7 **Read the passage below and answer the questions that follow.**

A piece of material is made in the form of a slab of material of thickness Δx and of area *A*. One face is maintained at a temperature θ and the other at $\theta+\Delta\theta$. The heat *Q* that flows perpendicular to the faces for a time *t* is measured. The experiment is repeated with other slabs of the same material but with different values of Δx and *A*. The results of such experiments show that, for a given value of $\Delta\theta$, *Q* is proportional to the time *t* and the area *A*. Also, for a given time and area, *Q* is proportional to the ratio $\frac{\Delta\theta}{\Delta x}$. These results may be written as

$$\frac{Q}{t} \propto A\frac{\Delta\theta}{\Delta x} \text{ where } \Delta\theta = \theta_1 - \theta_2 \text{ and } \theta_1 > \theta_2,$$

The equation above can be written as

$$\frac{Q}{t} = kA\frac{\theta_1 - \theta_2}{\Delta x} \ldots\ldots (1)$$

where *k* is known as the thermal conductivity.

Equation (1) is only approximately true. A more accurate version of the equation can be written as

$$\frac{dQ}{dt} = -kA\frac{d\theta}{dx} \ldots\ldots (2)$$

Suppose the conducting material lies between an inner cylinder of radius r_1 and an outer cylinder of radius r_2 both of length *L*. If the inner cylinder is maintained at a constant temperature θ_1 and the outer at θ_2, there will be a steady radial flow of heat at a constant rate $\frac{dQ}{dt}$. It can then be shown that

$$\frac{dQ}{dt} = \frac{k2\pi L(\theta_1 - \theta_2)}{ln\left(\frac{r_2}{r_1}\right)} \ldots\ldots (3)$$

Equation (3) refers only to heat flowing through the walls of the cylinder.

(a) By referring to equation (1), deduce the base units of *k*.

base units of k : ………………….. [2]

(b) In equation (2), suggest a reason for the negative sign.

………………………………………………………………………………

……………………………………………………………………………. [1]

(c) Jane is a connoisseur of fine teas. She is preparing her new tea with boiling water in her cylindrical mug with inner radius r_1 = 4.0 cm and length L = 10.0 cm not including the thickness of the base. The thickness of the mug walls as well as of the base is Δx = 5.0 mm. The mug has a lid for better insulation. The lid is also made of the same material as that of the mug. Assume it is flat and has the same thickness as the mug walls.

Jane pours her tea, filling her mug completely, and immediately places the lid on her mug.

As heat flows from the tea, the change in its temperature ΔT given by

$$Q = Mc\Delta T \text{ ……(4)}$$

where M is the mass of the tea water.

The following data are available:

Temperature of boiling water: θ_{boil} = 100 °C.

Thermal conductivity of mug: k = 1.0 in S.I. base units.

Room temperature: θ_{room} = 25 °C.

Density of tea-water: ρ = 1000 kg m^{-3}.

Specific heat capacity of tea-water: c = 4180 J kg^{-1} K^{-1}.

(i) Calculate the mass M of the water in the mug.

M = …………… kg [2]

(ii) Calculate a value for the *total* internal surface area A of the mug with the lid on.

A = ……………… m^2 [2]

(iii) Show that $\Delta T = (0.10)(\theta_{tea} - \theta_{room})$ after 30 s have elapsed, assuming equation (1) holds.

[2]

(d) The formula in **(c)(iii)** is used to evaluate the temperatures required to fill in the table shown in **Fig. 7.1**.

Time after pouring t/s	**Initial** $\theta_{tea}/^{o}C$	**Temperature change in 30 s** ΔT/ °C	**Final** $\theta_{tea}/^{o}C$
0	100	7.5	92.5
30	92.5	6.8	85.7
60	85.7	6.1	79.6
90			
120	74.1	4.9	69.2
150	69.2	4.4	64.8
180	64.8	4.0	60.8
210	60.8	3.6	57.2
240	57.2	3.2	54.0
270	54.0	2.9	51.1

Fig. 7.1

(i) Fill in the missing values in **Fig. 7.1** for t = 90 s. [2]

A graph to show the variation with time of the temperature of the tea is now drawn and shown in **Fig. 7.2**.

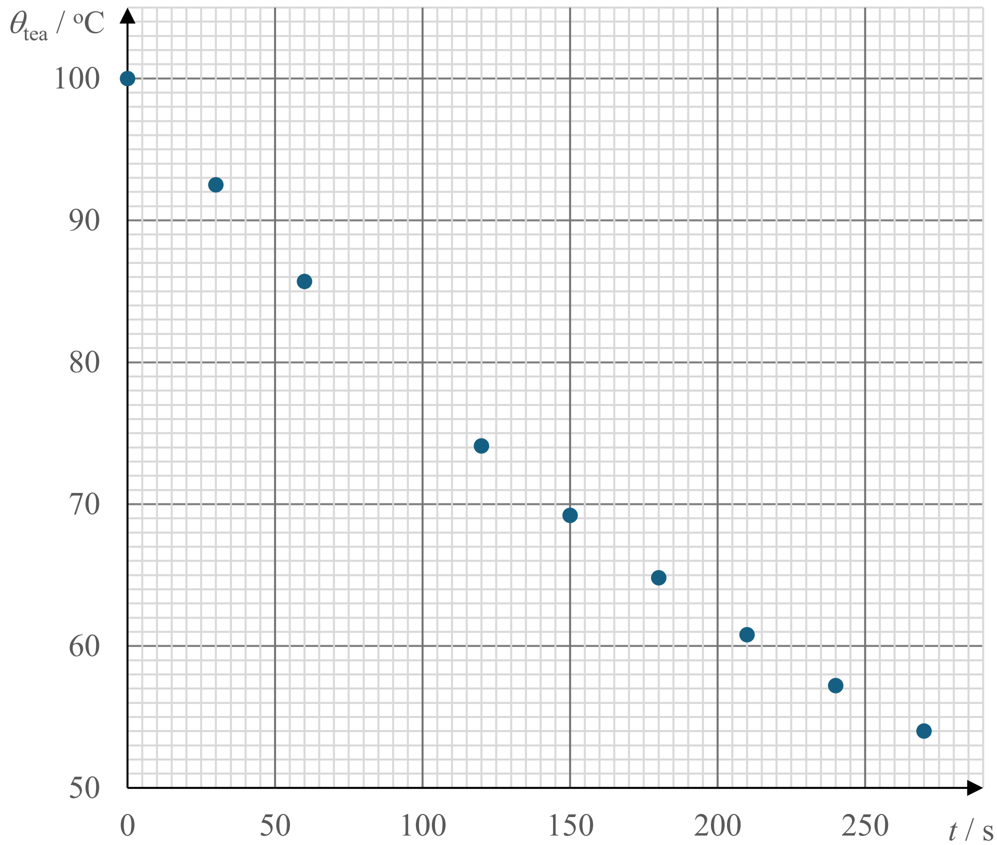

Fig. 7.2

(ii) Plot in the point corresponding to t = 90 s in **Fig. 7.1** and draw the best fit to the curve in **Fig. 7.2**. [2]

(iii) Determine the time taken for Jane's tea to drop from 100 °C to 55 °C.

t = ……….. s [1]

(e) **(i)** Calculate the rate of heat loss when the tea temperature is 90 ^{o}C using equation (1) *only for heat flowing through the walls of the container*.

Rate of heat loss = ……………….. W [2]

(ii) Calculate the rate of heat loss when the tea temperature is 90 ^{o}C using equation (3) *only for heat flowing through the walls of the container*.

Rate of heat loss = ……………….. W [1]

(iii) Hence calculate the percentage error incurred in using equation (1).

Percentage error = ………….. [1]

(f) Jane decides to reheat her tea from 55 ^{o}C to 70 ^{o}C with a small 1.1 kW heater she places into the tea. She wraps the whole mug with insulating material to prevent heat from escaping from the tea. Determine the time it takes for the tea to reach 70 ^{o}C. Assume all the energy supplied by the heater is absorbed by the water.

t = ……………… s [2]

********* END *********