33rd Singapore Physics Olympiad
Theory Paper — Full Solutions

Question 1 **[10 marks]**

A cylindrical rod of radius 1.0 cm and length 1.0 m consists of two equal sections: zinc and copper. The zinc end is pivoted at O. The rod is released from horizontal and swings to vertical. Find the angular velocity at the vertical position.

Given:

$$\rho_{\mathrm{Zn}} = 7135\,\mathrm{kg\,m^{-3}}, \quad \rho_{\mathrm{Cu}} = 8940\,\mathrm{kg\,m^{-3}}$$

Solution:

Volume of each half:

$$V = \pi r^2 \cdot \frac{L}{2} = \pi(0.01)^2(0.5) = 1.57 \times 10^{-4}\ \mathrm{m^3}$$

Masses:

$$m_{\mathrm{Zn}} = \rho_{\mathrm{Zn}} V \approx 7135 \times 1.57 \times 10^{-4} = 1.121\,\mathrm{kg}$$

$$m_{\mathrm{Cu}} = \rho_{\mathrm{Cu}} V \approx 8940 \times 1.57 \times 10^{-4} = 1.404\,\mathrm{kg}$$

Center of mass (from pivot at zinc end):

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{Zn}}(0.25) + m_{\mathrm{Cu}}(0.75)}{m_{\mathrm{Zn}} + m_{\mathrm{Cu}}} = \frac{1.121 \cdot 0.25 + 1.404 \cdot 0.75}{2.525} \approx 0.528\,\mathrm{m}$$

Loss in gravitational PE:

$$\Delta U = (m_{\mathrm{Zn}} + m_{\mathrm{Cu}}) g x_{\mathrm{CM}} \approx 2.525 \cdot 9.81 \cdot 0.528 \approx 13.06\,\mathrm{J}$$

Moment of inertia about pivot:

$$I_{\mathrm{Zn}} = \frac{1}{3} m_{\mathrm{Zn}} (0.5)^2 = \frac{1}{3}(1.121)(0.25) \approx 0.0934\,\mathrm{kgm^2}$$

$$I_{\mathrm{Cu}} = \frac{1}{12} m_{\mathrm{Cu}} (0.5)^2 + m_{\mathrm{Cu}}(0.75)^2 \approx 0.0293 + 0.7898 = 0.819\,\mathrm{kgm^2}$$

$$I = I_{\mathrm{Zn}} + I_{\mathrm{Cu}} \approx 0.912\,\mathrm{kgm^2}$$

Energy conservation:

$$\frac{1}{2} I \omega^2 = \Delta U \Rightarrow \omega = \sqrt{\frac{2\Delta U}{I}} = \sqrt{\frac{2 \cdot 13.06}{0.912}} \approx \sqrt{28.64} \approx 5.35\,\mathrm{rad\,s^{-1}}$$

$$\boxed{\omega \approx 5.35\,\mathrm{rad\,s^{-1}}}$$

Question 2(a) [5 marks]

Swing of length 5 m, amplitude 45°, sound frequency 400 Hz. Find max and min heard frequency.

Solution:

Max speed at bottom:

$$v_{\max} = \sqrt{2gL(1-\cos\theta_0)} = \sqrt{2\cdot 9.81\cdot 5\cdot(1-\cos 45^\circ)} \approx \sqrt{28.73} \approx 5.36\,\mathrm{m\,s^{-1}}$$

Doppler effect (observer moving):

$$f_{\max} = f_0\left(1+\frac{v_{\text{obs}}}{v_{\text{sound}}}\right) = 400\left(1+\frac{5.36}{330}\right) \approx 406.5\,\mathrm{Hz}$$

$$f_{\min} = 400\left(1-\frac{5.36}{330}\right) \approx 393.5\,\mathrm{Hz}$$

$$\boxed{f_{\max} \approx 406.5\,\mathrm{Hz}, \quad f_{\min} \approx 393.5\,\mathrm{Hz}}$$

Question 2(b) [5 marks]

Microscope: $f_o = 6.0\,\mathrm{mm}, f_e = 40.0\,\mathrm{mm}$, tube length $= 200\,\mathrm{mm}$, final image at $250\,\mathrm{mm}$ from eyepiece.

(i) Object distance from objective

For eyepiece:

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}, \quad v_e = -250\,\mathrm{mm}, \quad u_e = -(200 - v_o)$$

$$\frac{1}{40} = -\frac{1}{250} + \frac{1}{200-v_o} \Rightarrow \frac{1}{200-v_o} = \frac{1}{40} + \frac{1}{250} = \frac{29}{1000}$$

$$200 - v_o = \frac{1000}{29} \approx 34.48\,\mathrm{mm} \Rightarrow v_o \approx 165.52\,\mathrm{mm}$$

For objective:

$$\frac{1}{f_o} = \frac{1}{v_o} + \frac{1}{u_o} \Rightarrow \frac{1}{6.0} = \frac{1}{165.52} + \frac{1}{u_o} \Rightarrow u_o \approx 6.22\,\mathrm{mm}$$

$$\boxed{u_o \approx 6.22\,\mathrm{mm}}$$

(ii) Magnifying power

$$M_o = \frac{v_o}{u_o} \approx \frac{165.52}{6.22} \approx 26.6, \quad M_e = \frac{D}{|u_e|} = \frac{250}{34.48} \approx 7.25$$

$$M = M_o M_e \approx 26.6 \times 7.25 \approx 193$$

$$\boxed{M \approx 193}$$

Question 3 [10 marks]

Rotating conducting rod of length ℓ, resistance R, connected to resistor R_0, in uniform $\vec{B} \perp$ plane, angular speed ω.

(a) Induced emf

Element at radius r: $d\mathcal{E} = B(\omega r)dr$

$$\mathcal{E} = \int_0^\ell B\omega r\, dr = \frac{1}{2}B\omega\ell^2$$

$$\boxed{\mathcal{E} = \frac{1}{2}B\omega\ell^2}$$

(b) Power in R_0

Total resistance: $R + R_0$, current $I = \dfrac{\mathcal{E}}{R + R_0}$

$$P = I^2 R_0 = \frac{\mathcal{E}^2 R_0}{(R + R_0)^2} = \frac{B^2\omega^2\ell^4 R_0}{4(R + R_0)^2}$$

$$\boxed{P = \frac{B^2\omega^2\ell^4 R_0}{4(R + R_0)^2}}$$

(c) Origin of power

The power comes from mechanical work done by an external agent to maintain constant ω against the magnetic braking torque caused by induced current (Lenz's law). Thus, mechanical energy $\rightarrow$ electrical energy $\rightarrow$ heat.

$\boxed{\text{Mechanical work maintains rotation against magnetic damping; energy converted to Joule heating.}}$

Question 4 [10 marks]

Free electron collides with H atom (ground state). After collision, atom emits two photons; one has $\lambda_1 = 656.3\,\mathrm{nm}$ (Hα: $n = 3 \to 2$). Final electron de Broglie wavelength $\lambda_e = 1.915\,\mathrm{nm}$.

(a) Wavelength of other photon

656.3 nm corresponds to transition $n = 3 \to n = 2$. So atom was excited to $n = 3$. De-excitation path: $3 \to 2 \to 1$. Other photon is $2 \to 1$.

Energy of $2 \to 1$:

$$E = 13.6\left(1 - \frac{1}{4}\right) = 10.2\,\mathrm{eV}$$

$$\lambda = \frac{hc}{E} = \frac{1240\ \mathrm{eV\,nm}}{10.2\ \mathrm{eV}} \approx 121.6\,\mathrm{nm}$$

$$\boxed{\lambda \approx 121.6\,\mathrm{nm}}$$

(b) Initial electron speed

Final electron momentum:

$$p = \frac{h}{\lambda_e} = \frac{6.63 \times 10^{-34}}{1.915 \times 10^{-9}} \approx 3.46 \times 10^{-25}\,\mathrm{kg\,m\,s^{-1}}$$

$$v_f = \frac{p}{m_e} = \frac{3.46 \times 10^{-25}}{9.11 \times 10^{-31}} \approx 3.80 \times 10^{5}\,\mathrm{m\,s^{-1}}$$

$$K_f = \frac{1}{2} m_e v_f^2 \approx 65.7\,\mathrm{eV}$$

Energy lost to atom: excitation from $n = 1$ to $n = 3$:

$$\Delta E = 13.6\left(1 - \frac{1}{9}\right) = 12.09\,\mathrm{eV}$$

Initial kinetic energy: $K_i = K_f + \Delta E \approx 65.7 + 12.09 = 77.8\,\mathrm{eV}$

Initial speed:

$$v_i = \sqrt{\frac{2K_i e}{m_e}} = \sqrt{\frac{2 \cdot 77.8 \cdot 1.60 \times 10^{-19}}{9.11 \times 10^{-31}}} \approx 5.22 \times 10^{5}\,\mathrm{m\,s^{-1}}$$

$$\boxed{v_i \approx 5.22 \times 10^{5}\,\mathrm{m\,s^{-1}}}$$

Question 5(a) [7 marks]

Airplane speed: $540\,\mathrm{km\,h^{-1}} = 150\,\mathrm{m\,s^{-1}}$, height 1200 m. Fires projectile at speed u relative to plane $\rightarrow$ absolute speed $= 150 + u$.

Vehicle speed: $40\,\mathrm{m\,s^{-1}}$, initial horizontal distance $= d$ (km $\rightarrow 1000d$ m).

Time to fall: $t = \sqrt{2h/g} = \sqrt{2 \cdot 1200/9.81} \approx 15.64\,\mathrm{s}$

Horizontal distance traveled by projectile: $(150 + u)t$

By vehicle: $40t$

To hit: $(150 + u)t = 1000d + 40t \Rightarrow 1000d = (110 + u)t$

Minimum d occurs when $u = 0$ (minimum possible firing speed):

$$d_{\min} = \frac{110 \cdot 15.64}{1000} \approx 1.72\,\mathrm{km}$$

If $d = 5d_{\min} = 8.60\,\mathrm{km}$, then:

$$1000 \cdot 8.60 = (110 + u) \cdot 15.64 \Rightarrow u \approx \frac{8600}{15.64} - 110 \approx 439.3\,\mathrm{m\,s^{-1}}$$

Projectile speed when it hits = horizontal: $150 + u = 589.3\,\mathrm{m\,s^{-1}}$, vertical: $v_y = gt = 9.81 \cdot 15.64 \approx 153.4\,\mathrm{m\,s^{-1}}$

Speed magnitude:

$$v = \sqrt{(589.3)^2 + (153.4)^2} \approx 609\,\mathrm{m\,s^{-1}}$$

$$\boxed{d_{\min} \approx 1.72\,\mathrm{km}, \quad v \approx 609\,\mathrm{m\,s^{-1}}}$$

Question 5(b) [5 marks]

Satellite at $h = 400\,\mathrm{km}$, co-rotating with Earth.

Orbital radius: $r = R_E + h = 6370 + 400 = 6770\,\mathrm{km} = 6.77 \times 10^6\,\mathrm{m}$

Orbital period (Kepler):

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{r^3}{gR_E^2}} \quad (\text{since } g = GM_E/R_E^2)$$

$$T = 2\pi\sqrt{\frac{(6.77 \times 10^6)^3}{9.81 \cdot (6.37 \times 10^6)^2}} \approx 5540\,\mathrm{s} \approx 92.3\,\mathrm{min}$$

Earth rotates once per 24 h = 86 400 s. In time T, Earth rotates by angle $\theta = \omega_E T$, so satellite sees new point.

Number of photos in 24 h = number of orbits:

$$N = \frac{86400}{5540} \approx 15.6 \Rightarrow \boxed{15}$$

(Only full passes over distinct points count; typically integer number of revolutions.)

Question 6(a) [6 marks]

Uniformly charged insulating sphere, density ρ, radius R. Tunnel along diameter. Particle $-q$, mass m, released from surface.

Inside sphere, electric field:

$$E(r) = \frac{\rho r}{3\varepsilon_0}, \quad F = -qE = -\left(\frac{q\rho}{3\varepsilon_0}\right) r \Rightarrow \text{SHM}$$

Motion is simple harmonic about center.

Potential difference from surface to center:

$$\Delta V = V(0) - V(R) = \frac{\rho}{6\varepsilon_0}(3R^2 - R^2) - \frac{\rho R^2}{2\varepsilon_0} = \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho R^2}{2\varepsilon_0} = -\frac{\rho R^2}{6\varepsilon_0}$$

Better: use energy.

Energy conservation: loss in electric PE = gain in KE.

Total charge: $Q = \frac{4}{3}\pi R^3 \rho$

Potential at surface: $V(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$

Potential at center: $V(0) = \frac{3}{2}V(R) = \frac{\rho R^2}{2\varepsilon_0}$

So $\Delta V = V(0) - V(R) = \frac{\rho R^2}{6\varepsilon_0}$

KE gained:

$$\frac{1}{2}mv^2 = q\Delta V = \frac{q\rho R^2}{6\varepsilon_0} \Rightarrow v = \sqrt{\frac{q\rho R^2}{3m\varepsilon_0}}$$

$$\boxed{v = \sqrt{\frac{q\rho R^2}{3m\varepsilon_0}}, \quad \text{motion is simple harmonic}}$$

Question 6(b) [6 marks]

Forces: $F_A = -2m\mu x$ (if A at $x = -L/2$), $F_B = -m\mu(x - L)$. But better: let midpoint be origin, AB $= 2a$.

Let x = displacement from midpoint. Then $d_A = a + x$, $d_B = a - x$

Net force toward A: $F_A = -2m\mu(a + x)$ (negative if right of A)

But direction: F_A toward A, F_B toward B.

So net force:

$$F = -2m\mu(a + x) + m\mu(a - x) = -2m\mu a - 2m\mu x + m\mu a - m\mu x = -m\mu a - 3m\mu x$$

But equilibrium at net force $= 0 \to x_0 = -a/3$. However, problem states particle is at rest at midpoint and O is where net force = 0.

Alternative: set A at $x = 0$, B at $x = L$. Then $d_A = x$, $d_B = L - x$

$$F_{\text{net}} = -2m\mu x + m\mu(L - x) = m\mu(L - 3x)$$

Zero at $x = L/3 = O$. Let $y = x - L/3$, then:

$$F = m\mu(L - 3(y + L/3)) = -3m\mu y \Rightarrow m\ddot{y} = -3m\mu y \Rightarrow \ddot{y} + 3\mu y = 0$$

SHM with $\omega = \sqrt{3\mu}$, period $T = \dfrac{2\pi}{\sqrt{3\mu}}$
Amplitude: particle at rest at midpoint $x = L/2 = y = L/2 - L/3 = L/6$
So amplitude $A = L/6$
KE at O (y=0): max KE = total energy = $\frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m(3\mu)(L/6)^2 = \frac{m\mu L^2}{24}$

$$\boxed{T = \frac{2\pi}{\sqrt{3\mu}}, \quad A = \frac{L}{6}, \quad K = \frac{m\mu L^2}{24}}$$

Question 7(a) [6 marks]

Initial: $V = 8.0 \times 10^{-3}\,\mathrm{m}^3$, $P_1 = 1.14 \times 10^5\,\mathrm{Pa}$, T

Gas expands adiabatically (open lid) to $P_2 = 1.01 \times 10^5\,\mathrm{Pa}$, some gas escapes.
Then lid closed, gas reheated to T, new pressure $P_3 = 1.06 \times 10^5\,\mathrm{Pa}$

(i) Volume of remaining gas at original T, P_1?

After re-heating: $P_3 V = nRT$
Originally: $P_1 V = n_0 RT$
So ratio: $\frac{n}{n_0} = \frac{P_3}{P_1} = \frac{1.06}{1.14}$
At original P_1, T, volume of remaining gas:

$$V' = \frac{n}{n_0} V = \frac{1.06}{1.14} \times 8.0 \times 10^{-3} \approx 7.44 \times 10^{-3}\,\mathrm{m}^3$$

$$\boxed{V' \approx 7.44 \times 10^{-3}\,\mathrm{m}^3}$$

(ii) Find γ

Adiabatic expansion: $P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$, but gas that remains underwent $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$
But amount changed. Better: the gas that remains satisfied $T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}$
But after expansion, when lid closed, same n, then heated back to T: $P_2/T_2 = P_3/T \Rightarrow T_2 = T \cdot P_2/P_3$
So:

$$\frac{T_2}{T} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \Rightarrow \frac{P_2}{P_3} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}$$

Take logs:

$$\ln\left(\frac{P_2}{P_3}\right) = \frac{\gamma-1}{\gamma} \ln\left(\frac{P_2}{P_1}\right)$$

Plug: $P_1 = 1.14$, $P_2 = 1.01$, $P_3 = 1.06$ (in $\times 10^5$ Pa)

$$\frac{1.01}{1.06} = 0.9528, \quad \frac{1.01}{1.14} = 0.8860$$

$$\ln(0.9528) = -0.0484, \quad \ln(0.8860) = -0.1208$$

$$-0.0484 = \frac{\gamma-1}{\gamma}(-0.1208) \Rightarrow \frac{\gamma-1}{\gamma} = 0.4007 \Rightarrow \gamma = \frac{1}{1-0.4007} \approx 1.67$$

$$\boxed{\gamma \approx 1.67}$$

(iii) Atomicity

$\gamma = 5/3 \approx 1.67 \rightarrow$ monatomic gas.

$$\boxed{\text{Monatomic}}$$

Question 7(b) [6 marks]

Copper rod: $L = 0.5\,\text{m}$, $r = 0.01\,\text{m}$, $k = 400\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$

Hot end: $100\,^{\circ}\text{C}$, cold end in ice–water mix (0°C). Melt ice (200 g) and heat total water (300 g) to $20\,^{\circ}\text{C}$.

Total heat required:

$$Q = m_{\text{ice}} L_f + (m_{\text{ice}} + m_{\text{water}}) c \Delta T = 0.2 \cdot 3.34 \times 10^5 + 0.3 \cdot 4200 \cdot 20 = 66800 + 25200 = 92\,000\,\text{J}$$

Steady-state heat current:

$$\frac{dQ}{dt} = \frac{kA\Delta T}{L} = \frac{400 \cdot \pi (0.01)^2 \cdot 100}{0.5} = \frac{400 \cdot \pi \cdot 10^{-4} \cdot 100}{0.5} \approx 25.13\,\text{W}$$

Time:

$$t = \frac{Q}{dQ/dt} = \frac{92000}{25.13} \approx 3660\,\text{s} \approx 61\,\text{min}$$

$$\boxed{t \approx 3660\,\text{s (about } 61\,\text{min)}}$$

Question 8(a) [5 marks]

Source B accelerates from rest at $a = 0.5\,\mathrm{m\,s^{-2}}$ to point P at distance x, then moves at constant speed $v = \sqrt{2ax}$.

At that moment, frequency difference: $\Delta f = 8\,\mathrm{Hz}$, $f_0 = 256\,\mathrm{Hz}$, $v_{\text{sound}} = 330\,\mathrm{m\,s^{-1}}$

Observer hears:

$$f = f_0 \frac{v_{\text{sound}}}{v_{\text{sound}} - v} \quad \text{(source moving away)}$$

$$f_0 - f = 8 \Rightarrow f_0\left(1 - \frac{330}{330 - v}\right) = -8 \Rightarrow \frac{f_0 v}{330 - v} = 8$$

$$256v = 8(330 - v) \Rightarrow 256v + 8v = 2640 \Rightarrow v = \frac{2640}{264} = 10\,\mathrm{m\,s^{-1}}$$

Then $v^2 = 2ax \Rightarrow x = \frac{100}{2\cdot 0.5} = 100\,\mathrm{m}$

$$\boxed{x = 100\,\mathrm{m}}$$

Question 8(b) [7 marks]

Proton, $K = 10\,\mathrm{MeV}$, $B = 1.5\,\mathrm{T}$, field length $\Delta x = 2.0\,\mathrm{m}$

Non-relativistic? $m_p c^2 \approx 938\,\mathrm{MeV} \gg 10$ MeV $\rightarrow$ OK.

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 10 \cdot 10^6 \cdot 1.60 \times 10^{-19}}{1.67 \times 10^{-27}}} \approx 1.38 \times 10^7\,\mathrm{m\,s^{-1}}$$

Cyclotron radius:

$$r = \frac{mv}{qB} = \frac{1.67 \times 10^{-27} \cdot 1.38 \times 10^7}{1.60 \times 10^{-19} \cdot 1.5} \approx 0.96\,\mathrm{m}$$

Arc length in field: chord length $= 2.0\,\mathrm{m}$, but actually, particle traverses arc of circle with radius r, horizontal projection $= 2.0$ m.

So: $r \sin\alpha = 2.0 \Rightarrow \sin\alpha = 2.0/0.96 > 1 \rightarrow$ impossible!

Wait: magnetic field only changes direction, path is circular arc. The beam enters at $x = 0$, exits at $x = 2.0$, so the **horizontal displacement** is 2.0 m, which equals $r\sin\theta$, where $\theta =$ deflection angle.

But $r = 0.96$ m < 2.0 m $\rightarrow$ contradiction.

Thus, must treat **relativistically**.

Relativistic momentum:

$$K = (\gamma - 1)mc^2 = 10 \Rightarrow \gamma = 1 + \frac{10}{938} \approx 1.0107$$

$$p = \gamma m v = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} \approx \frac{1}{c}\sqrt{(10)^2 + 2 \cdot 10 \cdot 938} \approx \frac{\sqrt{18860}}{c} \approx \frac{137.3}{c}\ \mathrm{MeV/c}$$

In SI:

$$p = \frac{137.3 \cdot 10^6 \cdot 1.60 \times 10^{-19}}{3.00 \times 10^8} \approx 7.32 \times 10^{-20}\,\mathrm{kg\,m\,s^{-1}}$$

Then:

$$r = \frac{p}{qB} = \frac{7.32 \times 10^{-20}}{1.60 \times 10^{-19} \cdot 1.5} \approx 0.305\,\mathrm{m}$$

Now, horizontal distance: $x = r\sin\alpha = 2.0 \to$ still impossible since $r < 2$.

But actually: the particle moves in a **circular arc** of radius r. The maximum horizontal distance it can cover is $2r$ (diameter). Since $2r \approx 0.61\,\mathrm{m} < 2.0\,\mathrm{m}$, the proton **completes more than half a circle**.

Better: the angle swept θ satisfies:

$$\text{Arc length along x: } \Delta x = r\sin\theta + r\theta\text{? No.}$$

Actually, parametric: $x = r\sin\theta$, $y = r(1-\cos\theta)$. So $x = r\sin\theta = 2.0$

But $r = 0.305 \to \sin\theta = 2.0/0.305 \approx 6.56 \to$ impossible.

So **error**: the magnetic field is in z, motion in xy-plane. The **path is circular**, but the field region is only from $x = 0$ to $x = 2$. The proton **enters at (0,0) with velocity in x**, so trajectory: center at (0, r). Then:

$$x = r\sin\omega t, \quad y = r(1-\cos\omega t)$$

When $x = 2.0$, $r\sin\theta = 2.0 \Rightarrow \sin\theta = 2.0/r$

But this requires $r > 2.0$. So initial non-relativistic calc must have error.

Recheck non-relativistic v:

$$K = 10\text{ MeV} = 1.60\times10^{-12}\text{ J}$$

$$v = \sqrt{2K/m} = \sqrt{2\cdot 1.60\times10^{-12}/1.67\times10^{-27}} = \sqrt{1.916\times10^{15}} \approx 4.38\times10^{7}\,\mathrm{m\,s^{-1}}$$

(earlier used wrong conversion!)

Then:

$$r = \frac{mv}{qB} = \frac{1.67\times10^{-27}\cdot 4.38\times10^{7}}{1.60\times10^{-19}\cdot 1.5} \approx 3.05\,\mathrm{m}$$

Now, $x = r\sin\alpha = 2.0 \Rightarrow \sin\alpha = 2.0/3.05 = 0.6557 \Rightarrow \alpha = \arcsin(0.6557) \approx 41.0^\circ$

$$\boxed{\alpha \approx 41^\circ}$$

Question 9(a) [5 marks]

Light $\lambda = 122\,\text{nm}$ on metal $\to$ photoelectrons with max KE enter $B = 5 \times 10^{-5}\,\text{T}$, move in circle $r = 0.158\,\text{m}$.

Find work function ϕ.

From magnetic motion:

$$r = \frac{mv}{eB} \Rightarrow p = eBr$$

$$K = \frac{p^2}{2m} = \frac{(eBr)^2}{2m}$$

Compute:

$$p = 1.60 \times 10^{-19} \cdot 5 \times 10^{-5} \cdot 0.158 \approx 1.264 \times 10^{-24}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

$$K = \frac{(1.264 \times 10^{-24})^2}{2 \cdot 9.11 \times 10^{-31}} \approx 8.77 \times 10^{-19}\,\text{J} = \frac{8.77 \times 10^{-19}}{1.60 \times 10^{-19}} \approx 5.48\,\text{eV}$$

Photon energy:

$$E = \frac{1240}{122} \approx 10.16\,\text{eV}$$

Work function:

$$\phi = E - K \approx 10.16 - 5.48 = 4.68\,\text{eV}$$

$$\boxed{\phi \approx 4.68\,\text{eV}}$$

Question 9(b) [7 marks]

Muons: $v = 0.995c$, $\tau_{1/2} = 1.56\,\mu\text{s}$ (rest), mountain height 2000 m, 568 muons at top.

(i) Classical prediction at bottom

Time to travel: $t = \frac{2000}{0.995 \cdot 3.00 \times 10^8} \approx 6.70 \times 10^{-6}\,\text{s} = 6.70\,\mu\text{s}$

Number of half-lives: $n = 6.70/1.56 \approx 4.295$

Survival fraction: $(1/2)^{4.295} \approx 0.051$

Expected muons: $568 \cdot 0.051 \approx 29$

$$\boxed{29}$$

(ii) Why experiment sees 422?

Relativistic time dilation: in Earth frame, muon lifetime is $\gamma\tau$, so many survive.

$$\boxed{\text{Time dilation in special relativity extends muon lifetime in Earth frame.}}$$

(iii) Height in muon frame

Length contraction: $L' = L/\gamma$

$\gamma = 1/\sqrt{1 - 0.995^2} = 1/\sqrt{0.009975} \approx 10.01$

$$L' = 2000/10.01 \approx 200\,\text{m}$$

$$\boxed{200\,\text{m}}$$

(iv) Velocity of particle (0.9995c) in muon frame

Use relativistic velocity addition:

$$u' = \frac{u-v}{1-uv/c^2} = \frac{0.9995c - 0.995c}{1-(0.9995)(0.995)} = \frac{0.0045c}{1-0.9945025} = \frac{0.0045c}{0.0054975} \approx 0.819c$$

$$\boxed{u' \approx 0.819c}$$