

Question 2: Waves and Oscillations

(a) Tidal Motion

We model the sea depth $d(t)$ as a sinusoidal function.

- Mean Depth: $d_{\text{mean}} = \frac{30+20}{2} = 25$ m.
- Amplitude: $A = \frac{30-20}{2} = 5$ m.
- Period: $T = 12$ hours.
- Angular Frequency: $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$ rad/hr.

Since depth is minimum at $t = 0$ (12 noon), the model is $d(t) = d_{\text{mean}} - A \cos(\omega t)$.

$$d(t) = 25 - 5 \cos\left(\frac{\pi}{6}t\right)$$

We need to find the times when the depth is $d(t) = 22.5$ m.

$$22.5 = 25 - 5 \cos\left(\frac{\pi}{6}t\right) \implies \cos\left(\frac{\pi}{6}t\right) = 0.5$$

The principal value is $\frac{\pi}{6}t = \frac{\pi}{3}$, which gives $t = 2$ hours (2:00 pm). By symmetry, the second time is $t_2 = 12 - 2 = 10$ hours (10:00 pm). The ship can stay in the harbor from **2:00 pm to 10:00 pm**.

Assumption: The tidal motion is sinusoidal (Simple Harmonic Motion).