7. Inelastic Collision and Rotational Motion

1. **Block Sliding Down:** By energy conservation, $v = \sqrt{2gh} = \sqrt{2(9.80)(0.20)} \approx 1.98\,\mathrm{m/s}$.

2. **The Collision:** By conservation of angular momentum about the pivot O:

$$L_i = L_f$$
$$L(mv) = I_{total}\omega$$

The total moment of inertia is $I_{total} = I_{rod} + I_{block} = \frac{1}{3}ML^2 + mL^2 = \left(\frac{1}{3}M + m\right)L^2$.

$$\omega = \frac{mv}{\left(\frac{1}{3}M + m\right)L} = \frac{(0.050)(1.98)}{\left(\frac{1}{3}(0.100) + 0.050\right)(0.40)} \approx 2.97\,\mathrm{rad/s}$$

3. **The Swing Up:** By conservation of mechanical energy:

$$K_i = \Delta U_g$$
$$\frac{1}{2}I_{total}\omega^2 = \left(\frac{1}{2}M + m\right)gL(1 - \cos\theta)$$

Calculating the numerical values:

$$I_{total} = \left(\frac{1}{3}(0.1) + 0.05\right)(0.4)^2 \approx 0.01333\,\mathrm{kg \cdot m^2}$$
$$K_i = \frac{1}{2}(0.01333)(2.97)^2 \approx 0.0588\,\mathrm{J}$$
$$\Delta U_g = \left(\frac{1}{2}(0.1) + 0.05\right)(9.8)(0.4)(1 - \cos\theta) = 0.392(1 - \cos\theta)$$

Equating them: $0.0588 = 0.392(1 - \cos\theta) \implies 1 - \cos\theta \approx 0.15$.

$$\cos\theta = 0.85 \implies \theta = \arccos(0.85) \approx \mathbf{31.8}^\circ$$