SPhO 2022 Theory Paper 2 Solutions

1 Mechanics

1.1 Compound Pendulum Collision

This problem involves an inelastic collision followed by a pendulum swing. We apply the conservation of angular momentum for the collision and the conservation of mechanical energy for the swing.

1.1.1 1. Moment of Inertia Calculation

The total moment of inertia (I_{total}) of the system (rod + disc + object) about the pivot point A after the collision is calculated.

- **Rod:** $I_{\text{rod}} = \frac{1}{3}M_{\text{rod}}l^2 = \frac{1}{3}(1.5 \text{ kg})(0.50 \text{ m})^2 = 0.125 \text{ kg m}^2$
- **Disc (Parallel Axis Theorem):** $I_{\text{disc}} = I_{\text{cm}} + M_{\text{disc}}d^2 = \frac{1}{2}M_{\text{disc}}R^2 + M_{\text{disc}}l^2$ $I_{\text{disc}} = \frac{1}{2}(3.0)(0.075)^2 + (3.0)(0.50)^2 = 0.0084375 + 0.75 = 0.7584375 \text{ kg m}^2$
- **Object:** $I_{\text{obj}} = m_{\text{obj}}l^2 = (0.1 \text{ kg})(0.50 \text{ m})^2 = 0.025 \text{ kg m}^2$
- **Total Moment of Inertia:** $I_{\text{total}} = I_{\text{rod}} + I_{\text{disc}} + I_{\text{obj}} = 0.125 + 0.7584375 + 0.025 = 0.9084375 \text{ kg m}^2$

1.1.2 2. Conservation of Angular Momentum

Angular momentum about pivot A is conserved during the collision.

$$\begin{aligned} L_{\text{initial}} &= L_{\text{final}} \\ m_{\text{obj}}vl &= I_{\text{total}}\omega \\ (0.1)(30)(0.50) &= (0.9084375)\omega \\ \omega &= \frac{1.5}{0.9084375} \approx 1.6511 \text{ rad/s} \end{aligned}$$

1.1.3 3. Conservation of Energy

The rotational kinetic energy is converted into gravitational potential energy.

- **Center of Mass (from pivot A):** $y_{\text{cm}} = \frac{M_{\text{rod}}(l/2) + M_{\text{disc}}(l) + m_{\text{obj}}(l)}{M_{\text{total}}} = \frac{(1.5)(0.25) + (3.0)(0.5) + (0.1)(0.5)}{4.6} \approx 0.4185 \text{ m}$
- **Energy Conservation:**

$$\begin{aligned} \frac{1}{2}I_{\text{total}}\omega^2 &= M_{\text{total}}g\Delta h_{\text{cm}} = M_{\text{total}}gy_{\text{cm}}(1 - \cos\theta) \\ \frac{1}{2}(0.9084375)(1.6511)^2 &= (4.6)(9.81)(0.4185)(1 - \cos\theta) \\ 1.2396 &= 18.88(1 - \cos\theta) \\ 1 - \cos\theta &= \frac{1.2396}{18.88} \approx 0.06565 \\ \cos\theta &\approx 0.93435 \\ \theta &= \arccos(0.93435) \approx 20.9^\circ \end{aligned}$$

Note: The calculated answer is 20.9°*. The provided answer in the paper is* 17.5°*. This discrepancy may be due to a typo in the problem's given values.*