8 Read the passage below and answer the questions that follow.

Modern train system

With the advancement in technology, the train system has greatly evolved over the years to meet the need for speed, ride comfort and lower maintenance cost.

One main consideration when designing the train system is for train vehicles to navigate curves at high speeds without derailing or toppling over. To achieve this, the outer side of the track of a railway may be raised. This is similar to the banking of roads for cars to turn around a bend at high speed without skidding.

Fig. 8.1 shows the cross section of a railway track that is tilted by an angle θ. The quantity cant E is used to indicate the amount of banking of the railway track. Cant is the difference in vertical height between the two rails of the track. Another important quantity is the rail gauge w which is the distance between the two rails.

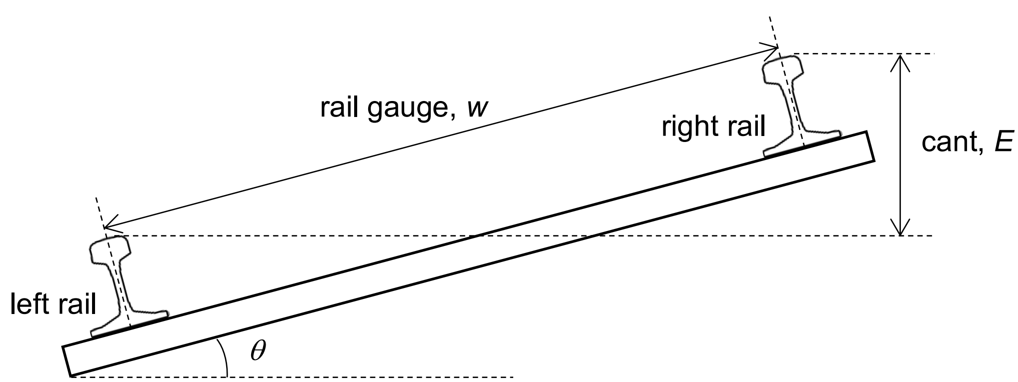

Fig. 8.1

When the train is on the track, the rails exert an effective contact force on the train.

For any banked track with cant E_0, the balance speed v_0 is the speed of the train such that this contact force is perpendicular to the bank.

However, trains do not always travel at the same speed. For example, compared to freight trains, passenger trains move at a higher speed and is usually higher than the balance speed.

At speeds above v_0, a larger cant E_v would be required to ensure that the contact force is perpendicular to the bank. The difference between the required cant E_v and actual cant E_0 is known as the cant deficiency, CD, where

$$CD = E_v - E_0$$

For example, a train moving on a track of cant E_0 = 120 mm has v_0 = 125 km h^{-1}.

When it moves at v = 140 km h^{-1} on the same track, the cant E_v required = 150 mm.

Hence,

$$CD = 150 \text{ mm} - 120 \text{ mm} = 30 \text{ mm}$$

In every railway design, there is a maximum allowable cant deficiency CD_{max} to ensure that trains do not derail and skid outwards.

The CD_{max} for common railway systems in the world are shown in Fig. 8.2.

type of railway	**rail gauge, *w* / mm**	**maximum allowable cant deficiency, CD_{max} / mm**

A	1435	110
B	1524	125
C	1668	135

Fig. 8.2

(a) A train is turning on a banked track at balance speed v_o.

(i) Explain why the train has an acceleration despite moving at a constant speed.

....

.... [1]

(ii) With reference to Fig. 8.1, show that the angle θ is related to the cant E and rail gauge w by the expression

$$\tan\theta = \frac{E}{\sqrt{\left(w^2 - E^2\right)}}$$

[1]

(iii) Show that the balance speed v_0 for the train is

$$v_0 = \left(\frac{Erg}{\sqrt{w^2 - E^2}} \right)^{\frac{1}{2}}$$

where r is the radius of curvature of the track and g is the acceleration due to gravity.

[2]

(b) A passenger train that runs on a type A railway system travels around a bend with a cant of 95 mm and a radius of curvature of 1500 m.

(i) Suggest why passenger trains travel at a higher speed compared to freight trains.

....

.... [1]

(ii) Using Fig. 8.2 and the equation in **(a)(iii)**, determine the maximum speed that the passenger train can travel.

maximum speed = ... m s^{-1} [3]

(iii) A luggage bag of mass 19.6 kg sits on the train carriage when it is turning at 150 km h^{-1}. Fig. 8.3 shows the frictional force between the luggage bag and the carriage floor.

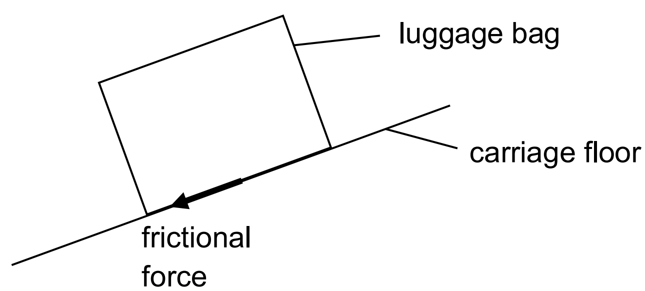

Fig. 8.3

1. On Fig. 8.3, draw and label arrows to show the other forces acting on the luggage bag.

[1]

2. Show that the centripetal force acting on the luggage bag is 23 N.

[1]

3. Show that the carriage floor is tilted at an angle of 3.8° with the horizontal.

[1]

4. The maximum friction between the bag and the carriage floor is 20 N.

   Determine whether the luggage will slide when the train is turning.

[4]

(c) Another safety consideration for a train going around a bend is to ensure that both train wheels are constantly in contact with the rails. Otherwise, the train might derail.

As the wheels on each side of the train are connected by an axle, they always rotate at the same angular speed when the train is moving. When the train goes around a bend, the outer rail traces a slightly longer arc length than the inner rail, as shown in Fig. 8.4.

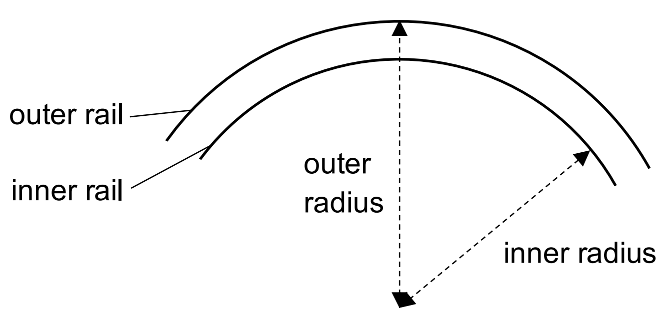

Fig. 8.4

This creates a problem as the outer wheel needs to cover more distance than the inner wheel. To overcome this problem, train wheels are designed to be slightly conical. When the train turns, it will move outward such that the effective diameter of the outer wheel becomes D_o and the effective diameter of the inner wheel becomes D_i where $D_o > D_i$ as shown in Fig. 8.5.

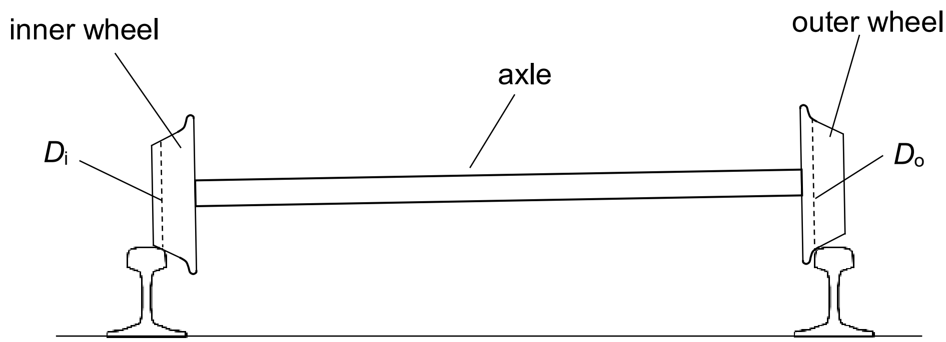

Fig. 8.5

(i) Explain how the conical shape of the wheels allows trains to stay in contact with the rails when going around a bend.

....

.... [1]

(ii) On a particular bend along a type B railway on level ground, the inner rail has a radius of 200 m and D_i is 1150 mm.

Determine D_o for the train to stay on track while turning.

D_o = ... m [2]

(d) Generally, train systems can run on ground level, elevated or underground.

Suggest one advantage of a train system that runs

(i) on ground level

....

[1]

(ii) underground.

[1]