8

(a) A rectangular coil PQRS of dimensions 14.0 cm by 12.0 cm, moves with a constant speed of 2.0 cm s^{-1} through a region of uniform magnetic field WXYZ of width 10.0 cm, as shown in Fig. 8.1. There is a magnetic flux density *B* of 1.5 T in WXYZ directed into the plane of the paper.

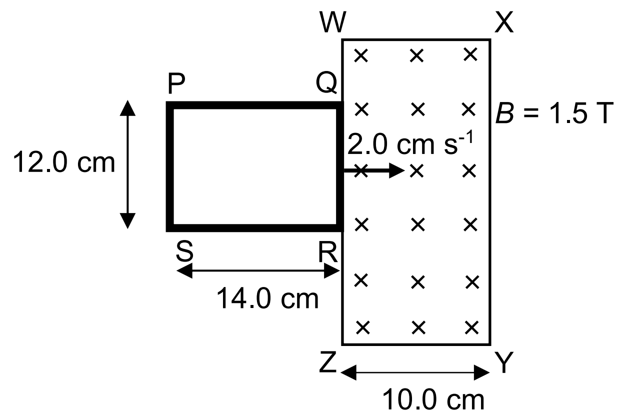


Fig. 8.1

(i) State and explain the direction of the induced current in coil PQRS when it enters the field.

....

....

....

....

....

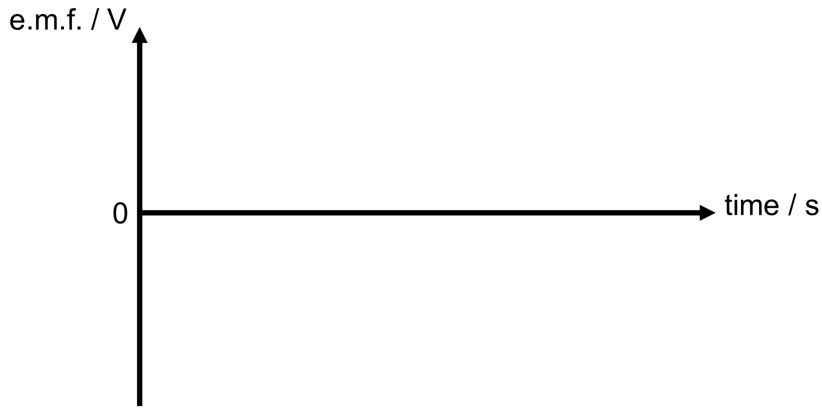
....

.... [3]

(ii) Calculate the maximum possible magnetic flux linkage during the motion.

maximum flux linkage = ... Wb [1]

(iii) The coil PQRS is moved from the position shown in Fig. 8.1 until side PS is aligned with WZ. On the axes below, sketch a graph to show the variation with time of induced e.m.f. in the coil. Label the axes with appropriate numerical values.



[2]

(b) In 1932, Ernest Lawrence built the first cyclotron, an early form of particle accelerator that accelerates charged particles in a confined space with a combination of electric field and magnetic field. Though it was succeeded by more powerful designs later, the cyclotron is still used in nuclear medicine today.

Fig. 8.2 shows the basic structure of a cyclotron. A pair of electromagnets generate a uniform magnetic field vertically through a pair of semicircular metal chambers, referred to as “dees”. The dees are hollow, allowing charged particles to move. An e.m.f. source that changes its polarity at regular intervals is connected to the dees.

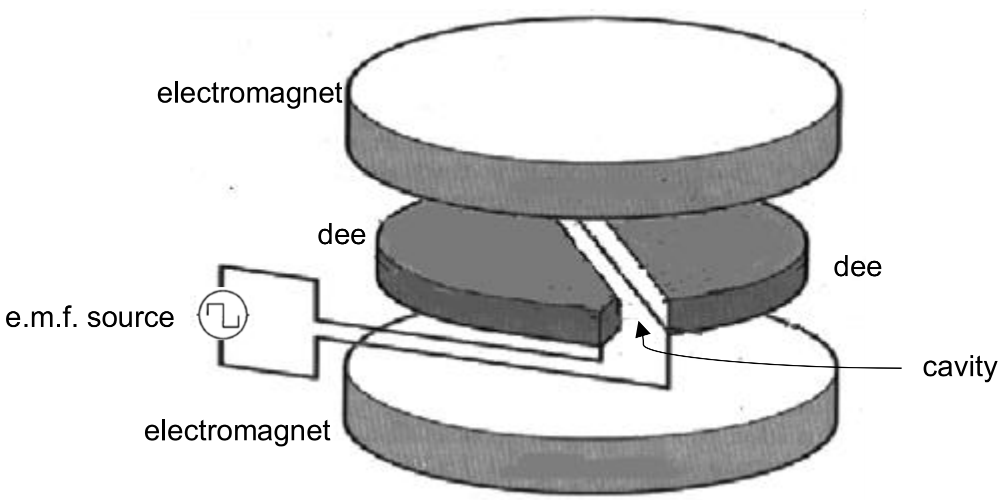


Fig. 8.2

Fig. 8.3 shows the plan view of the dees. A potential difference is applied across the dees to generate a uniform electric field as protons are injected at the edge of D2 at negligible initial speed. There is no electric field elsewhere.
The protons are accelerated to D1, in which it moves in a semi-circular path as indicated by the dashed line. The actual gap between the two dees is very small, so that the time taken for the proton to cross from one dee to another is negligible. The magnetic field also has negligible effect on the protons while they are moving between the dees.

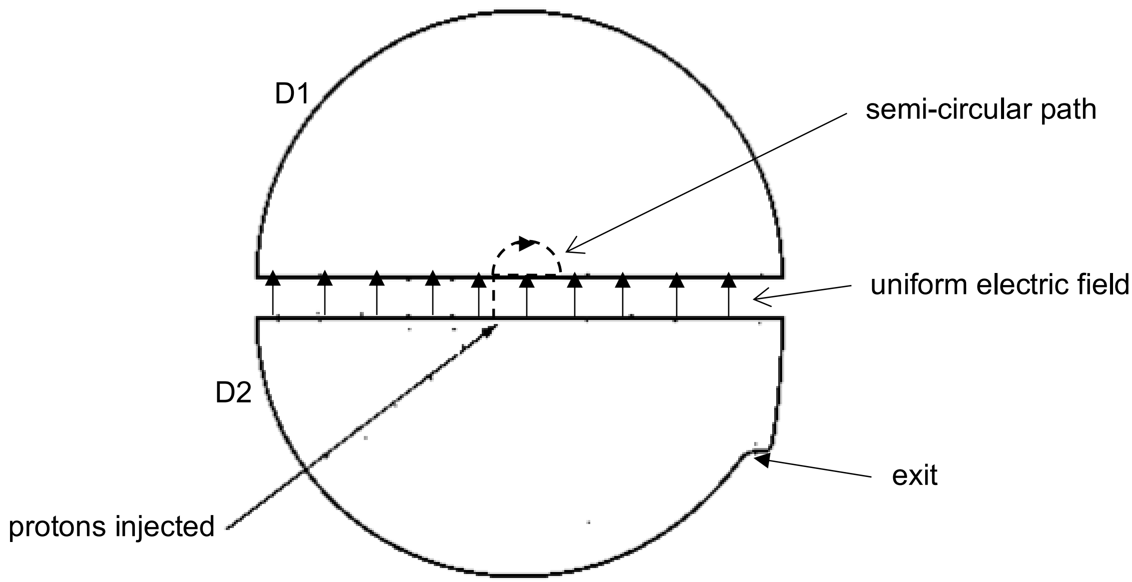


Fig 8.3 (plan view, not drawn to scale)

(i) Referring to Fig. 8.3, state the direction of the magnetic field.

.... [1]

(ii) Explain why the proton moves in the semi-circular path indicated.

....

....

.... [2]

(iii) Show that the radius of the semi-circular path of the proton, r, is

$$r = \frac{mv}{Be}$$

where m is the mass of proton,
v is the speed of proton,

B is the magnetic flux density,
e is the elementary charge

[2]

(iv) Hence, show that the time t to travel in D1 until it emerges into the gap is

$$t = \frac{\pi m}{Be}$$

[2]

(v) Hence, explain why the proton spends the same time in each dee.

....

....

.... [1]

(c) The e.m.f. source is set such that when the proton emerges from D1, the polarity of the applied potential difference changes while the magnitude remains the same. As a result, the emerging proton is accelerated towards D2. This process is repeated until the proton exits the cyclotron. A plausible path of a proton within the dees is depicted in Fig. 8.4.

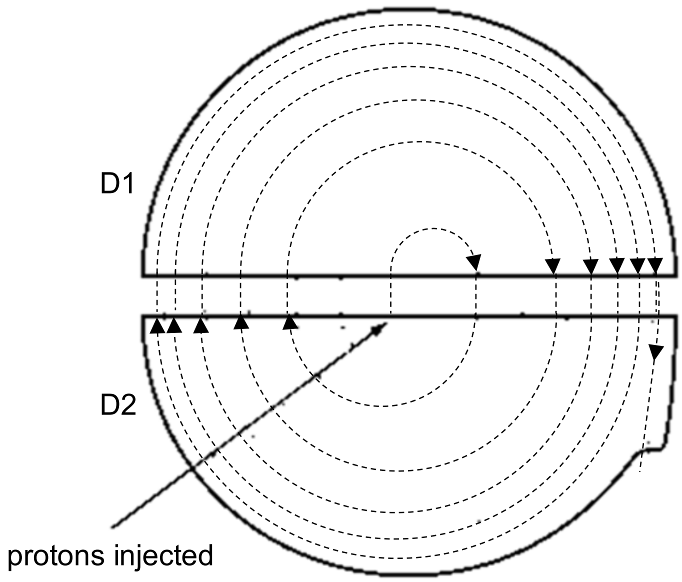


Fig. 8.4

(i) Sketch on Fig. 8.5, the variation of the speed of a proton with time as it goes through **four** semi-circles, starting from the moment it is injected.

Fig. 8.5

[3]

(ii) Show that the kinetic energy of the proton when it exits the cyclotron is

$$E = \frac{e^2B^2R^2}{2m}$$

where R is radius of the dees.

[3]

[Total: 20]