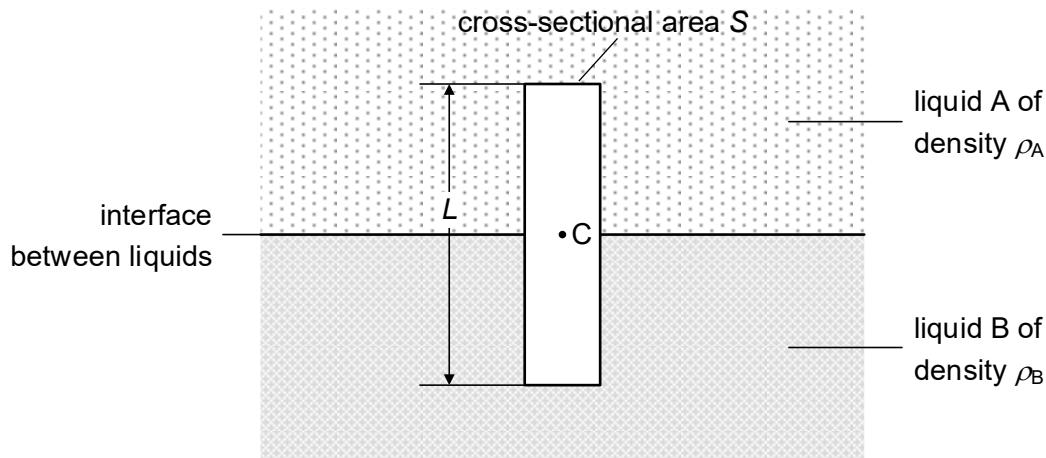


- 8 (a)** A rectangular block of length  $L$  and cross-sectional area  $S$  has an average density  $\rho$ . The block is immersed into a mixture of two liquids A and B of densities  $\rho_A$  and  $\rho_B$  respectively. The block floats vertically such that the midpoint C of the block is at the interface of the two liquids as shown in Fig. 8.1.



**Fig. 8.1**

- (i)** Show that the average density  $\rho$  of the block is

$$\rho = \frac{1}{2} \rho_A + \frac{1}{2} \rho_B.$$

[1]

- (ii)** The upthrust due to liquids A and B are  $U_A$  and  $U_B$ , respectively. State and explain whether  $U_A$  or  $U_B$  is larger.

.....

.....

.....

[2]

- (b) Due to some disturbance in the liquids, the block is now rotated slightly about its midpoint C as shown in Fig. 8.2. The block will return to its original vertical orientation.

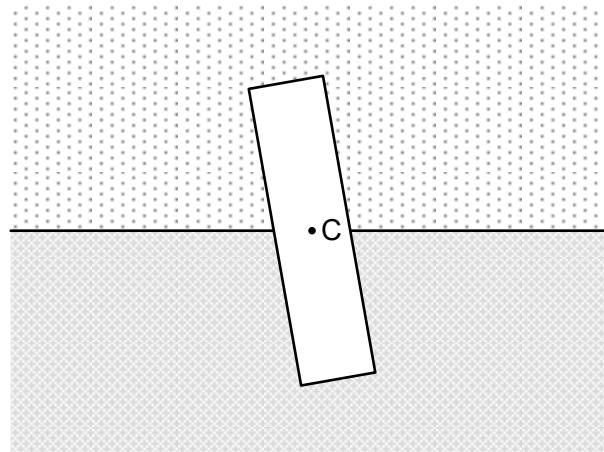


Fig. 8.2

- (i) Indicate in Fig. 8.2, the upthrust  $U_A$  due to liquid A and  $U_B$  due to liquid B. [2]
- (ii) Hence by considering the moment of force about C, deduce whether the centre of gravity of the block is above or below C. Explain your reasoning.

.....

.....

.....

.....

.....

.....

[2]

- (c) The block is now given a small displacement  $x$  upwards as shown in Fig. 8.2. The block will undergo simple harmonic motion when it is released.

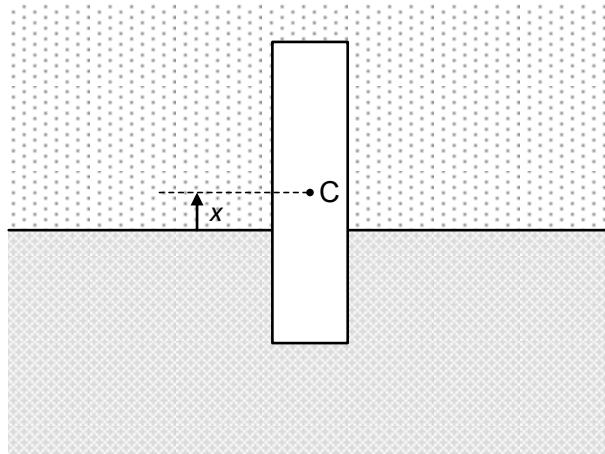


Fig. 8.2

(i) Deduce an expression, in terms of  $S$ ,  $L$ ,  $x$ ,  $\rho_A$ ,  $\rho_B$  and  $g$ , for

1. the upthrust in liquid A,

[1]

2. the upthrust in liquid B.

[1]

- (ii) Starting with Newton's second law, show that the acceleration  $a$  of the block when it is released is given by

$$a = -\frac{2(\rho_B - \rho_A)g}{(\rho_B + \rho_A)L} \cdot x$$

Assume that all viscous forces are negligible and take upwards as positive.

[3]

- (iii) Hence determine the period of oscillation, in terms of  $g$ ,  $L$ ,  $\rho_A$  and  $\rho_B$ .

[2]

- (d) While the block is oscillating, it was found that the speed of the block when the midpoint C crosses the interface between the liquids is  $0.40 \text{ m s}^{-1}$ .  
The length of the block is  $0.25 \text{ m}$  and the densities of liquids A and B are  $860 \text{ kg m}^{-3}$  and  $1300 \text{ kg m}^{-3}$ , respectively.

- (i) Determine the amplitude  $x_0$  of the oscillations.

$$x_0 = \dots \text{ m} \quad [2]$$

- (ii) Calculate the period  $T$  of oscillations.

$$T = \dots \text{ s} \quad [1]$$

- (iii) Determine the time taken for the midpoint C to travel from  $x = x_0$  to  $x = \frac{1}{2}x_0$ .

time taken = ..... s [3]

**End of Paper**