4. RL Circuit with Constant Current Source

(a) Current Through the Inductor as a Function of Time

By Kirchhoff's laws:

1. KCL: $I = I_R(t) + I_L(t)$
2. KVL: $V_R = V_L \implies I_R(t)R = L\frac{dI_L}{dt}$

Substituting $I_R = I - I_L$ gives the differential equation: $(I - I_L)R = L\frac{dI_L}{dt}$. Solving this equation with the initial condition $I_L(0) = 0$ yields:

$$I_L(t) = I(1 - e^{-(R/L)t})$$

(b) Time When Currents are Equal

We need to find the time t when $I_R(t) = I_L(t)$, which implies $I_L(t) = I/2$.

$$\begin{aligned}
\frac{I}{2} &= I(1 - e^{-(R/L)t}) \\
\frac{1}{2} &= 1 - e^{-(R/L)t} \\
e^{-(R/L)t} &= \frac{1}{2} \\
-\frac{R}{L}t &= \ln\left(\frac{1}{2}\right) = -\ln(2) \\
t &= \frac{L}{R}\ln(2)
\end{aligned}$$