(b) Radioactive Decay Chain

The quantity of nucleus B is maximum when its rate of change is zero: $\frac{dN_B}{dt} = 0$.

$$\frac{dN_B}{dt} = \lambda_A N_A(t) - \lambda_B N_B(t) = 0$$

At the time of maximum quantity, $t = T$: $\lambda_A N_A(T) = \lambda_B N_B(T)$. The solution for T is given by:

$$T = \frac{\ln(\lambda_A/\lambda_B)}{\lambda_A - \lambda_B}$$

With $t_{1/2,A} = 1$ hr and $t_{1/2,B} = 6$ hr, we have $\lambda_A = \ln(2)/1$ and $\lambda_B = \ln(2)/6$.

$$\frac{\lambda_A}{\lambda_B} = 6 \quad \text{and} \quad \lambda_A - \lambda_B = \frac{5}{6}\ln(2)$$

$$T = \frac{\ln(6)}{\frac{5}{6}\ln(2)} \approx \mathbf{3.10}\,\text{hours}$$

The maximum quantity $X = N_B(T)$. We use the relation $N_B(T) = \frac{\lambda_A}{\lambda_B} N_A(T) = 6N_A(T)$.

$$N_A(T) = N_0 e^{-\lambda_A T} = N_0 e^{-(\ln 2)T} = N_0 e^{-(\ln 2)\frac{6 \ln 6}{5 \ln 2}} = N_0 e^{-\frac{6}{5}\ln 6} = N_0(6^{-6/5})$$

$$X = 6N_A(T) = 6N_0(6^{-6/5}) = N_0 \cdot 6^{-1/5}$$

Given $N_0 = (3 \times 10^{-3}\,\text{mol}) \times (6.02 \times 10^{23}\,\text{mol}^{-1}) = 1.806 \times 10^{21}$.

$$X = \frac{1.806 \times 10^{21}}{6^{1/5}} \approx \frac{1.806 \times 10^{21}}{1.431} \approx \mathbf{1.26 \times 10^{21}}\ \text{nuclei}$$