Twentieth Singapore Physics Olympiad
Theoretical Paper

Tuesday 23 October 2007

Organized by

Institute of Physics, Singapore

In conjunction with

National University of Singapore,
National Institute of Education, Nanyang Technological University,
Ministry of Education, Singapore

And sponsored by

Micron Technology Foundation, Inc

Instructions to Candidates

1. This is a four-hour test.
2. This paper consists of TEN (10) questions printed on SIX (6) printed pages. Page SIX (6) is a Table of Fundamental Constants in Physics which may be useful for your calculations.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your name legibly on the top right hand corner of every answer sheet you submit.
5. Begin each answer on a fresh sheet of paper.
6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.
7. No books or documents relevant to the test may be brought into the examination hall.

1. You have five (5) blocks of dimension 1 meter by 1 meter and 25 mm thick. What is the furthest you can stack them from the edge of a table before they topple?

[8 marks]

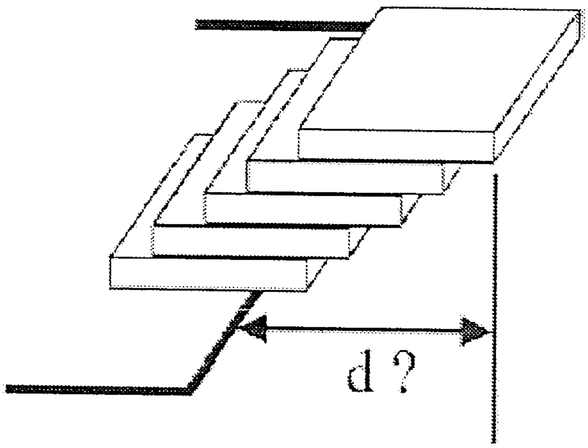

Figure 1: Five blocks stacked from the edge of a table.

2. A column is formed of two marble blocks each weighing 15 kN sitting atop each other as illustrated in the figure 2. A rope to which a force F is applied is attached at the top. The friction coefficient between the marble and the ground is $\mu_{M-G} = 0.15$ while the friction coefficient between marble and marble is $\mu_{M-M} = 0.20$. What is the maximum load $F_{\max}$ that can be applied? [8 marks]

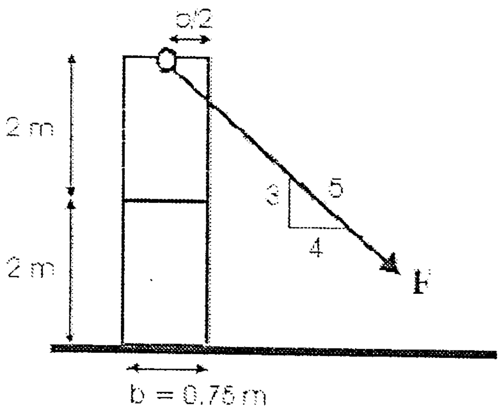

Figure 2: Two marble blocks.

3. A weighing scale (A) is fixed firmly to a wedge (B). A person of mass M is standing on the scale. The wedge is on a frictionless inclined ramp that makes an angle to the horizontal as shown in the figure. The system is sliding down the ramp with an acceleration a. The reading of the scale registers the apparent weight of the person. Derive an expression for the apparent weight W of the person as he slides down.

[6 marks]

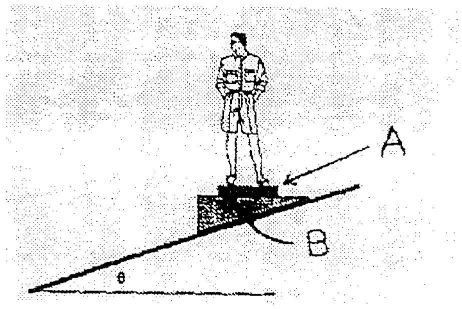

Figure 3: A man standing on a weighing machine and sliding down a ramp.

4. (a) A projectile is fired from ground level with an initial velocity of v_0 at an angle of θ above the horizontal. Show that the maximum height H is given by

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

where g is the acceleration of free fall. [4 marks]

(b) A rocket is projected upwards and explodes into three equally massive fragments just as it reaches the top of its flight. One of the fragments is observed to fall downwards in time t_1 as shown in Figure 4 while the other two land after a time t_2 after the burst. Determine the height $h(t_1, t_2)$ at which the fragmentation occurs. [8 marks]

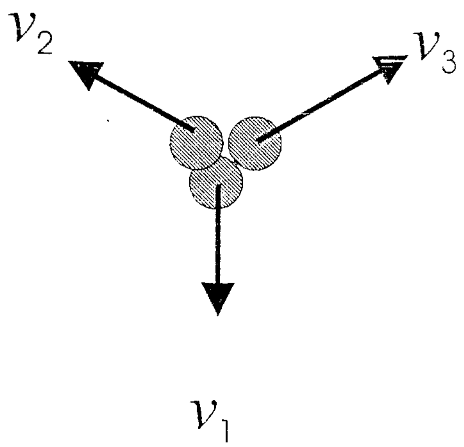

Figure 4: A rocket exploding into three fragments.

5. Explain the physical principles involved in the following:

   (a) A feather and a piece of metal of roughly the same size and shape fall with equal acceleration on the Moon but not the Earth. [3 marks]

   (b) Forces between the molecules of a gas cause deviations from ideal gas behavior but forces between the walls of the container and the gas molecules do not. [3 marks]

   (c) The sun appears elliptical as it is about to set over a distant horizon. [3 marks]

   (d) An electric motor in which the armature and the field windings are in series slows down more when the load is increased than a motor in which they are in parallel. [3 marks]

6. A circular coil has an inner radius R_1 and an outer radius R_2. The length of the coil is L.

   (i) Show that the magnetic induction B at the center when the coil carries a current I is

$$B = \frac{\mu_0 n I L}{2} \ln\left(\frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{1 + \sqrt{1 + \beta^2}}\right),$$

   where $\alpha = \frac{R_2}{R_1}$, $\beta = \frac{L}{2R_1}$ and n is the number of turns per square meter. [10 marks]

   (ii) Show that the length of the wire is

$$l = Vn = 2\pi n(\alpha^2 - 1)\beta R_1^3,$$

   where V is the volume occupied by the wire. [5 marks]

7. A ship in a calm port nearing a shore radio station receives a 200 MHz signal from the station's antenna. Both station's antenna and the ship's receiving antenna are located 20 m above the sea surface. A succession of maxima and minima are heard in the signal received at the ship. How far is the ship from the station the first time the signal passes through a minimum? How fast is the ship moving if the time between the first minimum and the next one is 50 s? [10 marks]

8. To account for the finite size of molecules and the attractive forces between molecules, Van der Waal devised an equation to describe the gas law for "real" gases

$$(P + \frac{a}{V^2})(V - b) = RT$$

   where P, V and T are the pressure, volume and temperature of the gas; and R is the universal gas constant.

   (a) What are the dimensions of the constants a and b? [4 marks]

   (b) Show that the critical point temperature and pressure are given by

$$T_c = \frac{8a}{27bR}; \qquad P_c = \frac{a}{27b^2}$$

   [7 marks]

9. (a) Consider an oscillating element of air of cross sectional area A and thickness Δx whose center is displaced from its equilibrium position by a distance x. We can write the pressure variation of the displaced element as $\Delta p = -B\frac{\Delta V}{V}$ where $V = A\Delta x$ and B is the bulk modulus. The volume change as a result unequal displacements of the two faces of the element differing by an amount Δs. By applying Newton's law of motion to the element, show that

$$\rho v^2 = B$$

where v is the speed of the element. [4 marks]

(b) A period of pulsating star may be modeled as a star whose radius R varies periodically with time, performing radial longitudinal pulsations in the fundamental standing wave mode. By analogy with a pipe with one open end, determine the period of the pulsation T in terms of the radius and the average speed of sound in the star matter. For white dwarf stars composed of material with a bulk modulus of 1.33×10^{22} Pa and a density of 10^{10} kgm^{-3} and a radius of 9.0×10^{-3} solar radius, what is the approximate pulsation period of this star? [6 marks]

10. A photon of energy E is scattered from a free stationary electron of rest mass m_0. Show that the maximum kinetic energy of the recoiling electron is given by

$$K_{max} = \frac{E^2}{E + m_0c^2}$$

[8 marks]

END OF PAPER

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational value
Avogadro's number	N	6.023×10^{23} mole^{-1}
Boltzmann constant	k	1.38×10^{23} JK^{-1}
Elementary charge	e	1.6×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Neutron rest mass	m_n	1.68×10^{-27} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} Js
Permittivity constant	ϵ_0	8.85×10^{-12} Fm^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan's constant	σ	5.68×10^{-8} Wm^{-2} K^4
Speed of light in vacuum	c	2.997×10^8 ms^{-1}
Unified atomic mass unit	u	1.66×10^{-27} kg
Universal gas constant	R	8.31 J mol^{-1} K^{-1}