SINGAPORE JUNIOR PHYSICS OLYMPIAD
SPECIAL ROUND

1. This paper **comprises 8 printed pages (inclusive of cover page.)**
2. Indicate all answers in the foolscap provided.
3. Scientific calculators are allowed in this examination.
4. You are to start and stop the paper as instructed.

CONSTANTS AND CONVERSION FACTORS

1 unified atomic mass unit,	$1\,u = 1.66 \times 10^{-27}$ kg
	$= 931$ MeV$/c^2$
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg
Magnitude of the electron charge,	$e = 1.60 \times 10^{-19}$ C
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}
Universal gas constant,	$R = 8.31$ J/(mol · K)
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K
Speed of light,	$c = 3.00 \times 10^8$ m/s
Planck's constant,	$h = 6.63 \times 10^{-34}$ J · s
	$= 4.14 \times 10^{-15}$ eV · s
	$hc = 1.99 \times 10^{-25}$ J · m
	$= 1.24 \times 10^3$ eV · nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N · m^2
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9$ N · m^2/C^2
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T · m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 10^{-7}$ (T · m)/A
Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg · s^2
Acceleration due to gravity at the Earth's surface,	$g = 9.8$ m/s^2
1 atmosphere pressure,	1 atm $= 1.0 \times 10^5$ N/m^2
	$= 1.0 \times 10^5$ Pa
1 electron volt,	1 eV $= 1.60 \times 10^{-19}$ J
1 angstrom,	1 Å $= 1 \times 10^{-10}$ m

UNITS

Name	Symbol
meter	m
kilogram	kg
second	s
ampere	A
kelvin	K
mole	mol
hertz	Hz
newton	N
pascal	Pa
joule	J
watt	W
coulomb	C
volt	V
ohm	Ω
henry	H
farad	F
tesla	T
degree Celsius	°C
electron-volt	eV

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
53°	4/5	3/5	4/3
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

1. [18 points] A ball is dropped vertically, falls through a height h and strikes a ramp that is inclined 45° to the horizontal. The ball undergoes perfectly elastic collision.

(a) How far down the ramp does the ball land after the first bounce?

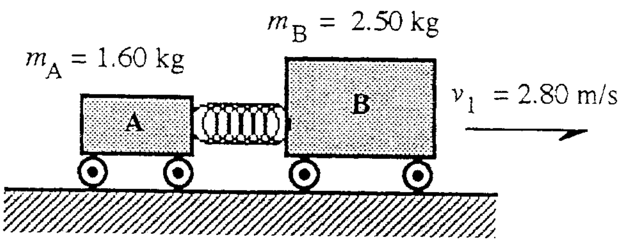
(b) Calculate the time t between the first bounce and the second bounce.

(c) Calculate the speed of impact, v_o and the angle θ that the ball makes with the vertical at the second impact.

(d) For the second and third bounces, find the distance the ball travels down the ramp (in term of h), the time in the air (in terms of t), and the speed at impact (in term of v_o) and the tangent of the angle with the vertical at impact.

(e) Generalize your answers to the *nth* bounce for the quantities in part (e).

2. [10 points] A spring is compressed between two carts and is temporarily clamped so that carts and spring move as a single unit. Initially, the carts move towards the right on a level surface with a velocity of 2.80 m.s/The masses of the carts are 1.60kg and 2.50kg, respectively as shown. The mass of the spring is much smaller than that of either cart and can be neglected. At a certain instant the clamp is suddenly released, and the carts separate, with cart B moving to the right at 3.20m/s.



(a) Calculate the velocity of cart A immediately after decompression of the spring.

(b) As a result of this interaction, do you expect the final total kinetic energy of the system to be equal to, greater than or less than the total initial kinetic energy? Explain your reasoning carefully and check your answer by making the relevant calculations.

(c) Is this event to be described as involving an elastic or an inelastic collision? Explain your reasoning.

3. [8 points] Assume that we have isolated a 1-g sample of carbon from a frozen animal and that the atmospheric ratio of the two carbon isotopes was the same when the animal died as it is now. (half-life of carbon is 5730 yrs)

(a) What was the decay rate in decays per minute of the ^{14}C shortly after the animal die?

(b) If the current decay rate is 1 per minute, how many years ago did the animal die?

Unfortunately, the ratio of the two isotopes of carbon has not been constant throughout time.

(c) How does the age of the sample change if the ratio is varied linearly in the past? Assume that the ratio decreases by 0.1% of the current value for each century that we go back in time.

4. [8 points] Hafele and Keating experiment

In 1972, two atomic clocks are put on an eastward bound and westward bound flight and flown around the Earth. They are then compared with the atomic clock which stays fixed on Earth. We consider a simplified version of the experiment.

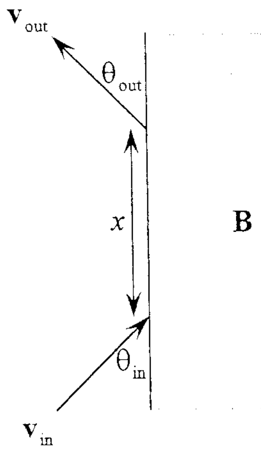
A clock is flown once around the circumference of the Earth (approximately 4.0×10^7m) and the typical airline plane speed is about 300m/s.

(a) What is the flight time T_o as measured by the ground clock?
Would the clock on the ground measure a longer time or shorter time for the flight time as compared to the flown clock?

(b) Show that the time difference between the two clocks can be given by $\frac{1}{2}\beta T_o$ where $\beta=v/c$. Show your steps clearly.

5. [8 points]

A proton moving with speed $v = 2.00\times10^5\ \mathrm{m\,s^{-1}}$ in a free field abruptly enters a uniform magnetic field $\mathbf{B} = -(0.850\ \mathrm{T})\hat{\mathbf{k}}$. The proton enters the magnetic field region at $\theta_{in} = 45.0^\circ$ as shown below, i.e., $\mathbf{v}_{in} = v\cos\theta_{in}\hat{\mathbf{i}} + v\sin\theta_{in}\hat{\mathbf{j}}$.



Here, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors in the direction of increasing *x*, *y*, and *z* respectively.

(a) At what velocity $\mathbf{v}_{out}$ does the proton leave the magnetic field region?

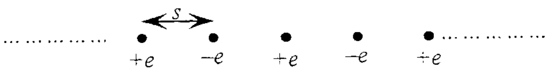
(b) Calculate the time interval *T* when it is in the field.

(c) At what distance *x* does it exit the field?

6. [8 points]

(a) In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (masses of $6.64\times10^{-27}\mathrm{kg}$) were fired toward a gold nucleus with charge $+79e$. An alpha particle, initially very far from the gold nucleus, is fired at $2.00\times10^7\mathrm{m\,s^{-1}}$ directly toward the nucleus. How close does the alpha particle get to the gold nucleus before turning around? For simplicity, assume the gold nucleus has infinite mass.

(b) Consider a one-dimensional infinite string of ions, of charges of magnitude *e* and alternating sign.



Let the distance between the ions be *s*. Calculate the electric potential energy per ion. [*Hint:* $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$]

7. [8 points]
Capacitors in networks cannot always be grouped into simple series or parallel combinations. As an example, Figure 1 shows three capacitors C_x, C_y, and C_z in a delta network, so called because of its triangular shape. This network has three terminals *a*, *b*, and *c* and hence cannot be transformed into a single equivalent capacitor.

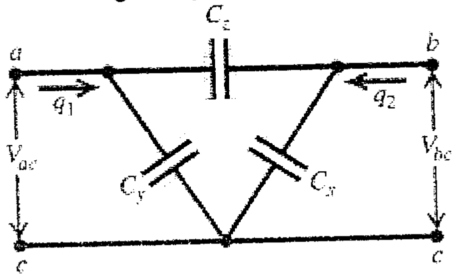


Figure 1

It can be shown that as far as any effect on the external circuit is concerned, a delta network is equivalent to what is called a Y network. For example, the delta network of Figure 1 can be replaced by the Y network of Figure 2.

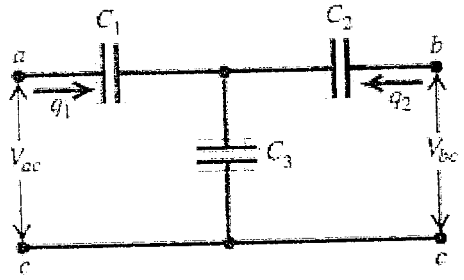
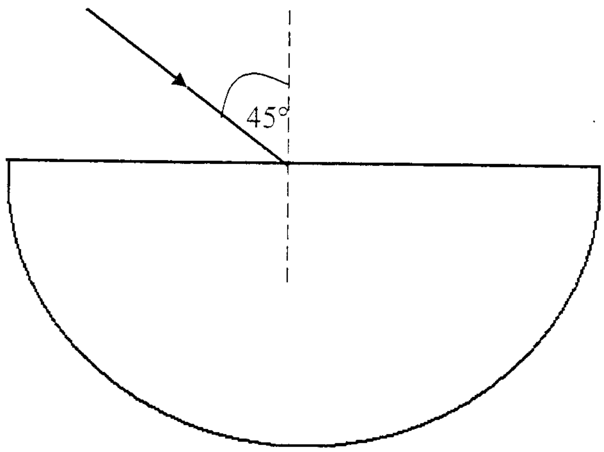


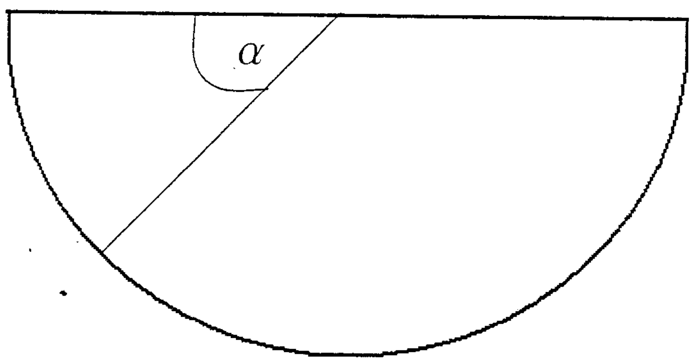
Figure 2

Determine C_1, C_2 and C_3 in terms of C_x, C_y, and C_z.

8. [8 points]
A ray of light falls on the plane surface of a half cylindrical glass at an angle of 45° in the plane perpendicular to the axis as shown in the figure. If the index of refraction of the glass is 1.4, copy this diagram and complete the path of the ray in the figure, indicating all relevant angles.



Let α be the angle that outgoing rays make with the plane surface of the half cylinder. Determine the range of possible angles of α.



9. [8 points]
A boiler contains 1.00 kg of liquid water at 100°C which needs to be converted to steam at 100° C by boiling at standard atmospheric pressure (1.00 atm or 1.01×10^5 Pa).
The volume of the water changes from an initial value of 1.00×10^{-3} m^3 as a liquid to a volume of 1.671 m^3 as steam. Taking the latent heat of vaporization of water is 2260 kJ kg^{-1},

(a) how much work is done by the system during this process?

(b) how much heat must be added to the system during the process?

(c) determine the change in the internal energy of the system during the boiling process

10. [8 points]

Stephen Hawking calculated the entropy of a non-rotating, uncharged black hole and obtained the expression

$$S = \frac{kc^3 A}{4G\hbar},$$

where $A \equiv 4\pi R_S^2$ is the surface area of the black hole and $R_S \equiv 2GM/c^2$ is the radius of a black hole of mass M (as predicted in general relativity). The internal energy of a black hole is given by its rest-mass energy, $E = Mc^2$.

(a) Calculate the temperature of the black hole. Express your final result in terms of the mass of the black hole and fundamental constants (Boltzmann's constant k, Planck's constant $\hbar = h/2\pi$, the speed of light c and Newton's gravitational constant G).

(b) Calculate the heat capacity of the black hole. Is it positive or negative? What happens to the black hole temperature as it loses energy?

11. [8 points]

A short-wave (HF) radio receiver receives simultaneously two signals from a transmitter 500 km away, one by a path along the surface of the Earth and one by reflection from a portion of the ionospheric layer situated at a height of 200 km. The layer acts as a perfect horizontal reflector. When the frequency of the transmitted wave is 10 MHz it is observed that the combined signal strength varies from the maximum to minimum and back to maximum 8 times per second. With what vertical speed is the ionosphere layer moving? You may assume that the Earth is flat and ignore disturbances.

END