7 A beam PQ is clamped at end P so that the beam is horizontal. A load of mass M is hung from end Q as shown in Fig. 7.1.

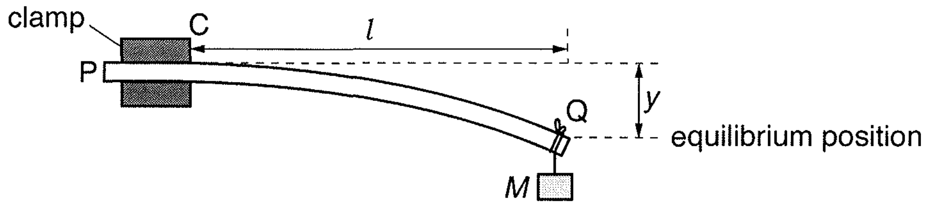

Fig. 7.1

The length of the beam from the edge C of the clamp to the end Q of the beam is l. The thickness of the beam is d and the width of the beam is b.

The beam bends due to the load of mass M. The vertical displacement of the end Q is y.

(a) The variation of y is given by the expression

$$y = k\,Mg\,l^{\mathrm{r}}\,d^{\mathrm{s}}\,b^{-1}$$

where k is a constant, g is the acceleration of free fall, and r and s are **integers**.

An experiment is carried out to determine k, r and s. The values of M and b are kept constant.

Fig. 7.2 shows the readings obtained.

y_1/m	y_2/m	l/m	$\ln(y_1/\mathrm{m})$	$\ln(l/\mathrm{m})$
0.257	0.148	0.900	−1.359	−0.105
0.178	0.103	0.800	−1.726	−0.223
0.118	0.068	0.700	−2.137	−0.357
0.073	0.042	0.600	−2.617	−0.511
0.042	0.024	0.500		
0.021	0.012	0.400	−3.863	−0.916

Fig. 7.2

The set of readings for y_1 is for a beam thickness d of 5.00×10^{-3} m.

The set of readings for y_2 is for a beam thickness d of 6.00×10^{-3} m.

(i) Theory suggests that the value of s is −3. Use Fig. 7.2 to show that the data supports this suggestion.

[2]

(ii) Complete Fig. 7.2 for $l = 0.500\,\text{m}$. [1]

(iii) A graph of some of the data showing the variation of $\ln(y_1/\text{m})$ with $\ln(l/\text{m})$ is shown in Fig. 7.3.

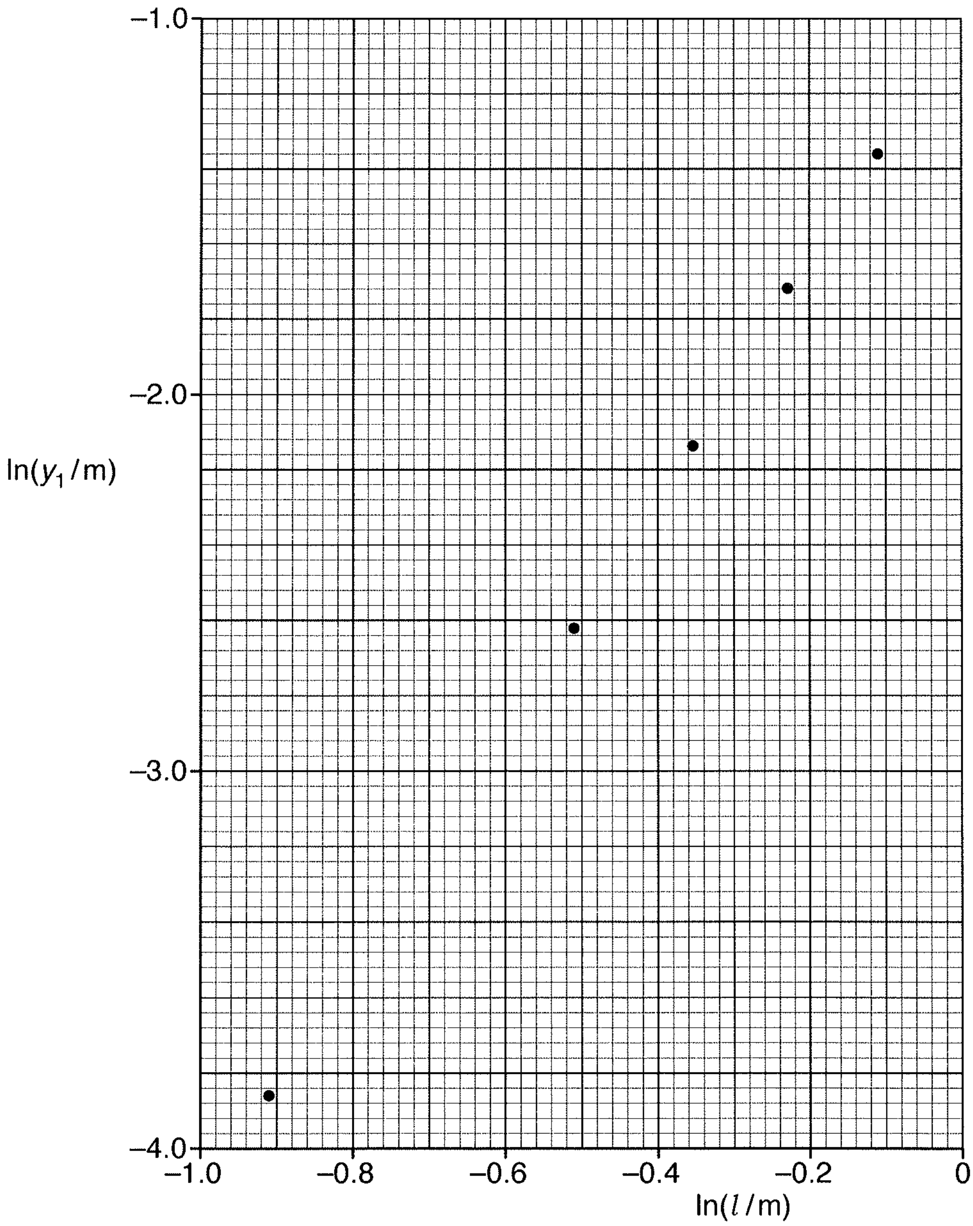

Fig. 7.3

On Fig. 7.3,

1. plot the point corresponding to $l = 0.500\,\text{m}$,

2. draw the line of best fit for all the points. [2]

(iv) Determine the gradient of the line you have drawn on Fig. 7.3.

gradient = ... [2]

(v) Explain why the graph of Fig. 7.3 supports the expression given in **(a)**.

....

.... [2]

(vi) Determine integer r.

r = ... [1]

(b) The fixed value of mass M is 0.500 kg and the fixed value of width b is 3.00×10^{-2} m.
Use Fig. 7.3 and the expression given in **(a)** to show that the constant k is about 3×10^{-10} when expressed in SI base units.

[2]

(c) The end Q of the beam is displaced from the equilibrium position, shown in Fig. 7.1, and then released so that the beam oscillates.
The vertical acceleration of the load is given by the equation

$$a = -\frac{b}{kMd^{s}l^{r}}\,x$$

where x is the vertical displacement of the end Q of the beam from the equilibrium position.

(i) Explain why the motion of the load is simple harmonic.

....

.... [1]

(ii) Calculate the time period of the oscillations of the load for a beam of length l of 0.600 m.

time period = ... s [2]