8 In 1798, Henry Cavendish performed an experiment to determine a value for the average density of the Earth.

The experiment can also be used to determine G, the gravitational constant.

Cavendish carried out the measurements using the method and torsion balance apparatus devised by John Michell in 1783.

Fig. 8.1 shows the torsion balance in its equilibrium position.

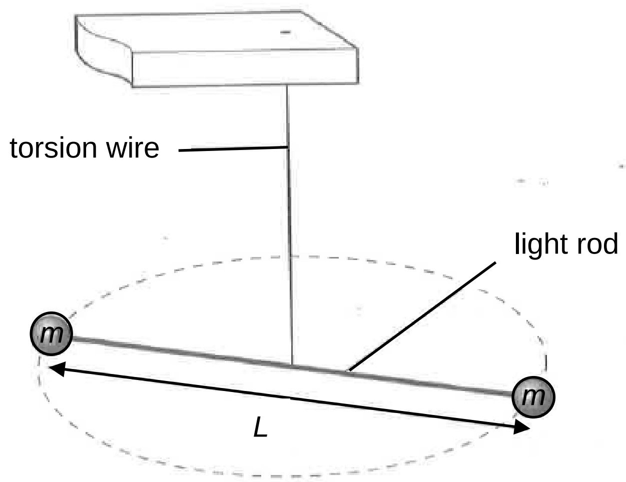

Fig. 8.1 (not to scale)

A stiff torsion wire was used to suspend a light horizontal rod from its midpoint.

Small lead spheres of mass m = 0.730 kg and diameter d = 50 mm were attached to the ends of the light rod.

The centre-to-centre separation of the spheres was L = 1.80 m.

(a) State what is meant by a *gravitational field*.

……

…… [1]

(b) **(i)** Determine the density ρ of lead.

ρ = …………… kg m^{-3} [2]

(ii) The rod was turned by a small angle θ from its equilibrium position in a horizontal plane, as shown in Fig. 8.2.

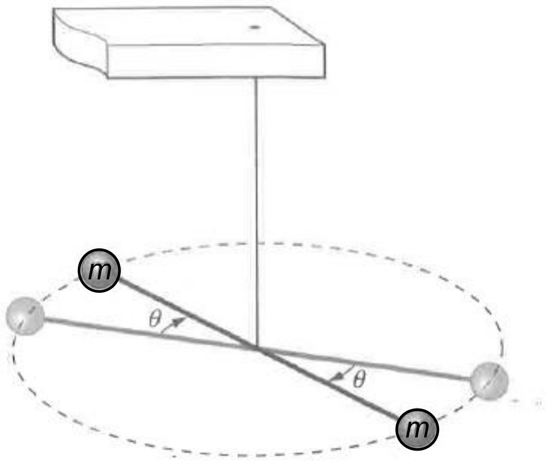

Fig. 8.2 (not to scale)

When released, the rod oscillated with simple harmonic motion about the equilibrium position with a period T.

The torque τ exerted on the rod by the torsion wire was given by the equation:

$$\tau = \frac{2\pi^2 m L^2 \theta}{T^2}$$

In an experiment conducted by Cavendish, the pendulum has a period of oscillation of T = 14.0 min when τ = 1.2 x 10^{-5} N m.

Determine the corresponding value of θ.

θ = …………… ° [3]

(iii) The rod was returned to its equilibrium position.

Next, Cavendish placed large lead spheres of mass M = 158 kg and diameter D = 30.0 cm near the small lead spheres.

A gravitational attraction force F between each pair of spheres caused the rod to rotate through an angle θ_1 as shown in Fig. 8.3.

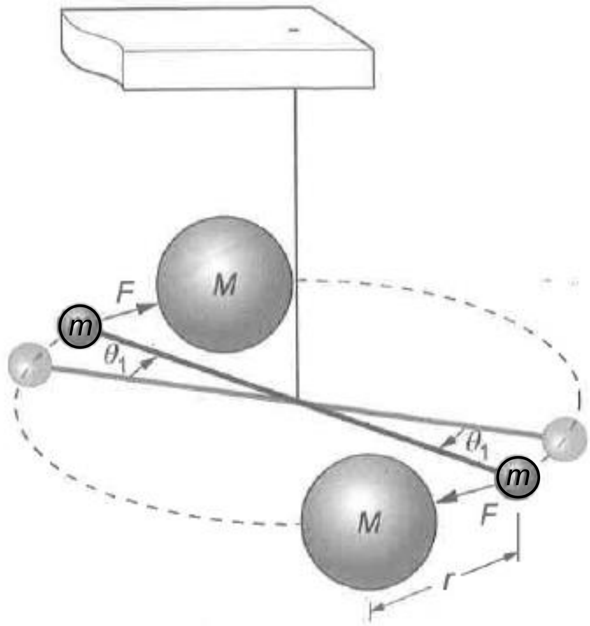

Fig. 8.3 (not to scale)

At this angle, the torque caused by the gravitational attraction was equal to the opposing torque caused by the torsion in the wire.

The centre-to-centre separation of one of the large lead spheres and the small lead sphere next to it was r.

The air gap between each large lead sphere and the small lead sphere next to it was 1.0 mm.

Show that r = 17.6 cm.

[2]

(iv) The angle of rotation θ_1 was too small to be measured directly.

Using a vernier scale, Cavendish was able to determine the displacement of the smaller lead spheres to be 4.1 mm ± 0.1 mm.

The centre-to-centre separation of the two small lead spheres, *L* was measured to be (1.80 ± 0.01) m.

Determine the angle θ_1 and its corresponding uncertainty.

θ_1 = …………… ± …………… rad [3]

(v) The large lead spheres were then removed, and the system oscillated with simple harmonic motion as before.

Show that the gravitational constant *G* is related to the period *T* by the relationship:

$$G = \frac{2\pi^2 L r^2 \theta_1}{M T^2}$$

[2]

(c) During a lecture, a professor performs a modern version of the Cavendish experiment. The professor uses the same torsion balance method but adds a mirror, laser and screen to measure the rotation.

Fig. 8.4 shows the torsion balance in its equilibrium position.

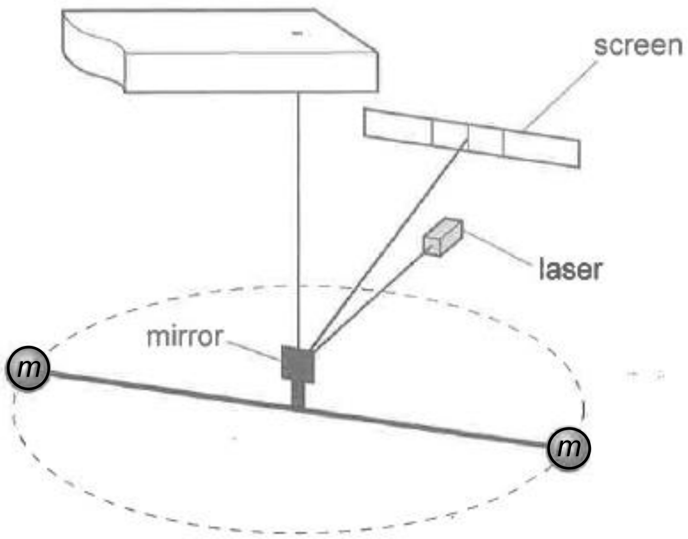

Fig. 8.4 (not to scale)

When the professor brings the large lead spheres near to the small lead spheres, the light rod rotates and the laser beam is deflected, as shown in Fig. 8.5.

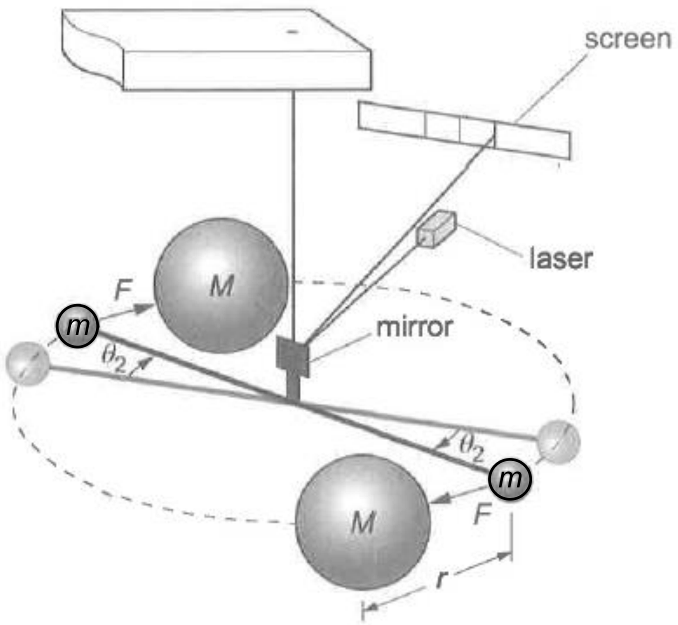

Fig. 8.5 (not to scale)

(i) When the rod rotates by an angle of θ_2, the laser beam deflects by an angle of $2\theta_2$ since the angle of incidence of the laser beam is equal to its angle of reflection.

The mirror is a distance 12.00 m from the screen. The professor measures the deflection of the laser beam on the screen as 15.6 cm.

Show that the corresponding angle of rotation θ_2 = 0.372 °.

[1]

(ii) The professor removes the large lead spheres and simultaneously starts a timer.

The professor records a time of 10 minutes 22 seconds for the laser beam to move from the maximum deflection, through the equilibrium position to the opposite maximum deflection, and back to the equilibrium position again.

Use this measurement and the expression in **(b)(v)** to determine a value and the S.I. base unit for the gravitational constant *G*.

G = ………………………… [3]

S.I. base unit = ………………………… [1]

(iii) The professor switches off the laser but leaves the system oscillating and the timer running overnight.

The next morning, the professor switches the laser back on and records the time when the laser beam next passes through the equilibrium position, traveling from the right, as *t* = 13 hours 48 minutes 18 seconds.

By considering uncertainty, explain qualitatively why using this value of *t* will lead to a more accurate determination of the value for the period *T*.

....

.... [1]

(iv) The professor calculates a new value for *G* using the value for *T* from **(b)(iii)**.

This new value for *G* is slightly lower than the accepted value of *G*.

Suggest the main reason for this difference.

....

.... [1]

[Total: 20]