Solutions to 35th Singapore Physics Olympiad Theory Paper 1

Question 1: Projectile Motion

(a) Smooth Inclined Plane

1. Analyze the Projectile Motion

Let v be the speed of the particle as it leaves the top of the inclined plane at an angle $\theta = 30°$. The vertical displacement is $y = -h = -50\sin(30°) = -25$ m. The horizontal displacement is $x = 153.3$ m.

The equations of projectile motion are:

$$x = (v\cos\theta)t$$
$$y = (v\sin\theta)t - \frac{1}{2}gt^2$$

Substitute known values:

1. $153.3 = (v\cos 30°)t \implies t = \frac{153.3}{v\cos 30°}$
2. $-25 = (v\sin 30°)t - \frac{1}{2}(9.81)t^2$

Substitute t from (1) into (2):

$$-25 = (v\sin 30°)\left(\frac{153.3}{v\cos 30°}\right) - 4.905\left(\frac{153.3}{v\cos 30°}\right)^2$$

$$-25 = 153.3\tan 30° - \frac{4.905 \times (153.3)^2}{v^2(\cos 30°)^2}$$

$$-25 \approx 88.51 - \frac{153258}{v^2}$$

$$\frac{153258}{v^2} \approx 113.51 \implies v^2 \approx 1350.2 \implies v \approx 36.74\,\text{m/s}$$

2. Analyze Motion on the Inclined Plane

Using the kinematic equation $v^2 = u^2 + 2aL$:

- Final speed, $v = 36.74$ m/s
- Distance, $L = 50$ m
- Acceleration, $a = -g\sin 30° = -9.81 \times 0.5 = -4.905\,\text{m/s}^2$

$$(36.74)^2 = u^2 + 2(-4.905)(50)$$
$$1350.2 = u^2 - 490.5 \implies u^2 = 1840.7 \implies u \approx 42.9\,\text{m/s}$$

The initial speed is $\mathbf{u} \approx \mathbf{43\ m/s}$.

(b) Rough Inclined Plane

The speed at the top of the plane must still be $v = 36.74$ m/s. With a coefficient of kinetic friction $\mu_k = 0.25$, the new acceleration a' is:

$$a' = -(g \sin 30^\circ + \mu_k g \cos 30^\circ)$$

$$a' = -9.81(\sin 30^\circ + 0.25 \cos 30^\circ) \approx -7.029 \,\mathrm{m/s}^2$$

Using $v^2 = (u')^2 + 2a'L$:

$$(36.74)^2 = (u')^2 + 2(-7.029)(50)$$

$$1350.2 = (u')^2 - 702.9 \implies (u')^2 = 2053.1 \implies u' \approx 45.31 \,\mathrm{m/s}$$

The percentage change in u is:

$$\%\,\mathrm{Change} = \frac{u' - u}{u} \times 100\% = \frac{45.31 - 42.9}{42.9} \times 100\% \approx 5.62\%$$

Question 2: Waves and Oscillations

(a) Tidal Motion

We model the sea depth $d(t)$ as a sinusoidal function.

- Mean Depth: $d_{\mathrm{mean}} = \frac{30+20}{2} = 25$ m.
- Amplitude: $A = \frac{30-20}{2} = 5$ m.
- Period: $T = 12$ hours.
- Angular Frequency: $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$ rad/hr.

Since depth is minimum at $t = 0$ (12 noon), the model is $d(t) = d_{\mathrm{mean}} - A\cos(\omega t)$.

$$d(t) = 25 - 5\cos\left(\frac{\pi}{6}t\right)$$

We need to find the times when the depth is $d(t) = 22.5$ m.

$$22.5 = 25 - 5\cos\left(\frac{\pi}{6}t\right) \implies \cos\left(\frac{\pi}{6}t\right) = 0.5$$

The principal value is $\frac{\pi}{6}t = \frac{\pi}{3}$, which gives $t = 2$ hours (2:00 pm). By symmetry, the second time is $t_2 = 12 - 2 = 10$ hours (10:00 pm). The ship can stay in the harbor from **2:00 pm to 10:00 pm**. **Assumption:** The tidal motion is sinusoidal (Simple Harmonic Motion).

(b) Doppler Effect and Beats

(i) Stationary Source, Moving Object

The frequency detected by the source after reflection, f_r, from an object moving away at speed v_{obj} is:

$$f_r = f_s\left(\frac{v_{\mathrm{sound}} - v_{\mathrm{obj}}}{v_{\mathrm{sound}} + v_{\mathrm{obj}}}\right)$$

Given $f_s = 256$ Hz, $v_{\mathrm{sound}} = 340$ m/s, and $f_{\mathrm{beat}} = 7$ Hz. The reflected frequency is $f_r = f_s - f_{\mathrm{beat}} = 256 - 7 = 249$ Hz.

$$249 = 256\left(\frac{340 - v_{\mathrm{obj}}}{340 + v_{\mathrm{obj}}}\right)$$

Solving for v_{obj} gives: $v_{\mathrm{obj}} \approx 4.71 \,\mathrm{m/s}$.

(ii) Moving Source, Moving Object

The source moves towards the object with $v_s = 5.0$ m/s. The reflected frequency f_r' is:

$$f_r' = f_s \left(\frac{v_{\text{sound}} - v_{\text{obj}}}{v_{\text{sound}} - v_s} \right) \left(\frac{v_{\text{sound}} + v_s}{v_{\text{sound}} + v_{\text{obj}}} \right)$$

Using $v_{\text{obj}} = 4.71$ m/s from our calculation:

$$f_r' = 256 \left(\frac{340 - 4.71}{340 - 5} \right) \left(\frac{340 + 5}{340 + 4.71} \right) \approx 256.44\,\text{Hz}$$

The new beat frequency is $f_{\text{beat}}' = |f_r' - f_s| = |256.44 - 256| = 0.44$ Hz.

Question 3: Electrostatics

(a) Time to Reach the Center

The total charge on the ring is $Q = \lambda(2\pi R) = (10 \times 10^{-9}) \times (2\pi \times 0.5) = 10\pi \times 10^{-9}$ C. For small displacements, $x_0 \ll R$, the force is approximately $F(x) \approx -kx$, indicating Simple Harmonic Motion.

$$k = \frac{|q|Q}{4\pi\epsilon_0 R^3} = \frac{(5 \times 10^{-9})(10\pi \times 10^{-9})}{4\pi(8.85 \times 10^{-12})(0.5)^3} \approx 1.129 \times 10^{-5}\,\text{N/m}$$

With mass $m = 1 \times 10^{-6}$ kg, the angular frequency is:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.129 \times 10^{-5}}{1 \times 10^{-6}}} \approx 3.36\,\text{rad/s}$$

The period is $T = \frac{2\pi}{\omega} \approx 1.87$ s. The time to reach the center is one-quarter of the period:

$$t = \frac{T}{4} \approx 0.467\,\text{s}$$

(b) Kinetic Energy at the Center

By conservation of energy, $K_f = U_i - U_f = qV(x_0) - qV(0)$.

$$K_f = \frac{qQ}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{R^2 + x_0^2}} - \frac{1}{R} \right)$$

$$K_f = (9 \times 10^9)(-5 \times 10^{-9})(10\pi \times 10^{-9}) \left(\frac{1}{\sqrt{0.5^2 + 0.005^2}} - \frac{1}{0.5} \right)$$

$$K_f \approx 1.414 \times 10^{-10}\,\text{J}$$

Question 4: Thermodynamics

(a) Thermal Conduction and Entropy

(i) Thermal Conductivity

The rate of heat flow P is:

$$P = \frac{dm}{dt} L_f = \left(\frac{0.1683}{60} \right) (3.36 \times 10^4) = 94.248\,\text{W}$$

From the heat conduction equation, $P = kA\frac{\Delta T}{L}$:

$$k = \frac{PL}{A\Delta T} = \frac{(94.248)(0.2)}{(\pi(0.01)^2)(150)} \approx 400\,\text{W m}^{-1}\text{K}^{-1}$$

(ii) Rate of Entropy Change

$$\frac{dS_{\text{total}}}{dt} = \frac{-P}{T_H} + \frac{P}{T_C} = 94.248 \left(\frac{1}{273.15} - \frac{1}{423.15} \right) \approx 0.1223 \, \text{J K}^{-1}\text{s}^{-1}$$

(b) Radiation Equilibrium

At equilibrium, power absorbed equals power emitted: $I(\pi R^2) + \sigma(4\pi R^2)T_{\text{env}}^4 = \sigma(4\pi R^2)T_{\text{eq}}^4$.

$$T_{\text{eq}}^4 = \frac{I}{4\sigma} + T_{\text{env}}^4$$

$$T_{\text{eq}} = \left(\frac{2400}{4(5.67 \times 10^{-8})} + (300.15)^4 \right)^{1/4} \approx 369.7 \, \text{K}$$

Question 5: Photoelectric Effect and Atomic Physics

(a) Linear Momentum of Photoelectrons

Maximum kinetic energy is $K_{\max} = eV_s = (1.60 \times 10^{-19})(4.25) = 6.80 \times 10^{-19}$ J. The momentum is $p = \sqrt{2m_e K_{\max}}$.

$$p = \sqrt{2(9.11 \times 10^{-31})(6.80 \times 10^{-19})} \approx 1.114 \times 10^{-24} \, \text{Ns}$$

(b) Ratio of Emitted Electrons to Incident Photons (N_e/N_p)

Rate of photoelectron emission:

$$N_e = \frac{I_{\text{sat}}}{e} = \frac{21.7 \times 10^{-9}}{1.60 \times 10^{-19}} = 1.356 \times 10^{11} \, \text{electrons/s}$$

Rate of photon incidence:

$$N_p = \frac{P_{\text{in}}}{E_p} = \frac{I \times A}{hc/\lambda} = \frac{(0.7)(10 \times 10^{-4})}{(6.63 \times 10^{-34})(3.00 \times 10^8)/(60 \times 10^{-9})} = 2.112 \times 10^{14} \, \text{photons/s}$$

The ratio is:

$$\frac{N_e}{N_p} = \frac{1.356 \times 10^{11}}{2.112 \times 10^{14}} \approx 6.42 \times 10^{-4}$$

(c) Collision with Hydrogen Atom

The electron's kinetic energy is $K_{\max} = 4.25$ eV. The minimum energy to excite a hydrogen atom from its ground state ($n = 1$) to the first excited state ($n = 2$) is:

$$\Delta E = E_2 - E_1 = \left(-\frac{13.6}{2^2} \right) - \left(-\frac{13.6}{1^2} \right) = 10.2 \, \text{eV}$$

Since the electron's energy (4.25 eV) is less than the required excitation energy (10.2 eV), the electron cannot excite the hydrogen atom. Therefore, no photons will be emitted.