4 A test-tube is partially loaded with small ball bearings such that it is able to float upright in water of density ρ as shown in Fig 4.1. The bottom of the test-tube is a distance H below the water surface.

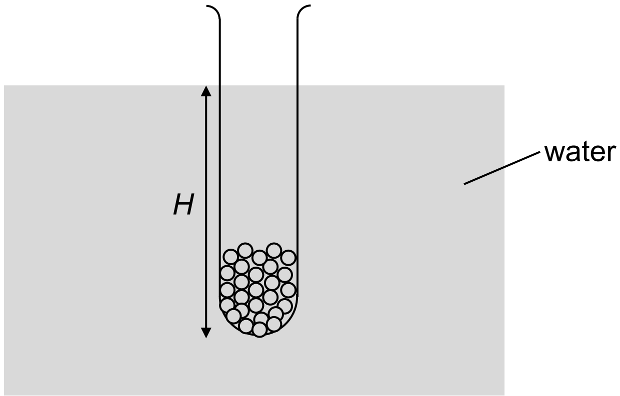

Fig 4.1

Ignoring its rounded bottom, the test-tube may be regarded as a cylinder of cross sectional area A and mass m. The mass of the ball bearings added is M.

(a) Derive an expression that relates H to A, ρ, M and m. [2]

(b) The test-tube is displaced vertically by displacement y and then released. Ignoring dissipative forces, and by considering the net force acting on the loaded test tube, show that the acceleration of the test-tube is given by

$$a = -\left(\frac{\rho A g}{M + m}\right) y$$

where g is the acceleration of free fall. [2]

(c) It is given that $\rho = 1.00 \times 10^3$ kg m^{-3}

$A = 6.0 \times 10^{-4}$ m^2

$M = 0.012$ kg

$m = 0.025$ kg

Show that the period of oscillation of the test-tube is 0.50 s. [3]

In practice, it is observed that the variation with time *t* of the vertical displacement *y* of the test-tube is as shown in Fig 4.2.

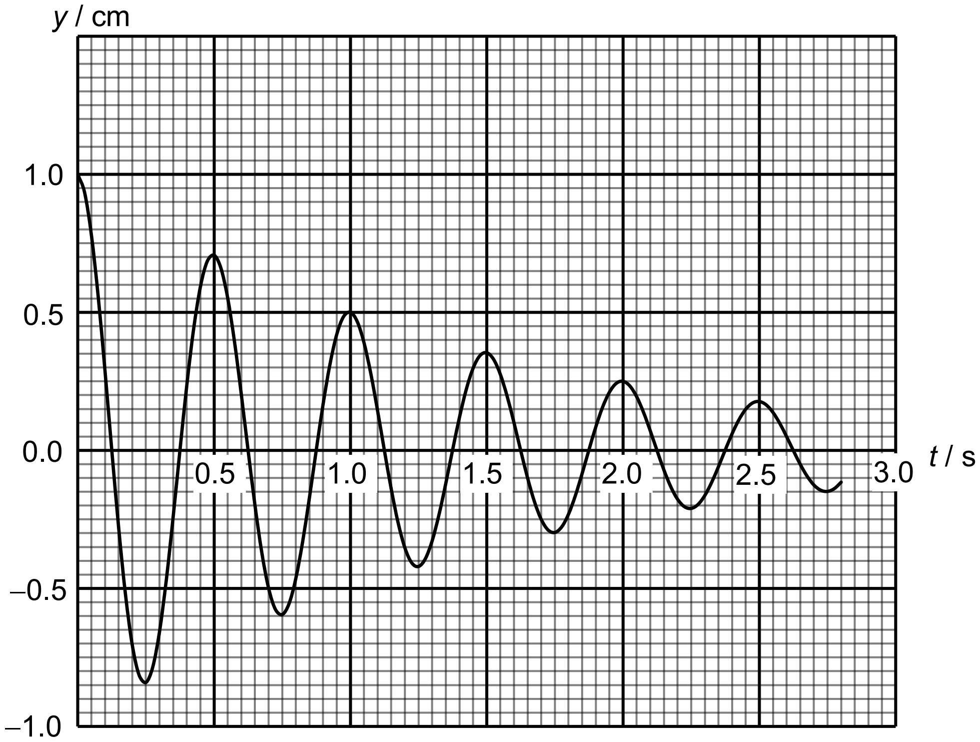

Fig. 4.2

(d) Explain why the amplitude of the oscillations decreases gradually over time. [1]

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(e) To sustain the oscillations of the test-tube, low-amplitude water waves of frequency 0.30 Hz are generated on the surface of the water.

(i) Sketch a graph to show the variation with time *t* of the vertical displacement *y* of the test-tube when it is oscillating steadily. Show appropriate numerical values on the time axis of the graph. [2]

(ii) It is observed that the amplitude of the vertical oscillations of this test-tube is rather small. Without changing the water waves, suggest with reasoning how the amplitude of the oscillations of this test-tube may be increased. [2]

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