7 Read the following article then answer the questions that follow.

Compton Scattering

Besides the photoelectric effect, a number of other experiments were carried out in the early twentieth century which also supported the photon theory. One of these was the Compton effect named after its discoverer. Compton scattered X-rays from various materials. He found that the scattered light had a slightly longer wavelength than the incident light. He was able to explain this result based on the quantum theory of light. Light is seen as particles colliding with the electrons of the material (Fig. 7.1).

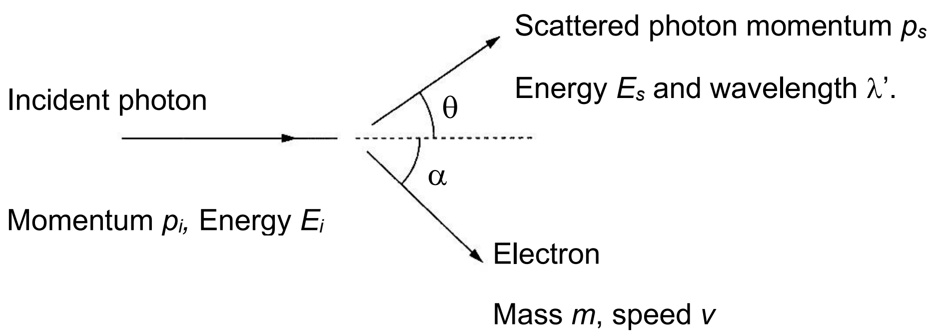

Fig 7.1

The incident photon has momentum p_i and energy E_i The photon is scattered through an angle θ and, after scattering, has momentum p_s and energy E_s. The electron of mass m which was originally stationary, moves off with speed v at an angle α to the original direction of the incident photon.

Compton applied the laws of conservation of energy and momentum to the collision and obtained the following equation for the wavelength of the scattered photons.

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where m is the mass of the electron. The quantity $\frac{h}{mc}$ has the dimension of length and is called the Compton wavelength of the electron, whose accepted value is 2.43×10^{-12} m.

The predicted wavelength of the scattered photons depends on the angle θ at which they are detected. Compton's measurements of 1923 were consistent with this formula.

In an experiment to provide evidence to justify Compton's theory, measurements were made of the wavelength λ of the incident photon, the wavelength λ' of the scattered photon and the angle θ of scattering. Some data from this experiment are given in Fig. 7.2.

λ / 10^{-12} m	λ' / 10^{-12} m	θ
191.92	193.27	59^{o}
965.04	966.84	75^{o}

Fig. 7.2

The wave theory of light does not predict such a shift: an incoming EM wave of frequency *f* should set electrons into oscillation at frequency *f*, and such oscillating electrons would re-emit EM waves of this same frequency. Hence Compton effect adds to the firm experimental foundation for the photon theory of light.

(a) Explain what is meant by a *photon.*

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………[1]

(b) The inelastic collision between a photon and a stationary electron may be represented by Fig. 7.1.

(i) Write down equations (in terms of p_i, p_s E_i, E_s, m, v, θ and α) that represent

1. the conservation of energy. [1]

2. the conservation of momentum along the direction of the incident photon. [1]

(ii) Using quantum theory of light, explain why a scattered photon has a wavelength longer than that of the incident photon.

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.... [2]

(c) In the Compton scattering experiment, the uncertainty in the measurement of θ is ± 5°. Determine the value of cos θ with its uncertainty, for the angle θ = 75° ± 5°.

cos θ = …………………… ± ………………… [2]

(d) Given that $\Delta\lambda = \lambda' - \lambda$, additional data for the variation of $\Delta\lambda$ against cos θ are shown in Fig. 7.3.

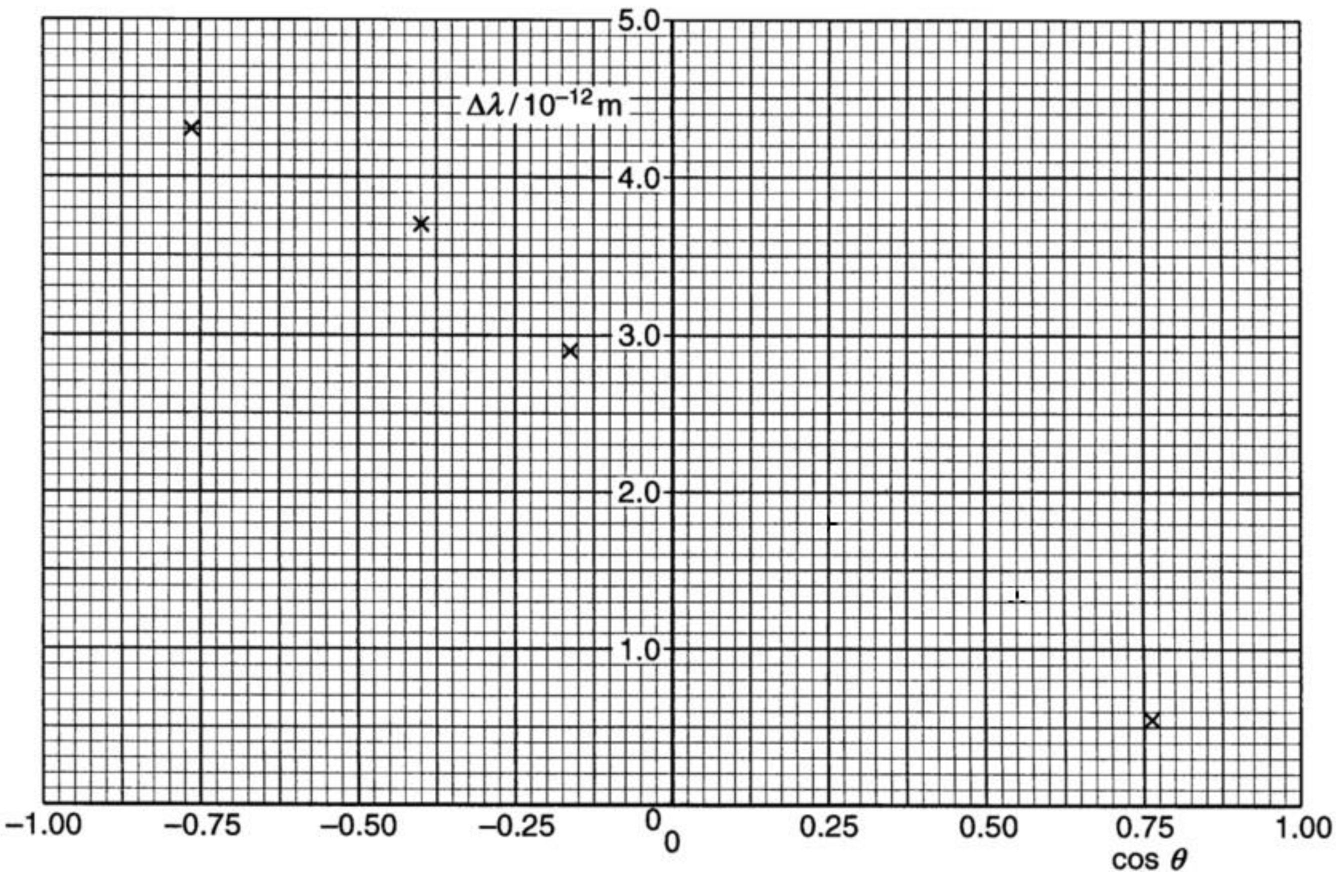

Fig. 7.3

(i) Plot the data given in Fig 7.2 on Fig 7.3. [3]

(ii) Draw the best-fit line for the points on Fig. 7.3. [1]

(iii) State and explain one way to determine the Compton's wavelength from Fig. 7.3.

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……… [2]

(iv) Determine the Compton's wavelength using the method described in **d (iii)**.

Compton's wavelength = …………………… [2]

(e) In another Compton scattering experiment, 19.0 keV X-ray photons scatter off a carbon target.

(i) Find the wavelength of the scattered photon if the scattered angle is 30°.

wavelength = ………………………………….. [2]

(ii) For a carbon atom, the binding energy of an electron is of the order of a few eV.

Compton's theory assumes that the electrons are not bound in the atoms but are free. Suggest whether this assumption is justified.

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……... [2]