32nd Singapore Physics Olympiad
Theory Paper 1 — Full Solutions

Question 1(a) [7 marks]

A projectile is fired with initial speed v_0 at angle θ to the horizontal. It passes through two points at the same height h above ground. Find the horizontal separation D between these points.

Solution:

Trajectory equation:

$$y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

Set $y = h$:

$$h = x\tan\theta - \frac{gx^2}{2v_0^2}(1+\tan^2\theta)$$

This is a quadratic in x: $Ax^2 - Bx + h = 0$, where

$$A = \frac{g}{2v_0^2\cos^2\theta}, \quad B = \tan\theta$$

The two roots x_1, x_2 correspond to the two points. Horizontal separation:

$$D = |x_2 - x_1| = \frac{\sqrt{B^2 - 4Ah}}{A}$$

But better: from standard projectile motion, time to reach height h:

$$h = v_0\sin\theta\, t - \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - v_0\sin\theta\, t + h = 0$$

Roots t_1, t_2; then $D = v_0\cos\theta\,|t_2 - t_1|$

$$|t_2 - t_1| = \frac{\sqrt{(v_0\sin\theta)^2 - 2gh}}{g/2}\cdot\frac{1}{2} = \frac{2\sqrt{v_0^2\sin^2\theta - 2gh}}{g}$$

So

$$D = v_0\cos\theta\cdot\frac{2\sqrt{v_0^2\sin^2\theta - 2gh}}{g} = \frac{2v_0\cos\theta}{g}\sqrt{v_0^2\sin^2\theta - 2gh}$$

$$\boxed{D = \frac{2v_0\cos\theta}{g}\sqrt{v_0^2\sin^2\theta - 2gh}}$$

Question 1(b) [3 marks]

Given $v_0 = 200\,\mathrm{m\,s^{-1}}$, angle adjusted for maximum range $\rightarrow \theta = 45°$. Find D.

Solution:

Maximum range occurs at $\theta = 45°$, but h is not specified! However, in part (a), h is a given parameter. Since h is not provided, we assume the question implies **general expression evaluated at $\theta = 45°$**—but that still depends on h.

But likely, the problem intends you to **express D in terms of h** for this case. However, since it asks to "calculate the value of D", and no h is given, we suspect a missing detail.

But re-examining: in many Olympiad problems, if not specified, they might mean the **maximum possible D** for a given h, but that doesn't help.

Alternatively—perhaps in part (b), **h is arbitrary**, and you just plug $\theta = 45°$ into the expression:

$$\cos\theta = \sin\theta = \frac{\sqrt{2}}{2}$$

$$D = \frac{2 \cdot 200 \cdot \frac{\sqrt{2}}{2}}{9.81}\sqrt{(200)^2 \cdot \frac{1}{2} - 2 \cdot 9.81 \cdot h} = \frac{200\sqrt{2}}{9.81}\sqrt{20000 - 19.62h}$$

But without h, no numerical answer.

However, checking standard interpretation: often in such problems, part (b) assumes the same h as would yield real roots, but since none is given, **the problem likely expects you to leave it in terms of h** or there's a typo.

But given the marks (3), and context, perhaps they expect you to **recognize that for maximum range, the trajectory is symmetric**, and the expression is as above.

Since the problem says "calculate the value", yet provides no h, we conclude it's an oversight. **We'll assume h is given implicitly**, but as it's not, we present the simplified expression:

$$\boxed{D = \frac{200\sqrt{2}}{9.81}\sqrt{20000 - 19.62\,h} \quad \text{(in metres)}}$$

¿ *Note: If the original exam included a specific h, it is missing in the provided text.*

Question 2(a) [4 marks]

A mass $m = 0.25\,\mathrm{kg}$ is attached to a spring ($k = 20\,\mathrm{N\,m^{-1}}$), released from unstretched position, oscillates with decreasing amplitude (damped), and comes to rest. Surroundings at $T = 27\,^\circ\mathrm{C} = 300\,\mathrm{K}$. Find entropy change of surroundings.

Solution:

Initially, spring is unstretched, mass is released $\rightarrow$ it falls under gravity until equilibrium.

But the process is **irreversible**: mechanical energy is dissipated as heat into surroundings.

Total mechanical energy lost = initial gravitational potential energy relative to final equilibrium.

At equilibrium: $kx_0 = mg \Rightarrow x_0 = \frac{mg}{k}$

Initial energy: only gravitational PE (set final equilibrium as zero PE for spring + gravity combined).

Better: total initial mechanical energy (just after release): spring PE = 0, gravitational PE = mgx_0 (if we measure from equilibrium).

At final rest: all energy dissipated as heat $Q = \frac{1}{2}kx_0^2 - mgx_0$? No.

Actually, when released from **unstretched** position (spring at natural length), the mass falls distance $x_0 = mg/k$ to equilibrium, but due to damping, it doesn't oscillate—it settles.

The loss in gravitational PE: mgx_0

Gain in spring PE: $\frac{1}{2}kx_0^2$

So energy dissipated:

$$Q = mgx_0 - \frac{1}{2}kx_0^2 = mg\left(\frac{mg}{k}\right) - \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{k} - \frac{1}{2}\frac{m^2g^2}{k} = \frac{1}{2}\frac{m^2g^2}{k}$$

This heat flows into surroundings at constant T, so entropy change:

$$\Delta S = \frac{Q}{T} = \frac{m^2g^2}{2kT}$$

Plug in:

$$m = 0.25, \quad g = 9.81, \quad k = 20, \quad T = 300$$

$$Q = \frac{(0.25)^2(9.81)^2}{2 \cdot 20} = \frac{0.0625 \cdot 96.236}{40} \approx \frac{6.0148}{40} \approx 0.1504\,\mathrm{J}$$

$$\Delta S = \frac{0.1504}{300} \approx 5.01 \times 10^{-4}\,\mathrm{J\,K^{-1}}$$

$$\boxed{\Delta S \approx 5.0 \times 10^{-4}\,\mathrm{J\,K^{-1}}}$$

Question 2(b)(i) [3 marks]

Solar constant: $I = 1.37 \times 10^3\,\mathrm{W/m^2}$

Sun radius: $R_s = 6.957 \times 10^8\,\mathrm{m}$

Earth orbit radius: $r = 1.496 \times 10^{11}\,\mathrm{m}$
Find Sun's surface temperature.
Solution:
Power radiated by Sun: $P = 4\pi R_s^2 \sigma T^4$
At Earth: $I = \frac{P}{4\pi r^2} = \frac{R_s^2 \sigma T^4}{r^2}$
So:

$$T^4 = \frac{Ir^2}{\sigma R_s^2} \Rightarrow T = \left(\frac{Ir^2}{\sigma R_s^2}\right)^{1/4}$$

Compute:

$$\frac{r}{R_s} = \frac{1.496 \times 10^{11}}{6.957 \times 10^8} \approx 215.0$$

$$\left(\frac{r}{R_s}\right)^2 \approx 46225$$

$$T^4 = \frac{1370 \cdot 46225}{5.67 \times 10^{-8}} \approx \frac{6.33 \times 10^7}{5.67 \times 10^{-8}} \approx 1.117 \times 10^{15}$$

$$T = (1.117 \times 10^{15})^{1/4} \approx 5770\,\mathrm{K}$$

$$\boxed{T \approx 5800\,\mathrm{K}} \quad \text{(commonly accepted value)}$$

Question 2(b)(ii) [3 marks]

Mars orbit radius: $r_M = 2.280 \times 10^{11}\,\mathrm{m}$. Find equilibrium temperature of Mars.
Solution:
Assume Mars absorbs as blackbody, radiates as blackbody. At equilibrium:
Absorbed power = emitted power

$$I_M \pi R_M^2 = \sigma T_M^4 (4\pi R_M^2) \Rightarrow T_M = \left(\frac{I_M}{4\sigma}\right)^{1/4}$$

But $I_M = I\left(\frac{r}{r_M}\right)^2 = 1370\left(\frac{1.496}{2.280}\right)^2 \approx 1370 \cdot (0.656)^2 \approx 1370 \cdot 0.430 \approx 589\,\mathrm{W/m^2}$
Then:

$$T_M = \left(\frac{589}{4 \cdot 5.67 \times 10^{-8}}\right)^{1/4} = \left(\frac{589}{2.268 \times 10^{-7}}\right)^{1/4} \approx (2.60 \times 10^9)^{1/4}$$

$$(2.60 \times 10^9)^{1/4} = (2.60)^{1/4} \cdot (10^9)^{1/4} \approx 1.27 \cdot 177.8 \approx 226\,\mathrm{K}$$

$$\boxed{T_M \approx 226\,\mathrm{K}}$$

Question 3(a) [4 marks]

Mass $m = 0.1\,\mathrm{kg}$, force $F = -10x$ (N), so $k = 10\,\mathrm{N\,m^{-1}}$

Initial: $x_0 = 0.05\,\mathrm{m}$, $v_0 = \frac{\sqrt{3}}{2}\ \mathrm{m\,s^{-1}} \approx 0.866\,\mathrm{m\,s^{-1}}$ away from O.

(i) Amplitude

$\omega = \sqrt{k/m} = \sqrt{10/0.1} = \sqrt{100} = 10\,\mathrm{rad\,s^{-1}}$

Energy: $\frac{1}{2}kA^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2$

$$A^2 = x_0^2 + \frac{m}{k}v_0^2 = (0.05)^2 + \frac{0.1}{10}\cdot\left(\frac{3}{4}\right) = 0.0025 + 0.01\cdot 0.75 = 0.0025 + 0.0075 = 0.01$$

$$A = 0.1\,\mathrm{m}$$

$$\boxed{A = 0.10\,\mathrm{m}}$$

(ii) Initial phase angle

General solution: $x = A\cos(\omega t + \phi)$

At $t = 0$: $x_0 = A\cos\phi = 0.05 = 0.1\cos\phi \Rightarrow \cos\phi = 0.5 \Rightarrow \phi = \pm\frac{\pi}{3}$

Velocity: $v = -A\omega\sin(\omega t + \phi) \Rightarrow v_0 = -A\omega\sin\phi$

$$0.866 = -0.1\cdot 10\cdot\sin\phi = -\sin\phi \Rightarrow \sin\phi = -0.866$$

So $\phi = -\frac{\pi}{3}$ (or $5\pi/3$)

$$\boxed{\phi = -\frac{\pi}{3}}$$

(iii) Max speed and acceleration

$$v_{\max} = A\omega = 0.1\cdot 10 = 1.0\,\mathrm{m\,s^{-1}}, \quad a_{\max} = A\omega^2 = 0.1\cdot 100 = 10\,\mathrm{m/s^2}$$

$$\boxed{v_{\max} = 1.0\,\mathrm{m\,s^{-1}}, \quad a_{\max} = 10\,\mathrm{m/s^2}}$$

Question 3(b) [6 marks]

Rod of length L, floats vertically with h above liquid $\rightarrow$ submerged length $= L - h$

Show SHM for small vertical displacement; find period.

Solution:

Let cross-section area $= A$, density of rod $= \rho$, liquid $= \rho_l$

At equilibrium: weight = buoyancy

$$\rho ALg = \rho_l A(L-h)g \Rightarrow \rho L = \rho_l(L-h)$$

Displace downward by small y: new submerged length $= L - h + y$

Net upward force:

$$F = -[\text{buoyancy} - \text{weight}] = -[\rho_l A(L-h+y)g - \rho A L g] = -\rho_l A g y$$

(using equilibrium condition)

So $F = -(\rho_l A g)y = -k_{\text{eff}} y$, so SHM with

$$\omega = \sqrt{\frac{\rho_l A g}{m}} = \sqrt{\frac{\rho_l A g}{\rho A L}} = \sqrt{\frac{\rho_l g}{\rho L}}$$

But from equilibrium: $\frac{\rho_l}{\rho} = \frac{L}{L-h}$, so

$$\omega = \sqrt{\frac{g}{L-h} \cdot \frac{L}{L} \cdot \frac{\rho_l}{\rho}} = \sqrt{\frac{g}{L-h} \cdot \frac{L}{L-h} \cdot \frac{L-h}{L}} \Rightarrow \text{better:}$$

$$\omega = \sqrt{\frac{\rho_l g}{\rho L}} = \sqrt{\frac{g}{L-h}} \quad (\text{since } \rho_l/\rho = L/(L-h))$$

Thus period:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L-h}{g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L-h}{g}}}$$

Question 4(a) [5 marks]

Spherical oil drop: potential at surface = 1000 V. Two identical drops merge. Find new potential.

Solution:

For isolated sphere: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

Let each drop have charge Q, radius R

After merging: charge = $2Q$, volume = $2 \cdot \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3 \Rightarrow R' = R \cdot 2^{1/3}$

New potential:

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R \cdot 2^{1/3}} = \frac{2}{2^{1/3}} \cdot \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 2^{2/3}V$$

$$2^{2/3} \approx 1.5874, \quad V' \approx 1.5874 \cdot 1000 \approx 1587\,\mathrm{V}$$

$$\boxed{V' = 2^{2/3} \times 1000\,\mathrm{V} \approx 1590\,\mathrm{V}}$$

Question 4(b) [5 marks]

Bainbridge mass spectrometer: velocity selector with $E = 100\,\mathrm{V\,cm^{-1}} = 10\,000\,\mathrm{V\,m^{-1}}$, $B = 0.2\,\mathrm{T}$

(i) Speed of ion that passes through

In velocity selector: $qE = qvB \Rightarrow v = E/B = 10000/0.2 = 5.0 \times 10^4\,\mathrm{m\,s^{-1}}$

$$\boxed{v = 5.0 \times 10^4\,\mathrm{m\,s^{-1}}}$$

(ii) Can it resolve ^{3}He and ^{4}He? Slit width = 1 mm

After selector, ions enter magnetic field (same B?), move in circular paths.

Radius: $r = \frac{mv}{qB}$

For same q, v, B: $\Delta r = r_4 - r_3 = \frac{v}{qB}(m_4 - m_3)$

Masses: $m_3 = 3u$, $m_4 = 4u$, $u = 1.66 \times 10^{-27}\,\mathrm{kg}$

$$\Delta r = \frac{5.0 \times 10^4}{1.60 \times 10^{-19} \cdot 0.2} \cdot (1.66 \times 10^{-27}) = \frac{5.0 \times 10^4 \cdot 1.66 \times 10^{-27}}{3.2 \times 10^{-20}} \approx \frac{8.3 \times 10^{-23}}{3.2 \times 10^{-20}} \approx 2.59 \times 10^{-3}\,\mathrm{m} = 2.59\,\mathrm{mm}$$

Slit width = 1 mm, so $\Delta r >$ slit width → **can resolve**

$$\boxed{\text{Yes, since } \Delta r \approx 2.6\,\mathrm{mm} > 1\,\mathrm{mm}}$$

Question 5(a) [5 marks]

Light intensity $I = 50\,\mathrm{W/m^2}$ incident normally on **perfect reflector**. Find radiation pressure.

Solution:

For perfect reflection: pressure $P = \frac{2I}{c}$

$$P = \frac{2 \cdot 50}{3.00 \times 10^8} = \frac{100}{3 \times 10^8} \approx 3.33 \times 10^{-7}\,\mathrm{Pa}$$

$$\boxed{P = \frac{2I}{c} \approx 3.33 \times 10^{-7}\,\mathrm{Pa}}$$

Question 5(b) [5 marks]

Positronium: electron and positron (mass m, opposite charges). Find shortest wavelength in Lyman series.

Solution:

Reduced mass: $\mu = \frac{m \cdot m}{m+m} = \frac{m}{2}$

Energy levels scale with reduced mass:

$$E_n = -\frac{\mu e^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2} = -\frac{1}{2} \cdot \frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2} = \frac{1}{2} E_n^{(\mathrm{H})}$$

So ionization energy $= \frac{1}{2} \times 13.6\,\mathrm{eV} = 6.8\,\mathrm{eV}$

Lyman series: $n \geq 2 \to n = 1$

Shortest wavelength = highest energy = $n = \infty \to n = 1$

$$\Delta E = 0 - (-6.8) = 6.8\,\mathrm{eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\ \mathrm{eV\,nm}}{6.8\ \mathrm{eV}} \approx 182\,\mathrm{nm}$$

$$\boxed{\lambda \approx 182\,\mathrm{nm}}$$