(b) Motion in an Electric Field

Inside the sphere, the electric field at distance r from the center is found using Gauss's Law:

$$E = \frac{\rho r}{3\epsilon_0}$$

The force on the particle with charge q is $F = qE$. Since q is negative, the force is a restoring force directed towards the center:

$$F = q\frac{\rho r}{3\epsilon_0} = -\left(\frac{|q|\rho}{3\epsilon_0}\right) r$$

This is Simple Harmonic Motion, $F = -k_{eff} r$, with $k_{eff} = \frac{|q|\rho}{3\epsilon_0}$. The angular frequency is:

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{|q|\rho}{3\epsilon_0 m}}$$

$$\omega = \sqrt{\frac{(0.05 \times 10^{-9})(5 \times 10^{-9})}{3(8.85 \times 10^{-12})(1.0 \times 10^{-9})}} = \sqrt{9.416} \approx 3.068\,\mathrm{rad/s}$$

(i) The time to move from one end to the other is half a period:

$$t = \frac{T}{2} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega} = \frac{\pi}{3.068} \approx 1.024\,\mathrm{s}$$

(ii) The speed is maximum at the center, $v_{max} = A\omega$, where amplitude $A = R = 0.5\,\mathrm{m}$.

$$v_{max} = (0.5)(3.068) \approx 1.534\,\mathrm{m/s}$$

Answers: (i) The time taken is **1.024 s**. (ii) The speed at the center is **1.534 m/s**.
