**7** Read the passage below and answer the questions that follow.

## Law of Gravitation and Celestial Mechanics

For thousands of years, Man has studied the night sky and some ancient buildings provide evidence of careful and patient astronomical observations by people of many different cultures. As instrumentation has improved, so has the precision with which astronomical observations could be made. Between 1576 and 1597, Danish astronomer Tycho Brahe wanted to determine how the heavens were constructed and pursued a project to determine the positions of both stars and planets.

German astronomer Johannes Kepler was Brahe's assistant for a short while before Brahe's death, whereupon he acquired his mentor's astronomical data and spent 16 years trying to deduce a mathematical model for the motion of the planets.

Kepler deduced three laws:

1. All planets move in elliptical orbits with the Sun at one focus.
2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.
3. The square of the circular orbital period of any planet is proportional to the cube of the radius of the circle.

As a result of Kepler's work, Newton formulated the law of gravitation.

**(a)** **(i)** *Newton's law of gravitation* partially states that the force between two masses is inversely proportional to the square of their distance apart.
Using concepts on an object moving in *circular motion*, explain how this statement can be deduced from Kepler's third law.

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**(ii)** Using Newton's laws, show that, for a circular orbit of an object about a planet,

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

where $T$ is the orbital period of the body,
$M$ is the mass of the planet, and
$r$ is the distance between the centre of mass of the body and the planet.

[3]

**(b)** The planet Jupiter has several moons. Data for some of these moons are shown in Table 7.1.

**Table 7.1**

| **moon** | **period** $T$ **/ days** | **mean distance from centre of Jupiter** $r$ **/ $10^9$m** | **$\log_{10}$ ($T$ / days)** | **$\log_{10}$ ($r$ / m)** |
|---|---|---|---|---|
| Sinope | 758 | 23.7 | 2.88 | 10.37 |
| Leda | 239 | 11.1 | | |
| Callisto | 16.7 | 1.88 | | |
| Io | 1.77 | 0.422 | | |
| Metis | 0.295 | 0.128 | -0.53 | 8.11 |

**(i)** Complete Table 7.1 for the moons – Leda, Callisto and Io. [2]

**(ii)** On the axes of Fig. 7.1, plot a graph of $\log_{10}$ ($T$/days) against $\log_{10}$ ($r$/m). [3]

**(iii)** Determine the gradient of the graph in Fig. 7.1.

gradient = …………………………..[2]

**(iv)** Hence, discuss whether the data in Table 7.1 support the relation given in **(a)(ii)**.

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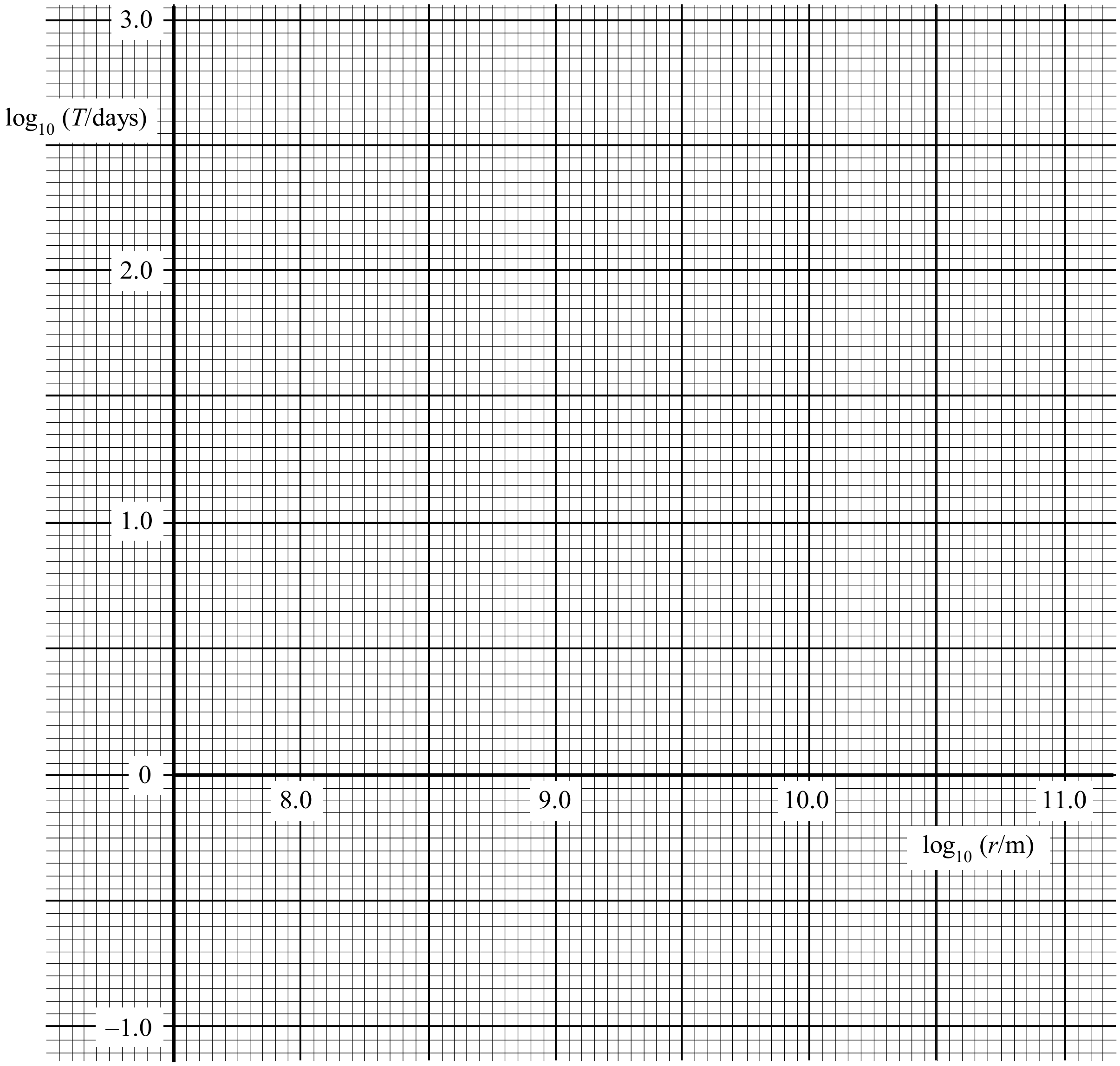


**Fig. 7.1**

**(c)** Observation shows that the moon Ganymede orbits Jupiter with a period of 7.16 days.

Use Fig. 7.1 to estimate the orbital radius of Ganymede.

orbital radius = ………………………… m [2]

**(d)** It is reported in the media that the moon Thebe is discovered to orbit Jupiter once every 16.2 hours at a height of $2.22 \times 10^5$ km above the surface of Jupiter.

Comment on the accuracy of this media report.

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**(e)** Suggest whether Fig. 7.1 could be used to check data on the orbital radii and periods of the moons of another planet (e.g. Saturn).

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[Total: 20]

**End of Paper**