

# SPhO 2024 Theory Paper

## Comprehensive Solutions

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## 4 Electromagnetic Induction

1. **Induced EMF ( $\mathcal{E}$ ):** At distance  $x$ , the rod length is  $L = 2x \tan(\frac{\alpha}{2})$ . The loop area is  $A = x^2 \tan(\frac{\alpha}{2})$ . The magnetic flux is  $\Phi_B = Bx^2 \tan(\frac{\alpha}{2})$ .

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = B \tan\left(\frac{\alpha}{2}\right) \cdot 2x \frac{dx}{dt} = 2Bxv \tan\left(\frac{\alpha}{2}\right)$$

2. **Induced Current ( $I$ ):** The rod resistance is  $R_{\text{rod}} = R \cdot L = 2Rx \tan(\frac{\alpha}{2})$ .

$$I = \frac{|\mathcal{E}|}{R_{\text{rod}}} = \frac{2Bxv \tan\left(\frac{\alpha}{2}\right)}{2Rx \tan\left(\frac{\alpha}{2}\right)} = \frac{Bv}{R}$$

3. **Magnetic Braking Force ( $F_m$ ):**

$$F_m = ILB = \left( \frac{Bv}{R} \right) \left( 2x \tan\left(\frac{\alpha}{2}\right) \right) B = \frac{2B^2 vx \tan\left(\frac{\alpha}{2}\right)}{R}$$

4. **Equation of Motion:**  $ma = -F_m$ . Use  $a = v \frac{dv}{dx}$ :

$$mv \frac{dv}{dx} = -\frac{2B^2 vx \tan\left(\frac{\alpha}{2}\right)}{R} \implies m \frac{dv}{dx} = -\frac{2B^2 x \tan\left(\frac{\alpha}{2}\right)}{R}$$

5. **Integration:** Integrate from initial state ( $x = x_0, v = v_0$ ) to final state ( $x = x_f, v = 0$ ).

$$\begin{aligned} \int_{v_0}^0 m dv &= \int_{x_0}^{x_f} -\frac{2B^2 \tan\left(\frac{\alpha}{2}\right)}{R} x dx \\ [mv]_{v_0}^0 &= -\frac{2B^2 \tan\left(\frac{\alpha}{2}\right)}{R} \left[ \frac{x^2}{2} \right]_{x_0}^{x_f} \\ -mv_0 &= -\frac{B^2 \tan\left(\frac{\alpha}{2}\right)}{R} (x_f^2 - x_0^2) \\ x_f^2 - x_0^2 &= \frac{mv_0 R}{B^2 \tan\left(\frac{\alpha}{2}\right)} \\ x_f &= \sqrt{x_0^2 + \frac{mv_0 R}{B^2 \tan\left(\frac{\alpha}{2}\right)}} \end{aligned}$$