

2.2 Springs System

2.2.1 (i) Bar in Horizontal Equilibrium

- **Equilibrium Conditions:** $\sum F_y = F_1 + F_2 - W = 0 \implies k_1\Delta l + k_2\Delta l = W \implies (30 + 50)\Delta l = 20 \implies \Delta l = 0.25 \text{ m}$. $\sum \tau_A = F_2(0.6 - x) - W(0.3) = 0$.
- **Solving for x:** $F_2 = k_2\Delta l = 50(0.25) = 12.5 \text{ N}$. $12.5(0.6 - x) - 20(0.3) = 0 \implies 12.5(0.6 - x) = 6 \implies 0.6 - x = 0.48 \implies x = \mathbf{0.12} \text{ m}$.

2.2.2 (ii) Particle Oscillating Between Springs

- **Equilibrium Position:** $k_1(l_1 - l_0) = k_2(l_2 - l_0)$ and $l_1 + l_2 = 0.5$. $30(l_1 - 0.2) = 50(0.5 - l_1 - 0.2) \implies 30l_1 - 6 = 15 - 50l_1 \implies 80l_1 = 21 \implies l_1 = \mathbf{0.2625} \text{ m}$. $l_2 = 0.5 - 0.2625 = \mathbf{0.2375} \text{ m}$.
- **Frequency of Oscillation:** The system behaves as two springs in parallel, so $k_{\text{eff}} = k_1 + k_2 = 80 \text{ N/m}$. The angular frequency is $\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{80}{0.05}} = 40 \text{ rad/s}$. The frequency is $f = \frac{\omega}{2\pi} = \frac{40}{2\pi} \approx \mathbf{6.37} \text{ Hz}$.