Question 4: Thermodynamics and Statistical Mechanics

(a) Entropy Change

For an isobaric (constant pressure) process, the change in entropy is:

$$\Delta S = nC_P \ln\left(\frac{T_2}{T_1}\right)$$

From the ideal gas law at constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, so $\frac{T_2}{T_1} = \frac{V_2}{V_1}$. Given $V_2 = 2V_1$, we have $\frac{T_2}{T_1} = 2$. For a diatomic ideal gas, $C_P = \frac{7}{2}R$.

$$\Delta S = nC_P \ln\left(\frac{V_2}{V_1}\right) = (1)\left(\frac{7}{2}R\right)\ln(2)$$

$$\Delta S = \frac{7}{2}(8.31)\ln(2) \approx 20.16\,\mathrm{J/K}$$

Answer: The change in entropy is **20.2 J/K**.

(b) Number of Gas Particles

The probability of a particle having energy E is $p(E) \propto e^{-E/kT}$. Taking the ratio of probabilities for the first two states:

$$\frac{p(E_2)}{p(E_1)} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-(E_2-E_1)/kT}$$

$$\frac{0.23}{0.63} = e^{-(0.0129-0.0043)/kT} = e^{-0.0086/kT}$$

$$\ln\left(\frac{0.23}{0.63}\right) = -1.0076 = -\frac{0.0086}{kT}$$

$$kT = \frac{0.0086}{1.0076} \approx 0.008535\,\mathrm{eV}$$

Now find the probability for the third state, $p(E_3)$:

$$\frac{p(E_3)}{p(E_1)} = e^{-(E_3-E_1)/kT}$$

$$p(E_3) = p(E_1)e^{-(0.0215-0.0043)/kT} = 0.63 \cdot e^{-0.0172/0.008535}$$

$$p(E_3) = 0.63 \cdot e^{-2.015} \approx 0.63 \times 0.1333 \approx 0.0840$$

For 1 mole of gas, $N_{total} = N_A = 6.02 \times 10^{23}$ particles.

$$N_3 = N_{total} \cdot p(E_3) = (6.02 \times 10^{23})(0.0840) \approx 5.06 \times 10^{22}$$

Answer: The number of particles is approximately $\mathbf{5.06 \times 10^{22}}$.
