

5. Standing Waves on a String

Derivation

The speed of a wave on a string with tension $T = mg$ and linear density μ is $v = \sqrt{mg/\mu}$. For a standing wave with n loops on a string of length L , the wavelength is $\lambda = 2L/n$, and the speed is $v = f\lambda = 2Lf/n$. Equating the two expressions for speed:

$$\sqrt{\frac{mg}{\mu}} = \frac{2Lf}{n} \implies m = \left(\frac{4L^2 f^2 \mu}{g} \right) \frac{1}{n^2}$$

Let m_1 correspond to $n + 1$ loops and m_2 correspond to n loops.

$$\frac{m_1}{m_2} = \frac{n^2}{(n+1)^2} \implies \sqrt{\frac{m_1}{m_2}} = \frac{n}{n+1}$$

$$n = \frac{\sqrt{m_1}}{\sqrt{m_2} - \sqrt{m_1}} = \frac{\sqrt{0.2861}}{\sqrt{0.4470} - \sqrt{0.2861}} = \frac{0.5349}{0.6686 - 0.5349} \approx 4.00$$

So, $n = 4$. Now we solve for μ using the equation for m_2 :

$$\mu = \frac{m_2 g n^2}{4 L^2 f^2} = \frac{(0.4470)(9.80)(4^2)}{4(1.20)^2(120)^2} \approx 8.45 \times 10^{-4} \text{ kg/m}$$

The linear density is **0.845 g/m**.