3 **(a)** Two planets, A and B, have the same diameter but different masses. The masses of planets A and B are 5.07×10^{24} kg and 3.23×10^{24} kg respectively, and the distance between the centres of both planets is 3.85×10^{8} m.

(i) Determine the resultant gravitational field strength at the midpoint between planets A and B.

gravitational field strength = ... N kg^{-1} [2]

(ii) On Fig. 3.1 below, sketch the variation of the resultant gravitational potential Φ with the displacement along a straight line from the surface of planet A to the surface of planet B.

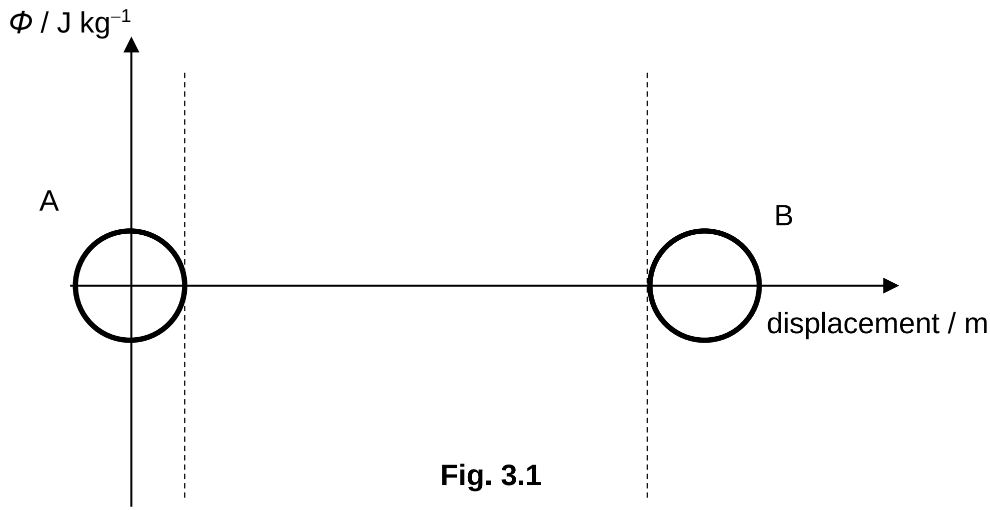

Fig. 3.1

[1]

(iii) On Fig. 3.2 below, sketch the variation of the resultant gravitational field strength g with the displacement along a straight line from the surface of planet A to the surface of planet B.

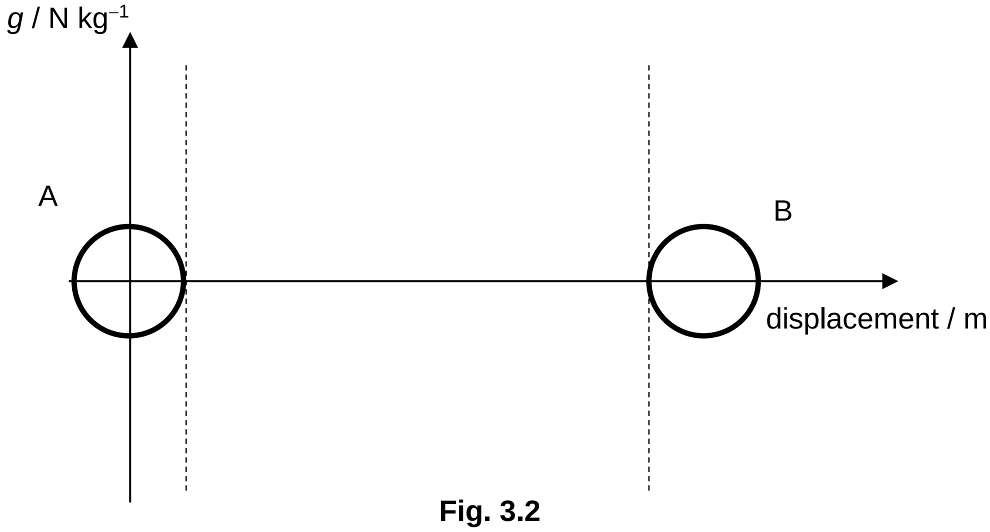

Fig. 3.2

[2]

(b) Fig. 3.3 shows a binary star system with stars C and D orbiting at different distances about a common point P with the same angular velocity in a circular path. The mass of star C, M_C, is twice of that of star D. The distances from point P to stars C and D are r_C and r_D respectively.

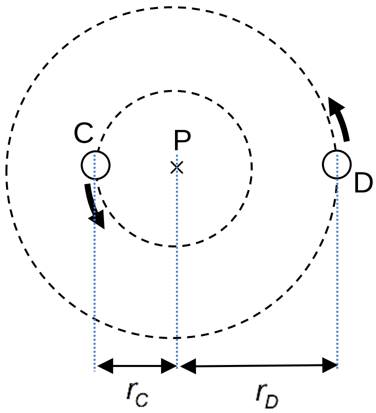

Fig. 3.3

(i) Show that $\frac{r_C}{r_D} = \frac{1}{2}$.

[1]

(ii) Show that the gravitational force F_G on star C is given by the expression

$$F_G = \frac{GM_C^{\,2}}{18r_C^{\,2}}$$

[1]

(iii) Given that the time taken for star C to complete one rotation of the circle is 3.84×10^9 s and the distance between the two stars is 7.20×10^{12} m, determine the mass of star C, M_C.

M_C = …………………………… kg [3]