5 Relativity

5.1 Relativistic Density

5.1.1 Method 1: Using Relativistic Mass

$\rho = \frac{m}{V} = \frac{\gamma m_0}{V_0/\gamma} = \gamma^2 \rho_0 = \frac{\rho_0}{1-v^2/c^2}$.

$$1.25 = \frac{1}{1-\beta^2} \implies 1-\beta^2 = 0.8 \implies \beta^2 = 0.2$$
$$v = \sqrt{0.2}c \approx \mathbf{0.447c}$$

5.1.2 Method 2: Using Invariant Mass

$\rho = \frac{m_0}{V} = \frac{m_0}{V_0/\gamma} = \gamma \rho_0 = \frac{\rho_0}{\sqrt{1-v^2/c^2}}$.

$$1.25 = \frac{1}{\sqrt{1-\beta^2}} \implies \sqrt{1-\beta^2} = 0.8 \implies 1-\beta^2 = 0.64$$
$$\beta^2 = 0.36 \implies v = \mathbf{0.6c}$$

5.2 Time Intervals Between Spaceships

5.2.1 Part 1: Time Interval in Earth Frame (Δt_S)

- Contracted lengths in Earth frame: $L_A = L_{0A}\sqrt{1-0.8^2} = 200(0.6) = 120$ m. $L_B = L_{0B}\sqrt{1-(-0.6)^2} = 150(0.8) = 120$ m.
- Time for tails to pass: $\Delta t_S(v_A - v_B) = L_A + L_B$ $\Delta t_S = \frac{120+120}{0.8c-(-0.6c)} = \frac{240}{1.4c} \approx \mathbf{5.71 \times 10^{-7}}$ s.

5.2.2 Part 2: Time Interval in Spaceship A's Frame (Δt_A)

- Use Lorentz transformation $t' = \gamma_A(t - v_A x/c^2)$ with $v_A = 0.8c$ and $\gamma_A = 5/3$.
- Event 1 (noses pass) in S: $(t_1, x_1) = (0, 0)$.
- Event 2 (tails pass) in S: $t_2 = \Delta t_S$, $x_2 = v_A t_2 - L_A = \frac{120}{7}$ m.
- Transforming event times to A's frame (S'): $t_1' = 0$.

$$
\begin{aligned}
t_2' = \gamma_A(t_2 - v_A x_2/c^2) &= \frac{5}{3}\left(\frac{240}{1.4c} - \frac{0.8c(120/7)}{c^2}\right) \\
&= \frac{5}{3}\left(\frac{1200}{7c} - \frac{96}{7c}\right) = \frac{5}{3}\frac{1104}{7c} = \frac{1840}{7c}
\end{aligned}
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- Time interval in A's frame: $\Delta t_A = t_2' - t_1' = \frac{1840}{7(3\times10^8)} \approx \mathbf{8.76 \times 10^{-7}}$ s.

Note: The calculated time in frame S matches the provided answer, but the time in frame A (8.76×10^{-7} *s) differs from the paper's answer of* 1.715×10^{-7} *s.*