6 Read the following article and then answer the questions that follow.

Inertial Navigation Systems

An Inertial Measurement System (INS) makes use of inertial navigation, a self-contained navigation technique in which measurements provided by accelerometers and gyroscopes are used to track the position and orientation of an object relative to a known starting point, orientation and velocity. Each system contains gyroscopes and accelerometers, measuring angular velocity and linear acceleration respectively. By processing signals from these sensors it is possible to track the position and orientation of a moving vehicle.

Inertial navigation is used in a wide range of applications including the navigation of aircraft, tactical and strategic missiles, spacecraft, submarines and ships.

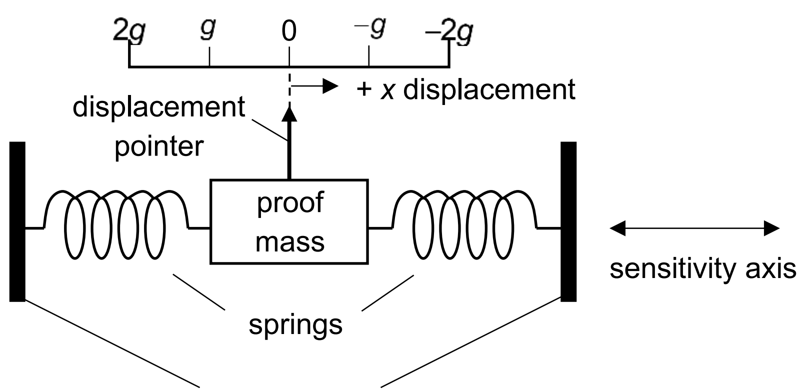

Fig. 6.1 (not to scale)

A linear accelerometer provides acceleration data along one axis. An accelerometer can be thought of as a “proof mass” suspended between two fixed sides with springs, as shown in Fig 6.1. The "proof mass" is allowed to move along what is known as the sensitivity axis. The displacement of the proof mass is measured using a displacement pick-off, giving a signal that is proportional to the force *F* acting on the mass in the direction of the input axis. Newton’s Second Law is then used to calculate the acceleration acting on the vehicle along the sensitivity axis.

Fig. 6.2 shows the algorithm for generating the change in velocity and change in position of the moving vehicle through “double integration” of the accelerometer data with respect to time.

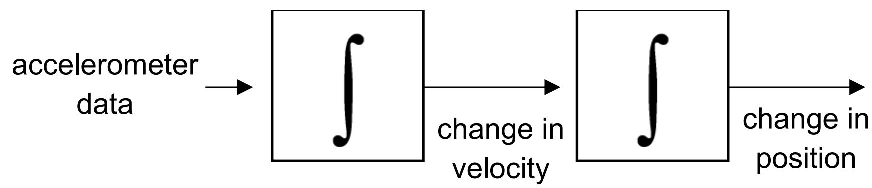

Fig. 6.2

The accelerometer data therefore is unable to give the velocity or position unless provided with both the initial velocity and initial position of the moving vehicle.

(a) **(i)** State Newton's First Law.

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(ii) Hence, explain why the initial values of velocity and position are required to prevent errors in determining the position of a moving vehicle equipped with INS.

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(b) **(i)** State Newton's Second Law.

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(ii) Suggest the number of accelerometers installed in the INS found in aircrafts and explain your answer.

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The Allan Variance is a method of analysing a sequence of data that is captured at regular time intervals, averaged across a duration of time. It is a measure of the stability of a time-varying signal due to noise and errors.

A linear accelerometer is made stationary so that there is no drift in position and velocity in real life. Acceleration data from an accelerometer, a_i , is repeatedly recorded at a frequency known as the *sampling rate*. The variance measures how far each individual data point is from the average value, squared:

$$\text{Allan Variance} = \frac{\sum_{i=1}^{n}\left(a_i - \langle a \rangle\right)^2}{n}$$

However, the aim of deploying an accelerometer in an INS is to measure change in velocity and change in position of a moving vehicle, so having a stationary accelerometer defeats the purpose.

In characterizing the noise in the output data from an accelerometer, it is desirable to seek the minimal time-window for which raw data should be averaged, so that the variance of the averaged data is “good enough”. Fig. 6.3. illustrates the process.

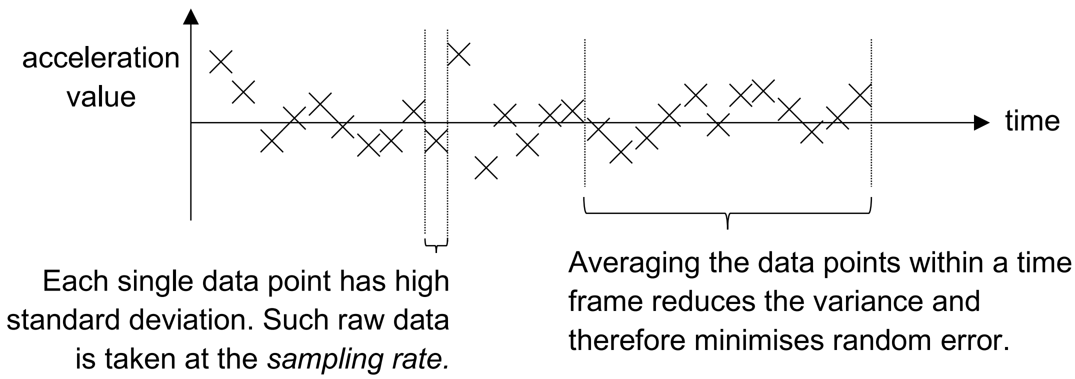

Fig. 6.3

To obtain the Allan Deviation plot, the variation of square-root of variance ($\text{deviation} = \sqrt{\text{variance}}$) is taken against the averaging time. For accelerometers, the two important error processes are:

- **White Noise**. White noise appears on a Allan Deviation plot as a slope with negatively sloping straight-line gradient. These are mainly random errors, which increasingly smoothed out by averaging the data points across longer time-intervals.

- **Bias Stability**. Bias stability appears on a Allan Deviation plot as a flat region around the minimum. This represents the base case scenario for generating accelerometer data and minimising white noise through the selection of appropriate time-periods for averaging.

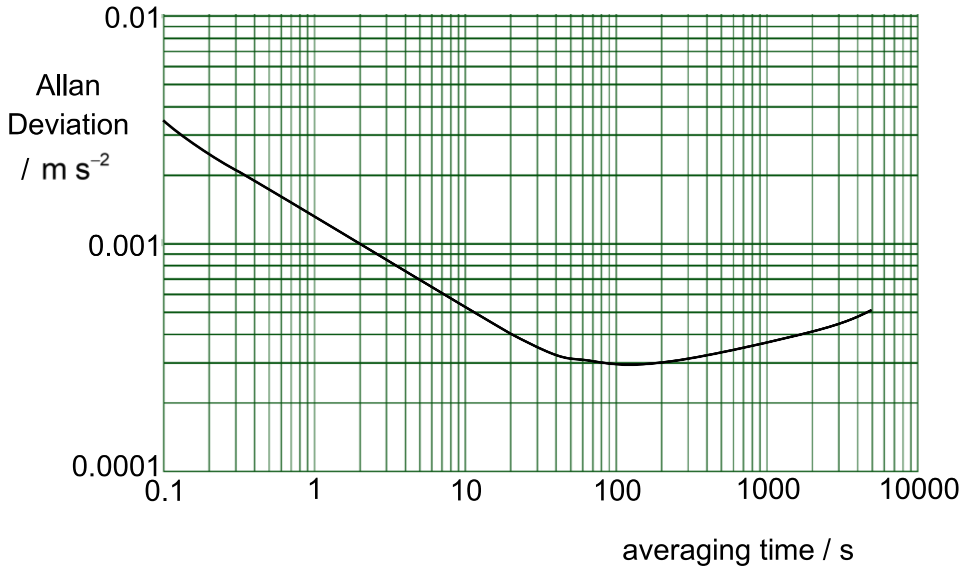

Fig. 6.4

(c) **(i)** Describe the advantage of representing the variation of a quantity on a logarithmic scale over a linear scale.

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(ii) Using Fig. 6.4,

1. deduce the sampling rate,

sampling rate = ………………………………… Hz [1]

2. state the Allen deviation associated with a single data point.

deviation = ………………………………… m s^{-2} [1]

(iii) It is suggested that the Allan Deviation for white noise decreases with an inverse square-root relationship with the average time. This suggestion is intended for averaging times of between 1 s to 10 s.

Using Fig. 6.4, verify the suggested relationship.

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(iv) **1.** Suggest a suitable time duration over which the accelerometer output should be averaged for a least-noise scenario.

time duration = ………………………………… s [1]

2. State the deviation associated with the least-noise scenario.

deviation = ………………………………… ms^{-2} [1]

3. Determine the drift in position data if the stationary INS is operated under least-noise scenario for an hour.

drift = ………………………………… m [2]

An INS is very important to a submarine because Global Position System (GPS) signals cannot reach underwater. A submarine will rely on absolute positioning systems, like the GPS, to obtain initial position and velocity data before submerging, where tracking provided by an INS then will take over.

The U.S. Navy is currently developing a system known as the Positioning System for Deep Ocean Navigation (POSYDON). The operating principles of POSYDON resembles that of GPS via a network of stationary beacons . A submarine will receive multiple signals which allow it to triangulate its position underwater. Unlike GPS which uses electromagnetic radiation, POSYDON will use acoustic signals.

(d) **(i)** Fig. 6.6 shows a scale diagram of 2 beacons on the surface of water which are placed 900 m away from each other. They simultaneously emit an acoustic signal at t = 0 s. Fig. 6.5 shows the times at which the submarine receives the acoustic signals.

	signal from beacon 1	signal from beacon 2
time of receipt	0.324 s	0.346 s

Fig. 6.5

The average speed of sound in water is $1500\ \mathrm{ms^{-1}}$.

By means of a scale drawing, mark on Fig. 6.6 the location of a submarine and state the actual depth of the submarine.

Fig. 6.6

depth = ……………………………….. m [2]

(ii) A network of POSYDON beacons will saturate the worldwide underwater eco-habitat with sound waves. Suggest an environmental concern as a result of the sound waves.

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[Total: 17]

Section B

Answer **one** question from this section.