8

Consider a small rock of mass m at a distance r from the center of the Earth. The mass of Earth is M.

The variation of gravitational potential energy U of this system with r is shown in Fig. 8.1 where the radius of Earth R is 6400 km. Resistive forces due to the Earth's atmosphere is negligible.

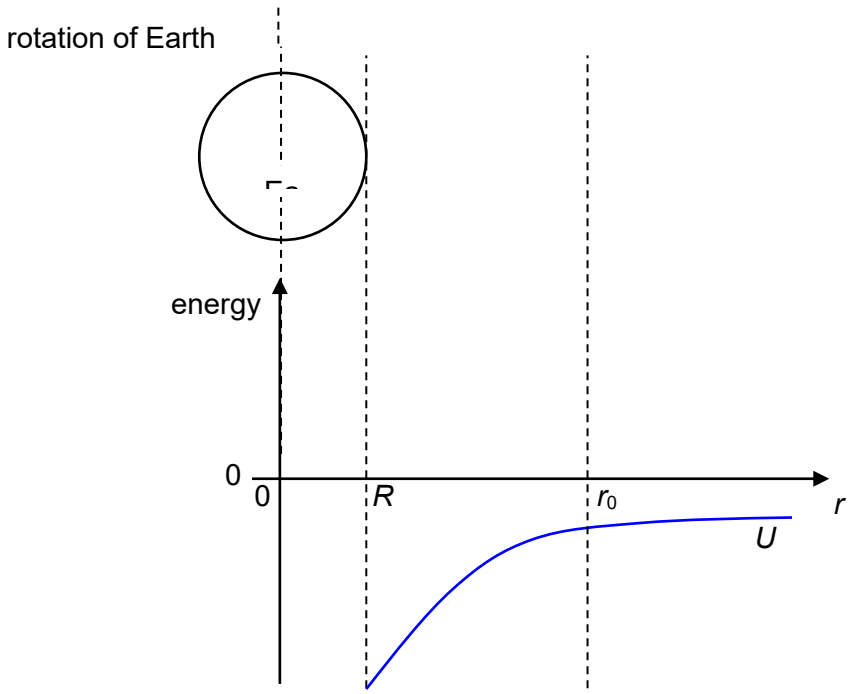

Fig. 8.1

(a)

(i)

State how the magnitude of U varies with r.

.[1]

(ii)

Explain why *U* is a negative quantity.

.

.

..

.

.[3]

(b)

The rock is released from rest at a distance r_0 from the center of the Earth, above a point P on Earth at the equator.

(i)

On Fig. 8.1, sketch the graph of

[redacted]

1. the total energy E

2. the kinetic energy K

[redacted]

from $r = r_0$ to $r = R$ as the rock falls towards the Earth. Label each graph clearly.

[3]

(ii)

Explain how the acceleration of the rock changes as it approaches the Earth.

[redacted]

._____.

.._____.

._____.

._____.[2]

(iii)

It takes the rock a time t_0 of one hour to reach the surface of the Earth.

By considering the angular velocity of the Earth, calculate the angular displacement of the Earth, in radian, during t_0.

angular displacement = _____ rad

[2]

[redacted]

(iv)

Hence, calculate the distance away from P when the rock reaches the Earth.

[redacted]

distance = _____ m

[1]

[redacted]

(c)

Instead of releasing the rock from rest at $r = r_0$, it is given a speed v_0 such that it can orbit the Earth in uniform circular motion as shown in Fig. 8.2.

[redacted]

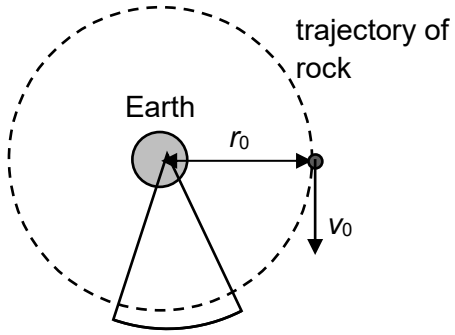

Fig. 8.2

[redacted]

(i)

Derive the expression for the speed v_0 of the rock in terms of G, M and r_0.

[2]

(ii)

Derive the expression for the period T of the revolution of the rock in terms of G, M and r_0.

[1]

(d)

Suppose the rock is launched at the same location with a speed v_1 so that it traverses an elliptical trajectory shown in Fig. 8.3 via the bold line.

Point Q is the common point for the two different trajectories.

[redacted]

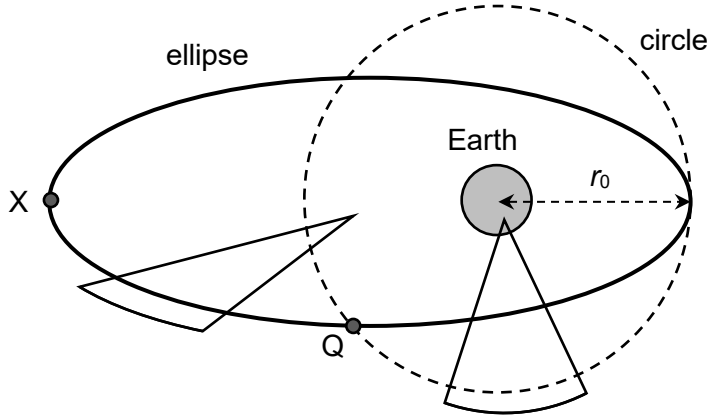

Fig. 8.3

(i)

The launching speed v_1 of the rock is less than the speed v_0 of the rock in a circular orbit.

State how the speed of the rock at Q, when it is undergoing elliptical motion, compares to the speed of the rock when it was in a circular orbit.

Explain your answer.

[redacted]

.

.

.

..

.

.[3]

(ii)

Hence predict how the speed will change as the rock moves from Q to X where X is the point where it is furthest away from the Earth.

Explain your answer.

.

..

.

.[2]

[Total: 20]