2 Rotational Motion and Friction

2.1 (a) Calculate the maximum static frictional force that can act on the block.

The normal force $N = mg = 2.5 \text{ kg} \times 9.81 \text{ m/s}^2 = 24.525 \text{ N}$. The maximum static frictional force $f_{s,max}$ is:

$$f_{s,max} = \mu_s N = 0.4 \times 24.525 \text{ N} = 9.81 \text{ N}$$

2.2 (b) Determine whether the block will slide or stay in place.

The required centripetal force is $F_c = m\omega^2 r$:

$$F_c = 2.5 \text{ kg} \times (3 \text{ rad/s})^2 \times 0.3 \text{ m} = 6.75 \text{ N}$$

Since $F_c = 6.75 \text{ N} < f_{s,max} = 9.81 \text{ N}$, the block will **stay in place**.

2.3 (c) Calculate the minimum coefficient of friction required for the block to stay in place at the edge.

At the edge, $r_{\text{edge}} = 0.5 \text{ m}$. The required centripetal force is:

$$F_{c,\text{edge}} = m\omega^2 r_{\text{edge}} = 2.5 \times (3)^2 \times 0.5 = 11.25 \text{ N}$$

For the block to stay, $f_{s,max,min} = \mu_{s,min} mg \geq F_{c,edge}$.

$$\mu_{s,min} \geq \frac{11.25 \text{ N}}{2.5 \text{ kg} \times 9.81 \text{ m/s}^2} \approx 0.4587$$

The minimum coefficient is approximately **0.459**.

2.4 (d) Determine the speed of the block when it begins to slip.

Slipping begins when $\frac{mv^2}{r} = f_{s,max} = \mu_s mg$.

$$v = \sqrt{\mu_s g r} = \sqrt{0.4 \times 9.81 \times 0.5} = \sqrt{1.962} \approx 1.40 \text{ m/s}$$

3 Spring and Conservation Laws

3.1 (a) Derive an expression for the speed of the sphere when the block is fixed.

By conservation of energy (initial spring potential energy to final kinetic energy):

$$\frac{1}{2}kd^2 = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{k}{m}}d$$

3.2 (b) Derive an expression for the speed of the sphere when the block is free to move.

Let v_m be the sphere's velocity and v_M be the block's velocity. Conservation of Momentum: $mv_m + Mv_M = 0 \implies v_M = -\frac{m}{M}v_m$. Conservation of Energy: $\frac{1}{2}kd^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2$. Substituting for v_M:

$$\begin{aligned} kd^2 &= mv_m^2 + M\left(-\frac{m}{M}v_m\right)^2 = mv_m^2 + \frac{m^2}{M}v_m^2 \\ kd^2 &= v_m^2\left(m + \frac{m^2}{M}\right) = v_m^2\frac{m(M+m)}{M} \\ v_m &= d\sqrt{\frac{kM}{m(M+m)}} \end{aligned}$$

3.3 (c) Derive an expression for the distance the block has travelled.

The center of mass (CM) of the system remains stationary. Let the block's displacement be D_M and the sphere's be D_m. $mD_m + MD_M = 0$. The initial compression is the sum of the magnitudes of their displacements relative to the uncompressed position: $d = |D_m| + |D_M|$. From the first equation, $|D_m| = \frac{M}{m}|D_M|$. Substituting this into the second:

$$d = \frac{M}{m}|D_M| + |D_M| = |D_M|\left(\frac{M+m}{m}\right)$$

The distance travelled by the block is $|D_M| = \frac{md}{m+M}$.