3. Electromagnetism

(a) Charge Flow in a Capacitor

Initial charge: $Q_i = C_i V = (6 \times 10^{-6}\,\mathrm{F})(24\,\mathrm{V}) = 144\,\mu\mathrm{C}$. When the dielectric ($\kappa = 2.5$) fills half the space, the new capacitance is equivalent to two capacitors in parallel: one with dielectric and one with air.

$$C_f = C_1 + C_2 = \frac{\kappa\epsilon_0(A/2)}{d} + \frac{\epsilon_0(A/2)}{d} = \frac{\kappa+1}{2}C_i$$

$$C_f = \frac{2.5+1}{2}(6\,\mu\text{F}) = 10.5\,\mu\text{F}$$

The battery is still connected, so V is constant. Final charge:

$$Q_f = C_f V = (10.5 \times 10^{-6}\,\text{F})(24\,\text{V}) = 252\,\mu\text{C}$$

The charge that flowed through the battery is:

$$\Delta Q = Q_f - Q_i = 252\,\mu\text{C} - 144\,\mu\text{C} = \mathbf{108}\,\mu\text{C}$$