2. Waves on a Sonometer

(a) Distance Between Wedges

For the fundamental mode, the frequency is $f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$. Tension $T = mg = (2.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 19.62\,\mathrm{N}$. Linear mass density $\mu = \rho A = \rho(\pi r^2) = (8830)\pi(0.75\times10^{-3})^2 \approx 0.0156\,\mathrm{kg/m}$. Rearranging for length L:

$$L = \frac{1}{2f_1}\sqrt{\frac{T}{\mu}} = \frac{1}{2(22.0)}\sqrt{\frac{19.62}{0.0156}} \approx \mathbf{0.806}\,\mathrm{m}$$

(b) Change in Distance with Cylinder in Water

The buoyant force F_B reduces the tension. Volume of cylinder $V = \frac{m}{\rho_{\mathrm{cyl}}} = \frac{2.0}{8400} \approx 2.381\times10^{-4}\,\mathrm{m}^3$. Buoyant force $F_B = \rho_{\mathrm{water}} g V = (1000)(9.81)(2.381\times10^{-4}) \approx 2.336\,\mathrm{N}$. New tension $T' = T - F_B = 19.62 - 2.336 = 17.284\,\mathrm{N}$. To keep the frequency constant, the new length L' must satisfy $\frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L'}\sqrt{\frac{T'}{\mu}}$.

$$L' = L\sqrt{\frac{T'}{T}} = 0.806\sqrt{\frac{17.284}{19.62}} \approx 0.757\,\mathrm{m}$$

The change in distance is $\Delta L = L - L' = 0.806 - 0.757 = 0.049\,\mathrm{m}$. The distance must be **decreased by 4.9 cm**.

(c) Beat Frequency

The cylinder is now in brine ($\rho_{\mathrm{brine}} = 1220\,\mathrm{kg/m^3}$). New buoyant force $F'_B = \rho_{\mathrm{brine}} g V = (1220)(9.81)(2.381\times10^{-4}) \approx 2.850\,\mathrm{N}$. New tension $T'' = T - F'_B = 19.62 - 2.850 = 16.77\,\mathrm{N}$. New fundamental frequency with length $L = 0.806$ m:

$$f_1'' = \frac{1}{2L}\sqrt{\frac{T''}{\mu}} = \frac{1}{2(0.806)}\sqrt{\frac{16.77}{0.0156}} \approx 20.33\,\mathrm{Hz}$$

The frequency of the second overtone (3rd harmonic) is $f_3'' = 3f_1'' = 3\times20.33 \approx 61.0\,\mathrm{Hz}$. The beat frequency with a 64 Hz piano note is:

$$f_{\mathrm{beat}} = |f_3'' - f_{\mathrm{piano}}| = |61.0 - 64| = \mathbf{3.0}\,\mathrm{Hz}$$