6. Relativistic Electron in a Nucleus

Calculation Steps

1. **Estimate Minimum Momentum:** From Heisenberg's Uncertainty Principle, with $\Delta x \approx r = 6 \times 10^{-15}\,\mathrm{m}$:
$$p \approx \Delta p \approx \frac{\hbar}{\Delta x} = \frac{1.054 \times 10^{-34}\,\mathrm{J \cdot s}}{6 \times 10^{-15}\,\mathrm{m}} \approx 1.757 \times 10^{-20}\,\mathrm{kg \cdot m/s}$$

2. **Calculate Energy Terms in MeV:**

   - Rest Energy of Electron: $m_e c^2 = 0.511\,\mathrm{MeV}$.
   - Momentum Energy: $pc = (1.757 \times 10^{-20})(3.00 \times 10^8) = 5.271 \times 10^{-12}\,\mathrm{J}$.

$$pc = \frac{5.271 \times 10^{-12}\,\mathrm{J}}{1.602 \times 10^{-13}\,\mathrm{J/MeV}} \approx 32.9\,\mathrm{MeV}$$

3. **Calculate Kinetic Energy:** Using the relativistic energy-momentum relation $E^2 = (pc)^2 + (m_e c^2)^2$:

$$E = \sqrt{(32.9\,\mathrm{MeV})^2 + (0.511\,\mathrm{MeV})^2} \approx 32.9\,\mathrm{MeV}$$

The minimum kinetic energy is $K = E - m_e c^2$:

$$K = 32.9\,\mathrm{MeV} - 0.511\,\mathrm{MeV} \approx 32.4\,\mathrm{MeV}$$

4. **Convert Back to Joules:**

$$K = (32.4\,\mathrm{MeV})(1.602 \times 10^{-13}\,\mathrm{J/MeV}) \approx \mathbf{5.19 \times 10^{-12}}\,\mathrm{J}$$