9. Mass on a Rotating Tilted Spring System

(a) Effective Spring Constant (k_t) for $\omega = 0$

The two spring segments act in parallel, so the effective constant is $k_t = k_1 + k_2$.

$$k_1 = k\frac{L}{L_1}, \quad k_2 = k\frac{L}{L - L_1}$$

$$k_t = kL\left(\frac{1}{L_1} + \frac{1}{L - L_1}\right) = \frac{kL^2}{L_1(L - L_1)}$$

(b) Equilibrium Position (r_0) with Rotation

At equilibrium, the net inward spring force balances the outward centrifugal force: $F_{spring} = m\omega^2 r_0$. Solving the force balance equation for r_0 gives:

$$r_0 = \frac{kLR_0}{(L - L_1)(k_t - m\omega^2)} = \frac{kLR_0L_1}{kL^2 - m\omega^2 L_1(L - L_1)}$$

(c) Lowest Angular Frequency ω to Reach the Edge

The mass reaches the edge ($r_0 = R_0$) when the total outward force (centrifugal plus the maximum radial gravity component) is balanced by the spring force at that position.

$$F_{spring}(R_0) = F_{centrifugal}(R_0) + F_{gravity,max}$$

$$\frac{kLR_0}{L_1} = m\omega^2 R_0 + mg\sin\phi$$

Solving for ω:

$$\omega = \sqrt{\frac{kL}{mL_1} - \frac{g\sin\phi}{R_0}}$$

Substituting values:

$$\omega = \sqrt{\frac{(10)(0.5)}{(1)(0.2)} - \frac{(9.8)\sin(0.4)}{1}} = \sqrt{25 - 3.816} = \sqrt{21.184} \approx \mathbf{4.60}\,\mathrm{rad/s}$$