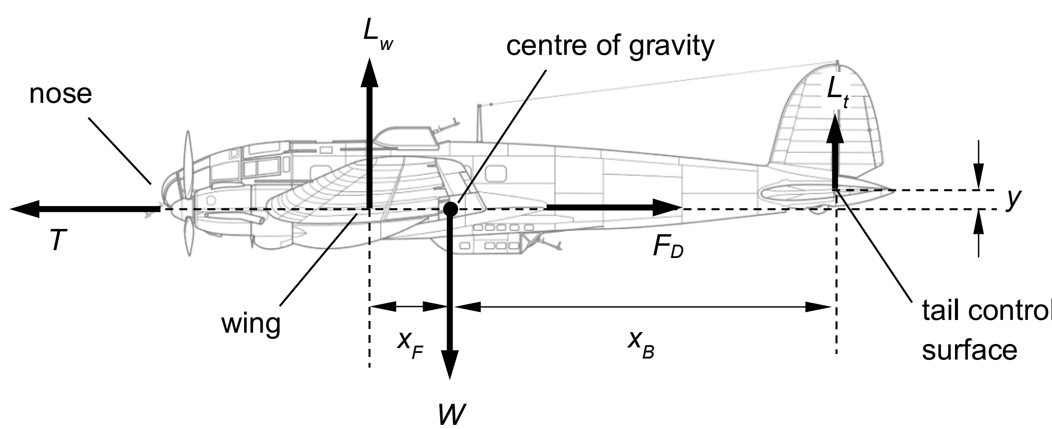

Fig. 7.4

T is the thrust from the propellers, L_w is the lift from the wings, L_t is the lift from the tail control surfaces, W is the weight of the aircraft, and F_D is the drag force.

The centre of gravity is a distance x_F away from the centre of lift of the wings and a distance x_B from the tail control surfaces.

The centre of lift of the tail control surfaces is at a perpendicular distance y above the centre of gravity.

(i) Take moments about the centre of gravity, to show that, when the aircraft is level and flying at constant speed and altitude,

$$L_t = \frac{Wx_F}{x_F + x_B}$$

[2]

(ii) Explain clearly why the drag force F_D does not appear in the relationship in **(c)(i).**

………

…… [1]

(iii) Using data from Table 7.1, determine the magnitude of the drag force F_D on the bomber when it is travelling at a constant speed of 200 km h^{-1}.

F_D = …………………………… N [4]

[Total: 21]

End of paper