Question 3: Electrostatics

(a) Time to Reach the Center

The total charge on the ring is $Q = \lambda(2\pi R) = (10 \times 10^{-9}) \times (2\pi \times 0.5) = 10\pi \times 10^{-9}$ C. For small displacements, $x_0 \ll R$, the force is approximately $F(x) \approx -kx$, indicating Simple Harmonic Motion.

$$k = \frac{|q|Q}{4\pi\epsilon_0 R^3} = \frac{(5 \times 10^{-9})(10\pi \times 10^{-9})}{4\pi(8.85 \times 10^{-12})(0.5)^3} \approx 1.129 \times 10^{-5}\,\text{N/m}$$

With mass $m = 1 \times 10^{-6}$ kg, the angular frequency is:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.129 \times 10^{-5}}{1 \times 10^{-6}}} \approx 3.36\,\text{rad/s}$$

The period is $T = \frac{2\pi}{\omega} \approx 1.87$ s. The time to reach the center is one-quarter of the period:

$$t = \frac{T}{4} \approx 0.467\,\text{s}$$

(b) Kinetic Energy at the Center

By conservation of energy, $K_f = U_i - U_f = qV(x_0) - qV(0)$.

$$K_f = \frac{qQ}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{R^2 + x_0^2}} - \frac{1}{R}\right)$$

$$K_f = (9 \times 10^9)(-5 \times 10^{-9})(10\pi \times 10^{-9})\left(\frac{1}{\sqrt{0.5^2 + 0.005^2}} - \frac{1}{0.5}\right)$$

$$K_f \approx 1.414 \times 10^{-10}\,\text{J}$$