37th Singapore Physics Olympiad Theory Paper

Organised by

Institute of Physics

In conjunction with
National Institute of Education Singapore, Nanyang Technological University Singapore

National University of Singapore

Ministry of Education Singapore

And sponsored by
Micron Technology Foundation, Inc

Instructions to Candidates

1. This is a **3-hour** paper.
2. This paper consists of **Ten (10)** questions printed on **Twenty-Eight (28)** pages. The second page is a General Information Sheet.
3. Attempt all questions.
4. Write your answers in the space provided in the question booklet. Full working must be shown. Correct answer without proper working will **NOT** be awarded marks.
5. You may request working paper from the invigilators.
6. You may not refer to any books or documents relevant to the competition.

NAME:____________________________________ INDEX NO: ______________

SCHOOL: _________________________________

For Examiner's Use

Question No.	**Marks Awarded**
1	**/ 10**
2	**/ 10**
3	**/ 10**
4	**/ 10**
5	**/ 10**
6	**/ 10**
7	**/ 10**
8	**/ 10**
9	**/ 5**
10	**/ 5**
Total	**/ 90**

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.81 \text{ m s}^{-2} = \lvert\vec{g}\rvert$
Radius of the Earth,	$R_E = 6.371 \times 10^6 \text{m}$
Universal gas constant,	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$
Atomic mass unit,	$u = 1.66 \times 10^{-27}$ kg
Speed of light in vacuum,	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Charge of electron,	$e = 1.60 \times 10^{-19}$ C
Planck's constant,	$h = 6.63 \times 10^{-34}$ J s
Mass of electron,	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.000549u$
Mass of proton,	$m_p = 1.67 \times 10^{-27} \text{ kg} = 1.007u$
Rest mass of alpha particle,	$m_\alpha = 4.003\text{u}$
Boltzmann constant,	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number,	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5$ Pa
Density of water,	$\rho_w = 1000 \text{ kg m}^{-3}$
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0 + a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$
Area under the curve of $y = x^n$ for x between 0 and x_0	$\int_0^{x_0} x^n \, dx = \frac{1}{n+1} x_0^{n+1}$

1. (a)(i) Figure 1.1 shows how the velocity $v/(\mathrm{ms}^{-1})$ varies with the displacement x/m for a particle undergoing simple harmonic motion.

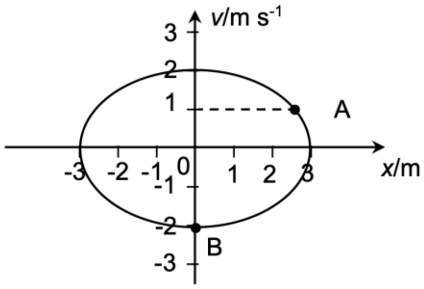

Figure 1.1

Determine the period of the motion. [2 marks]

(a)(ii) Calculate the time taken for the particle to move from the state represented by the point A to the state represented by the point B. [2 marks]

(b)(i) By using the expression $v = \pm\omega\sqrt{x_0^2 - x^2}$, show that the potential energy E of a particle of mass m undergoing simple harmonic motion of angular frequency ω is given by $E = \frac{1}{2}m\omega^2x^2$. [3 marks]

(b)(ii) Figure 1.2 shows a pendulum bob of mass m suspended from a string of length L.

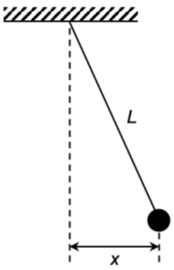

Figure 1.2

Show that the potential energy E of the pendulum is given by (with respect to the equilibrium position, $E = mgL - mg\sqrt{L^2 - x^2}$. [1 mark]

(b)(iii) By combining the results of **(b)(i)** and **(b)(ii)**, show that the period of a pendulum of length L is $2\pi\sqrt{\frac{L}{g}}$ for small-angle oscillations. [2 marks]

2. (a) A 2.5 kg block is placed on a horizontal rotating disk with a radius of 0.5 m. The disk rotates at a constant angular speed of 3 rad/s. The block is located 0.3 m from the centre of the disk. The coefficient of static friction between the block and the disk is 0.4.

Calculate the maximum static frictional force that can act on the block. [2 marks]

(b) Determine whether the block will slide or stay in place as the disk rotates. [3 marks]

(c) If the block remains stationary relative to the disk, calculate the minimum coefficient of friction required for the block to stay in place if it were moved to the edge of the disk (0.5 m from the centre). [2 marks]

(d) If the block is now moved to the edge (0.5 m from the centre) and the coefficient of friction remains 0.4, determine the speed of the block when it begins to slip relative to the rotating disk. [3 marks]

3. (a) As shown in Figure 3, a spring of spring constant k is placed horizontally on the floor. One end of the spring is attached to a solid block of mass M and the other end of the spring is compressed by a sphere of mass m by a distance d. Neglect all friction and neglect the mass of the spring as well as the mass of the plate between the sphere and the spring.

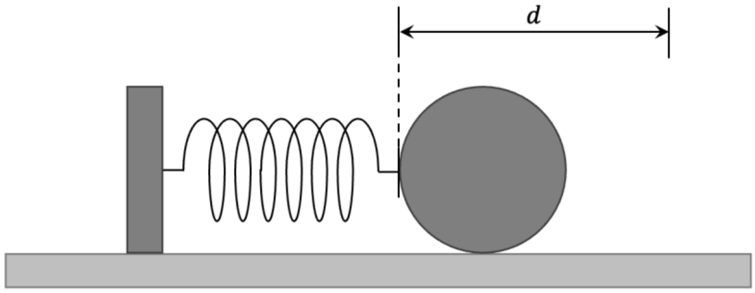

Figure 3

The block is fixed on the floor. Derive an expression for the speed of the sphere when it is at a distance d from the initial position. [2 marks]

(b) The block is now free to move. Derive an expression for the speed of the sphere relative to the floor after it loses contact with the spring. [4 marks]

(c) The block is still free to move. Derive an expression for the distance the block has travelled when the sphere just loses contact with the spring. [4 marks]

4. Figure 4 shows a long, straight wire of negligible resistance bent into a V shape, with its two arms making an angle α with each other, and placed horizontally in a vertical uniform magnetic field of strength B. A rod of total mass m, and resistance R per unit length, is placed on the V-shaped conductor, at a distance x_0 from its vertex A, and perpendicular to the bisector of the angle α.

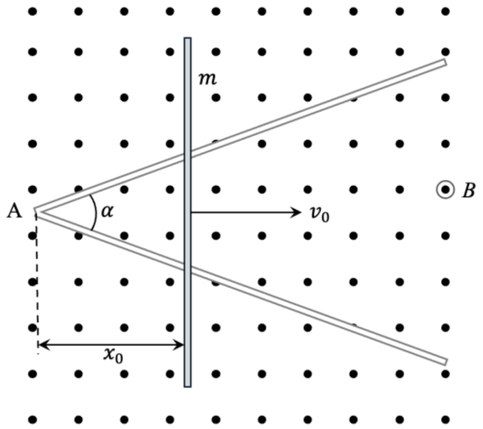

Figure 4 (Top view)

The rod started off with an initial velocity v_0 in the direction along the bisector, and away from A. The rod is long enough not to fall off the wire during the subsequent motion, and the electrical contact between the two is perfect, while friction between them is negligible.

Show that the rod will stop at $x = \sqrt{x_0^2 + \frac{mv_0R}{B^2 \tan\frac{\alpha}{2}}}$. [10 marks]

5 (a)(i) A cargo spacecraft of mass m has an initial circular orbit of radius R_1 about a planet of mass M. A Hohmann transfer orbit is used to transfer it to a final circular orbit of radius R_2 using two engine burns.

Determine an expression for the minimum speed of the spacecraft required to enter the transfer orbit from the initial circular orbit in terms of m, M, R_1 and R_2.

[3 marks]

(a)(ii) Show that the increase in total energy of the spacecraft for the first burn is given by

$$\frac{GMm}{2R_1}\left(\frac{R_2 - R_1}{R_2 + R_1}\right)$$

[2 marks]

(a)(iii) Determine an expression for the time of flight for the transfer from the initial to the final orbit in terms of m, M, R_1 and R_2. [2 marks]

(b) Figure 5.1 shows a satellite with three solar panels folded in close to the satellite's axis for the journey into space in the hold of the cargo spacecraft.

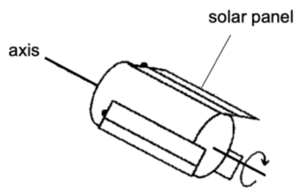

Figure 5.1

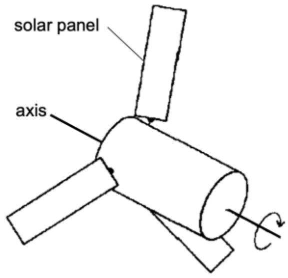

Figure 5.2

Just before it is released into space, the satellite is spun to rotate at $5.2\ \text{rad s}^{-1}$. Once released, the solar panels are extended as shown in Figure 5.2.

moment of inertia of the satellite about its axis with panels folded = 110 kg m^2
moment of inertia of the satellite about its axis with panels extended = 230 kg m^2

Calculate the decrease in rotational kinetic energy after the solar panels have been extended. [3 marks]

6 (a)(i) A 5.0 kg mass is hung with a long string from a frictionless, massless pulley of radius 0.10 m as shown in Figure 6. The axis of the pulley is attached to a 10-turns, rectangular loop in the horizontal plane which carries a current of 5.0 A in the direction shown. The entire loop has negligible mass and is in a uniform, horizontal magnetic field of flux density 0.50 T directed towards the left.

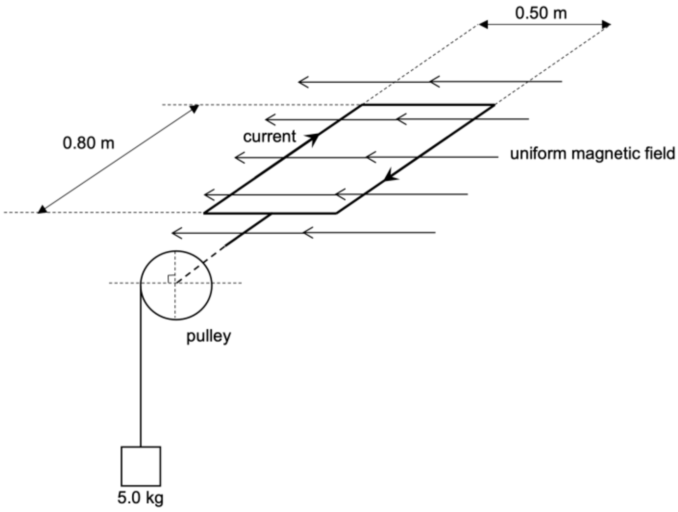

Figure 6

Determine the magnitude of the magnetic dipole moment. [2 marks]

(a)(ii)

Determine the magnitude of the initial magnetic torque on the loop.

[2 marks]

(b) Calculate the net torque acting on the pulley. [2 marks]

(c)(i) The magnetic torque causes the rectangular loop to rotate to a vertically upright position and the mass to move upwards.

Calculate the change in potential energy of the magnetic dipole. [2 marks]

(c)(ii) Calculate the speed of the mass at this position. [2 marks]

7 (a) Consider a circuit consisting of a resistor of resistance R, inductor of inductance L and a capacitor of capacitance C as shown in Figure 7. The capacitor is initially fully charged at time $t = 0$.

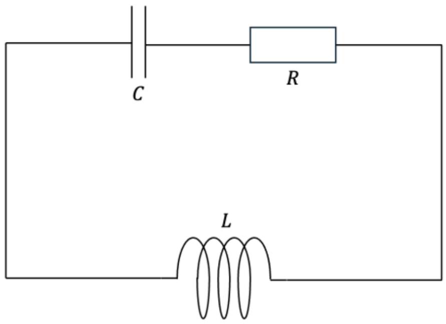

Figure 7

Given that the potential difference V in the capacitor is governed by the differential equation

$$\frac{d^2V}{dt^2} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \left(\frac{1}{LC}\right)V = 0$$

Show that $V = Ae^{-\alpha t}\cos\omega t$ is a solution to the differential equation above where $\alpha = \frac{R}{2L}$. Hence, write down an equation for ω in terms of R, L and C. [5 marks]

(b)(i) Given that $C = 15.0\ \text{nF}$, $L = 0.22\ \text{mH}$ and $R = 75.0\ \Omega$, calculate the oscillation frequency of the circuit and the time taken for the amplitude of oscillation to decay to 10.0 % of its original value. [3 marks]

(b)(ii) Calculate the value of R that would result in a critically damped circuit. [2 marks]

8 (a) A spacecraft is moving at a constant velocity v relative to an observer on Earth. The spacecraft is traveling in a straight line from point A to point B, which are 10 light-years apart as measured in Earth's frame.

Calculate the time it takes for the spacecraft to travel from point A to point B as measured by the observer on Earth if the velocity of the spacecraft is $0.8c$, where c is the speed of light. [2 marks]

(b) Determine the time experienced by a clock on the spacecraft for the journey from A to B. [3 marks]

(c) Suppose a light signal is sent from point A to point B just as the spacecraft passes point A. How much time does it take for the signal to reach point B as measured by:

i. The observer on Earth.

ii. The astronaut on the spacecraft.

[3 marks]

(d) Assume that when the spacecraft reaches point B, a signal is sent back to point A. According to the observer on Earth, do the events "spacecraft reaches point B" and "signal reaches point A" occur simultaneously? Explain your reasoning. [2 marks]

9 (a) One mole of helium gas, initially at state A where pressure is 101.3 kPa and temperature is 5.0 °C, undergoes a process in which pressure falls to half its initial value at constant volume to state B. It then expands at the same pressure, doubling its volume, to state C.

Calculate the total work done by the gas. [2 marks]

(b) Suppose the gas undergoes a process from state A to state C at constant temperature, determine the work done by the gas. [3 marks]

10 (a) Figure 10 shows an opaque screen in which there are two narrow parallel slits, P and Q, separated by a distance d.

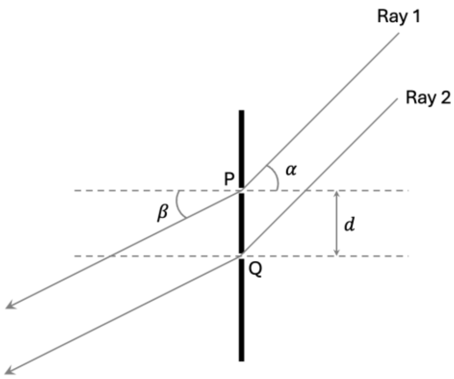

Figure 10

A beam of monochromatic light of wavelength λ is incident on the screen at an angle α to the normal. Consider light emerging from the slits at an angle β to the normal.

Find an expression for the total path difference between rays 1 and 2 as a result of passing through the slits. Hence find the condition for the light emerging at angle β to be of maximum intensity. [2 mark]

(b) Show that, for small values of the angle β, the angular separation between adjacent maxima in the emergent light is independent of the angle of incidence α.

[3 marks]

- End of paper -