

- 8 (a) A rectangular block of length L and cross-sectional area S has an average density ρ . The block is immersed into a mixture of two liquids A and B of densities ρ_A and ρ_B respectively. The block floats vertically such that the midpoint C of the block is at the interface of the two liquids as shown in Fig. 8.1.

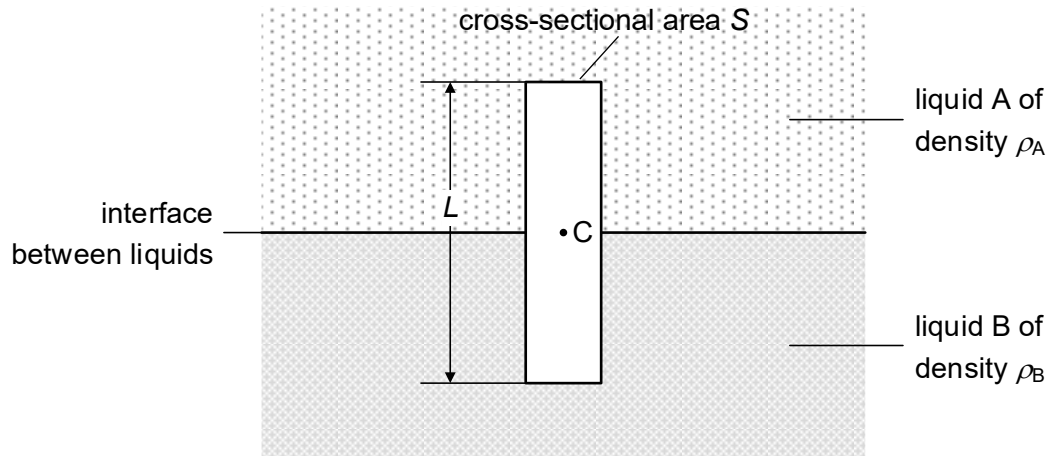


Fig. 8.1

- (i) Show that the average density ρ of the block is

$$\rho = \frac{1}{2}\rho_A + \frac{1}{2}\rho_B.$$

[1]

- (ii) The upthrust due to liquids A and B are U_A and U_B , respectively. State and explain whether U_A or U_B is larger.

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..... [2]

- (b) Due to some disturbance in the liquids, the block is now rotated slightly about its midpoint C as shown in Fig. 8.2. The block will return to its original vertical orientation.

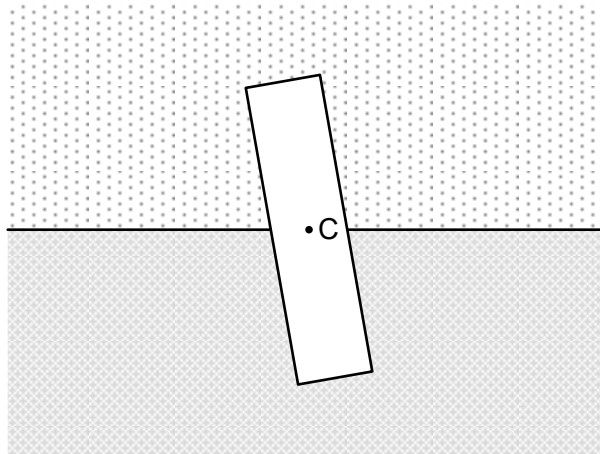


Fig. 8.2

- (i) Indicate in Fig. 8.2, the upthrust U_A due to liquid A and U_B due to liquid B.

[2]

- (ii) Hence by considering the moment of force about C, deduce whether the centre of gravity of the block is above or below C. Explain your reasoning.

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..... [2]

- (c) The block is now given a small displacement x upwards as shown in Fig. 8.2. The block will undergo simple harmonic motion when it is released.

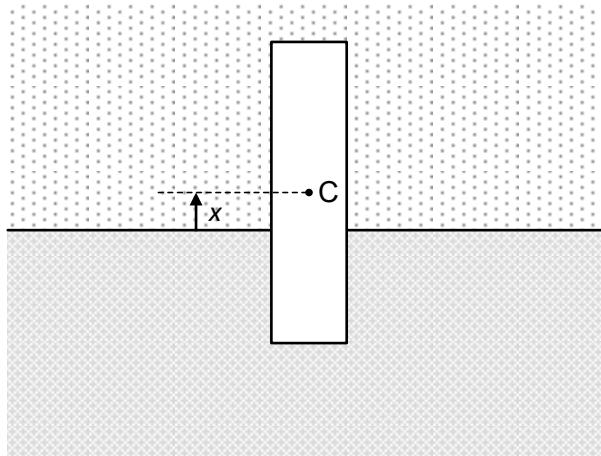


Fig. 8.2

- (i) Deduce an expression, in terms of S , L , x , ρ_A , ρ_B and g , for
1. the upthrust in liquid A,

[1]

2. the upthrust in liquid B.

[1]

- (ii) Starting with Newton's second law, show that the acceleration a of the block when it is released is given by

$$a = -\frac{2(\rho_B - \rho_A)g}{(\rho_B + \rho_A)L} \cdot x$$

Assume that all viscous forces are negligible and take upwards as positive.

[3]

- (iii) Hence determine the period of oscillation, in terms of g , L , ρ_A and ρ_B .

[2]

- (d) While the block is oscillating, it was found that the speed of the block when the midpoint C crosses the interface between the liquids is 0.40 m s^{-1} .
The length of the block is 0.25 m and the densities of liquids A and B are 860 kg m^{-3} and 1300 kg m^{-3} , respectively.

- (i) Determine the amplitude x_0 of the oscillations.

$$x_0 = \dots\dots\dots \text{ m} \quad [2]$$

- (ii) Calculate the period T of oscillations.

$$T = \dots\dots\dots \text{ s} \quad [1]$$

- (iii) Determine the time taken for the midpoint C to travel from $x = x_0$ to $x = \frac{1}{2}x_0$.

time taken = s [3]

End of Paper