Question 5: Modern Physics

(a) Recoil of a Hydrogen Atom

The energy of the emitted photon is approximately the energy difference between the states:

$$\Delta E = E_3 - E_1 = -13.6\text{eV}\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = -13.6\left(\frac{1}{9} - 1\right) = 13.6 \times \frac{8}{9} \approx 12.09\,\text{eV}$$

$$E_\gamma \approx \Delta E = 12.09\,\text{eV} = 1.934 \times 10^{-18}\,\text{J}$$

The photon's momentum is $p_\gamma = E_\gamma / c$:

$$p_\gamma = \frac{1.934 \times 10^{-18}}{3.00 \times 10^8} \approx 6.447 \times 10^{-27}\,\text{kg m/s}$$

By conservation of momentum, the recoil momentum of the atom is $p_H = p_\gamma$. The recoil speed is:

$$v_H = \frac{p_H}{M_H} = \frac{6.447 \times 10^{-27}}{1.66 \times 10^{-27}} \approx 3.88\,\text{m/s}$$

The recoil energy is:

$$E_{recoil} = \frac{p_H^2}{2M_H} = \frac{(6.447 \times 10^{-27})^2}{2(1.66 \times 10^{-27})} \approx 1.25 \times 10^{-26}\,\text{J}$$

Answers: Recoil speed is $\mathbf{3.88\,m/s}$. Recoil energy is $\mathbf{1.25 \times 10^{-26}\,J}$.