Twenty-Seventh Singapore Physics Olympiad
Theoretical Paper

Instructions to Candidates

1. This is a **four-hour** test.

2. This paper consists of **TEN** (10) questions printed on **SEVEN** (7) printed pages. Page **SEVEN** (7) is a Table of Fundamental Constants in Physics which may be useful for your calculations.

3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [   ].

4. Write your name legibly on the top right hand corner of every answer sheet you submit.

5. Begin each answer on a fresh sheet of paper.

6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.

7. No books or documents relevant to the test may be brought into the examination hall.

1. **Yo-Yo!** A yo-yo of mass M lies on a smooth horizontal table as shown below. The moment of inertia about the center may be taken as $\frac{1}{2}MA^2$. A string is pulled with force F from the inner radius B as indicated below.
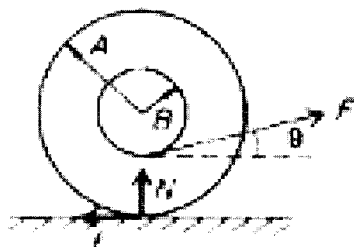

Figure 1: Yo-yo on a smooth table

(a) Draw diagrams to show which direction the yo-yo will roll if $\theta = 0$, $\pi/2$, π. [2]

(b) For what value of θ will the yo-yo slide without rolling independent of the roughness (coefficient of friction) of the table or the magnitude of F? [3]

(c) At what angle θ will the yo-yo roll, independent of the smoothness of the table? [5]

2. **Yo-yo continued** A yo-yo is pulled by its string along a horizontal surface without slipping. The horizontal velocity of the end of the string remains equal to v. A bar is pivoted as shown and remains supported by the yo-yo. Find the angular speed of the bar ω as a function of the angle θ. The outer and the inner radii of the yo-yo are R and r respectively. [10]
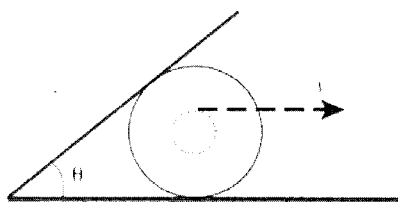

Figure 2: Pulled yo-yo and supported bar

3. **Cylindrical capacitors**

   (a) A long cylindrical conductor has a radius a and a linear charge density $+\lambda$. It is surrounded by a coaxial cylindrical conducting shell with inner radius b and linear charge density $-\lambda$. Calculate the capacitance per unit length for this capacitor, assuming that there is vacuum in the space between the cylinders. [4]

   (b) Consider a cylindrical capacitor of length l and of radii a and b as defined in (a), write down the conditions for which increasing l by 10% would be more effective in increasing its capacitance than increasing a by 10%. [6]

4. **Spring with mass** Problems involving springs often consider the springs to be massless. Of course, that is not true in reality. Here, we consider a spring with mass M, equilibrium length L_o, and spring constant k.

   (a) The spring above has one end fixed and the other end moving with speed v. Assume the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the spring. Express the kinetic energy of the spring in terms of M and v. [5]

   (b) Write down the conservation of energy equation of a spring-mass system where the mass m is moving at the end of a *massless* spring. What is the angular frequency ω for such a system? [2]

   (c) By considering the procedure in (b), what is the angular frequency ω of the spring mentioned in (a)? If the effective mass M' of the spring is defined as $\omega = \sqrt{\frac{k}{M'}}$, what is M' in terms of M? [3]

5. **Catching the wave**

   The speed of of a travelling wave v along a string is a function of its tension T and its linear mass density ρ in the form $v = T^\alpha \rho^\beta$.

   (a) Find α and β. [2]

   (b) A uniform, almost inextensible string of length l and total mass M is suspended vertically and tapped at the top end so that a transverse impulse runs down it. At the same moment, a body is released from rest and falls freely from the top of the string. How far from the bottom does the body pass the impulse? [8]

6. **Playing with Horizontal Plates** A capacitor system is made of 2 horizontal square plates of sides l separated by a distance d.

(a) If the uniform electric force field between them is E. what are the charges on the respective plates? Express your answer in terms of E. l and other fundamental constansts. [3]

(b) What is the force which the plates exert on each other when the potential difference between then is V? [2]

(c) The upper plate is fixed and the lower plate is resting on a table top and is free to move. The potential difference V is gradually increased till the lower plate suddenly jumps up. Calculate the value of V at which this occurs. given that the lower plate is made of aluminium (density 2.71×10^3kg m^{-3}) of thickness 8.00μm and that the initial separation is 12.7mm. [3]

(d) What is the energy per unit volume stored in the capacitor just before the lower plate jumps up? [2]

7. (a) At low temperatures . the heat capacity of a substance is dependent on its temperature. A thermally insulated piece of the substance is heated under atmospheric pressure by an electric cuurent so that it receives electric energy at a constant power P. This leads to an increase of the absolute temperature T of the substance with time t as $T(t) = T_\circ[1 + a(t - t_\circ)]^{\frac{1}{3}}$. Here. a, $t_\circ$ and $T_\circ$ are constants. Determine the heat capacity C_p of the substance as a function of temperature. [5]

(b) The index of refraction of glass can be increased by diffusing in impurities. It is then possible to make a lens of constant thickness. Given a disk of radius a and thickness d, express the index of refraction $n(r)$ as a function of $n_\circ$ (refractive index at the centre of the lens), r (radius from the centre of the lens), d and F. You may assume $d << a$. [5]

8. (a) After falling a time t in the atmosphere. an outer shell of a meteorite of thickness x will have been heated significantly. The thickness can be estimated by dimensional analysis as the following thermodynamic parameters: $x = t^\alpha \rho^\beta c^\gamma k^\sigma$.

ρ is the meteorite's density.

c is the metorite's specific heat capacity.

k is the meteorite's thermal conductivity.

Determine by dimensional analysis the value of the four powers α. β. γ and σ. [5]

(b) A beam of 10^6 K_l mesons per second with $\beta = \frac{v}{c} = \frac{1}{\sqrt{2}}$ is observed to interact with a lead brick such that the K_l mesons are being converted to K_s mesons with the internal state of the lead brick identical before and after the reaction. The direction of motion of the incoming K_l and outgoing K_s may also be considered to be identical. Using

$m(K_l) = 5 \times 10^8 eV/c^2$.
$m(K_l) - m(K_s) = 3.5 \times 10^{-6} eV/c^2$.

give the magnitude and direction of the avearge force exerted on the brick by this process. [5]

9. Among the first successes of the interpretation by Ampere of magnetic phenomena, we have the computation of the magnetic field B generated by wires carrying an electric current. as compared to early assumptions originally made by Biot and Savart.

A particularly interesting case is that of a very long thin wire, carrying a constant current i. made out of two rectilinear sections and bent in the form of a "V", with angular half-span α(see figure). According to Ampere's computations, the magnitude B of the magnetic field in a given point P lying on the axis of the "V", outside of it and at a distance d from its vertex. is proportional to $\tan(\frac{\alpha}{2})$. Ampere's work was later embodied in Maxwell's electromagnetic theory. and is universally accepted.
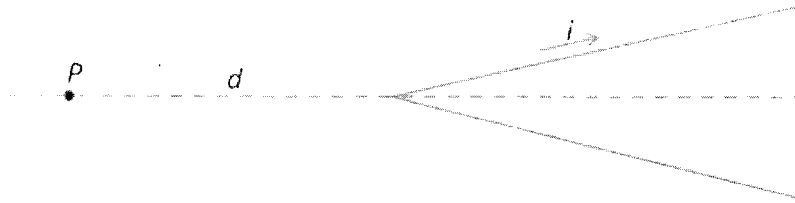

Figure 3: Configuration of V-shaped wire

Using our knowledge of electromagnetism

(a) find an expression of the magnetic field B in terms of the given quantities and any other relevant constants. Also, indicate the direction of the magnetic field [4]

(b) Compute the field at a point P^* symmetric to P with respect to the vertex i.e along the axis and at the same distance d but inside the "V". [4]

10. **Radioactive dating of sample** ^{87}Rb (element no 37) decays into the stable isotope ^{87}Sr (element no. 38) with a half life of $T_{\frac{1}{2}} = 4.9 \times 10^{10}$ year, relative to the stable isotope ^{86}Sr. At the point of formation, the ratio $^{87}Sr/^{86}Sr$ was identical for all minerals in the sample while the ratio $^{87}Rb/^{86}Sr$ was quite different. As time passes on, the amount of ^{87}Rb decreases by decay, while consequently the amount of ^{87}Sr increases. As a result, the ratio of $^{87}Sr/^{86}Sr$ will be different today. In the following figure(a), the points on the horizontal line refer to the ratio $^{87}Rb/^{86}Sr$ in different minerals at the formation of the sample.

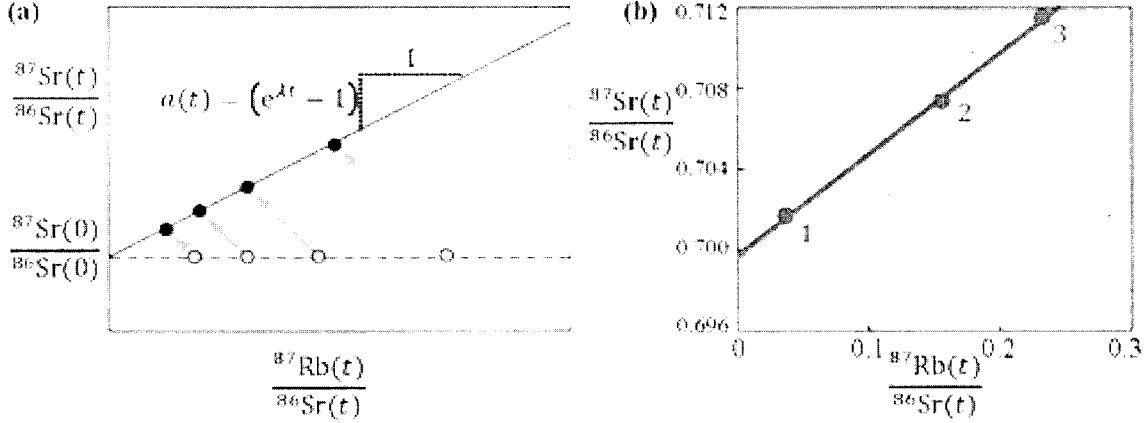Figure 4: **(a)** The ratio $^{87}Sr/^{86}Sr$ in different minerals at the time of formation, $t = 0$ (open circles) and at present time (filled circles) **(b)** The isochron-line for three different minerals taken from the sample at present time

(a) Write down the nuclear reaction depicting the decay scheme for the transformation of $^{87}_{37}Rb$ to $^{87}_{38}Rb$. [2]

(b) Show that the present-time ratio $^{87}Sr/^{86}Sr$ was plotted versus the present-time ratio $^{87}Rb/^{86}Sr$ in different mineral samples from the same sample forms a straight line (the isochron line), with slope $a(t) = (e^{\lambda t} - 1)$. Here t is the time since the formation of the minerals, while λ is the decay constant inversely proportional to half-life $T_{\frac{1}{2}}$. [5]

(c) Determine the age τ_m of the sample using the isochron line as in the above diagram. [3]

END OF PAPER

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational value
Avogadro's number	N	6.023×10^{23} mole^{-1}
Boltzmann constant	k	1.38×10^{23} JK^{-1}
Elementary charge	e	1.6×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Neutron rest mass	m_n	1.68×10^{-27} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} Js
Permittivity constant	ϵ_0	8.85×10^{-12} Fm^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan's constant	σ	5.68×10^{-8} Wm^{-2} K^4
Speed of light in vacuum	c	2.997×10^8 ms^{-1}
Unified atomic mass unit	u	1.66×10^{-27} kg
Universal gas constant	R	8.31 J mol^{-1} K^{-1}