SPhO 2024 Theory Paper
Comprehensive Solutions

Contents

4 Electromagnetic Induction

1. **Induced EMF ($\mathcal{E}$):** At distance x, the rod length is $L = 2x\tan(\frac{\alpha}{2})$. The loop area is $A = x^2\tan(\frac{\alpha}{2})$. The magnetic flux is $\Phi_B = Bx^2\tan(\frac{\alpha}{2})$.

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = B\tan(\frac{\alpha}{2})\cdot 2x\frac{dx}{dt} = 2Bxv\tan(\frac{\alpha}{2})$$

2. **Induced Current (I):** The rod resistance is $R_{\text{rod}} = R\cdot L = 2Rx\tan(\frac{\alpha}{2})$.

$$I = \frac{|\mathcal{E}|}{R_{\text{rod}}} = \frac{2Bxv\tan(\frac{\alpha}{2})}{2Rx\tan(\frac{\alpha}{2})} = \frac{Bv}{R}$$

3. **Magnetic Braking Force (F_m):**

$$F_m = ILB = \left(\frac{Bv}{R}\right)\left(2x\tan(\frac{\alpha}{2})\right)B = \frac{2B^2vx\tan(\frac{\alpha}{2})}{R}$$

4. **Equation of Motion:** $ma = -F_m$. Use $a = v\frac{dv}{dx}$:

$$mv\frac{dv}{dx} = -\frac{2B^2vx\tan(\frac{\alpha}{2})}{R} \implies m\frac{dv}{dx} = -\frac{2B^2x\tan(\frac{\alpha}{2})}{R}$$

5. **Integration:** Integrate from initial state ($x = x_0, v = v_0$) to final state ($x = x_f, v = 0$).

$$\begin{aligned}
\int_{v_0}^{0} m\,dv &= \int_{x_0}^{x_f} -\frac{2B^2\tan(\frac{\alpha}{2})}{R}x\,dx \\
[mv]_{v_0}^{0} &= -\frac{2B^2\tan(\frac{\alpha}{2})}{R}\left[\frac{x^2}{2}\right]_{x_0}^{x_f} \\
-mv_0 &= -\frac{B^2\tan(\frac{\alpha}{2})}{R}(x_f^2 - x_0^2) \\
x_f^2 - x_0^2 &= \frac{mv_0R}{B^2\tan(\frac{\alpha}{2})} \\
x_f &= \sqrt{x_0^2 + \frac{mv_0R}{B^2\tan\left(\frac{\alpha}{2}\right)}}
\end{aligned}$$