7 (a) A dish is made from a section of a hollow glass sphere.

The dish, fixed to a horizontal table, contains a small solid ball of mass 45 g, as shown in Fig. 7.1.
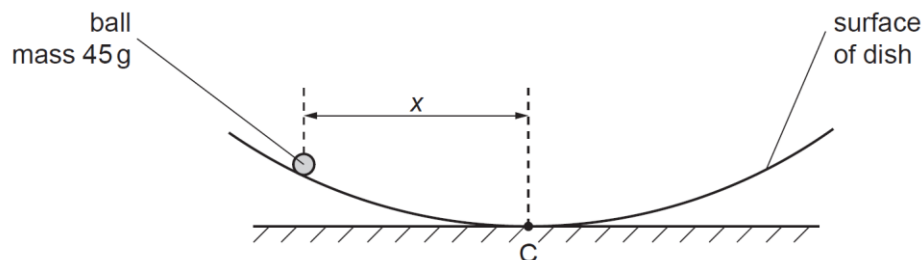

Fig. 7.1

The horizontal displacement of the ball from the centre C of the dish is x.

Initially, the ball is held at rest with distance $x = 3.0$ cm.

The ball is then released. The variation with time t of the horizontal displacement x of the ball from point C is shown in Fig. 7.2.
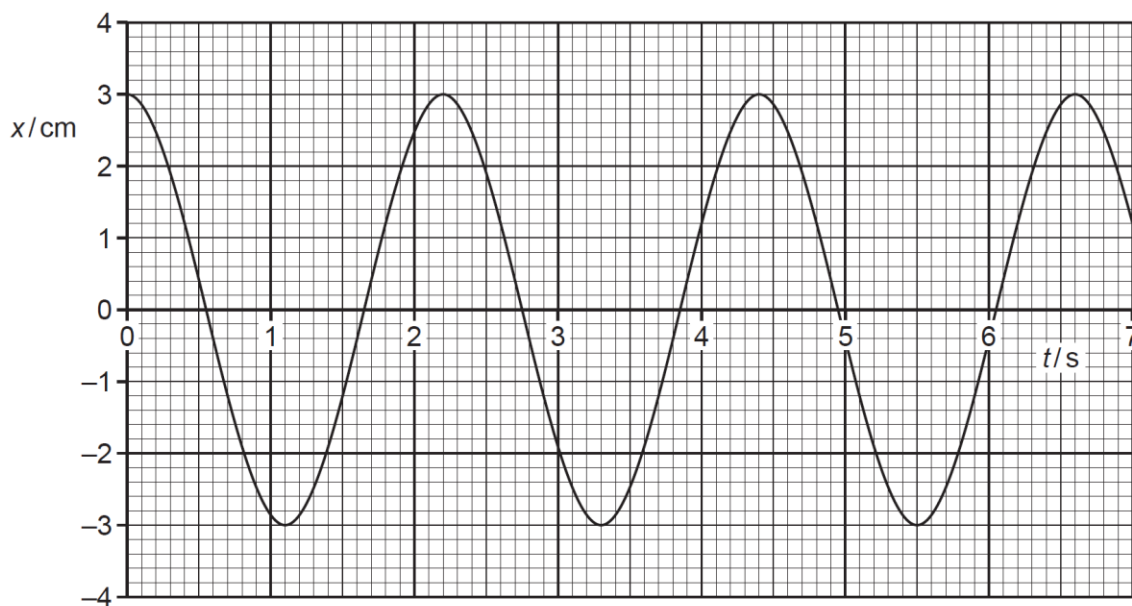

Fig. 7.2

The motion of the ball in the dish is simple harmonic with its acceleration a given by the expression

$$a = -\left(\frac{g}{R}\right)x$$

where g is the acceleration of free fall and R is a constant that depends on the dimensions of the dish and the ball.

(i) Explain how the expression $a = -\left(\dfrac{g}{R}\right)x$ suggests that the motion is simple harmonic?

....

....

....

....[2]

(ii) Use Fig. 7.2 to show that the angular frequency ω of the oscillation of the ball in the dish is 2.9 rad s⁻¹.

[1]

(iii) Determine R.

R = ... m [1]

(iv) Calculate the speed of the ball when x = 1.5 cm.

speed = ... m s^{-1} [2]

(v) Calculate the total energy of the oscillation.

total energy = ... J [3]

(vi) Determine the kinetic energy E_k of the ball in terms of time t.
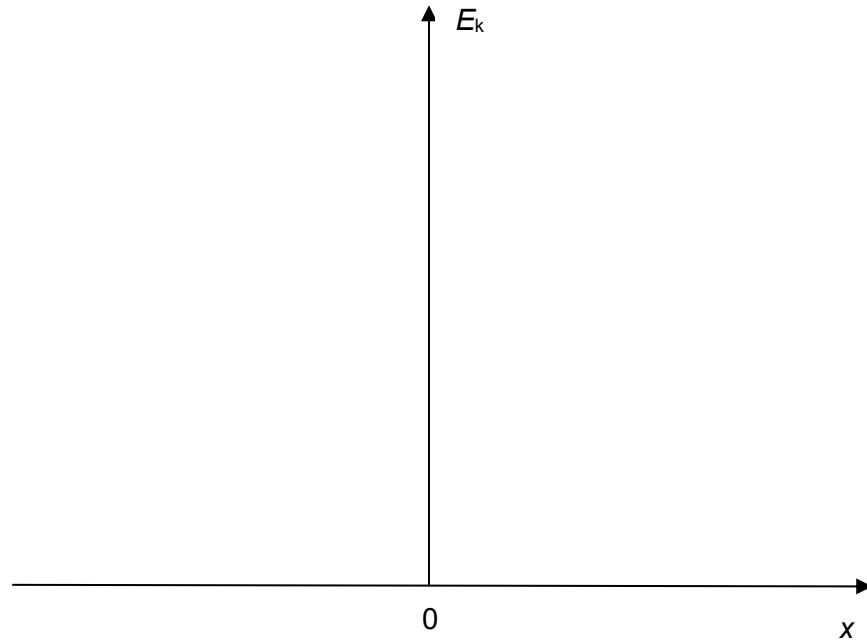
E_k = ... J [2]

(vii) Some moisture collects on the surface of the dish so that the motion of the ball becomes damped.

On the axes of Fig. 7.3, draw a graph to show variation of x with E_k of the damped motion of the ball for the first 2.2 s after the release of the ball.
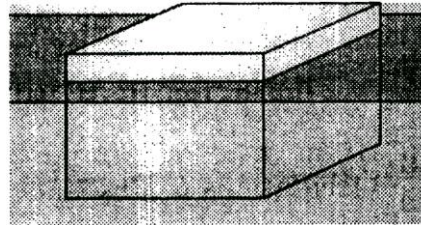
No numerical values are required.

[1]

Fig. 7.3

(b) A block of wood of mass m floats in still water, as shown in Fig. 7.4 below.

Fig. 7.4

When the block is pushed down into the water, without totally submerging it, and is then released, it bobs up and down in the water with a frequency f given by the expression

$$f = \frac{1}{2\pi} \sqrt{\frac{28}{m}}$$

where f is measured in Hz and m in kg.

Surface water waves of speed 0.90 m s^{-1} and wavelength 0.30 m are then incident on the block. These cause resonance in the up-and-down motion of the block.

(i) Calculate

    1. the frequency of the water waves,

frequency = …………….…………. Hz [1]

2. the mass of the block.

mass = …………….…………. kg [1]

(ii) Describe and explain what happens to the amplitude of the vertical oscillations of the block after the following changes are made independently:

1. water waves of larger amplitude are incident on the block,

....

....

....[2]

2. the distance between the wave crests increases,

....

....

....[2]

3. the block has absorbed some water.

....

....

....[2]

[Total: 20]