CHAPTER 1

Project

By Christophe Prud'homme and Mourad Ismail

Chapter ref: **[prudhomm-project]**

1.1 Projects

1.1.1 Cooling of electronic components

This test case has been proposed by Annabelle Le-Hyaric[1] and Michel Fouquembergh[2] both from EADS IW.

We consider a 2D model representative of the neighboring of an electronic component submitted to a cooling air flow. It is described by four geometrical domains in $\mathbb{R}^2$ named $\Omega_i, i = 1, 2, 3, 4$, see figure 1.1 on the following page. We suppose the velocity v is known in each domain — for instance in Ω_4 it is the solution of previous Navier-Stokes computations. — The temperature T of the domain $\Omega = \cup_{i=1}^4 \Omega_i$ is then solution of heat transfer equation :

$$\rho C_i \Big(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \Big) - \nabla \cdot (k_i \nabla T) = Q_i, \quad i = 1, 2, 3, 4 \tag{1.1}$$

where t is the time and in each sub-domain Ω_i, ρC_i is the volumic thermal capacity , k_i is thermal conductivity and Q_i is a volumic heat dissipated.

One should notice that the convection term in heat transfer equation may lead to spatial oscillations which can be overcome by Petrov-Galerkin type or continuous interior penalty stabilization techniques.

[1] Anabelle.Le-Hyaric@eads.net
[2] michel.fouquembergh@eads.net

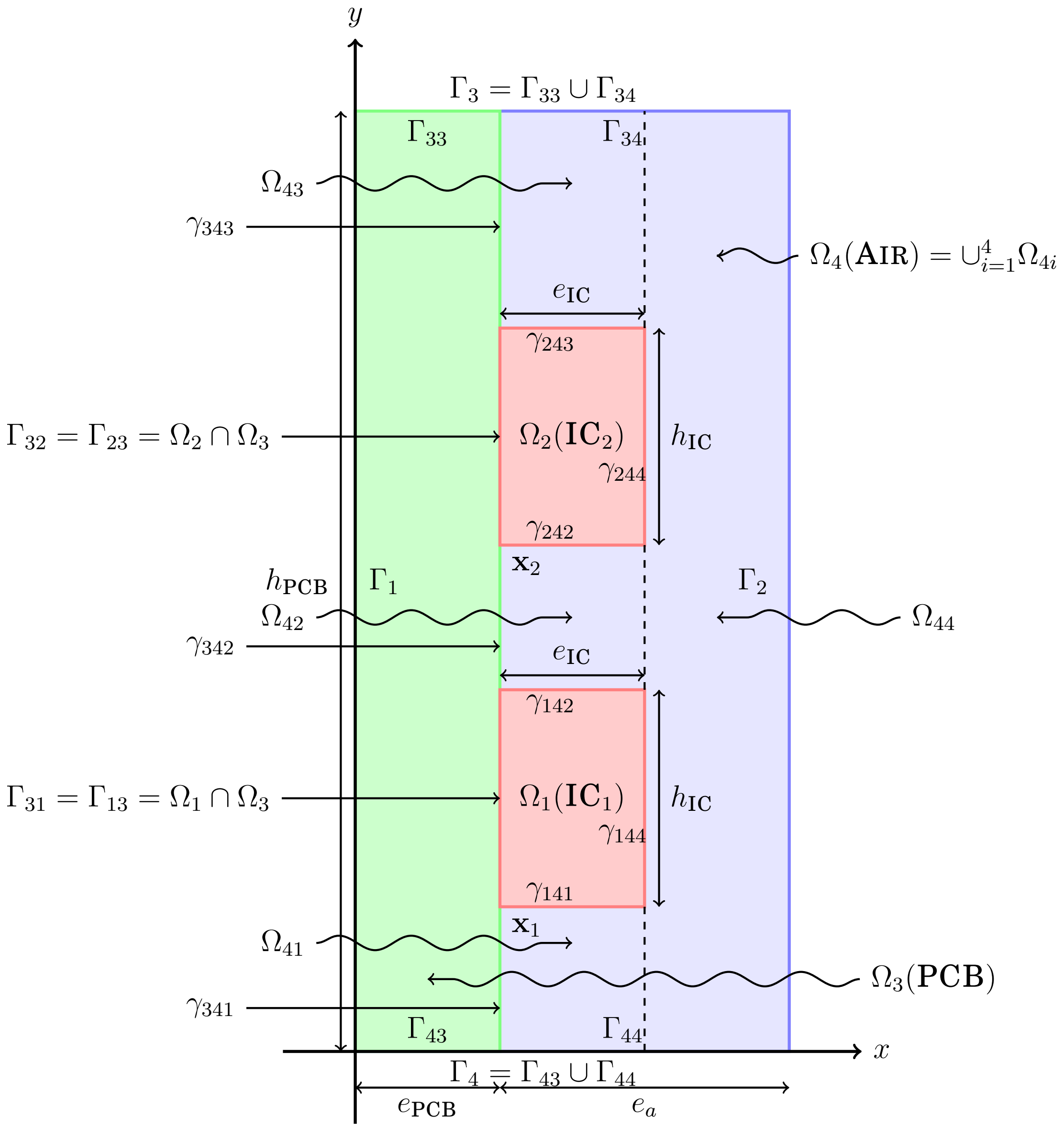

Figure 1.1: Geometry of $\Omega = \cup_{i=1}^4 \Omega_i$ with $\partial\Omega = \cup_{i=1}^4 \Gamma_i$

Integrated circuits (ICS) (domains Ω_1 and Ω_2) are respectively soldered on PCB at $\mathbf{x1} = (e_{\text{PCB}}, h_1)$ and $\mathbf{x_2} = (e_{\text{PCB}}, h_2)$. They are considered as rectangles with width e_{IC} and height h_{IC}. The printed circuit board (PCB) is a rectangle Ω_3 of width e_{PCB} and height h_{PCB}. The air(AIR) is flowing along the PCB in domain Ω_4. Speed in the air channel Ω_4 is supposed to have a parabolic profile function of x coordinate. Its expression is simplified as follows (notice that v is normally solution of Navier-Stokes equations; the resulting temperature and velocity fields should be quite different from that simplified model), we have for all $0 \leq y \leq h_{PCB}$

$$\begin{aligned} e_{Pcb} + e_{Ic} \leq x \leq e_{Pcb} + e_a, \quad \mathbf{v} &= \frac{3}{2(e_a - e_{Ic})}\, D \left(1 - \left(\frac{x - \left(\frac{e_a + e_{Ic}}{2} + e_{Pcb}\right)}{\frac{(e_a - e_{Ic})}{2}}\right)^2\right) f(t)\, \mathbf{y} \\ e_{Pcb} \leq x \leq e_{Pcb} + e_{Ic}, \quad \mathbf{v} &= 0 \end{aligned} \tag{1.2}$$

where f is a function of time modelling the starting of the PCB ventilation, *i.e.*

$$f(t) = 1 - \exp(-\frac{t}{3}), \tag{1.3}$$

D is the air flow rate, see table 1.1 on page 5 and $\mathbf{y} = (0, 1)^T$ is the unit vector along the y axis. A quick verification shows that

$$\int_{\Gamma_4 \cap \Omega_4} \mathbf{v} \cdot \mathbf{n} = \int_{\Gamma_4 \cap \Omega_4} v_y = D \tag{1.4}$$

The medium velocity $\mathbf{v}_i = \mathbf{0}, i = 1, 2, 3$ in the solid domains $\Omega_i, i = 1, 2, 3$.
ICS dissipate heat, we have respectively

$$\begin{array}{lll} Q_1 & = Q\big(1 - \exp(-t)\big) & \text{in } \Omega_1 \\ Q_2 & = Q\big(1 - \exp(-t)\big) & \text{in } \Omega_2 \end{array} \tag{1.5}$$

where Q is defined in table 1.1 on page 5.
We shall denote $\mathbf{n}_{|\Omega_i} = \mathbf{n}_i$ denotes the unit outward normal to Ω_i and $\mathbf{n}_{|\Omega_j} = \mathbf{n}_j$ denotes the unit outward normal to Ω_j.

Boundary conditions We set

(i) on $\Gamma_3 \cap \Omega_3$, a zero flux (Neumann-like) condition

$$-k_3 \, \nabla T \cdot \mathbf{n}_3 \; = \; 0; \tag{1.6}$$

(ii) on $\Gamma_3 \cap \Omega_4$, a zero flux (Robin-like) condition

$$(-k_4 \, \nabla T + \rho C_4 T \mathbf{v}) \cdot \mathbf{n}_4 \; = \; 0; \tag{1.7}$$

(iii) on $\Gamma_4, (0 \leq x \leq e_{\text{PCB}} + e_a, y = 0)$ the temperature is set (Dirichlet condition)

$$T \; = T_0; \tag{1.8}$$

(iv) between Γ_1 and Γ_2, periodic conditions

$$\begin{aligned} T_{\big|\mathbf{x}=0} &= T_{\big|\mathbf{x}=e_{Pcb}+e_a} \\ -k_3\ \nabla T\cdot \mathbf{n}_3{}_{\big|\mathbf{x}=0} &= k_4\ \nabla T\cdot \mathbf{n}_4{}_{\big|\mathbf{x}=e_{Pcb}+e_a}; \end{aligned} \tag{1.9}$$

(v) at interfaces between the ICs and PCB, there is a thermal contact conductance:

$$\begin{aligned} -k_1\ \nabla T\cdot \mathbf{n}_1 - k_3\ \nabla T\cdot \mathbf{n}_3 = r_{13}\big(T_{\partial\Omega_1} - T_{\partial\Omega_3}\big) \\ -k_2\ \nabla T\cdot \mathbf{n}_2 - k_3\ \nabla T\cdot \mathbf{n}_3 = r_{23}\big(T_{\partial\Omega_2} - T_{\partial\Omega_3}\big); \end{aligned} \tag{1.10}$$

(vi) on other internal boundaries, the coontinuity of the heat flux and temperature, on $\Gamma_{ij} = \Omega_i \cap \Omega_j \neq \emptyset$

$$\begin{aligned} T_i &= T_j \\ k_i\ \nabla T\cdot \mathbf{n}_i &= -k_j\ \nabla T\cdot \mathbf{n}_j. \end{aligned} \tag{1.11}$$

Initial condition At $t = 0s$, we set $T = T_0$.

Inputs

The table 1.1 on the next page displays the various fixed and variables parameters of this test-case.

Outputs

The outputs are *(i)* the mean temperature $s_1(\mu)$ of the hottest IC

$$s_1(\mu) = \frac{1}{e_{\text{IC}} h_{\text{IC}}} \int_{\Omega_2} T \tag{1.12}$$

and *(ii)* mean temperature $s_2(\mu)$ of the air at the outlet

$$s_2(\mu) = \frac{1}{e_a} \int_{\Omega_4 \cap \Gamma_3} T \tag{1.13}$$

both depends on the solution of (1.1) and are dependent on the parameter set μ.

We need to monitor $s_1(\mu)$ and $s_2(\mu)$ because $s_1(\mu)$ is the hottest part of the model and the IC can't have a temperature above $340K$. $s_2(\mu)$ is the outlet of the air and in an industrial system we can have others components behind this outlet. So the temperature of the air doesn't have to be high to not interfere the proper functioning of these.

Name	Description	Nominal Value	Range	Units
	Parameters			
t	time		$[0, 1500]$	s
Q	heat source	10^6	$[0, 10^6]$	$W \cdot m^{-3}$
	IC Parameters			
$k_1 = k_2 = k_{\text{IC}}$	thermal conductivity	2	$[0.2, 150]$	$W \cdot m^{-1} \cdot K^{-1}$
$r_{13} = r_{23} = r$	thermal conductance	100	$[10^{-1}, 10^2]$	$W \cdot m^{-2} \cdot K^{-1}$
ρC_{IC}	heat capacity	$1.4 \cdot 10^6$		$J \cdot m^{-3} \cdot K^{-1}$
e_{IC}	thickness	$2 \cdot 10^{-3}$		m
$h_{\text{IC}} = L_{\text{IC}}$	height	$2 \cdot 10^{-2}$		m
h_1	height	$2 \cdot 10^{-2}$		m
h_2	height	$7 \cdot 10^{-2}$		m
	PCB Parameters			
$k_3 = k_{\text{PCB}}$	thermal conductivity	0.2		$W \cdot m^{-1} \cdot K^{-1}$
ρC_3	heat capacity	$2 \cdot 10^6$		$J \cdot m^{-3} \cdot K^{-1}$
e_{PCB}	thickness	$2 \cdot 10^{-3}$		m
h_{PCB}	height	$13 \cdot 10^{-2}$		m
	Air Parameters			
T_0	Inflow temperature	300		K
D	Inflow rate	$7 \cdot 10^{-3}$	$[5 \cdot 10^{-4}, 10^{-2}]$	$m^2 \cdot s^{-1}$
k_4	thermal conductivity	$3 \cdot 10^{-2}$		$W \cdot m^{-1} \cdot K^{-1}$
ρC_4	heat capacity	1100		$J \cdot m^{-3} \cdot K^{-1}$
e_a	thickness	$4 \cdot 10^{-3}$	$[2.5 \cdot 10^{-3}, 5 \cdot 10^{-2}]$	m

Table 1.1: Table of fixed and variable parameters

1.1.2 *Guidline for implementation*

Using the simplified model

For sake of simplicity we will suppose that the temperature is continuous everywhere in the domain Ω. So, the thermal contact conductance (1.10) will be replaced by continuity conditions for heat flow and temperature. The periodic boundary conditions on Γ_1 and Γ_2 will be also replaced by zero flux conditions.

The problem in which we are interested will be now :

$$\rho C_i\Big(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\Big) - \nabla\cdot(k_i\nabla T) = Q_i \text{ in } \Omega = \cup_{i=1}^4\Omega_i, \quad i = 1,2,3,4 \tag{1.14}$$

subject to the following boundary conditions :

$$\begin{aligned}
T &= T_0 && \text{on} \quad \Gamma_4 && (1.15)\\
-k_3\nabla T\cdot\mathbf{n}_3 &= 0 && \text{on} \quad \Gamma_1\cup\Gamma_{33} && (1.16)\\
-k_4\nabla T\cdot\mathbf{n}_4 &= 0 && \text{on} \quad \Gamma_2 && (1.17)\\
(-k_4\nabla T + \rho C_4 T\mathbf{v})\cdot\mathbf{n}_4 &= 0 && \text{on} \quad \Gamma_{34} && (1.18)\\
-k_i\nabla T\cdot\mathbf{n}_i &= k_3\nabla T\cdot\mathbf{n}_3 && \text{on} \quad \Gamma_{3i}, \quad i=1,2 && (1.19)\\
-k_1\nabla T\cdot\mathbf{n}_1 &= k_4\nabla T\cdot\mathbf{n}_4 && \text{on} \quad \gamma_{141}\cup\gamma_{144}\cup\gamma_{142} && (1.20)\\
-k_2\nabla T\cdot\mathbf{n}_2 &= k_4\nabla T\cdot\mathbf{n}_4 && \text{on} \quad \gamma_{242}\cup\gamma_{244}\cup\gamma_{243} && (1.21)
\end{aligned}$$

In order to solve numerically this problem, we will follow these steps :

(i) write the Variational Formulation of problem (1.14) with the boundary conditions (1.15), (1.16), (1.17), (1.18), (1.19), (1.20) and (1.21),

(ii) create a mesh of the domain Ω (figure 1.1) using Gmsh. Remember to take different references for each subdomain and for each part of the boundary (this step was already done in **PS2**. See appendix A.1 on page 10).

(iii) Write the corresponding FreeFem++ application following these steps :

  (a) Start by assuming that $\mathbf{v} = 0$ and by omitting the time derivative.

  (b) Take into account the advection term by using the formula (1.2). We assume that we are in a steady regime so the time variable t goes to infinity.

  (c) Add the time derivative of the temperature T and discretize it using the explicit Euler scheme
$$\frac{\partial T}{\partial t} \simeq \frac{T^{n+1} - T^n}{\delta t},$$
  where T^n is an approximation of T at time $t^n = n\delta t$ and δt is the time step.

Using Navier-Stokes equations

As mentioned above, the air flow is governed by the incompressible Navier-Stokes equations :

$$\begin{aligned}
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} - \nu\Delta\mathbf{v} + \frac{1}{\rho_f}\nabla p &= \mathbf{F} \quad \text{in } \Omega_4, && (1.22)\\
\nabla\cdot\mathbf{v} &= 0 \quad \text{in } \Omega_4, && (1.23)\\
\mathbf{v} &= 0 \quad \text{on } \partial\Omega_4. && (1.24)
\end{aligned}$$

Where :

- $\mathbf{v}$: the velocity field
- p : the pressure
- $\mathbf{F}$: some external force. We could take $\mathbf{F} = \begin{pmatrix} 0 \\ T - T_0 \end{pmatrix}$ for example
- ρ_f : the fluid density
- ν : the cinematic viscosity. It is given by the ratio between the dynamic viscosity η and the density ρ_f. ($\nu = \dfrac{\eta}{\rho_f}$)
- $\mathbf{v} = 0$ in Ω_4 at $t = 0$.

As the velocity and the pressure are coupled, one can decouple them to resolve Navier-Stokes equations. To do this, we could use a projection scheme allowing to treat separately the viscous effect and the incompressibility constraint of the flow. More precisely, we consider a finite time interval $[0, T]$ and we introduce the discretization $t^n = n\delta t$ for $0 \le n \le N$ where $\delta t = \frac{T}{N}$ is the time step. Then, we introduce a sequence $(\mathbf{v}^n)$ for the approximation of the velocity $\mathbf{v}$ at time t^n.

Concerning the pressure p, we use (p^n) and an intermediate sequence (ψ^n). We propose the following projection scheme (see **[guermond:06]** for more details).

$$\begin{aligned}
\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\delta t} - \nu\Delta\mathbf{v}^{n+1} + \frac{1}{\rho}\nabla\psi^n &= \mathbf{G}^{n+1}, \quad \mathbf{v}^{n+1}|_{\partial\Omega_4} = 0 && (1.25)\\
\Delta\psi^{n+1} &= \frac{\nabla\cdot\mathbf{v}^{n+1}}{\delta t}, \quad \frac{\partial\psi^{n+1}}{\partial\mathbf{n}}|_{\partial\Omega_4} = 0 && (1.26)\\
p^{n+1} &= \psi^{n+1} - \nu\nabla\cdot\mathbf{v}^{n+1} && (1.27)
\end{aligned}$$

where

$$\mathbf{G}^{n+1} = \mathbf{F}^{n+1} - \mathbf{v}^n\cdot\nabla\mathbf{v}^n \tag{1.28}$$