9 A uniform sphere of radius R has mass M. The mass of the sphere may be assumed to be a point mass at the centre of the sphere, as illustrated in Fig. 9.1.

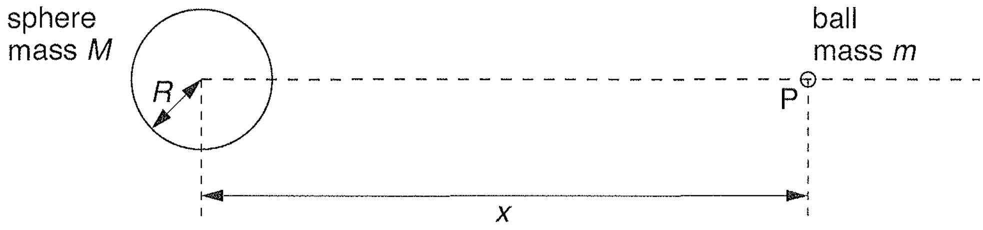

Fig. 9.1

A small ball of mass m is situated at point P, a distance x from the centre of the large mass. The sphere and the ball may be considered to be isolated in space.

(a) State expressions (one in each case) in terms of M, m, x and the gravitational constant G for

(i) the gravitational field strength g_P at point P,

.... [1]

(ii) the potential energy E_P of the small ball at point P.

.... [1]

(b) (i) The gravitational field strength at the surface of the sphere illustrated in Fig. 9.1 is g_s.

On the axes of Fig. 9.2, sketch a graph to show the variation with distance x of the gravitational field strength g of the sphere of mass M for values of x from $x = R$ to $x = 4R$.

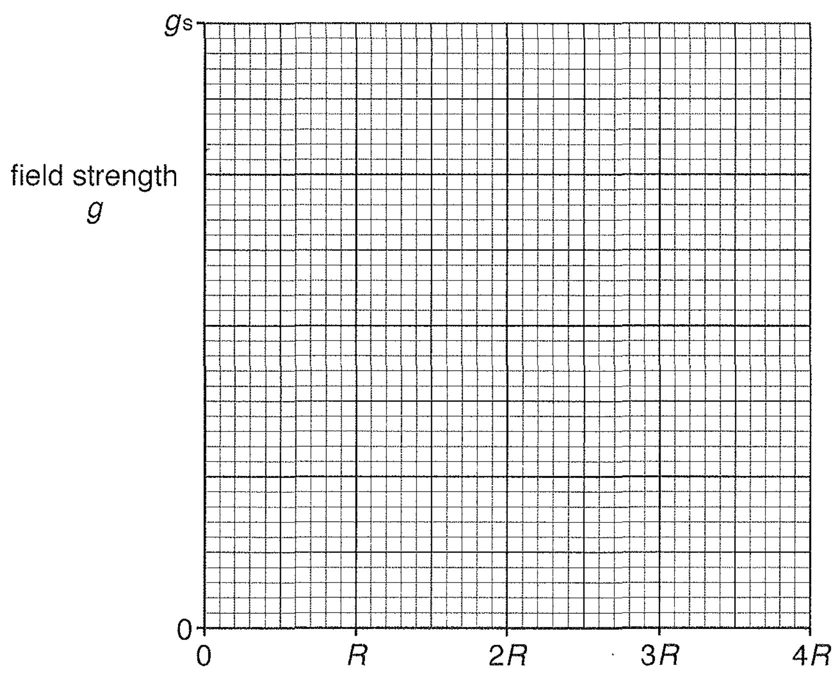

Fig. 9.2 [3]

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(ii) State and explain the effect, if any, on the graph you have sketched on Fig. 9.2 when mass is lost uniformly from the surface of the sphere.

....

....

.... [2]

Question 9 continues over the page

(c) A binary star consists of two stars A and B. The two stars may be considered to be isolated in space. The centres of the two stars are separated by a constant distance d, as illustrated in Fig. 9.3.

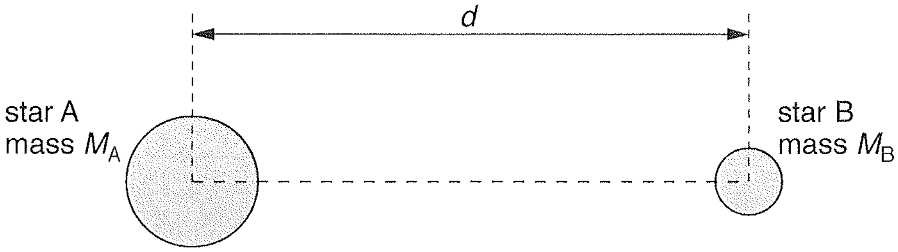

Fig. 9.3

Star A, of mass M_A, has a larger mass than star B of mass M_B, such that $\frac{M_A}{M_B} = 3.0$.

The stars are in circular orbits about each other such that the centre of their orbits is at a fixed point.
Viewed from Earth over a period of time equal to the period T of the orbits, the appearance of the stars is shown in Fig. 9.4.

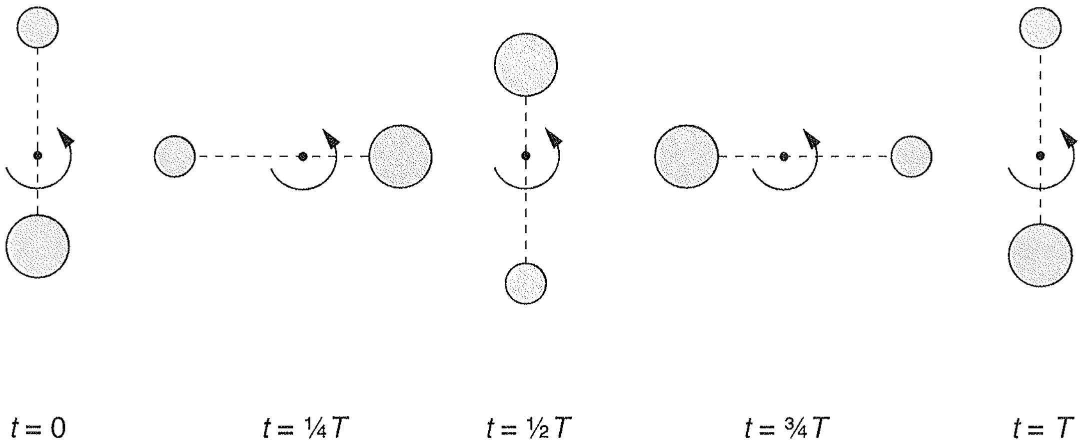

Fig. 9.4

The period T of each orbit is 4.0 years.

The separation d of the centres of the stars is 3.0×10^{11} m.

(i) Explain why the centripetal forces acting on the two stars are equal in magnitude.

....

....

.... [2]

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(ii) Calculate the angular speed ω of star A.

ω = ... rad s^{-1} [2]

(iii) Determine the radius of the orbit of star A. Explain your working.

radius = ... m [4]

(d) Use data from **(c)** and your answers in **(c)** to determine the mass of each star.

mass M_A of star A = ... kg

mass M_B of star B = ... kg [3]

(e) The plane of the orbits of the binary star in **(c)** is normal to the line of sight from Earth to the binary star.

A second binary star has the plane of its orbits parallel to the line of sight from Earth. This binary star is so far from Earth that the individual stars cannot be distinguished.

Suggest and explain what observation can be made to determine the period of the orbits of the stars.

....

....

.... [2]

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