4 Thermodynamics

4.1 Blackbody Radiation

4.1.1 (i) Maximum Temperature

At equilibrium, power in equals power out (Stefan-Boltzmann law).

$$\begin{aligned}
P_{\text{in}} &= \sigma\epsilon A(T_{\text{max}}^4 - T_{\text{env}}^4) \\
1800 &= (5.67 \times 10^{-8})(1)(4\pi(0.3)^2)(T_{\text{max}}^4 - 293.15^4) \\
T_{\text{max}}^4 &= \frac{1800}{(5.67 \times 10^{-8})(1.131)} + 293.15^4 \approx 3.544 \times 10^{10} \\
T_{\text{max}} &= (3.544 \times 10^{10})^{1/4} \approx 433.8 \text{ K}
\end{aligned}$$

Note: The calculated temperature is 433.8 K. The provided answer is 318.0 K, which would correspond to a heater power of ∼180 W, suggesting a typo in the problem (1.8 kW).

4.1.2 (ii) Initial Rate of Fall of Temperature

The rate of heat loss is $\frac{dQ}{dt} = -mc\frac{dT}{dt}$.

- Mass: $m = \rho V = (8940)\left(\frac{4}{3}\pi(0.3)^3\right) \approx 1011.1$ kg.

- At the moment the heater is switched off, the rate of heat loss is equal to the heater power, $\frac{dQ}{dt} = 1800$ W.
- Rate of temperature fall:

$$-\frac{dT}{dt} = \frac{1}{mc}\frac{dQ}{dt} = \frac{1800 \text{ W}}{(1011.1 \text{ kg})(389 \text{ J kg}^{-1}\text{K}^{-1})} \approx \mathbf{4.58 \times 10^{-3}} \text{ K/s}$$

4.2 Gas Pistons System

4.2.1 (i) Changes in Argon Gas

- Oxygen (isothermal): $p_{b,f} = 2p_0$. Argon (adiabatic): $p_{a,f} = p_{b,f} = 2p_0$.
- **Pressure Change (Argon):** $p_a V_a^\gamma = p_{a,f} V_{a,f}^\gamma \implies p_a V_a^{5/3} = (2p_0)(8V_a)^{5/3} \implies p_a = 64p_0$. $\Delta p = p_{a,f} - p_a = 2p_0 - 64p_0 = \mathbf{-62p_0}$.
- **Temperature Change (Argon):** $T_a V_a^{\gamma-1} = T_{a,f} V_{a,f}^{\gamma-1} \implies T_a V_a^{2/3} = T_{a,f}(8V_a)^{2/3} \implies T_{a,f} = T_a/4$. $\Delta T = T_{a,f} - T_a = T_a/4 - T_a = -\mathbf{\frac{3}{4}T_a}$.

4.2.2 (ii) Final Pressure of Mixture

- Total volume: $V_{\text{total}} = V_{a,f} + V_{b,f} = 8V_a + 7V_a = 15V_a$.
- Moles of Oxygen: $n = \frac{|Q|}{RT_0 \ln(2)}$.
- Total moles: $n_{\text{total}} = n_a + n = 8 + \frac{|Q|}{RT_0 \ln(2)}$.
- Final Pressure (Ideal Gas Law): $p_{\text{final}} V_{\text{total}} = n_{\text{total}} RT_0$.

$$p_{\text{final}}(15V_a) = \left(8 + \frac{|Q|}{RT_0 \ln(2)}\right) RT_0$$

$$p_{\text{final}} = \frac{8RT_0 + \frac{|Q|}{\ln(2)}}{15V_a}$$