

7 RLC Circuit

7.1 (a) Show $V = Ae^{-\alpha t} \cos(\omega t)$ is a solution and find ω .

Given: $\frac{d^2V}{dt^2} + \frac{R}{L} \frac{dV}{dt} + \frac{1}{LC} V = 0$. Substituting $V(t) = Ae^{-\alpha t} \cos(\omega t)$ and its derivatives into the equation yields:

$$\cos(\omega t) \left[\alpha^2 - \omega^2 - \frac{R}{L} \alpha + \frac{1}{LC} \right] + \sin(\omega t) \left[2\alpha\omega - \frac{R}{L} \omega \right] = 0$$

For this to be true for all t , the coefficients must be zero. From the $\sin(\omega t)$ term: $2\alpha\omega - \frac{R}{L}\omega = 0 \implies \alpha = \frac{R}{2L}$. From the $\cos(\omega t)$ term, substituting $\alpha = R/2L$:

$$\begin{aligned} \left(\frac{R}{2L} \right)^2 - \omega^2 - \frac{R}{L} \left(\frac{R}{2L} \right) + \frac{1}{LC} &= 0 \\ \omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2} &\implies \omega = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L} \right)^2} \end{aligned}$$

7.2 (b)(i) Calculate oscillation frequency and decay time.

Given: $C = 15.0 \times 10^{-9}$ F, $L = 0.22 \times 10^{-3}$ H, $R = 75.0 \Omega$. $\alpha = \frac{R}{2L} = \frac{75.0}{2 \times 0.22 \times 10^{-3}} \approx 1.705 \times 10^5 \text{ s}^{-1}$. $\omega = \sqrt{\frac{1}{(0.22 \times 10^{-3})(15.0 \times 10^{-9})} - (1.705 \times 10^5)^2} \approx 5.234 \times 10^5 \text{ rad/s}$. Oscillation frequency $f = \frac{\omega}{2\pi} \approx \frac{5.234 \times 10^5}{2\pi} \approx 8.33 \times 10^4 \text{ Hz} = \mathbf{83.3 \text{ kHz}}$. Decay to 10%: $e^{-\alpha t} = 0.10 \implies t = \frac{\ln(10)}{\alpha} \approx \frac{2.3026}{1.705 \times 10^5} \approx 1.35 \times 10^{-5} \text{ s} = \mathbf{13.5 \mu s}$.

7.3 (b)(ii) Calculate the value of R for a critically damped circuit.

Critical damping occurs when $\omega = 0$.

$$\frac{1}{LC} = \left(\frac{R_{\text{crit}}}{2L} \right)^2 \implies R_{\text{crit}} = 2\sqrt{\frac{L}{C}}$$

$$R_{\text{crit}} = 2\sqrt{\frac{0.22 \times 10^{-3}}{15.0 \times 10^{-9}}} \approx 242.2 \Omega$$

The resistance is approximately **242 Ω** .