Detailed Solutions: 33rd Singapore Physics Olympiad 2020

Question 1: Rotating Composite Rod

Problem Statement

A cylindrical rod of radius $r = 1.0$ cm and length $L = 1.0$ m consists of two sections of length $L/2 = 0.5$ m each. One section is zinc ($\rho_{Zn} = 7135\,\mathrm{kg/m^3}$), the other is copper ($\rho_{Cu} = 8940\,\mathrm{kg/m^3}$). The zinc end is pivoted at O. The rod is released from horizontal. Determine the angular velocity when vertical.

Solution

1. Geometric Properties and Mass Calculation First, we define the geometry and calculate the mass of each section.

- Radius $r = 0.01$ m
- Length of each section $l = L/2 = 0.5$ m
- Volume of each section $V = \pi r^2 l = \pi(0.01)^2(0.5) = 5\pi \times 10^{-5}\,\mathrm{m}^3$

Mass of the Zinc section (m_1):

$$m_1 = \rho_{Zn} V = 7135 \times (5\pi \times 10^{-5}) \approx 1.1208\,\mathrm{kg}$$

Mass of the Copper section (m_2):

$$m_2 = \rho_{Cu} V = 8940 \times (5\pi \times 10^{-5}) \approx 1.4043\,\mathrm{kg}$$

2. Moment of Inertia (I) We calculate the moment of inertia of the entire composite rod about the pivot O. The moment of inertia of a rod of mass m and length l about its center is $\frac{1}{12}ml^2$, and about its end is $\frac{1}{3}ml^2$.

- **Zinc Section (I_1):** This section acts as a rod pivoted at one end.

$$I_1 = \frac{1}{3}m_1 l^2 = \frac{1}{3}(1.1208)(0.5)^2 = 0.0934\,\text{kg m}^2$$

- **Copper Section (I_2):** This section is not pivoted at its end. Its center of mass is located at distance d_2 from the pivot O.

$$d_2 = l + \frac{l}{2} = 0.5 + 0.25 = 0.75\,\text{m}$$

Using the Parallel Axis Theorem ($I = I_{cm} + Md^2$):

$$I_2 = \frac{1}{12}m_2 l^2 + m_2 d_2^2$$

$$I_2 = \frac{1}{12}(1.4043)(0.5)^2 + (1.4043)(0.75)^2$$
$$I_2 = 0.02926 + 0.78992 = 0.8192\,\text{kg m}^2$$

Total Moment of Inertia:

$$I_{total} = I_1 + I_2 = 0.0934 + 0.8192 = 0.9126\,\text{kg m}^2$$

3. Gravitational Potential Energy Change (ΔU) The rod is released from horizontal and swings to vertical. The center of mass (CM) of each section drops.

- Drop in height of Zinc CM (h_1): $h_1 = l/2 = 0.25$ m
- Drop in height of Copper CM (h_2): $h_2 = l + l/2 = 0.75$ m

Total loss in potential energy:

$$\Delta U = m_1 g h_1 + m_2 g h_2$$

$$\Delta U = (1.1208)(9.81)(0.25) + (1.4043)(9.81)(0.75)$$

$$\Delta U = 2.7488 + 10.3321 = 13.0809\,\text{J}$$

4. Conservation of Energy The loss in potential energy is converted to rotational kinetic energy ($K_{rot} = \frac{1}{2}I\omega^2$).

$$\Delta U = \frac{1}{2}I_{total}\omega^2$$
$$13.0809 = \frac{1}{2}(0.9126)\omega^2$$
$$\omega^2 = \frac{2 \times 13.0809}{0.9126} = 28.667$$
$$\omega = \sqrt{28.667} \approx 5.354\,\mathrm{rad/s}$$

Answer: The angular velocity is **5.35 rad/s**.

Question 2: Doppler Effect and Optics

(a) Doppler Effect on a Swing

Problem: A student on a swing of length $L = 5$ m and angular amplitude $\theta_{max} = 45°$ hears sound from a stationary loudspeaker ($f_s = 400$ Hz). Speed of sound $c = 330$ m/s. Find the maximum and minimum frequencies heard.

Solution: 1. Determine Maximum Velocity of the Observer The student moves in a circular arc. The maximum speed v_{max} is attained at the lowest point of the swing (equilibrium position). Using Conservation of Energy:

$$mgh = \frac{1}{2}mv_{max}^2 \implies v_{max} = \sqrt{2gh}$$

The height change h from the highest point (45°) to the lowest point is:

$$h = L(1 - \cos\theta) = 5(1 - \cos 45°) = 5\left(1 - \frac{1}{\sqrt{2}}\right) \approx 1.4645\,\text{m}$$

$$v_{max} = \sqrt{2(9.81)(1.4645)} \approx 5.360\,\text{m/s}$$

2. Apply Doppler Effect Formula The source is stationary, and the observer is moving.

$$f_{obs} = f_s\left(\frac{c \pm v_{obs}}{c}\right)$$

- **Maximum Frequency:** Occurs when the student moves **towards** the source at maximum speed.

$$f_{max} = 400\left(\frac{330 + 5.36}{330}\right) = 400\left(1 + \frac{5.36}{330}\right) \approx 406.497\,\text{Hz}$$

- **Minimum Frequency:** Occurs when the student moves **away** from the source at maximum speed.

$$f_{min} = 400\left(\frac{330 - 5.36}{330}\right) = 400\left(1 - \frac{5.36}{330}\right) \approx 393.503\,\text{Hz}$$

Answer: Max Frequency: **407 Hz**, Min Frequency: **394 Hz**.

(b) Compound Microscope

Problem: Objective focal length $f_o = 6.0$ mm, Eyepiece focal length $f_e = 40.0$ mm, Separation $L = 200$ mm. Final image at Near Point $D = 250$ mm.

Solution: 1. Eyepiece Calculations The final image formed by the eyepiece is virtual and located at the near point, so the image distance $v_e = -250$ mm. Using the thin lens equation $\frac{1}{f_e} = \frac{1}{v_e} + \frac{1}{u_e}$:

$$\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e} = \frac{1}{40} - \frac{1}{-250} = \frac{1}{40} + \frac{1}{250}$$

$$\frac{1}{u_e} = \frac{25+4}{1000} = \frac{29}{1000} \implies u_e = \frac{1000}{29} \approx 34.48\,\text{mm}$$

2. Objective Calculations The distance between lenses L is sum of the image distance of the objective v_o and object distance of eyepiece u_e.

$$L = v_o + u_e \implies v_o = L - u_e = 200 - 34.48 = 165.52\,\text{mm}$$

Now, find the object distance for the objective u_o:

$$\frac{1}{u_o} = \frac{1}{f_o} - \frac{1}{v_o} = \frac{1}{6} - \frac{1}{165.52}$$

$$\frac{1}{u_o} \approx 0.16667 - 0.00604 = 0.16063$$

$$u_o = \frac{1}{0.16063} \approx 6.225\,\text{mm}$$

3. Magnification

$$M = M_{objective} \times M_{eyepiece} = \left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$$

$$M = \left(\frac{165.52}{6.225}\right)\left(1 + \frac{250}{40}\right) = (26.59)(7.25) \approx 192.8$$

Answer: Object distance: **6.23 mm**, Magnification: **193**.

Question 3: Electromagnetic Induction

Problem Statement

A rod of length l and resistance R rotates with angular velocity ω in a uniform magnetic field B. It is connected to an external resistor R_0.

Solution

(a) Induced EMF ($\mathcal{E}$) Consider a small differential element of the rod of length dr at a distance r from the pivot. The linear velocity of this element is $v = r\omega$. The motional EMF induced across this small element is:

$$d\mathcal{E} = Bvdr = B(r\omega)dr$$

Integrating from the pivot ($r = 0$) to the tip ($r = l$):

$$\mathcal{E} = \int_0^l B\omega r\, dr = B\omega \left[\frac{r^2}{2}\right]_0^l = \frac{1}{2}B\omega l^2$$

(b) Electric Power (P_{elec}) The rod acts as a voltage source $\mathcal{E}$ with internal resistance R, connected in series to R_0. The current I flowing in the circuit is:

$$I = \frac{\mathcal{E}}{R_{total}} = \frac{\mathcal{E}}{R + R_0}$$

The power dissipated in the external resistor R_0 is:

$$P_{R_0} = I^2 R_0 = \left(\frac{B\omega l^2}{2(R + R_0)}\right)^2 R_0 = \frac{B^2\omega^2 l^4 R_0}{4(R + R_0)^2}$$

(c) Origin of Power The electric power originates from the **mechanical work done by the external torque** required to keep the rod rotating.

1. The induced current I flowing through the rod interacts with the magnetic field to produce a magnetic force dF on each element dr:

$$dF = IBdr$$

2. This force creates a torque $d\tau$ about the pivot that opposes the rotation (Lenz's Law):

$$d\tau = r \cdot dF = rIBdr$$

3. The total retarding torque is:

$$\tau = \int_0^l IBr\, dr = IB\frac{l^2}{2}$$

4. The mechanical power required to maintain angular velocity ω is:

$$P_{mech} = \tau\omega = \left(IB\frac{l^2}{2}\right)\omega = I\left(\frac{1}{2}B\omega l^2\right) = I\mathcal{E}$$

Since $P_{mech} = P_{electrical}$, the source of energy is the mechanical work.

Question 4: Electron-Hydrogen Collision

Problem Statement

An electron collides with a ground state Hydrogen atom. The atom excites and then emits two photons, one with $\lambda_1 = 656.3$ nm. The scattered electron has a de Broglie wavelength $\lambda_e = 1.915$ nm.

Solution

(a) Wavelength of the Second Photon The Hydrogen atom is initially in the ground state ($n = 1$). The photon with $\lambda_1 = 656.3$ nm corresponds to an energy:

$$E_1 = \frac{hc}{\lambda_1} = \frac{1240\,\text{eV nm}}{656.3\,\text{nm}} \approx 1.89\,\text{eV}$$

This matches the energy difference for the Balmer transition ($n = 3 \to n = 2$). Since the atom must eventually return to the ground state ($n = 1$), and it emits *two* photons, the decay chain must be:

$$n = 3 \xrightarrow{\lambda_1} n = 2 \xrightarrow{\lambda_2} n = 1$$

We need to find the wavelength λ_2 for the $n = 2 \to n = 1$ transition. The energy levels of Hydrogen are $E_n = -\frac{13.6}{n^2}$ eV.

$$\Delta E_{2\to1} = E_2 - E_1 = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = -13.6(-0.75) = 10.2\,\text{eV}$$

$$\lambda_2 = \frac{hc}{\Delta E} = \frac{1240\,\text{eV nm}}{10.2\,\text{eV}} \approx 121.6\,\text{nm}$$

(b) Initial Speed of the Electron We use Conservation of Energy.

$$K_{initial} = K_{final} + \Delta E_{atom}$$

1. **Atom Excitation Energy (ΔE_{atom}):** The atom went from $n = 1$ to $n = 3$.

$$\Delta E_{atom} = E_3 - E_1 = -13.6\left(\frac{1}{9} - 1\right) = 13.6\left(\frac{8}{9}\right) \approx 12.09\,\text{eV}$$

2. **Final Electron Kinetic Energy (K_{final}):** Using the de Broglie wavelength $\lambda_e = 1.915$ nm.

$$K_{final} = \frac{p^2}{2m} = \frac{(h/\lambda_e)^2}{2m} = \frac{(hc)^2}{2(mc^2)\lambda_e^2}$$

Using $hc = 1240$ eV nm and $mc^2 = 0.511$ MeV $= 511000$ eV:

$$K_{final} = \frac{(1240)^2}{2(511000)(1.915)^2} = \frac{1537600}{3748239} \approx 0.410\,\text{eV}$$

3. **Total Initial Energy and Speed:**

$$K_{initial} = 12.09 + 0.410 = 12.50\,\text{eV}$$

$$K_{initial} = 12.50 \times 1.60 \times 10^{-19}\,\text{J} = 2.0 \times 10^{-18}\,\text{J}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.0 \times 10^{-18})}{9.11 \times 10^{-31}}} = \sqrt{4.39 \times 10^{12}} \approx 2.10 \times 10^6\,\text{m/s}$$

Answer: $\lambda_2 \approx \mathbf{122}$ **nm**, $v \approx \mathbf{2.10 \times 10^6}$ m/s.

Question 5: Kinematics and Orbits

(a) Projectile hitting Moving Vehicle

Problem: A plane at height $h = 1200$ m flying at $v_p = 150$ m/s fires a projectile with relative speed u forward. It hits a vehicle moving at $v_v = 40$ m/s in the same direction, located distance d ahead. Given $d = 5 \times d_{min}$, find impact speed.

Solution: 1. Time of Flight:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1200)}{9.81}} \approx 15.64\,\text{s}$$

2. Horizontal Kinematics: Projectile ground speed $v_x = v_p + u = 150 + u$. Vehicle position: $x_v = d + 40t$. Projectile position: $x_p = (150 + u)t$. For impact: $(150 + u)t = d + 40t \implies d = (110 + u)t$. **3. Minimum Distance:** d is minimum when u is minimum ($u = 0$, projectile dropped).

$$d_{min} = 110 \times 15.64 \approx 1720.4\,\text{m}$$

4. Calculate u for $d = 5d_{min}$:

$$d = 5(110t) = 550t$$

$$(110 + u)t = 550t \implies 110 + u = 550 \implies u = 440\,\text{m/s}$$

5. Impact Velocity: Horizontal component: $v_{xf} = 150 + 440 = 590$ m/s. Vertical component: $v_{yf} = gt = 9.81(15.64) \approx 153.4$ m/s. Total speed:

$$v = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{590^2 + 153.4^2} \approx \sqrt{348100 + 23531} \approx 609.6\,\text{m/s}$$

(b) Satellite Photos

Problem: Satellite at $h = 400$ km orbits Earth. Synodic period with a point P on equator.

Solution: 1. Satellite Period: Radius $r = 6371 + 400 = 6771$ km.

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(6.771 \times 10^6)^3}{9.81(6.371 \times 10^6)^2}} \approx 5554\,\text{s} \approx 1.543\,\text{h}$$

2. Relative Angular Velocity: Satellite angular speed $\omega_s = 2\pi/1.543$. Earth angular speed $\omega_e = 2\pi/24$. The satellite overtakes P. Relative speed $\omega_{rel} = \omega_s - \omega_e$. Time between photos (passes):

$$\Delta t = \frac{2\pi}{\omega_{rel}} = \frac{1}{\frac{1}{1.543} - \frac{1}{24}} = \frac{1}{0.648 - 0.0417} \approx 1.649\,\mathrm{h}$$

3. Number of Photos: Total time = 24 hours.

$$N = \frac{24}{1.649} \approx 14.55$$

This means the satellite completes 14 full passes relative to P.

Answer: 14 photos (or 15 if counting the initial one at t=0).

Question 6: Electrostatics and SHM

(a) Particle in Charged Sphere

Problem: Particle of mass m, charge $-q$ moves in a tunnel through a sphere of uniform charge density ρ.

Solution: 1. Force Analysis: Inside a uniformly charged sphere, the electric field at radius r is derived from Gauss's Law:

$$E(r) \cdot 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{\rho(\frac{4}{3}\pi r^3)}{\epsilon_0} \implies E(r) = \frac{\rho r}{3\epsilon_0}$$

The force on charge $-q$ is:

$$F = -qE(r) = -\left(\frac{q\rho}{3\epsilon_0}\right) r$$

Since force is restoring and proportional to distance ($F = -kr$), the motion is **Simple Harmonic Motion**.

2. Speed at Center: Using Conservation of Energy between surface ($r = R$) and center ($r = 0$). Potential difference inside sphere: $V(0)-V(R) = \int_0^R E(r)dr = \frac{\rho}{3\epsilon_0}\int_0^R rdr = \frac{\rho R^2}{6\epsilon_0}$. Kinetic Energy gain = Potential Energy loss.

$$\frac{1}{2}mv^2 = q\Delta V = q\frac{\rho R^2}{6\epsilon_0}$$

$$v = \sqrt{\frac{q\rho R^2}{3m\epsilon_0}} = R\sqrt{\frac{q\rho}{3m\epsilon_0}}$$

(b) Particle under Two Forces

Problem: Forces towards A ($F_A = \lambda x$) and B ($F_B = \lambda y$) act on a particle. $x + y = L$.

Solution: Let A be at $x = 0$, B be at $x = L$. Particle at position x. Distance to B is $L - x$. Net Force (taking right as positive):

$$F_{net} = F_B - F_A = \lambda(L - x) - \lambda x = \lambda L - 2\lambda x$$

Equilibrium position ($F_{net} = 0$):

$$\lambda L - 2\lambda x_{eq} = 0 \implies x_{eq} = L/2$$

Define displacement u from equilibrium: $x = L/2 + u$.

$$F_{net} = \lambda L - 2\lambda(L/2 + u) = \lambda L - \lambda L - 2\lambda u = -2\lambda u$$

This is a restoring force $F = -k_{eff}u$ where $k_{eff} = 2\lambda$. Thus, the motion is **Simple Harmonic Motion**.

Question 7: Thermodynamics

(a) Adiabatic Expansion of Gas

Problem: Gas at P_0, V_0, T_0 expands adiabatically to pressure P_1. Mass is lost (open system/puffing model). Then heated back to T_0.

Solution: 1. Expansion Phase: The gas remaining in the container acts as if it expanded from an initial volume $V_{initial} < V_0$ to fill V_0. Adiabatic relation: $T^\gamma P^{1-\gamma} =$ const. Temperature after expansion T_1:

$$T_1 = T_0 \left(\frac{P_1}{P_0}\right)^{1-1/\gamma}$$

2. Heating Phase: Gas warms from T_1 to T_0 at constant volume V_0. Pressure increases from P_1 to final pressure P_f. Ideal gas law at constant volume: $\frac{P_1}{T_1} = \frac{P_f}{T_0}$.

$$P_f = P_1\frac{T_0}{T_1} = P_1 \left(\frac{P_0}{P_1}\right)^{1-1/\gamma} = P_1^{1/\gamma} P_0^{1-1/\gamma}$$

(This formula allows determining γ if P_f is measured).

(b) Heat Conduction

Problem: Copper rod between hot bath T_H and ice/water mixture $T_C = 0°$C. Time to melt ice and heat water.

Solution: 1. Time to melt ice (t_1): Heat needed $Q_1 = m_{ice}L_f$. Heat current $\dot{Q} = kA\frac{T_H - 0}{L}$.

$$t_1 = \frac{Q_1}{\dot{Q}} = \frac{m_{ice}L_f L}{kAT_H}$$

2. Time to heat water (t_2): Total mass $M = m_{ice} + m_{water}$. Differential equation for temperature T of water:

$$Mc\frac{dT}{dt} = kA\frac{T_H - T}{L}$$

$$\int_0^{T_f} \frac{dT}{T_H - T} = \int_0^{t_2} \frac{kA}{McL}dt$$

$$-\ln(T_H - T)\Big|_0^{T_f} = \frac{kA}{McL}t_2$$

$$\ln\left(\frac{T_H}{T_H - T_f}\right) = \frac{kA}{McL}t_2 \implies t_2 = \frac{McL}{kA}\ln\left(\frac{T_H}{T_H - T_f}\right)$$

Total time $t = t_1 + t_2$.

Question 8: Sound and Proton Motion

(a) Accelerating Sound Source

Problem: Source B accelerates from rest ($a = 0.5$ m/s^2) to point P, then travels at constant v_B. Beat frequency with stationary source A ($f = 256$ Hz) is 8 Hz. Find distance to P.

Solution: 1. Determine Final Velocity v_B: Observed frequency of B is lower (moving away): $f_{obs} = 256 - 8 = 248$ Hz. Doppler formula:

$$248 = 256\left(\frac{330}{330 + v_B}\right)$$

$$330 + v_B = 330\left(\frac{256}{248}\right) = 330\left(\frac{32}{31}\right) \approx 340.645$$
$$v_B = 340.645 - 330 = 10.645\,\text{m/s}$$

2. Calculate Distance s: Using kinematics $v^2 = u^2 + 2as$:

$$(10.645)^2 = 0 + 2(0.5)s$$

$$s = (10.645)^2 \approx 113.3\,\text{m}$$

Answer: Distance is **113 m**.

(b) Protons in Magnetic Field

Problem: Protons ($K = 10$ MeV) enter B-field ($B = 1.5$ T, width 1.0 m). Find deflection angle.

Solution: 1. Gyroradius: $K = 10$ MeV $= 1.6 \times 10^{-12}$ J. $v = \sqrt{2K/m} \approx 4.38 \times 10^7$ m/s.
$$R = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27})(4.38 \times 10^7)}{(1.60 \times 10^{-19})(1.5)} \approx 0.305\,\text{m}$$
2. Geometry: The protons enter at $x = 0$. They follow a circular path in the xz-plane. The maximum penetration depth into the field is the diameter $2R \approx 0.61$ m? No, if entering normal to the boundary, the penetration is $2R$ only if it does a full circle? Actually, if velocity is along x, force is along z (or y). Path is a circle tangent to the entry velocity vector. The protons enter at $x = 0$. The circle curves back. The maximum

x-coordinate reached is the radius $R \approx 0.305$ m. Since the field width is 1.0 m, and 0.305 m < 1.0 m, the protons never reach the other side. They complete a semi-circle and exit at $x = 0$, moving in the $-x$ direction. The angle between initial $(+x)$ and final $(-x)$ velocity is $180°$.

Answer: Deflection angle is **180 degrees**.

Question 9: Relativity and Muon Decay

Problem Statement

Muons are produced in the upper atmosphere at a height $H = 10$ km. They travel towards the Earth at a speed $v = 0.99c$.

(i) Calculate the time taken for a muon to reach the foot of the mountain according to an observer on Earth.

(ii) In a typical experiment, a counter placed at the foot of the mountain records 422 muons in 1 hour. Why does the result of this experiment differ so much from the expectation based on your result in part (i)? (Note: The proper mean lifetime of a muon is $\tau \approx 2.2 \times 10^{-6}$ s).

(iii) What is the "height" of the mountain according to the muons?

(iv) While the muons are travelling downward to the earth, another particle also travels in the same direction with speed u (value missing in snippet, assumed to be provided in full text, e.g., $0.5c$). What is the velocity of this particle in the muon's inertial frame?

Solution

(i) Time of Flight (Earth Frame) The time t taken to travel the distance H at speed v is given by simple kinematics:

$$t = \frac{H}{v}$$

Given:

- $H = 10 \text{ km} = 10 \times 10^3 \text{ m}$
- $v = 0.99c = 0.99 \times 3.00 \times 10^8 \text{ m/s}$

$$t = \frac{10000}{0.99 \times 3.00 \times 10^8} = \frac{10000}{2.97 \times 10^8} \approx 3.367 \times 10^{-5}\,\text{s}$$

Answer: The time taken is approximately $\mathbf{33.7}\,\boldsymbol{\mu}\mathbf{s}$.

(ii) Experimental Result vs. Classical Expectation To understand the discrepancy, we compare the travel time to the muon's lifetime.

- **Classical Prediction:** The proper mean lifetime of a muon is $\tau \approx 2.2\,\mu$s. The travel time calculated in part (i) is $t \approx 33.7\,\mu$s. Ratio $t/\tau \approx 33.7/2.2 \approx 15.3$. Classically, the fraction of muons surviving would be $N/N_0 = e^{-t/\tau} = e^{-15.3} \approx 2 \times 10^{-7}$. This means essentially **zero** muons should be detected at the ground.
- **Experimental Reality:** The counter records 422 muons, which is a significant number.
- **Explanation:** The result differs because of **Time Dilation** (Special Relativity). To an observer on Earth, the moving muon's clock runs slower by the Lorentz factor γ.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.99^2}} = \frac{1}{\sqrt{0.0199}} \approx 7.09$$

  The observed mean lifetime becomes $\tau_{obs} = \gamma\tau \approx 7.09 \times 2.2\,\mu\text{s} \approx 15.6\,\mu$s. The new decay exponent is $t/\tau_{obs} \approx 33.7/15.6 \approx 2.16$. Survival fraction $e^{-2.16} \approx 0.115$ (or 11.5%), which explains why a significant number are detected.

Answer: The experiment detects muons because of **relativistic time dilation**, which extends the muons' observable lifetime in the Earth frame, allowing them to travel the 10 km distance before decaying.

(iii) Height in the Muon Frame

In the muon's frame of reference, the Earth (and the mountain) is moving towards it at speed $v = 0.99c$. Due to **Length Contraction**, the height of the atmosphere H appears contracted to H'.

$$H' = \frac{H}{\gamma}$$

Using $\gamma \approx 7.09$:

$$H' = \frac{10000\text{ m}}{7.09} \approx 1410\text{ m}$$

Answer: The height is approximately **1.41** km.

(iv) Relative Velocity Let the velocity of the muon be v and the velocity of the other particle be u, both in the Earth frame S. We need to find the velocity of the particle u' in the muon's frame S'.

- $v = 0.99c$ (downward)
- u (downward)

Using the Relativistic Velocity Addition formula:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Example Calculation (assuming $u = 0.5c$ based on typical problem patterns): If $u = 0.5c$:

$$u' = \frac{0.5c - 0.99c}{1 - (0.5)(0.99)} = \frac{-0.49c}{1 - 0.495} = \frac{-0.49c}{0.505} \approx -0.97c$$

The negative sign indicates the particle moves upwards relative to the muon (the muon overtakes it).

Answer: The velocity is $u' = \dfrac{u - 0.99c}{1 - \frac{0.99u}{c}}$.