Solutions to Singapore Physics Olympiad 2020 Theory Paper

1 Rotating Rod

This problem can be solved using the **principle of conservation of mechanical energy**. The initial potential energy of the rod in the horizontal position is converted into rotational kinetic energy when it reaches the vertical position.

1. Calculate Masses and Centers of Mass

- Radius of the rod, $r = 1.0$ cm $= 0.01$ m.
- Length of each section, $l_{\text{section}} = 0.5$ m.
- Volume of each section, $V = \pi r^2 l_{\text{section}} = \pi(0.01)^2(0.5) = 5\pi \times 10^{-5}$ m^3.
- Mass of the zinc section: $m_{\text{Zn}} = \rho_{\text{Zn}} V = (7135 \text{ kg/m}^3)(5\pi \times 10^{-5} \text{ m}^3) \approx 1.121$ kg.
- Mass of the copper section: $m_{\text{Cu}} = \rho_{\text{Cu}} V = (8940 \text{ kg/m}^3)(5\pi \times 10^{-5} \text{ m}^3) \approx 1.404$ kg.

The center of mass of the zinc section is at 0.25 m from the pivot O, and the center of mass of the copper section is at $0.5 + 0.25 = 0.75$ m from O.

2. Calculate Moment of Inertia

The total moment of inertia (I_{total}) about the pivot O is the sum of the moments of inertia of the two sections.

- For the zinc section (a rod rotating about its end):

$$I_{\text{Zn}} = \frac{1}{3} m_{\text{Zn}} l_{\text{section}}^2 = \frac{1}{3}(1.121)(0.5)^2 \approx 0.0934 \text{ kg m}^2$$

- For the copper section, we use the parallel axis theorem, $I = I_{\text{cm}} + md^2$. The section extends from 0.5 m to 1.0 m. Its center of mass is at $d = 0.75$ m from the pivot.

$$I_{\text{cm, Cu}} = \frac{1}{12} m_{\text{Cu}} l_{\text{section}}^2 = \frac{1}{12}(1.404)(0.5)^2 \approx 0.0292 \text{ kg m}^2$$

$$I_{\text{Cu}} = I_{\text{cm, Cu}} + m_{\text{Cu}} d^2 = 0.0292 + (1.404)(0.75)^2 \approx 0.0292 + 0.7898 = 0.819 \text{ kg m}^2$$

- Total moment of inertia:

$$I_{\text{total}} = I_{\text{Zn}} + I_{\text{Cu}} = 0.0934 + 0.819 = 0.9124 \text{ kg m}^2$$

3. Apply Conservation of Energy

Let the pivot point O be the reference level for gravitational potential energy ($h = 0$).

- **Initial State (Horizontal):** $PE_i = 0$, $KE_i = 0$.
- **Final State (Vertical):** $PE_f = -m_{\text{Zn}} g(0.25) - m_{\text{Cu}} g(0.75)$, $KE_f = \frac{1}{2} I_{\text{total}} \omega^2$.

By conservation of energy, Loss in PE = Gain in KE:

$$\begin{aligned}
m_{\text{Zn}}g(0.25) + m_{\text{Cu}}g(0.75) &= \frac{1}{2}I_{\text{total}}\omega^2 \\
g[1.121(0.25) + 1.404(0.75)] &= \frac{1}{2}(0.9124)\omega^2 \\
9.81[0.2803 + 1.053] &= 0.4562\omega^2 \\
13.079 &= 0.4562\omega^2 \\
\omega &= \sqrt{\frac{13.079}{0.4562}} \approx 5.35 \text{ rad/s}
\end{aligned}$$

The angular velocity of the rod when it is in the vertical position is **5.35 rad/s**.

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2 Doppler Effect and Optics

2.1 Sound Frequency on a Swing

1. Find the Maximum Speed of the Swing

Using conservation of energy:

$$\Delta h = L - L\cos\theta_{\max} = 5(1 - \cos 45^\circ) \approx 1.464 \text{ m}$$
$$v_{\max} = \sqrt{2g\Delta h} = \sqrt{2(9.81)(1.464)} \approx 5.36 \text{ m/s}$$

2. Apply the Doppler Effect Formula

The formula is $f' = f_0\left(\frac{v \pm v_o}{v}\right)$.

- **Maximum frequency** (student moves towards the source):

$$f_{\max} = 400\left(\frac{330 + 5.36}{330}\right) \approx 406.5 \text{ Hz}$$

- **Minimum frequency** (student moves away from the source):

$$f_{\min} = 400\left(\frac{330 - 5.36}{330}\right) \approx 393.5 \text{ Hz}$$

2.2 Compound Microscope

i. The distance of the object from the objective lens (u_o)

1. **Analyze the Eyepiece:** Find the intermediate image position (u_e).

$$\frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{v_e} \implies \frac{1}{40.0} = \frac{1}{u_e} - \frac{1}{250} \implies u_e \approx 34.48 \text{ mm}$$

2. **Find the Objective's Image Distance (v_o):**

$$v_o = L - u_e = 200 - 34.48 = 165.52 \text{ mm}$$

3. **Analyze the Objective:** Find the object distance (u_o).

$$\frac{1}{f_o} = \frac{1}{u_o} + \frac{1}{v_o} \implies \frac{1}{6.0} = \frac{1}{u_o} + \frac{1}{165.52} \implies u_o \approx 6.226 \text{ mm}$$

The distance is **6.23 mm**.

ii. The magnifying power of the microscope (M)

- $m_o = -\frac{v_o}{u_o} = -\frac{165.52}{6.226} \approx -26.59$
- $m_e = 1 + \frac{D}{f_e} = 1 + \frac{250}{40} = 7.25$
- $M = |m_o| \times m_e = 26.59 \times 7.25 \approx 192.8$

The magnifying power is approximately **193**.

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3 Rotating Conductor in a Magnetic Field

3.1 Induced EMF

The motional EMF is given by:

$$\mathcal{E} = \int_0^l (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_0^l B(\omega r)dr = B\omega \left[\frac{r^2}{2}\right]_0^l = \frac{1}{2}B\omega l^2$$

3.2 Power in the Resistor

- Total resistance: $R_{\text{total}} = R + R_0$.
- Current: $I = \frac{\mathcal{E}}{R_{\text{total}}} = \frac{B\omega l^2}{2(R+R_0)}$.
- Power in R_0:

$$P_{R_0} = I^2 R_0 = \left(\frac{B\omega l^2}{2(R+R_0)}\right)^2 R_0 = \frac{B^2\omega^2 l^4 R_0}{4(R+R_0)^2}$$

3.3 Origin of Electric Power

A magnetic torque τ_m opposes the rotation. To maintain constant ω, an external agent must supply mechanical power $P_{\text{mech}} = \tau_{\text{ext}}\omega$ that equals the total electrical power dissipated, $P_{\text{elec}} = I^2 R_{\text{total}}$. The origin is the **mechanical work done by an external agent**.

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4 Hydrogen Atom Excitation

4.1 Wavelength of the Other Photon

The emission of $\lambda_1 = 656.3$ nm is the $n = 3 \to n = 2$ transition. The other photon is from the $n = 2 \to n = 1$ transition.

- $\Delta E_{2\to 1} = E_2 - E_1 = -13.6(\frac{1}{2^2} - \frac{1}{1^2}) = 10.2$ eV.
- $\lambda_2 = \frac{hc}{\Delta E_{2\to 1}} = \frac{1240 \text{ eV}\cdot\text{nm}}{10.2 \text{ eV}} \approx 121.6$ nm.

The wavelength is **121.6 nm**.

4.2 Speed of the Free Electron Before Collision

- Energy absorbed by H atom ($n = 1 \to n = 3$): $\Delta E_H = -13.6(\frac{1}{3^2} - \frac{1}{1^2}) = 12.09$ eV.
- KE of electron after collision ($\lambda_{e,f} = 1.915$ nm):

$$KE_{e,f} = \frac{p_{e,f}^2}{2m_e} = \frac{(h/\lambda_{e,f})^2}{2m_e} \approx 0.41 \text{ eV}$$

- KE of electron before collision (by energy conservation):

$$KE_{e,i} = \Delta E_H + KE_{e,f} = 12.09 \text{ eV} + 0.41 \text{ eV} = 12.50 \text{ eV}$$

- Initial speed:

$$v_{e,i} = \sqrt{\frac{2KE_{e,i}}{m_e}} = \sqrt{\frac{2(12.50\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} \approx 2.10\times10^{6}\text{ m/s}$$

The initial speed was 2.10×10^6 **m/s**.

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5 Projectile Motion and Orbital Mechanics

5.1 Projectile Motion

1. Time of Flight

$$t_{\text{flight}} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1200)}{9.81}} \approx 15.64\text{ s}$$

2. Minimum Value of d

$d = (v_{\text{plane}} - v_{\text{veh}} + u)t_{\text{flight}}$. Minimum d is when $u = 0$.

$$d_{\min} = (150 - 40) \times 15.64 \approx 1720.4\text{ m} = \mathbf{1.72\text{ km}}$$

3. Speed of Projectile at Impact

Given $d' = 5d_{\min}$, we find the required u:

$$u = \frac{d'}{t_{\text{flight}}} - 110 = \frac{5\times1720.4}{15.64} - 110 \approx 440\text{ m/s}$$

Velocity components at impact:

- $v_x = v_{\text{plane}} + u = 150 + 440 = 590$ m/s.
- $v_y = gt_{\text{flight}} = 9.81 \times 15.64 \approx 153.4$ m/s.

$$v_{\text{final}} = \sqrt{v_x^2 + v_y^2} = \sqrt{590^2 + 153.4^2} \approx \mathbf{610\text{ m/s}}$$

5.2 Satellite Orbits

1. **Satellite's Angular Velocity (ω_s):**

$$\omega_s = \sqrt{\frac{GM_E}{r^3}} = \sqrt{\frac{gR_E^2}{(R_E+h)^3}} \approx 1.132\times10^{-3}\text{ rad/s}$$

2. **Earth's Angular Velocity (ω_E):**

$$\omega_E = \frac{2\pi}{86400\text{ s}} \approx 7.27\times10^{-5}\text{ rad/s}$$

3. **Number of Photos:**

$$N = \frac{T_{\text{day}}}{T_{\text{rel}}} = \frac{T_{\text{day}}}{2\pi/(\omega_s - \omega_E)} = \frac{86400\times(1.059\times10^{-3})}{2\pi} \approx 14.56$$

The satellite can take **14** photos.

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6 Simple Harmonic Motion

6.1 Particle in a Charged Sphere

1. Describe the Motion

Inside the sphere, $E = \frac{\rho r}{3\epsilon_0}$. The force on charge $-q$ at position x from the center is $F = -qE = -\left(\frac{q\rho}{3\epsilon_0}\right)x$. This is a linear restoring force ($F = -kx$), so the motion is **Simple Harmonic Motion (SHM)**.

2. Speed at the Center

Using conservation of energy:

$$\frac{1}{2}mv_{\max}^2 = q\Delta V = q\int_0^R E dr = q\frac{\rho R^2}{6\epsilon_0}$$

$$v_{\max} = \sqrt{\frac{q\rho R^2}{3m\epsilon_0}} = R\sqrt{\frac{q\rho}{3m\epsilon_0}}$$

6.2 Particle Between Two Points

1. Show the Motion is SHM

The net force on the particle at position x from A is $F_{\text{net}} = -kx + k(L-x) = kL - 2kx = -2k(x - L/2)$. Since the force is a linear restoring force about the equilibrium point $x = L/2$, the motion is **SHM**.

2. Period of Motion

The effective spring constant is $K = 2k$.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{2k}}$$

3. Amplitude and Kinetic Energy

Assuming the particle is at rest at the midpoint of the equilibrium point ($L/2$) and B (L), the turning point is $x = 3L/4$.

- Amplitude: $A = |\frac{3L}{4} - \frac{L}{2}| = \frac{L}{4}$.
- Max Kinetic Energy (at equilibrium point):

$$KE_{\max} = \frac{1}{2}KA^2 = \frac{1}{2}(2k)\left(\frac{L}{4}\right)^2 = \frac{kL^2}{16}$$

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7 Thermodynamics

7.1 Ideal Gas

1. **Intermediate Temperature and γ:**
   - Isochoric heating from state 2 to 3: $T_2 = T_3(P_2/P_3) = 300(1.0/1.1) \approx 272.7$ K.
   - Adiabatic expansion from state 1 to 2 for the remaining gas: $T_1 P_1^{(1-\gamma)/\gamma} = T_2 P_2^{(1-\gamma)/\gamma}$.

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} \implies 1.1 = (1.2)^{(\gamma-1)/\gamma}$$

$$\ln(1.1) = \frac{\gamma - 1}{\gamma}\ln(1.2) \implies \gamma \approx \mathbf{2.10}$$

2. **Initial Volume of Remaining Gas:**

$$P_1 V_i^\gamma = P_2 V_0^\gamma \implies V_i = V_0 \left(\frac{P_2}{P_1}\right)^{1/\gamma} = (1.0)\left(\frac{1.0}{1.2}\right)^{1/2.095} \approx \mathbf{0.917\ m^3}$$

3. **Atomicity of the Gas:** The value $\gamma \approx 2.10$ is physically unrealistic for an ideal gas ($\gamma \leq 5/3$). No conclusion can be drawn.

7.2 Heat Conduction

1. **Stage 1: Melting Ice (0°C to 0°C):**

$$\frac{dQ}{dt} = kA\frac{\Delta T}{L} = (385)(\pi \times 10^{-4})\frac{100}{0.5} \approx 24.19 \text{ W}$$

$$t_1 = \frac{m_{\text{ice}} L_f}{dQ/dt} = \frac{(0.2)(3.34 \times 10^5)}{24.19} \approx 2761.5 \text{ s}$$

2. **Stage 2: Heating Water (0°C to 10°C):**

$$\int_0^{10} \frac{dT_C}{100 - T_C} = \int_0^{t_2} \frac{kA}{m_w c_w L} dt$$

$$t_2 = \frac{m_w c_w L}{kA} \ln\left(\frac{100}{90}\right) \approx 548.8 \text{ s}$$

3. **Total Time:**

$$t_{\text{total}} = t_1 + t_2 = 2761.5 + 548.8 = 3310.3 \text{ s} \approx \mathbf{3310\ s}$$

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8 Waves and Magnetism

8.1 Doppler Effect with Acceleration

1. **Find the Speed of Source B (v_B):** Beat frequency is 8 Hz. Assuming the frequency from B is higher (approaching):

$$f_B = 256 + 8 = 264 \text{ Hz}$$

$$f_B = f_A\left(\frac{v}{v - v_B}\right) \implies 264 = 256\left(\frac{330}{330 - v_B}\right) \implies v_B = 10 \text{ m/s}$$

2. **Find the Distance to Point P:**

$$v^2 = u^2 + 2as \implies 10^2 = 0 + 2(0.5)s \implies s = \mathbf{100\ m}$$

8.2 Proton in a Magnetic Field

1. **Radius of Path:**

$$p = \sqrt{2m_p KE} = \sqrt{2(1.672 \times 10^{-27})(10 \text{ MeV})} \approx 7.32 \times 10^{-20} \text{ kg m/s}$$

$$R = \frac{p}{qB} = \frac{7.32 \times 10^{-20}}{(1.602 \times 10^{-19})(1.5)} \approx 0.305 \text{ m}$$

2. **Exit Angle:** Since the field width ($L = 2.0$ m) is much larger than the radius ($R = 0.305$ m), the proton completes a half-circle and exits in the opposite direction. The angle is **180°**.

Solution to SPhO 2020 Q9(a)

Given:

- Wavelength of incident light: $\lambda = 122\,\text{nm} = 122 \times 10^{-9}\,\text{m}$
- Magnetic flux density: $B = 5.5 \times 10^{-5}\,\text{T}$
- Radius of electron path: $r = 15.8\,\text{cm} = 0.158\,\text{m}$
- Electron charge: $e = 1.602 \times 10^{-19}\,\text{C}$
- Electron mass: $m_e = 9.109 \times 10^{-31}\,\text{kg}$
- Planck constant: $h = 6.626 \times 10^{-34}\,\text{J s}$
- Speed of light: $c = 3.00 \times 10^{8}\,\text{m s}^{-1}$

Step 1: Determine maximum kinetic energy from magnetic field data

When an electron moves perpendicular to a uniform magnetic field, it follows a circular path due to the magnetic Lorentz force providing the centripetal force:

$$evB = \frac{m_e v^2}{r}$$

Solving for momentum $p = m_e v$:

$$p = eBr$$

Hence, the maximum kinetic energy is:

$$K_{\text{max}} = \frac{p^2}{2m_e} = \frac{(eBr)^2}{2m_e}$$

Substitute values:

$$K_{\text{max}} = \frac{(1.602 \times 10^{-19}\,\text{C} \cdot 5.5 \times 10^{-5}\,\text{T} \cdot 0.158\,\text{m})^2}{2 \cdot 9.109 \times 10^{-31}\,\text{kg}}$$

First compute the numerator:

$$eBr = (1.602 \times 10^{-19})(5.5 \times 10^{-5})(0.158) \approx 1.392 \times 10^{-24}\ \text{kg m s}^{-1}$$

$$(eBr)^2 \approx (1.392 \times 10^{-24})^2 = 1.938 \times 10^{-48}$$

Now divide by $2m_e$:

$$K_{\text{max}} = \frac{1.938 \times 10^{-48}}{2 \times 9.109 \times 10^{-31}} \approx \frac{1.938 \times 10^{-48}}{1.822 \times 10^{-30}} \approx 1.064 \times 10^{-18} \text{ J}$$

Convert to electronvolts (1 eV $= 1.602 \times 10^{-19}$ J):

$$K_{\text{max}} = \frac{1.064 \times 10^{-18}}{1.602 \times 10^{-19}} \approx 6.64\,\text{eV}$$

Step 2: Calculate photon energy

Photon energy is:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

$$E_{\text{photon}} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^{8})}{122 \times 10^{-9}} = \frac{1.988 \times 10^{-25}}{1.22 \times 10^{-7}} \approx 1.629 \times 10^{-18} \text{ J}$$

Convert to eV:

$$E_{\text{photon}} = \frac{1.629 \times 10^{-18}}{1.602 \times 10^{-19}} \approx 10.17\,\text{eV}$$

(Alternatively, use $E(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{122} \approx 10.16\,\text{eV}$.)

Step 3: Apply photoelectric equation to find work function

$$K_{\text{max}} = E_{\text{photon}} - \phi \quad \Rightarrow \quad \phi = E_{\text{photon}} - K_{\text{max}}$$

$$\phi = 10.17\,\text{eV} - 6.64\,\text{eV} = 3.53\,\text{eV}$$

Final Answer

$$\boxed{\phi \approx 3.5\,\text{eV}}$$

9.2 Muon Decay and Special Relativity

(i) Classical Muon Count

$$t = H/v = 2000/(0.998c) \approx 6.68 \times 10^{-6} \text{ s}$$

$$N = N_0(1/2)^{t/T_{1/2}} = 568(1/2)^{6.68/1.5} \approx \mathbf{26\ muons}$$

(ii) Reason for Discrepancy

The discrepancy is due to **time dilation**. From the Earth's frame, the muon's half-life is longer, so fewer decay.

(iii) Height of the Mountain According to Muons

The height is subject to **length contraction**.

$$\gamma = \frac{1}{\sqrt{1 - 0.998^2}} \approx 15.8$$

$$H_{\text{muon}} = H_{\text{earth}}/\gamma = 2000/15.8 \approx \mathbf{127\ m}$$

(iv) Velocity in Muon's Frame

Using the relativistic velocity addition formula:

$$u' = \frac{u - V}{1 - uV/c^2} = \frac{0.990c - 0.998c}{1 - (0.990)(0.998)} = \frac{-0.008c}{0.01198} \approx -0.668c$$

The velocity is **0.668c** in the direction opposite to the muon's travel.