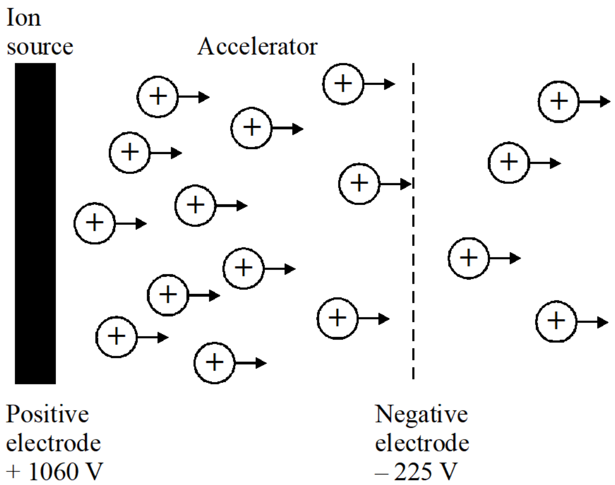
7. This question examines two ways to propel a spacecraft: using an ion engine and using a solar sail.

Using an ion engine: Deep Space 1

In 1998 NASA launched the probe called Deep Space 1. It was designed to test new technologies for future deep space and interplanetary missions. Once in orbit, this probe was the first to use an ion engine to propel it on its mission.

The diagram below simplifies the main features of the ion engine.



Atoms of xenon were ionised by the loss of a single electron and then accelerated until they were ejected out of the rear of the probe, providing the means of propulsion. The mass of a xenon ion is 2.2×10^{-25} kg. The positive and negative electrodes were operating at +1060 V and –225 V respectively.

The ion engine used only a very small amount of xenon at a time. It may take 4 days or more just to use up 1 kg of xenon. For its whole mission, Deep Space 1 used about 74 kg of xenon to accelerate to a speed of about 4.3 km s^{-1}. At that time, this was greater than any spacecraft had ever been able to change its speed. The ion engine thrusted for 678 days, far longer than any propulsion system had ever been operated.

Using a solar sail: LightSail 2

LightSail® is a crowdfunded project from The Planetary Society to demonstrate that solar sailing is a viable means of propulsion. The LightSail 2 spacecraft, launched on June 25, 2019, uses sunlight alone to change its orbit, and is currently operating under an extended mission to further advance solar sailing technology.

A solar sail, simply put, is a spacecraft propelled by sunlight. Whereas conventional rockets are propelled by the combustion of rocket fuel, a solar sail is pushed forward by light from the Sun. A solar sail does this by capturing the momentum of photons with sheets of large, reflective material such as Mylar.

The picture below shows the LightSail 2 spacecraft.

Size

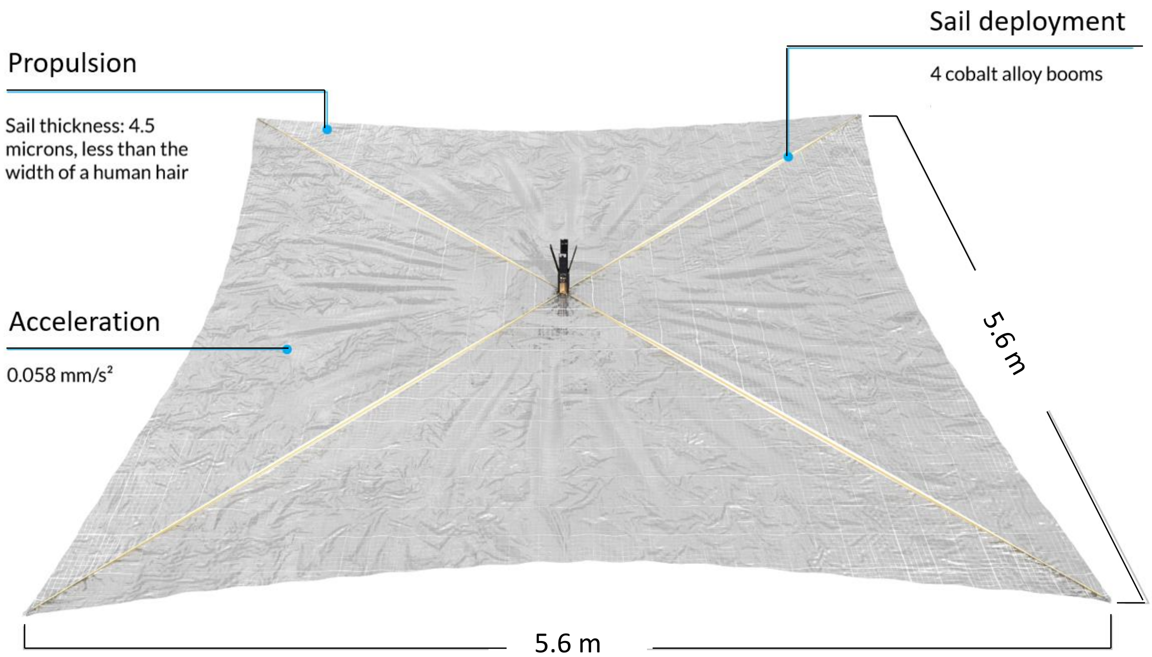
5.6 x 5.6 m

Boom length

4 m

Total sail area

$32\ m^2$



The sail is 4.5 μm thick and its total area is 32 m^2. The picture also suggests that the spacecraft experiences an acceleration of 0.058 mm s^{-2}.

LightSail 2 is currently orbiting Earth with an orbital radius of about 7020 km. The solar radiation flux (intensity) at that altitude is about 1400 W m^{-2}. The mass of Earth is 6.0×10^{24} kg.

(a) Show that the speed of a xenon ion after being accelerated is about 4.3×10^{4} m s^{-1}. [3]

(b)(i) Show that the maximum mass of xenon ejected by the engine per second is 2.9×10^{-6} kg. [2]

(ii) Calculate the maximum thrust (the force of propulsion) on Deep Space 1. [2]

(c) Assuming that the mass of Deep Space 1 remains constant, estimate its mass. [2]

(d) Simply firing xenon ions into space would leave Deep Space 1 negatively charged. Suggest a reason why this would lead to reduced thrust. [1]

(e) Chemical rockets eject their propellant at about a tenth of the velocity achieved by ion engines but produce much greater thrust by ejecting more than a thousand kilograms per second. Suggest why ion engines may be preferable for missions extending over long distances and periods of time. [2]

(f)(i) Show that the energy E and momentum p of a photon are related by the expression $E = pc$, where c is the speed of light. [2]

(ii) For LightSail 2, show that the total momentum of the photons striking 1 m^2 of sail in one second is 4.7×10^{-6} N s. Assume that the photons strike the sail at right angles. [2]

(iii) Hence, find the force exerted on the whole sail if it is completely reflective. [2]

(iv) Explain how this force in (iii) will change if the whole sail is completely non-reflective so that all of the incident light is absorbed. [2]

(g) Discuss whether the centripetal acceleration of LightSail 2 can be 0.058 mm s^{-2}. [2]

(h) Comparing the solar sail with the ion engine, suggest one advantage and one disadvantage of the solar sail. [2]

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