5 (a) A cube lies with one of its faces on the $x-y$ plane and three of its edges along the x, y & z axes. The cube starts to slide horizontally along the $x-$axis with constant speed v. It is found that as a result of the motion, the density of the material of the cube appears to increase by 25%. Calculate the value of v.

Let ℓ_0 be the length of the edge of the cube when it is stationary and m_0 be its mass when it is not moving.
The density of the material of the cube when it is stationary is

$$\rho_0 = \frac{m_0}{\ell_0^3}$$

When it is in motion, the edges parallel to the $y-$ and $z-$ axes remains unchanged in length since both edges are moving perpendicular to the direction of motion.
The length along the $x-$axis now becomes

$$\ell_x = \ell_0\sqrt{1-\beta^2} \qquad \text{where } \beta = \frac{v}{c}$$ [1]

The mass of the cube appears to become

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$ [1]

The density of the material of the cube in motion becomes

$$\rho = \frac{m}{\ell_0^2 \ell_x} = \frac{m_0}{\sqrt{1-\beta^2}} \times \frac{1}{\ell_0^3\sqrt{1-\beta^2}}$$

$$= \frac{m_0}{\ell_0^3\left(1-\beta^2\right)} = \frac{\rho_0}{1-\beta^2}$$ [1]

$$\frac{\rho}{\rho_0} = \frac{1}{1-\beta^2} = 1.25$$

$$1-\beta^2 = \frac{4}{5} \Rightarrow \beta = \sqrt{0.2}$$

$$\therefore v = 0.4472c$$ [1]

Alternative Solution (not involving relativistic mass)

Let ℓ_0 be the length of the edge of the cube when it is stationary and m be its n
The density of the material of the cube when it is stationary is

$$\rho_0 = \frac{m}{\ell_0^3}$$

When it is in motion, the edges parallel to the $y-$ and $z-$ axes rem unchanged in length since both edges are moving perpendicular to the directio motion. [1]

The length along the $x-$axis now becomes

$$\ell_x = \ell_0\sqrt{1-\beta^2} \qquad \text{where } \beta = \frac{v}{c}$$ [1]

The density of the material of the cube in motion becomes

$$\rho = \frac{m}{\ell_0^2 \ell_x} = \frac{m}{\ell_0^3\sqrt{1-\beta^2}}$$

$$= \frac{\rho_0}{\sqrt{1-\beta^2}}$$ [1]

$$\frac{\rho}{\rho_0} = \frac{1}{\sqrt{1-\beta^2}} = 1.25$$

$$\sqrt{1-\beta^2} = \frac{4}{5} \Rightarrow \beta^2 = \frac{9}{25}$$

$$\therefore v = 0.6c$$ [1]

[4 marks]

(b) An observer in the Earth frame observes two spaceships A and B pass each other in opposite directions. In the inertial frame in which both spaceships are at rest, the lengths of spaceship A and spaceship B are 200 m and 150 m respectively. The observer in the Earth frame measures that the speed of spaceship A is $0.8c$ and that of spaceship B is $0.6c$. As measured by this observer, what is the time interval between the instant when the nose of spaceship A passes the nose of spaceship B and the instant when the tail of spaceship A passes the tail of spaceship B? What is the corresponding time interval measured by an observer sitting at the nose of spaceship A?

[5.714×10^{-7} s; 1.715×10^{-7} s]

[6 marks]