8 **(a)** Define simple harmonic motion.

....

....

....

.... [2]

(b) Fig. 8.1 shows a U-tube containing a liquid that was initially at rest. The U-tube has a uniform cross-sectional area A. The density of the liquid is ρ and the total length of the liquid is L.

Due to a small disturbance, the liquid level on the right limb is displaced upwards by a distance x as shown in Fig. 8.2. The difference in the liquid levels between the two limbs will produce a pressure difference and this in turn will cause a restoring force to act on the length of liquid to accelerate it back towards its equilibrium position.

(i) In terms of A, L and ρ, write down the expression for the total mass M of the liquid.

[1]

(ii) Derive the expression for the magnitude of the restoring force F acting on the liquid in terms of ρ, g and x and A.

[2]

(iii) Using the relationship

$$F = Ma$$

where a is the acceleration of the liquid, and your answers in parts **(i)** and **(ii)**, show that the liquid will oscillate with simple harmonic motion after it is disturbed from its equilibrium state. You may assume that viscous forces in the liquid are negligible.

[3]

(iv) Hence, show that the angular frequency of the oscillations is $\omega = \sqrt{\frac{2g}{L}}$.

[1]

(c) The liquid shown in Fig. 8.2 has a length L of 0.92 m, mass M of 1.2 kg and an initial displacement of 0.080 m.

(i) Calculate the angular frequency ω.

ω = ... rad s^{-1} [1]

(ii) Determine the period T of the oscillations.

T = ... s [1]

(iii) Determine the maximum kinetic energy $E_{K,max}$ of the oscillating liquid.

$E_{K,max}$ = ... J [2]

(d) Viscous forces in the liquid in **(c)** are assumed to be negligible. In practice, there will be damping due to small viscous forces in the liquid.

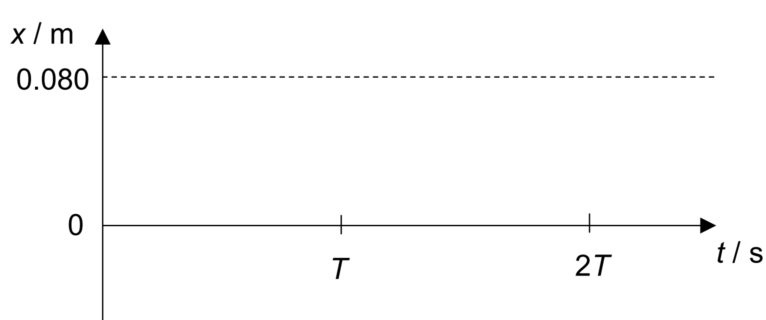
(i) Explain what is meant by damping.

....

.... [1]

(ii) On the axes in Fig. 8.3, sketch the variation with time t of the displacement x of the liquid in **(c)** for the first two cycles of oscillations.

Assume that the period of oscillations is unaffected by the viscous forces.



[2]

(iii) On the axes in Fig. 8.4, sketch the corresponding variation with time t of the kinetic energy of the liquid for the first two cycles of oscillation.

You need not label any numerical values on your graph.

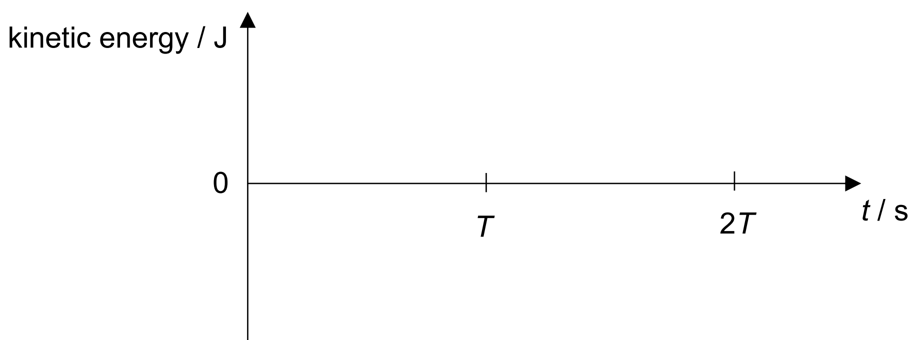


Fig. 8.4

[2]

(e) Damping is important in some applications. State one such application and explain why damping is important.

....

....

....

.... [2]

[Total: 20]