

Solutions to 35th Singapore Physics Olympiad Theory Paper 1

Question 1: Projectile Motion

(a) Smooth Inclined Plane

1. Analyze the Projectile Motion

Let v be the speed of the particle as it leaves the top of the inclined plane at an angle $\theta = 30^\circ$. The vertical displacement is $y = -h = -50 \sin(30^\circ) = -25$ m. The horizontal displacement is $x = 153.3$ m.

The equations of projectile motion are:

$$x = (v \cos \theta)t$$
$$y = (v \sin \theta)t - \frac{1}{2}gt^2$$

Substitute known values:

$$1. 153.3 = (v \cos 30^\circ)t \implies t = \frac{153.3}{v \cos 30^\circ}$$

$$2. -25 = (v \sin 30^\circ)t - \frac{1}{2}(9.81)t^2$$

Substitute t from (1) into (2):

$$\begin{aligned} -25 &= (v \sin 30^\circ) \left(\frac{153.3}{v \cos 30^\circ} \right) - 4.905 \left(\frac{153.3}{v \cos 30^\circ} \right)^2 \\ -25 &= 153.3 \tan 30^\circ - \frac{4.905 \times (153.3)^2}{v^2 (\cos 30^\circ)^2} \\ -25 &\approx 88.51 - \frac{153258}{v^2} \\ \frac{153258}{v^2} &\approx 113.51 \implies v^2 \approx 1350.2 \implies v \approx 36.74 \text{ m/s} \end{aligned}$$

2. Analyze Motion on the Inclined Plane

Using the kinematic equation $v^2 = u^2 + 2aL$:

- Final speed, $v = 36.74$ m/s
- Distance, $L = 50$ m
- Acceleration, $a = -g \sin 30^\circ = -9.81 \times 0.5 = -4.905 \text{ m/s}^2$

$$(36.74)^2 = u^2 + 2(-4.905)(50)$$

$$1350.2 = u^2 - 490.5 \implies u^2 = 1840.7 \implies u \approx 42.9 \text{ m/s}$$

The initial speed is $\mathbf{u} \approx 43 \text{ m/s}$.

(b) Rough Inclined Plane

The speed at the top of the plane must still be $v = 36.74$ m/s. With a coefficient of kinetic friction $\mu_k = 0.25$, the new acceleration a' is:

$$a' = -(g \sin 30^\circ + \mu_k g \cos 30^\circ)$$

$$a' = -9.81(\sin 30^\circ + 0.25 \cos 30^\circ) \approx -7.029 \text{ m/s}^2$$

Using $v^2 = (u')^2 + 2a'L$:

$$(36.74)^2 = (u')^2 + 2(-7.029)(50)$$

$$1350.2 = (u')^2 - 702.9 \implies (u')^2 = 2053.1 \implies u' \approx 45.31 \text{ m/s}$$

The percentage change in u is:

$$\% \text{ Change} = \frac{u' - u}{u} \times 100\% = \frac{45.31 - 42.9}{42.9} \times 100\% \approx 5.62\%$$