8 Special Relativity

8.1 (a) Calculate the time it takes for the spacecraft to travel from A to B as measured by the observer on Earth.

$$\Delta t_{\text{Earth}} = \frac{L_0}{v} = \frac{10 \text{ light-years}}{0.8c} = 12.5 \text{ years}$$

8.2 (b) Determine the time experienced by a clock on the spacecraft.

This is the proper time Δt_0. First, find the Lorentz factor γ:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-0.8^2}} = \frac{1}{0.6} = \frac{5}{3}$$

$$\Delta t_0 = \frac{\Delta t_{\text{Earth}}}{\gamma} = \frac{12.5 \text{ years}}{5/3} = 7.5 \text{ years}$$

8.3 (c) How much time does it take for the signal to reach point B as measured by:

i. The observer on Earth. The light signal travels 10 light-years at speed c. Time taken is **10 years**.

ii. The astronaut on the spacecraft. Use the Lorentz transformation for time. Earth frame is S, spacecraft is S'. Event 1 (Signal sent from A): $(t_1, x_1) = (0, 0)$ in S. Event 2 (Signal reaches B): $(t_2, x_2) = (10 \text{ yr}, 10 \text{ ly})$ in S. Time of Event 2 in S':

$$t_2' = \gamma \left(t_2 - \frac{vx_2}{c^2}\right) = \frac{5}{3}\left(10 \text{ yr} - \frac{(0.8c)(10 \text{ ly})}{c^2}\right) = \frac{5}{3}(10-8) \text{ yr} = \frac{10}{3} \text{ years}$$

Time taken is **3.33 years**.

8.4 (d) Do the events "spacecraft reaches point B" and "signal reaches point A" occur simultaneously according to the observer on Earth?

No.

- Spacecraft reaches B at $t_1 = 12.5$ years.
- The signal is sent from B at $t = 12.5$ years and travels 10 light-years back to A, which takes another 10 years. The signal reaches A at $t_2 = 12.5 + 10 = 22.5$ years.

Since $t_1 \neq t_2$, the events are not simultaneous.