35th Singapore Physics Olympiad Theory Paper 2 With Answers

Organised by

Institute of Physics

In conjunction with
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Ministry of Education Singapore

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Instructions to Candidates

1. This is a 2-hour paper.

2. This paper consists of five (5) questions printed on fourteen (14) pages.
   The second page is a General Information Sheet.

3. Attempt all questions. Write your index number on the top of all pages submitted.

4. Write your answers in the space provided in the question booklet. Full working must be shown. Correct answer without proper working will **NOT** be awarded marks.

5. You may request working paper from the invigilators.

6. You may not refer to any books or documents relevant to the competition.

NAME:____ INDEX NO: ______________

SCHOOL: ______________________________________

For Examiner's Use

Question No.	Marks Awarded	Paper 2 is graded out of a total of 58 marks as Q4 should be out of 12 marks instead of the stated 14 marks indicated.
1	/ 13	
2	/ 12	
3	/ 11	
4	/ ~~14~~ 12	
5	/ 10	
Total	/ ~~60~~ 58	

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.81$ m s^{-2} $= \lvert\vec{g}\rvert$
Radius of the Earth,	$R_E = 6.371 \times 10^6$m
Universal gas constant,	$R = 8.31$ J mol^{-1} K^{-1}
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
Atomic mass unit,	$u = 1.66 \times 10^{-27}$ kg
Speed of light in vacuum,	$c = 3.00 \times 10^8$ m s^{-1}
Charge of electron,	$e = 1.60 \times 10^{-19}$ C
Planck's constant,	$h = 6.63 \times 10^{-34}$ J s
Mass of electron,	$m_e = 9.11 \times 10^{-31}$ kg $= 0.000549u$
Mass of proton,	$m_p = 1.67 \times 10^{-27}$ kg $= 1.007u$
Rest mass of alpha particle,	$m_\alpha = 4.003$u
Boltzmann constant,	$k = 1.38 \times 10^{-23}$ J K^{-1}
Avogadro's number,	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5$ Pa
Density of water,	$\rho_w = 1000$ kg m^{-3}
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3$ J kg^{-1} K^{-1}
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5$ J kg^{-1}
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0+a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$
Area under the curve of $y = x^n$ for x between 0 and x_0	$\int_0^{x_0} x^n \, dx = \frac{1}{n+1} x_0^{n+1}$

1. (a) A compound pendulum consists of a uniform thin rod AB of length 50 cm and mass 1.5 kg and a thin disc with diameter 15 cm and mass 3.0 kg attached to the end B of the rod. The system can oscillate about a horizontal axis through the end A of the rod perpendicular to the plane of the disc. Initially, the system hangs in equilibrium with the rod vertical. A small object of mass 0.1 kg travelling horizontally with a speed of 30 ms^{-1} strikes the system at B and sticks to it. What is the greatest angle which the rod makes with the downward vertical in the subsequent motion?

[8 marks]

[17.5°]

[Moment of inertia of a disc, with mass M and radius R, about an axis through its centre is $\frac{1}{2}MR^2$;

Moment of inertia of a thin rod, with mass M and length ℓ, about an axis through its end is $\frac{1}{3}M\ell^2$]

1. (b) An alloy is made by combining different percentage **by mass** of gold and copper together. A block of such alloy has a weight of 10.0 kgf in air. When the block is completely immersed in a saturated salt solution which has a density of 1230 kg m^{-3}, its weight appears to become 9.326 kgf. Calculate the percentage **by mass** of gold and copper in the alloy.

[5 marks]

[The alloy contains 95% by mass of gold and 5% by mass of copper.]

[kgf = kilogram-force. It is the weight of a kilogram under standard gravity. One kgf = 9.81 N

Densities (in kg m^{-3}) : gold : 19 300 ; copper : 8 960]

2. (a) A wire is mounted on a sounding board and has a tension of F_1. When it is set into vibration in the fundamental mode, the sound waves produce a beat with frequency 5 Hz with a 256-Hz tuning fork. When the tension in the wire is increased to F_2, the beat frequency produced by the sound waves in the fundamental mode becomes 3 Hz with the tuning fork. Find the ratio $\frac{F_2}{F_1}$.

[$\frac{F_2}{F_1} = \left(\frac{259}{251}\right)^2 = 1.065 \ or \ \left(\frac{253}{251}\right)^2 = 1.016$]

[4 marks]

2. (b) (i) A uniform bar AB of length 60 cm and weight 20 N is suspended by two springs 1 and 2. Spring 1 has a force constant of 30 Nm^{-1} while the force constant of spring 2 is 50 Nm^{-1}. Both springs have a length of 20 cm when they are not stretched. Spring 1 is attached to the end A of the bar and spring 2 is attached to a point on the bar at a distance x from end B of the bar as shown in Fig. 1. What is the value of x so that the bar can remain horizontal while in equilibrium?
[0.12 m]

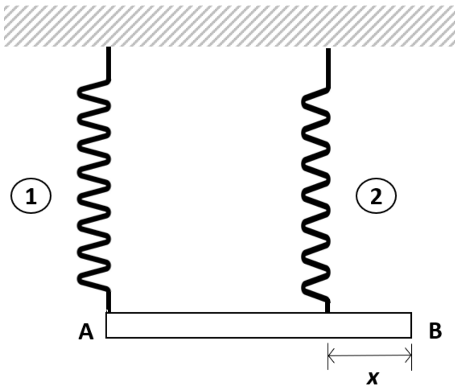


Fig. 1.

(ii) The two springs are now attached to two vertical walls at 50 cm apart. A small particle of mass 50g is attached to the free ends of both springs as shown in Fig. 2. The particle can move over a smooth horizontal surface between the two walls.

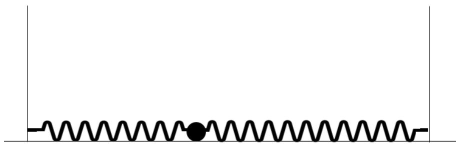


Fig. 2.

1. What are the lengths of both springs when the system is in equilibrium?
[Length of spring 1 and spring 2 are 0.2625 m and 0.2375 m respectively at equilibrium.]

2. If the particle is displaced through a small distance from the equilibrium position and released, what is the frequency of its oscillation?
[6.37 Hz]

[8 marks]

Note: Responses were graded based on erroneous values given in the original question. Corrections are in RED.

3. (a) A uniform thin wire XY is suspended by two inelastic strings attached to its end. The wire has a mass of 20 g and its resistance is 1.2 Ω. The length of the wire is 50 cm. The wire and the supporting strings are placed in a region where there is a uniform magnetic field of magnitude 40 mT and direction perpendicular to the plane containing the strings and the wire, as shown in Fig. 3. What is the direction and magnitude of the potential difference that must be applied to the wire so that the tension in the strings becomes zero?

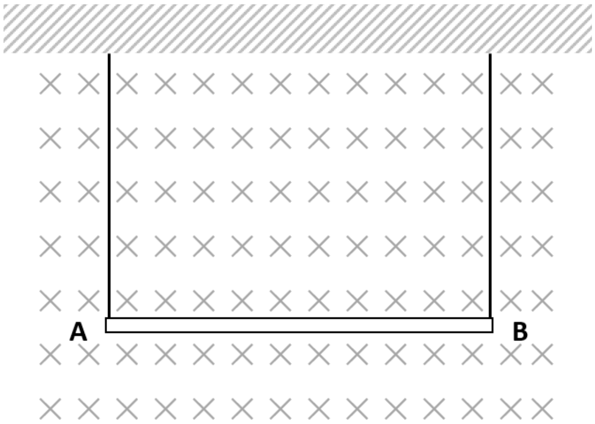


Fig. 3.

[From A to B, 11.77 V]

[4 marks]

3. (b) The surface charge density $\sigma(r)$ at a point on a circular thin disc, having diameter 20 cm, can be described by the equation $\sigma(r) = +1.05r \ \ \mu C\,\mathrm{m}^{-2}$ where r is the distance of the point from the centre of the disc. A point charge of $+10\,\mathrm{nC}$ is placed on the axis of the disc and at a distance 50 cm from the centre of the disc. What is the magnitude and direction of the electrostatic force acting on this point charge due to the positive charge on the disc?

[7 marks]

[$1.523 \times 10^{-6}\ \hat{\imath}$ N]

[For a uniformly charged ring with radius R, and carrying a total charge of $+Q$, the electric field on the axial point, at a distance x from the centre of the ring is $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{x}{\left(R^2+x^2\right)^{\frac{3}{2}}}\hat{i}$ where $\hat{i}$ is a unit vector along the axis of the ring and pointing away from the centre of the ring].

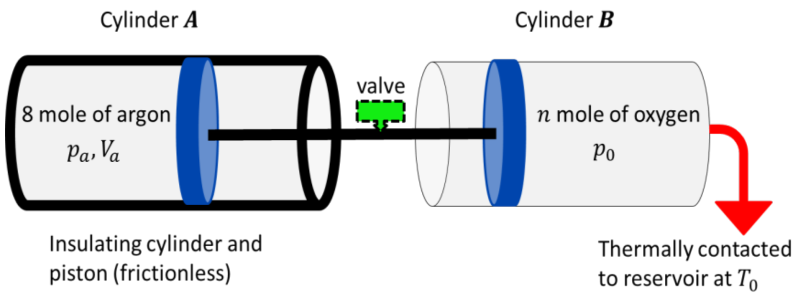
4. (a) A spherical blackbody has a radius of 30 cm and is placed in a constant temperature environment having temperature $20\,°C$. The blackbody is heated by an internal heater which has a heating power of 1.8 kW.

(i) What is the maximum temperature which the blackbody can attain?
[318.0 K]

(ii) After attaining the maximum temperature, the internal heater is switched off. If the density of the material of the blackbody is 8940 kgm^{-3} and its specific heat capacity is 389J $\text{kg}^{-1}\,\text{K}^{-1}$, what is the initial rate of fall of temperature of the blackbody?
[4.58×10^{-3} K s^{-1}]

[6 marks]

4. (b) Cylinder A and its piston are thermally insulated from the surrounding and cylinder B is in thermal contact with a heat reservoir at a constant temperature of T_0. The two frictionless pistons are rigidly connected by a thin hollow rod (with negligible volume) and a valve that is closed. Initially, the piston of cylinder A is fixed, and inside the cylinder it consists 8 moles of monoatomic argon gas at a pressure p_a higher than atmospheric pressure p_0 ($p_a > p_0$) and volume V_a. Inside cylinder B, there is an unknown amount (n moles) of diatomic oxygen gas at atmospheric pressure p_0. When the piston of cylinder A is freed, it moves very slowly until equilibrium is achieved with the volume of the argon gas being 8 times higher, and the density of the oxygen has increased by a factor of two. The heat reservoir receives heat transfer of $|Q|$ during the process.



(i) Show that the change of the pressure and temperature of the argon are given by

$$\Delta p = -62p_0 \quad ; \quad \Delta T = -\frac{3}{4}T_a$$

where p_0 is the atmospheric pressure and T_a is the initial temperature of the argon.

(ii) Finally, the valve on the thin hollow rod is opened. Calculate the final pressure of the mixture of the gases and express it in terms of $|Q|, T_0$ and V_a.

[$p_{\text{total}} = \dfrac{\frac{|Q|}{\ln 2} + 8RT_0}{15V_a}$]

[6 marks]

5 (a) A cube lies with one of its faces on the $x-y$ plane and three of its edges along the x, y & z axes. The cube starts to slide horizontally along the $x-$axis with constant speed v. It is found that as a result of the motion, the density of the material of the cube appears to increase by 25%. Calculate the value of v.

Let ℓ_0 be the length of the edge of the cube when it is stationary and m_0 be its mass when it is not moving.
The density of the material of the cube when it is stationary is

$$\rho_0 = \frac{m_0}{\ell_0^3}$$

When it is in motion, the edges parallel to the $y-$ and $z-$ axes remains unchanged in length since both edges are moving perpendicular to the direction of motion.
The length along the $x-$axis now becomes

$$\ell_x = \ell_0\sqrt{1-\beta^2} \qquad \text{where } \beta = \frac{v}{c}$$ [1]

The mass of the cube appears to become

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$ [1]

The density of the material of the cube in motion becomes

$$\rho = \frac{m}{\ell_0^2 \ell_x} = \frac{m_0}{\sqrt{1-\beta^2}} \times \frac{1}{\ell_0^3\sqrt{1-\beta^2}}$$

$$= \frac{m_0}{\ell_0^3\left(1-\beta^2\right)} = \frac{\rho_0}{1-\beta^2}$$ [1]

$$\frac{\rho}{\rho_0} = \frac{1}{1-\beta^2} = 1.25$$

$$1-\beta^2 = \frac{4}{5} \Rightarrow \beta = \sqrt{0.2}$$

$$\therefore v = 0.4472c$$ [1]

Alternative Solution (not involving relativistic mass)

Let ℓ_0 be the length of the edge of the cube when it is stationary and m be its n
The density of the material of the cube when it is stationary is

$$\rho_0 = \frac{m}{\ell_0^3}$$

When it is in motion, the edges parallel to the $y-$ and $z-$ axes rem
unchanged in length since both edges are moving perpendicular to the directio
motion. [1]
The length along the $x-$axis now becomes

$$\ell_x = \ell_0\sqrt{1-\beta^2} \qquad \text{where } \beta = \frac{v}{c}$$ [1]

The density of the material of the cube in motion becomes

$$\rho = \frac{m}{\ell_0^2 \ell_x} = \frac{m}{\ell_0^3\sqrt{1-\beta^2}}$$

$$= \frac{\rho_0}{\sqrt{1-\beta^2}}$$ [1]

$$\frac{\rho}{\rho_0} = \frac{1}{\sqrt{1-\beta^2}} = 1.25$$

$$\sqrt{1-\beta^2} = \frac{4}{5} \Rightarrow \beta^2 = \frac{9}{25}$$

$$\therefore v = 0.6c$$ [1]

[4 marks]

(b) An observer in the Earth frame observes two spaceships A and B pass each other in opposite directions. In the inertial frame in which both spaceships are at rest, the lengths of spaceship A and spaceship B are 200 m and 150 m respectively. The observer in the Earth frame measures that the speed of spaceship A is $0.8c$ and that of spaceship B is $0.6c$. As measured by this observer, what is the time interval between the instant when the nose of spaceship A passes the nose of spaceship B and the instant when the tail of spaceship A passes the tail of spaceship B? What is the corresponding time interval measured by an observer sitting at the nose of spaceship A?

[5.714×10^{-7} s; 1.715×10^{-7} s]

[6 marks]