

Solution to SPhO 2020 Q9(a)

Given:

- Wavelength of incident light: $\lambda = 122 \text{ nm} = 122 \times 10^{-9} \text{ m}$
- Magnetic flux density: $B = 5.5 \times 10^{-5} \text{ T}$
- Radius of electron path: $r = 15.8 \text{ cm} = 0.158 \text{ m}$
- Electron charge: $e = 1.602 \times 10^{-19} \text{ C}$
- Electron mass: $m_e = 9.109 \times 10^{-31} \text{ kg}$
- Planck constant: $h = 6.626 \times 10^{-34} \text{ J s}$
- Speed of light: $c = 3.00 \times 10^8 \text{ m s}^{-1}$

Step 1: Determine maximum kinetic energy from magnetic field data

When an electron moves perpendicular to a uniform magnetic field, it follows a circular path due to the magnetic Lorentz force providing the centripetal force:

$$evB = \frac{m_e v^2}{r}$$

Solving for momentum $p = m_e v$:

$$p = eBr$$

Hence, the maximum kinetic energy is:

$$K_{\max} = \frac{p^2}{2m_e} = \frac{(eBr)^2}{2m_e}$$

Substitute values:

$$K_{\max} = \frac{(1.602 \times 10^{-19} \text{ C} \cdot 5.5 \times 10^{-5} \text{ T} \cdot 0.158 \text{ m})^2}{2 \cdot 9.109 \times 10^{-31} \text{ kg}}$$

First compute the numerator:

$$eBr = (1.602 \times 10^{-19})(5.5 \times 10^{-5})(0.158) \approx 1.392 \times 10^{-24} \text{ kg m s}^{-1}$$

$$(eBr)^2 \approx (1.392 \times 10^{-24})^2 = 1.938 \times 10^{-48}$$

Now divide by $2m_e$:

$$K_{\max} = \frac{1.938 \times 10^{-48}}{2 \times 9.109 \times 10^{-31}} \approx \frac{1.938 \times 10^{-48}}{1.822 \times 10^{-30}} \approx 1.064 \times 10^{-18} \text{ J}$$

Convert to electronvolts ($1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$):

$$K_{\max} = \frac{1.064 \times 10^{-18}}{1.602 \times 10^{-19}} \approx 6.64 \text{ eV}$$

Step 2: Calculate photon energy

Photon energy is:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

$$E_{\text{photon}} = \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{122 \times 10^{-9}} = \frac{1.988 \times 10^{-25}}{1.22 \times 10^{-7}} \approx 1.629 \times 10^{-18} \text{ J}$$

Convert to eV:

$$E_{\text{photon}} = \frac{1.629 \times 10^{-18}}{1.602 \times 10^{-19}} \approx 10.17 \text{ eV}$$

(Alternatively, use $E(\text{eV}) = \frac{1240}{\lambda(\text{nm})} = \frac{1240}{122} \approx 10.16 \text{ eV}$.)

Step 3: Apply photoelectric equation to find work function

$$K_{\max} = E_{\text{photon}} - \phi \Rightarrow \phi = E_{\text{photon}} - K_{\max}$$

$$\phi = 10.17 \text{ eV} - 6.64 \text{ eV} = 3.53 \text{ eV}$$

Final Answer

$\phi \approx 3.5 \text{ eV}$