Twentieth-First Singapore Physics Olympiad
Theoretical Paper

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Organized by

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Instructions to Candidates

1. This is a four-hour test.
2. This paper consists of **TEN** (10) questions printed on **SIX** (6) printed pages. Page **SIX** (6) is a Table of Fundamental Constants in Physics which may be useful for your calculations.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your name legibly on the top right hand corner of every answer sheet you submit.
5. Begin each answer on a fresh sheet of paper.
6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.
7. No books or documents relevant to the test may be brought into the examination hall.

1. A point moves along the x-axis with an acceleration $a(t) = kt^2$, where t is the time the point has been in motion, and k is a constant. If the initial speed of the point is u, show that the distance travelled in time t is $x(t) = ut + \frac{1}{12}kt^4$. [6]

2. A hungry bear weighing 700 N walks out on a beam in an attempt to retrieve a basket of food hanging at the end of the beam (see Fig 1). The beam is uniform, weighs 200 N, and is 6.00 m long; the basket weighs 80.0 N. The beam is supported by a light wire inclined at an angle of 60° to the horizontal beam.

   (a) When the bear is at $x = 1.00$ m, find the tension in the wire and the components of the force exerted by the wall on the left end of the beam. [6]

   (b) If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks? [6]

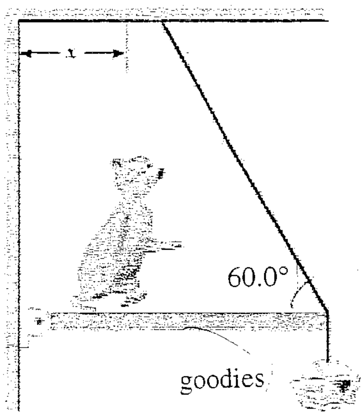

Figure 1: Bear on a beam.

3. A chain of mass M and length L is suspended vertically with its lower end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale when a length of the chain x has fallen? You may neglect the size of the individual links. [10]

4. In the Crab nebula there is a magnetic field of about 2×10^{-8} T and electrons whose energy is about 2×10^{14} eV.

   (a) find the radius of gyration of the electrons.. Compare this radius with the radius of the Earth. [5]

   (b) How long does it take for an electron to complete one turn, giving your anwsers in Earth days? [5]

5. A satellite of mass 1 000 kg is describing an elliptical orbit around the Earth. The minimum height of the orbit above the Earths surface is 250 km (at perigee) and the maximum height above the Earths surface is 1 200 km (at apogee).

   (a) Sketch the variation of the kinetic energy of the satellite with the height above the Earths surface from perigee to apogee. [4]

   (b) If the velocities of the satellite at apogee and at perigee are v_a and v_p respectively, use the conservation of angular momentum equation to find v_a in terms of v_p. [6]

   (c) By considering the conservation of energy equation, hence determine the value of v_a [4]

6. In the troposphere (lower part of the atmosphere), the temperature is not uniform but decreases with increasing height according to the relation

$$T = T_0 - \alpha y$$

   where T_0 is the temperature of the Earth's surface and T is the temperature at height y. The proportionality constant α is called the lapse rate of temperature.

   (a) Find the presure at height y in terms of the molecular mass of air M, p_0, the pressure at the Earth's surface and any other appropriate physical quantities. [6]

   (b) Obtain the expression for the pressure in (a) in the limit $\alpha \to 0$. [4]

   (c) If the lapse rate of temperature α is 6×10^{-3} K m^{-1}, calculate the value of p for a height of 8863 m. You may assume that the temperature on the Earth's surface is 300 K and $p_0 = 1$ atm. [4]

7. A flash of light is sent out from a point x_1 on the x-axis of an inertial frame **S**, and it is received at a point $x_2 = x_1 + \ell$. Consider another inertial frame, **S**$'$, moving with constant speed $V = bc$ along the x-axis; show that, in **S**$'$:

(a) the separation between the point of emission and the point of reception of the light is $\ell' = \ell \left(\frac{1-b}{1+b}\right)^{1/2}$ [6]

(b) the time interval between the emission and reception of the light is

$$\Delta t' = \frac{\ell}{c}\left(\frac{1-b}{1+b}\right)^{1/2}.$$

[4]

8. Two small spheres, each of mass 2.00 g, are suspended by light strings 10.0 cm in length (See Figure 2). A uniform electric field is applied in the x direction. The spheres have charges equal to -5.00×10^{-8} C and $+5.00 \times 10^{-8}$ C. Determine the electric field that enables the spheres to be in equilibrium at an angle $\theta = 10.0°$. [8]

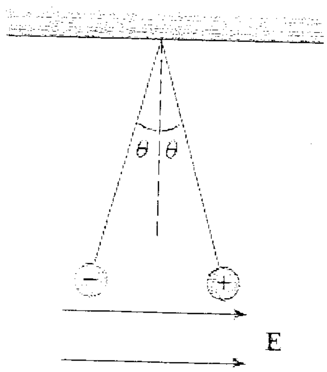

Figure 2: Two small spheres suspended by light strings

9. A conducting rod of length ℓ moves with velocity v parallel to a long wire carrying a steady current I. The axis of the rod is maintained perpendicular to the wire with the near end a distance r away, as shown in Figure 3. Show that the magnitude of the emf induced in the rod is

$$\mathcal{E} = \kappa I v \ln\left(1 + \frac{\ell}{r}\right),$$

where κ is a constant. Write the constant κ. [10]

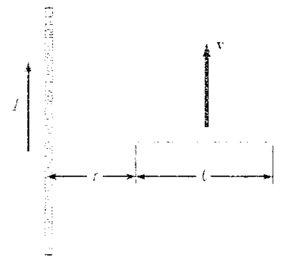

Figure 3: Conducting rod moving next to a long wire carrying a current.

10. Compact disc (CD) and digital video disc (DVD) players use interference to generate a strong signal from a tiny bump. The depth of a pit is chosen to be one quarter of the wavelength of the laser light used to read the disc. Then light reflected from the pit and light reflected from the adjoining flat differ in path length traveled by one-half wavelength, to interfere destructively at the detector. As the disc rotates, the light intensity drops significantly every time light is reflected from near a pit edge. The space between the leading and trailing edges of a pit determines the time between the fluctuations. The series of time intervals is decoded into a series of zeros and ones that carries the stored information. Assume that infrared light with a wavelength of 780 nm in vacuum is used in a CD player. The disc is coated with plastic having an index of refraction of 1.50. What should be the depth of each pit? A DVD player uses light of a shorter wavelength, and the pit dimensions are correspondingly smaller. This is one factor resulting in greater storage capacity on a DVD compared to a CD. [6]

END OF PAPER