Question 3: Electrical Circuits and Electrostatics

(a) Maximum Power

For a single resistor: $R = 3\,\Omega$, $P_{max} = 108\,\text{W}$. Max current: $I_{max} = \sqrt{P_{max}/R} = \sqrt{108/3} = 6\,\text{A}$. Max voltage: $V_{max} = \sqrt{P_{max} \cdot R} = \sqrt{108 \times 3} = 18\,\text{V}$.

Fig. 2(a): Two resistors in series ($R_{top} = 6\,\Omega$) are in parallel with one resistor ($R_{bot} = 3\,\Omega$). The voltage is the same across both. The bottom resistor is the limiting component. The max voltage across it is 18 V. At this voltage: $P_{bot} = V^2/R_{bot} = 18^2/3 = 108\,\text{W}$. $P_{top} = V^2/R_{top} = 18^2/6 = 54\,\text{W}$. Total power: $P_{total} = P_{top} + P_{bot} = 54 + 108 = 162\,\text{W}$. **Answer:** Maximum power for Fig. 2(a) is **162 W**.

Fig. 2(b): Two resistors in parallel ($R_{par} = 1.5\,\Omega$) are in series with one resistor ($R_{ser} = 3\,\Omega$). Total resistance is $R_{eq} = 1.5 + 3 = 4.5\,\Omega$. The single series resistor is the limiting component. The max current through it is $I_{max} = 6\,\text{A}$. This is the total current. Total power: $P_{total} = I_{total}^2 R_{eq} = 6^2 \times 4.5 = 36 \times 4.5 = 162\,\text{W}$. **Answer:** Maximum power for Fig. 2(b) is **162 W**.

(b) Motion in an Electric Field

Inside the sphere, the electric field at distance r from the center is found using Gauss's Law:

$$E = \frac{\rho r}{3\epsilon_0}$$

The force on the particle with charge q is $F = qE$. Since q is negative, the force is a restoring force directed towards the center:

$$F = q\frac{\rho r}{3\epsilon_0} = -\left(\frac{|q|\rho}{3\epsilon_0}\right) r$$

This is Simple Harmonic Motion, $F = -k_{eff} r$, with $k_{eff} = \frac{|q|\rho}{3\epsilon_0}$. The angular frequency is:

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{|q|\rho}{3\epsilon_0 m}}$$

$$\omega = \sqrt{\frac{(0.05 \times 10^{-9})(5 \times 10^{-9})}{3(8.85 \times 10^{-12})(1.0 \times 10^{-9})}} = \sqrt{9.416} \approx 3.068\,\text{rad/s}$$