4 A cylindrical tube, containing some sand, floats upright in a liquid of density ρ as shown in Fig. 4.1.

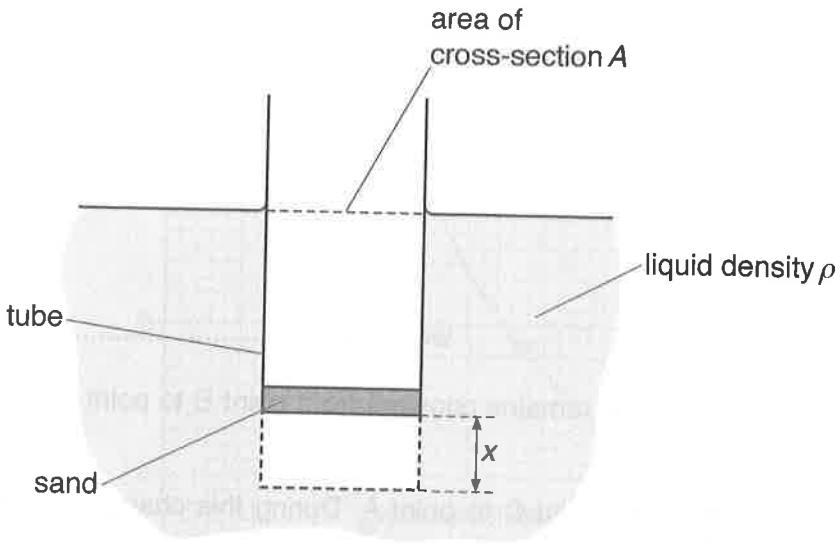

Fig. 4.1

The tube has cross-sectional area A. The total mass of the tube and sand is M.

The tube is displaced vertically downwards and then released. The tube oscillates vertically.

(a) (i) Derive an expression, in terms of ρ, A, x and the acceleration of free fall g, for the resultant force F acting vertically on the tube when the displacement of the tube from its equilibrium position is x. Explain your working.

[3]

(ii) Using the expression derived in **(a)(i)**, show that the acceleration a of the tube at displacement x is given by

$$a = -\left(\frac{\rho A g}{M}\right)x.$$

[2]

(b) The equation derived in **(a)(ii)** is a form of the expression for simple harmonic motion.

The mass M of the tube and sand is 130 g. The area of cross-section A of the tube is 5.3 cm^2.

Calculate the frequency of oscillation of the tube when floating in a liquid of density $1.2 \times 10^3 kg m^{-3}$.

frequency = ... Hz [4]

[Total: 9]