

important step in the knowledge discovery process, because quality decisions must be based on quality data. Detecting data anomalies, rectifying them early, and reducing the data to be analyzed can lead to huge payoffs for decision making.

## 2.2 Descriptive Data Summarization

For data preprocessing to be successful, it is essential to have an overall picture of your data. Descriptive data summarization techniques can be used to identify the typical properties of your data and highlight which data values should be treated as noise or outliers. Thus, we first introduce the basic concepts of descriptive data summarization before getting into the concrete workings of data preprocessing techniques.

For many data preprocessing tasks, users would like to learn about data characteristics regarding both central tendency and dispersion of the data. Measures of central tendency include *mean*, *median*, *mode*, and *midrange*, while measures of data dispersion include *quartiles*, *interquartile range (IQR)*, and *variance*. These descriptive statistics are of great help in understanding the distribution of the data. Such measures have been studied extensively in the statistical literature. From the data mining point of view, we need to examine how they can be computed efficiently in large databases. In particular, it is necessary to introduce the notions of *distributive measure*, *algebraic measure*, and *holistic measure*. Knowing what kind of measure we are dealing with can help us choose an efficient implementation for it.

### 2.2.1 Measuring the Central Tendency

In this section, we look at various ways to measure the central tendency of data. The most common and most effective numerical measure of the “center” of a set of data is the (*arithmetic*) *mean*. Let  $x_1, x_2, \dots, x_N$  be a set of  $N$  values or observations, such as for some attribute, like *salary*. The **mean** of this set of values is

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N} = \frac{x_1 + x_2 + \dots + x_N}{N}. \quad (2.1)$$

This corresponds to the built-in aggregate function, *average* (**avg()** in SQL), provided in relational database systems.

A **distributive measure** is a measure (i.e., function) that can be computed for a given data set by partitioning the data into smaller subsets, computing the measure for each subset, and then merging the results in order to arrive at the measure’s value for the original (entire) data set. Both **sum()** and **count()** are distributive measures because they can be computed in this manner. Other examples include **max()** and **min()**. An **algebraic measure** is a measure that can be computed by applying an algebraic function to one or more distributive measures. Hence, *average* (or **mean()**) is an algebraic measure because it can be computed by **sum()/count()**. When computing

data cubes<sup>2</sup>, `sum()` and `count()` are typically saved in precomputation. Thus, the derivation of *average* for data cubes is straightforward.

Sometimes, each value  $x_i$  in a set may be associated with a weight  $w_i$ , for  $i = 1, \dots, N$ . The weights reflect the significance, importance, or occurrence frequency attached to their respective values. In this case, we can compute

$$\bar{x} = \frac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_N x_N}{w_1 + w_2 + \dots + w_N}. \quad (2.2)$$

This is called the **weighted arithmetic mean** or the **weighted average**. Note that the weighted average is another example of an algebraic measure.

Although the mean is the single most useful quantity for describing a data set, it is not always the best way of measuring the center of the data. A major problem with the *mean* is its sensitivity to extreme (e.g., outlier) values. Even a small number of extreme values can corrupt the mean. For example, the mean salary at a company may be substantially pushed up by that of a few highly paid managers. Similarly, the average score of a class in an exam could be pulled down quite a bit by a few very low scores. To offset the effect caused by a small number of extreme values, we can instead use the **trimmed mean**, which is the mean obtained after chopping off values at the high and low extremes. For example, we can sort the values observed for *salary* and remove the top and bottom 2% before computing the mean. We should avoid trimming too large a portion (such as 20%) at both ends as this can result in the loss of valuable information.

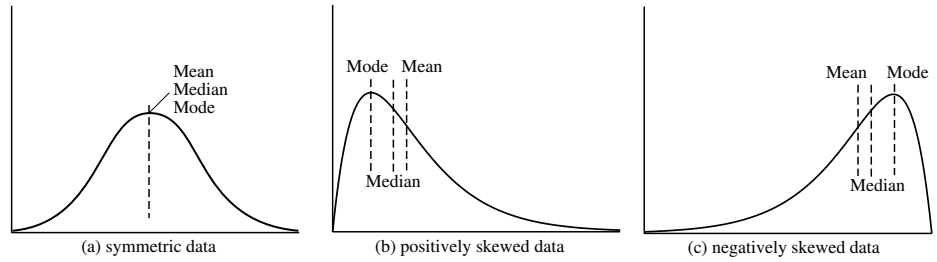
For skewed (asymmetric) data, a better measure of the center of data is the *median*. Suppose that a given data set of  $N$  distinct values is sorted in numerical order. If  $N$  is odd, then the **median** is the *middle value* of the ordered set; otherwise (i.e., if  $N$  is even), the median is the average of the middle two values.

A **holistic measure** is a measure that must be computed on the entire data set as a whole. It cannot be computed by partitioning the given data into subsets and merging the values obtained for the measure in each subset. The median is an example of a holistic measure. Holistic measures are much more expensive to compute than distributive measures such as those listed above.

We can, however, easily *approximate* the median value of a data set. Assume that data are grouped in intervals according to their  $x_i$  data values and that the frequency (i.e., number of data values) of each interval is known. For example, people may be grouped according to their annual salary in intervals such as 10–20K, 20–30K, and so on. Let the interval that contains the median frequency be the *median interval*. We can approximate the median of the entire data set (e.g., the median salary) by interpolation using the formula:

$$median = L_1 + \left( \frac{N/2 - (\sum freq)_l}{freq_{median}} \right) width, \quad (2.3)$$

<sup>2</sup>Data cube computation is described in detail in Chapters 3 and 4.



**Figure 2.2** Mean, median, and mode of symmetric versus positively and negatively skewed data.

where  $L_1$  is the lower boundary of the median interval,  $N$  is the number of values in the entire data set,  $(\sum freq)_l$  is the sum of the frequencies of all of the intervals that are lower than the median interval,  $freq_{median}$  is the frequency of the median interval, and  $width$  is the width of the median interval.

Another measure of central tendency is the *mode*. The **mode** for a set of data is the value that occurs most frequently in the set. It is possible for the greatest frequency to correspond to several different values, which results in more than one mode. Data sets with one, two, or three modes are respectively called **unimodal**, **bimodal**, and **trimodal**. In general, a data set with two or more modes is **multimodal**. At the other extreme, if each data value occurs only once, then there is no mode.

For unimodal frequency curves that are moderately skewed (asymmetrical), we have the following empirical relation:

$$mean - mode = 3 \times (mean - median). \quad (2.4)$$

This implies that the mode for unimodal frequency curves that are moderately skewed can easily be computed if the mean and median values are known.

In a unimodal frequency curve with perfect symmetric data distribution, the mean, median, and mode are all at the same center value, as shown in Figure 2.2(a). However, data in most real applications are not symmetric. They may instead be either positively skewed, where the mode occurs at a value that is smaller than the median (Figure 2.2(b)), or negatively skewed, where the mode occurs at a value greater than the median (Figure 2.2(c)).

The **midrange** can also be used to assess the central tendency of a data set. It is the average of the largest and smallest values in the set. This algebraic measure is easy to compute using the SQL aggregate functions, `max()` and `min()`.

## 2.2.2 Measuring the Dispersion of Data

The degree to which numerical data tend to spread is called the **dispersion**, or **variance** of the data. The most common measures of data dispersion are *range*, the *five-number summary* (based on *quartiles*), the *interquartile range*, and the *standard deviation*. Boxplots

can be plotted based on the five-number summary and are a useful tool for identifying outliers.

## Range, Quartiles, Outliers, and Boxplots

Let  $x_1, x_2, \dots, x_N$  be a set of observations for some attribute. The **range** of the set is the difference between the largest ( $\max()$ ) and smallest ( $\min()$ ) values. For the remainder of this section, let's assume that the data are sorted in increasing numerical order.

The  $k$ th **percentile** of a set of data in numerical order is the value  $x_i$  having the property that  $k$  percent of the data entries lie at or below  $x_i$ . The *median* (discussed in the previous subsection) is the 50th percentile.

The most commonly used percentiles other than the median are **quartiles**. The **first quartile**, denoted by  $Q_1$ , is the 25th percentile; the **third quartile**, denoted by  $Q_3$ , is the 75th percentile. The quartiles, including the median, give some indication of the center, spread, and shape of a distribution. The distance between the first and third quartiles is a simple measure of spread that gives the range covered by the middle half of the data. This distance is called the **interquartile range (IQR)** and is defined as

$$IQR = Q_3 - Q_1. \quad (2.5)$$

Based on reasoning similar to that in our analysis of the median in Section 2.2.1, we can conclude that  $Q_1$  and  $Q_3$  are holistic measures, as is *IQR*.

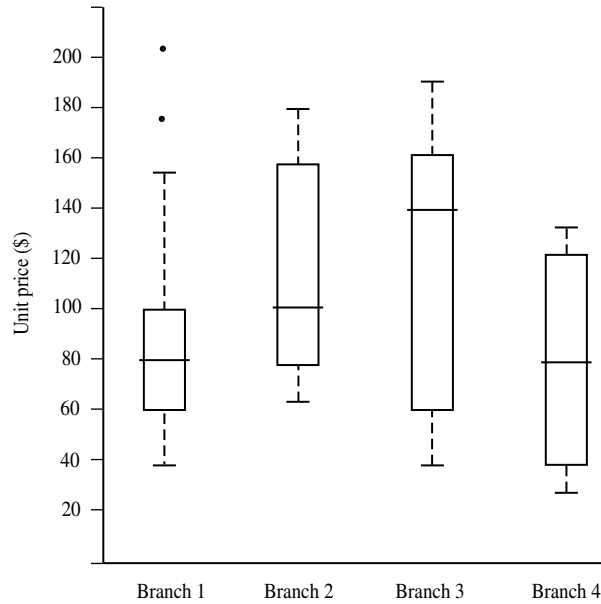
No single numerical measure of spread, such as *IQR*, is very useful for describing skewed distributions. The spreads of two sides of a skewed distribution are unequal (Figure 2.2). Therefore, it is more informative to also provide the two quartiles  $Q_1$  and  $Q_3$ , along with the median. A common rule of thumb for identifying suspected **outliers** is to single out values falling at least  $1.5 \times IQR$  above the third quartile or below the first quartile.

Because  $Q_1$ , the median, and  $Q_3$  together contain no information about the endpoints (e.g., tails) of the data, a fuller summary of the shape of a distribution can be obtained by providing the lowest and highest data values as well. This is known as the *five-number summary*. The **five-number summary** of a distribution consists of the median, the quartiles  $Q_1$  and  $Q_3$ , and the smallest and largest individual observations, written in the order *Minimum*,  $Q_1$ , *Median*,  $Q_3$ , *Maximum*.

**Boxplots** are a popular way of visualizing a distribution. A boxplot incorporates the five-number summary as follows:

- Typically, the ends of the box are at the quartiles, so that the box length is the interquartile range, *IQR*.
- The median is marked by a line within the box.
- Two lines (called *whiskers*) outside the box extend to the smallest (*Minimum*) and largest (*Maximum*) observations.

When dealing with a moderate number of observations, it is worthwhile to plot potential outliers individually. To do this in a boxplot, the whiskers are extended to



**Figure 2.3** Boxplot for the unit price data for items sold at four branches of *AllElectronics* during a given time period.

the extreme low and high observations *only if* these values are less than  $1.5 \times IQR$  beyond the quartiles. Otherwise, the whiskers terminate at the most extreme observations occurring within  $1.5 \times IQR$  of the quartiles. The remaining cases are plotted individually. Boxplots can be used in the comparisons of several sets of compatible data. Figure 2.3 shows boxplots for unit price data for items sold at four branches of *AllElectronics* during a given time period. For branch 1, we see that the median price of items sold is \$80,  $Q_1$  is \$60,  $Q_3$  is \$100. Notice that two outlying observations for this branch were plotted individually, as their values of 175 and 202 are more than 1.5 times the IQR here of 40. The efficient computation of boxplots, or even *approximate boxplots* (based on approximates of the five-number summary), remains a challenging issue for the mining of large data sets.

## Variance and Standard Deviation

The variance of  $N$  observations,  $x_1, x_2, \dots, x_N$ , is

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \frac{1}{N} \left[ \sum x_i^2 - \frac{1}{N} (\sum x_i)^2 \right], \quad (2.6)$$

where  $\bar{x}$  is the mean value of the observations, as defined in Equation (2.1). The **standard deviation**,  $\sigma$ , of the observations is the square root of the variance,  $\sigma^2$ .

The basic properties of the standard deviation,  $\sigma$ , as a measure of spread are

- $\sigma$  measures spread about the mean and should be used only when the mean is chosen as the measure of center.
- $\sigma = 0$  only when there is no spread, that is, when all observations have the same value. Otherwise  $\sigma > 0$ .

The variance and standard deviation are algebraic measures because they can be computed from distributive measures. That is,  $N$  (which is `count()` in SQL),  $\sum x_i$  (which is the `sum()` of  $x_i$ ), and  $\sum x_i^2$  (which is the `sum()` of  $x_i^2$ ) can be computed in any partition and then merged to feed into the algebraic Equation (2.6). Thus the computation of the variance and standard deviation is scalable in large databases.

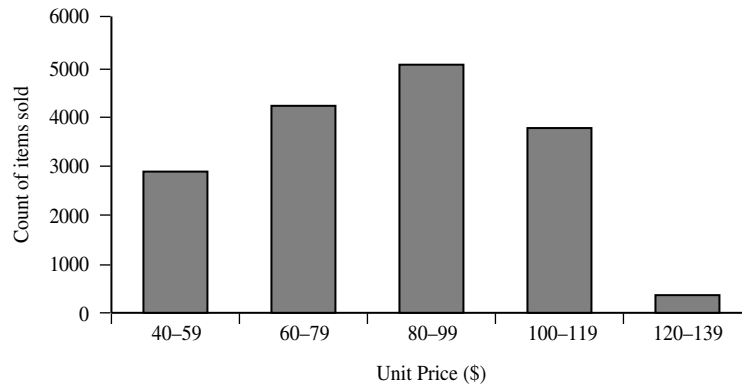
### 2.2.3 Graphic Displays of Basic Descriptive Data Summaries

Aside from the bar charts, pie charts, and line graphs used in most statistical or graphical data presentation software packages, there are other popular types of graphs for the display of data summaries and distributions. These include *histograms*, *quantile plots*, *q-q plots*, *scatter plots*, and *loess curves*. Such graphs are very helpful for the visual inspection of your data.

Plotting **histograms**, or **frequency histograms**, is a graphical method for summarizing the distribution of a given attribute. A histogram for an attribute  $A$  partitions the data distribution of  $A$  into disjoint subsets, or *buckets*. Typically, the width of each bucket is uniform. Each bucket is represented by a rectangle whose height is equal to the count or relative frequency of the values at the bucket. If  $A$  is categoric, such as *automobile\_model* or *item\_type*, then one rectangle is drawn for each known value of  $A$ , and the resulting graph is more commonly referred to as a **bar chart**. If  $A$  is numeric, the term *histogram* is preferred. Partitioning rules for constructing histograms for numerical attributes are discussed in Section 2.5.4. In an equal-width histogram, for example, each bucket represents an equal-width range of numerical attribute  $A$ .

Figure 2.4 shows a histogram for the data set of Table 2.1, where buckets are defined by equal-width ranges representing \$20 increments and the frequency is the count of items sold. Histograms are at least a century old and are a widely used univariate graphical method. However, they may not be as effective as the quantile plot, q-q plot, and boxplot methods for comparing groups of univariate observations.

A **quantile plot** is a simple and effective way to have a first look at a univariate data distribution. First, it displays all of the data for the given attribute (allowing the user to assess both the overall behavior and unusual occurrences). Second, it plots quantile information. The mechanism used in this step is slightly different from the percentile computation discussed in Section 2.2.2. Let  $x_i$ , for  $i = 1$  to  $N$ , be the data sorted in increasing order so that  $x_1$  is the smallest observation and  $x_N$  is the largest. Each observation,  $x_i$ , is paired with a percentage,  $f_i$ , which indicates that approximately 100 $f_i$ % of the data are below or equal to the value,  $x_i$ . We say “approximately” because



**Figure 2.4** A histogram for the data set of Table 2.1.

**Table 2.1** A set of unit price data for items sold at a branch of *AllElectronics*.

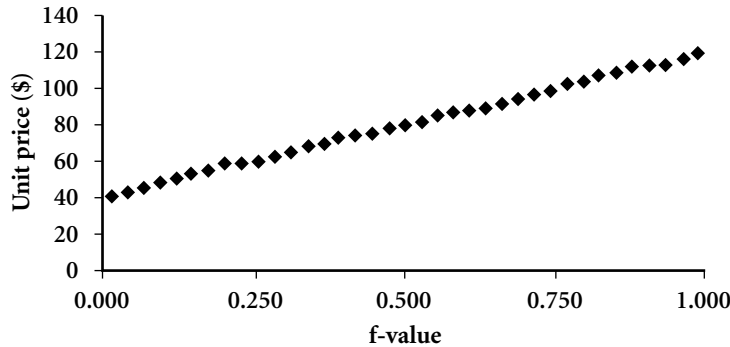
<i>Unit price (\$)</i>	<i>Count of items sold</i>
40	275
43	300
47	250
..	..
74	360
75	515
78	540
..	..
115	320
117	270
120	350

there may not be a value with exactly a fraction,  $f_i$ , of the data below or equal to  $x_i$ . Note that the 0.25 quantile corresponds to quartile  $Q_1$ , the 0.50 quantile is the median, and the 0.75 quantile is  $Q_3$ .

Let

$$f_i = \frac{i - 0.5}{N}. \quad (2.7)$$

These numbers increase in equal steps of  $1/N$ , ranging from  $1/2N$  (which is slightly above zero) to  $1 - 1/2N$  (which is slightly below one). On a quantile plot,  $x_i$  is graphed against  $f_i$ . This allows us to compare different distributions based on their quantiles. For example, given the quantile plots of sales data for two different time periods, we can



**Figure 2.5** A quantile plot for the unit price data of Table 2.1.

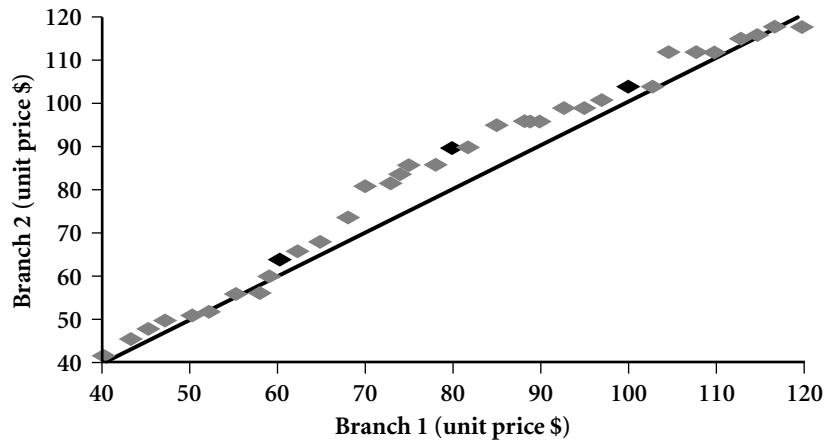
compare their  $Q_1$ , median,  $Q_3$ , and other  $f_i$  values at a glance. Figure 2.5 shows a quantile plot for the *unit price* data of Table 2.1.

A **quantile-quantile plot**, or **q-q plot**, graphs the quantiles of one univariate distribution against the corresponding quantiles of another. It is a powerful visualization tool in that it allows the user to view whether there is a shift in going from one distribution to another.

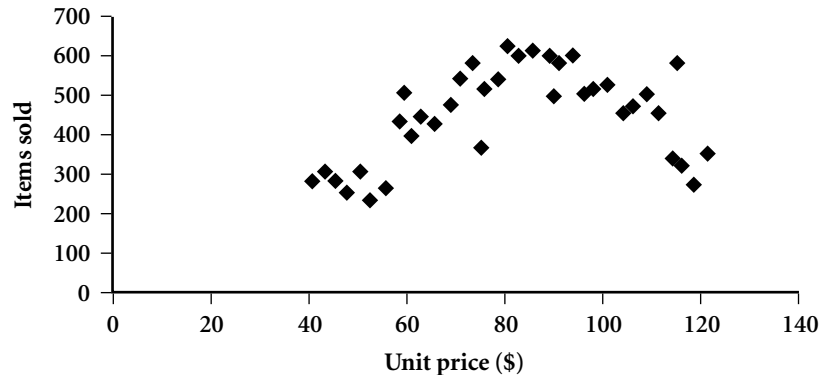
Suppose that we have two sets of observations for the variable *unit price*, taken from two different branch locations. Let  $x_1, \dots, x_N$  be the data from the first branch, and  $y_1, \dots, y_M$  be the data from the second, where each data set is sorted in increasing order. If  $M = N$  (i.e., the number of points in each set is the same), then we simply plot  $y_i$  against  $x_i$ , where  $y_i$  and  $x_i$  are both  $(i - 0.5)/N$  quantiles of their respective data sets. If  $M < N$  (i.e., the second branch has fewer observations than the first), there can be only  $M$  points on the q-q plot. Here,  $y_i$  is the  $(i - 0.5)/M$  quantile of the  $y$  data, which is plotted against the  $(i - 0.5)/M$  quantile of the  $x$  data. This computation typically involves interpolation.

Figure 2.6 shows a quantile-quantile plot for *unit price* data of items sold at two different branches of *AllElectronics* during a given time period. Each point corresponds to the same quantile for each data set and shows the unit price of items sold at branch 1 versus branch 2 for that quantile. For example, here the lowest point in the left corner corresponds to the 0.03 quantile. (To aid in comparison, we also show a straight line that represents the case of when, for each given quantile, the unit price at each branch is the same. In addition, the darker points correspond to the data for  $Q_1$ , the median, and  $Q_3$ , respectively.) We see that at this quantile, the unit price of items sold at branch 1 was slightly less than that at branch 2. In other words, 3% of items sold at branch 1 were less than or equal to \$40, while 3% of items at branch 2 were less than or equal to \$42. At the highest quantile, we see that the unit price of items at branch 2 was slightly less than that at branch 1. In general, we note that there is a shift in the distribution of branch 1 with respect to branch 2 in that the unit prices of items sold at branch 1 tend to be lower than those at branch 2.





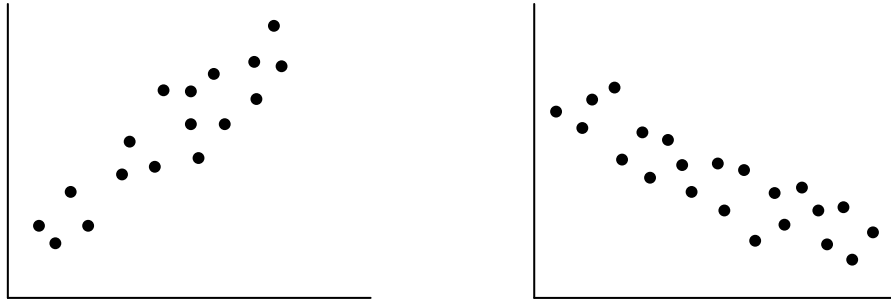
**Figure 2.6** A quantile-quantile plot for unit price data from two different branches.



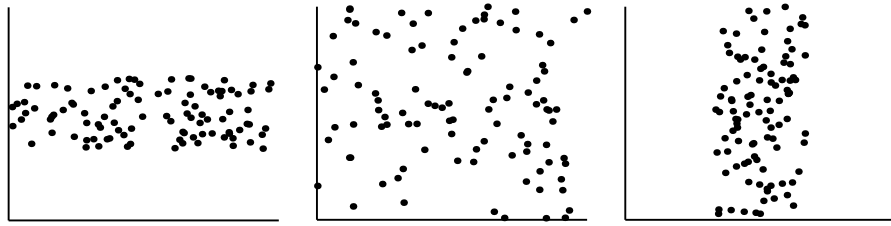
**Figure 2.7** A scatter plot for the data set of Table 2.1.

A **scatter plot** is one of the most effective graphical methods for determining if there appears to be a relationship, pattern, or trend between two numerical attributes. To construct a scatter plot, each pair of values is treated as a pair of coordinates in an algebraic sense and plotted as points in the plane. Figure 2.7 shows a scatter plot for the set of data in Table 2.1. The scatter plot is a useful method for providing a first look at bivariate data to see clusters of points and outliers, or to explore the possibility of correlation relationships.<sup>3</sup> In Figure 2.8, we see examples of positive and negative correlations between

<sup>3</sup> A statistical test for correlation is given in Section 2.4.1 on data integration (Equation (2.8)).



**Figure 2.8** Scatter plots can be used to find (a) positive or (b) negative correlations between attributes.



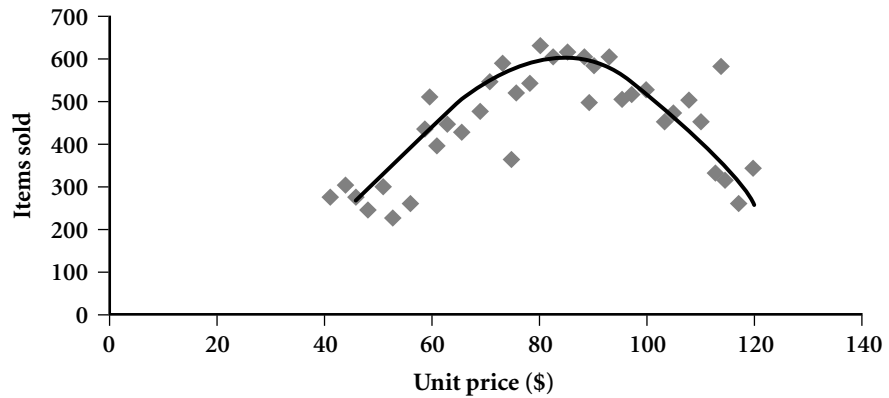
**Figure 2.9** Three cases where there is no observed correlation between the two plotted attributes in each of the data sets.

two attributes in two different data sets. Figure 2.9 shows three cases for which there is no correlation relationship between the two attributes in each of the given data sets.

When dealing with several attributes, the **scatter-plot matrix** is a useful extension to the scatter plot. Given  $n$  attributes, a scatter-plot matrix is an  $n \times n$  grid of scatter plots that provides a visualization of each attribute (or dimension) with every other attribute. The scatter-plot matrix becomes less effective as the number of attributes under study grows. In this case, user interactions such as zooming and panning become necessary to help interpret the individual scatter plots effectively.

A **loess curve** is another important exploratory graphic aid that adds a smooth curve to a scatter plot in order to provide better perception of the pattern of dependence. The word *loess* is short for “local regression.” Figure 2.10 shows a loess curve for the set of data in Table 2.1.

To fit a loess curve, values need to be set for two parameters— $\alpha$ , a smoothing parameter, and  $\lambda$ , the degree of the polynomials that are fitted by the regression. While  $\alpha$  can be any positive number (typical values are between  $1/4$  and  $1$ ),  $\lambda$  can be  $1$  or  $2$ . The goal in choosing  $\alpha$  is to produce a fit that is as smooth as possible without unduly distorting the underlying pattern in the data. The curve becomes smoother as  $\alpha$  increases. There may be some lack of fit, however, indicating possible “missing” data patterns. If  $\alpha$  is very small, the underlying pattern is tracked, yet overfitting of the data may occur where local “wiggles” in the curve may not be supported by the data. If the underlying pattern of the data has a



**Figure 2.10** A loess curve for the data set of Table 2.1.

“gentle” curvature with no local maxima and minima, then local linear fitting is usually sufficient ( $\lambda = 1$ ). However, if there are local maxima or minima, then local quadratic fitting ( $\lambda = 2$ ) typically does a better job of following the pattern of the data and maintaining local smoothness.

In conclusion, descriptive data summaries provide valuable insight into the overall behavior of your data. By helping to identify noise and outliers, they are especially useful for data cleaning.

## 2.3 Data Cleaning

Real-world data tend to be incomplete, noisy, and inconsistent. *Data cleaning* (or *data cleansing*) routines attempt to fill in missing values, smooth out noise while identifying outliers, and correct inconsistencies in the data. In this section, you will study basic methods for data cleaning. Section 2.3.1 looks at ways of handling missing values. Section 2.3.2 explains data smoothing techniques. Section 2.3.3 discusses approaches to data cleaning as a process.

### 2.3.1 Missing Values

Imagine that you need to analyze *AllElectronics* sales and customer data. You note that many tuples have no recorded value for several attributes, such as customer *income*. How can you go about filling in the missing values for this attribute? Let’s look at the following methods:

1. **Ignore the tuple:** This is usually done when the class label is missing (assuming the mining task involves classification). This method is not very effective, unless the tuple contains several attributes with missing values. It is especially poor when the percentage of missing values per attribute varies considerably.

2. **Fill in the missing value manually:** In general, this approach is time-consuming and may not be feasible given a large data set with many missing values.
3. **Use a global constant to fill in the missing value:** Replace all missing attribute values by the same constant, such as a label like “*Unknown*” or  $-\infty$ . If missing values are replaced by, say, “*Unknown*,” then the mining program may mistakenly think that they form an interesting concept, since they all have a value in common—that of “*Unknown*.” Hence, although this method is simple, it is not foolproof.
4. **Use the attribute mean to fill in the missing value:** For example, suppose that the average income of *AllElectronics* customers is \$56,000. Use this value to replace the missing value for *income*.
5. **Use the attribute mean for all samples belonging to the same class as the given tuple:** For example, if classifying customers according to *credit\_risk*, replace the missing value with the average *income* value for customers in the same credit risk category as that of the given tuple.
6. **Use the most probable value to fill in the missing value:** This may be determined with regression, inference-based tools using a Bayesian formalism, or decision tree induction. For example, using the other customer attributes in your data set, you may construct a decision tree to predict the missing values for *income*. Decision trees, regression, and Bayesian inference are described in detail in Chapter 6.

Methods 3 to 6 bias the data. The filled-in value may not be correct. Method 6, however, is a popular strategy. In comparison to the other methods, it uses the most information from the present data to predict missing values. By considering the values of the other attributes in its estimation of the missing value for *income*, there is a greater chance that the relationships between *income* and the other attributes are preserved.

It is important to note that, in some cases, a missing value may not imply an error in the data! For example, when applying for a credit card, candidates may be asked to supply their driver’s license number. Candidates who do not have a driver’s license may naturally leave this field blank. Forms should allow respondents to specify values such as “not applicable”. Software routines may also be used to uncover other null values, such as “don’t know”, “?”, or “none”. Ideally, each attribute should have one or more rules regarding the *null* condition. The rules may specify whether or not nulls are allowed, and/or how such values should be handled or transformed. Fields may also be intentionally left blank if they are to be provided in a later step of the business process. Hence, although we can try our best to clean the data after it is seized, good design of databases and of data entry procedures should help minimize the number of missing values or errors in the first place.

### 2.3.2 Noisy Data

“*What is noise?*” **Noise** is a random error or variance in a measured variable. Given a numerical attribute such as, say, *price*, how can we “smooth” out the data to remove the noise? Let’s look at the following data smoothing techniques:

Sorted data for *price* (in dollars): 4, 8, 15, 21, 21, 24, 25, 28, 34

Partition into (equal-frequency) bins:

Bin 1: 4, 8, 15  
 Bin 2: 21, 21, 24  
 Bin 3: 25, 28, 34

Smoothing by bin means:

Bin 1: 9, 9, 9  
 Bin 2: 22, 22, 22  
 Bin 3: 29, 29, 29

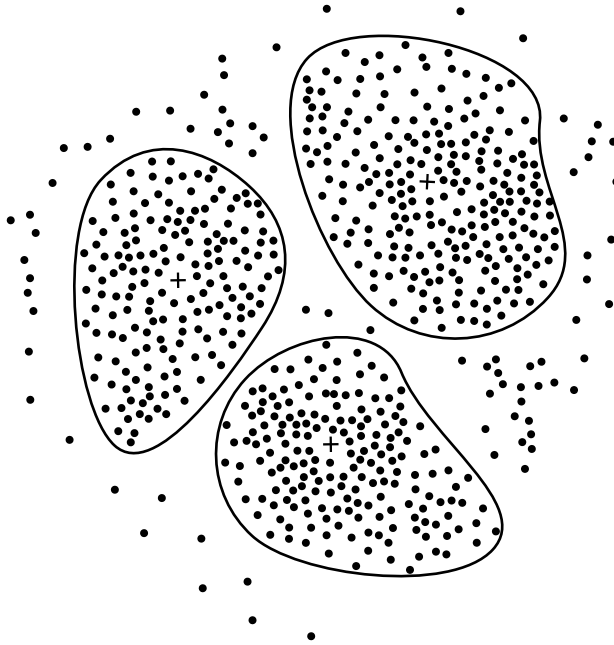
Smoothing by bin boundaries:

Bin 1: 4, 4, 15  
 Bin 2: 21, 21, 24  
 Bin 3: 25, 25, 34

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**Figure 2.11** Binning methods for data smoothing.

1. **Binning:** Binning methods smooth a sorted data value by consulting its “neighborhood,” that is, the values around it. The sorted values are distributed into a number of “buckets,” or *bins*. Because binning methods consult the neighborhood of values, they perform *local* smoothing. Figure 2.11 illustrates some binning techniques. In this example, the data for *price* are first sorted and then partitioned into *equal-frequency* bins of size 3 (i.e., each bin contains three values). In **smoothing by bin means**, each value in a bin is replaced by the mean value of the bin. For example, the mean of the values 4, 8, and 15 in Bin 1 is 9. Therefore, each original value in this bin is replaced by the value 9. Similarly, **smoothing by bin medians** can be employed, in which each bin value is replaced by the bin median. In **smoothing by bin boundaries**, the minimum and maximum values in a given bin are identified as the *bin boundaries*. Each bin value is then replaced by the closest boundary value. In general, the larger the width, the greater the effect of the smoothing. Alternatively, bins may be *equal-width*, where the interval range of values in each bin is constant. Binning is also used as a discretization technique and is further discussed in Section 2.6.
2. **Regression:** Data can be smoothed by fitting the data to a function, such as with regression. *Linear regression* involves finding the “best” line to fit two attributes (or variables), so that one attribute can be used to predict the other. *Multiple linear regression* is an extension of linear regression, where more than two attributes are involved and the data are fit to a multidimensional surface. Regression is further described in Section 2.5.4, as well as in Chapter 6.



**Figure 2.12** A 2-D plot of customer data with respect to customer locations in a city, showing three data clusters. Each cluster centroid is marked with a “+”, representing the average point in space for that cluster. Outliers may be detected as values that fall outside of the sets of clusters.

3. **Clustering:** Outliers may be detected by clustering, where similar values are organized into groups, or “clusters.” Intuitively, values that fall outside of the set of clusters may be considered outliers (Figure 2.12). Chapter 7 is dedicated to the topic of clustering and outlier analysis.

Many methods for data smoothing are also methods for data reduction involving discretization. For example, the binning techniques described above reduce the number of distinct values per attribute. This acts as a form of data reduction for logic-based data mining methods, such as decision tree induction, which repeatedly make value comparisons on sorted data. Concept hierarchies are a form of data discretization that can also be used for data smoothing. A concept hierarchy for *price*, for example, may map real *price* values into *inexpensive*, *moderately-priced*, and *expensive*, thereby reducing the number of data values to be handled by the mining process. Data discretization is discussed in Section 2.6. Some methods of classification, such as neural networks, have built-in data smoothing mechanisms. Classification is the topic of Chapter 6.

### 2.3.3 Data Cleaning as a Process

Missing values, noise, and inconsistencies contribute to inaccurate data. So far, we have looked at techniques for handling missing data and for smoothing data. *“But data cleaning is a big job. What about data cleaning as a process? How exactly does one proceed in tackling this task? Are there any tools out there to help?”*

The first step in data cleaning as a process is *discrepancy detection*. Discrepancies can be caused by several factors, including poorly designed data entry forms that have many optional fields, human error in data entry, deliberate errors (e.g., respondents not wanting to divulge information about themselves), and data decay (e.g., outdated addresses). Discrepancies may also arise from inconsistent data representations and the inconsistent use of codes. Errors in instrumentation devices that record data, and system errors, are another source of discrepancies. Errors can also occur when the data are (inadequately) used for purposes other than originally intended. There may also be inconsistencies due to data integration (e.g., where a given attribute can have different names in different databases).<sup>4</sup>

*“So, how can we proceed with discrepancy detection?”* As a starting point, use any knowledge you may already have regarding properties of the data. Such knowledge or “data about data” is referred to as **metadata**. For example, what are the domain and data type of each attribute? What are the acceptable values for each attribute? What is the range of the length of values? Do all values fall within the expected range? Are there any known dependencies between attributes? The descriptive data summaries presented in Section 2.2 are useful here for grasping data trends and identifying anomalies. For example, values that are more than two standard deviations away from the mean for a given attribute may be flagged as potential outliers. In this step, you may write your own scripts and/or use some of the tools that we discuss further below. From this, you may find noise, outliers, and unusual values that need investigation.

As a data analyst, you should be on the lookout for the inconsistent use of codes and any inconsistent data representations (such as “2004/12/25” and “25/12/2004” for *date*). **Field overloading** is another source of errors that typically results when developers squeeze new attribute definitions into unused (bit) portions of already defined attributes (e.g., using an unused bit of an attribute whose value range uses only, say, 31 out of 32 bits).

The data should also be examined regarding unique rules, consecutive rules, and null rules. A **unique rule** says that each value of the given attribute must be different from all other values for that attribute. A **consecutive rule** says that there can be no missing values between the lowest and highest values for the attribute, and that all values must also be unique (e.g., as in check numbers). A **null rule** specifies the use of blanks, question marks, special characters, or other strings that may indicate the null condition (e.g., where a value for a given attribute is not available), and how such values should be handled. As mentioned in Section 2.3.1, reasons for missing values may include (1) the person originally asked to provide a value for the attribute refuses and/or finds

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<sup>4</sup>Data integration and the removal of redundant data that can result from such integration are further described in Section 2.4.1.

that the information requested is not applicable (e.g., a *license-number* attribute left blank by nondrivers); (2) the data entry person does not know the correct value; or (3) the value is to be provided by a later step of the process. The null rule should specify how to record the null condition, for example, such as to store zero for numerical attributes, a blank for character attributes, or any other conventions that may be in use (such as that entries like “don’t know” or “?” should be transformed to blank).

There are a number of different commercial tools that can aid in the step of discrepancy detection. **Data scrubbing tools** use simple domain knowledge (e.g., knowledge of postal addresses, and spell-checking) to detect errors and make corrections in the data. These tools rely on parsing and fuzzy matching techniques when cleaning data from multiple sources. **Data auditing tools** find discrepancies by analyzing the data to discover rules and relationships, and detecting data that violate such conditions. They are variants of data mining tools. For example, they may employ statistical analysis to find correlations, or clustering to identify outliers. They may also use the descriptive data summaries that were described in Section 2.2.

Some data inconsistencies may be corrected manually using external references. For example, errors made at data entry may be corrected by performing a paper trace. Most errors, however, will require *data transformations*. This is the second step in data cleaning as a process. That is, once we find discrepancies, we typically need to define and apply (a series of) transformations to correct them.

Commercial tools can assist in the data transformation step. **Data migration tools** allow simple transformations to be specified, such as to replace the string “gender” by “sex”. **ETL (extraction/transformation/loading) tools** allow users to specify transforms through a graphical user interface (GUI). These tools typically support only a restricted set of transforms so that, often, we may also choose to write custom scripts for this step of the data cleaning process.

The two-step process of discrepancy detection and data transformation (to correct discrepancies) iterates. This process, however, is error-prone and time-consuming. Some transformations may introduce more discrepancies. Some *nested discrepancies* may only be detected after others have been fixed. For example, a typo such as “20004” in a year field may only surface once all date values have been converted to a uniform format. Transformations are often done as a batch process while the user waits without feedback. Only after the transformation is complete can the user go back and check that no new anomalies have been created by mistake. Typically, numerous iterations are required before the user is satisfied. Any tuples that cannot be automatically handled by a given transformation are typically written to a file without any explanation regarding the reasoning behind their failure. As a result, the entire data cleaning process also suffers from a lack of interactivity.

New approaches to data cleaning emphasize increased interactivity. Potter’s Wheel, for example, is a publicly available data cleaning tool (see <http://control.cs.berkeley.edu/abc>) that integrates discrepancy detection and transformation. Users gradually build a series of transformations by composing and debugging individual transformations, one step at a time, on a spreadsheet-like interface. The transformations can be specified graphically or by providing examples. Results are shown immediately on the records that are visible on the screen. The user can choose to undo the transformations, so that transformations



that introduced additional errors can be “erased.” The tool performs discrepancy checking automatically in the background on the latest transformed view of the data. Users can gradually develop and refine transformations as discrepancies are found, leading to more effective and efficient data cleaning.

Another approach to increased interactivity in data cleaning is the development of declarative languages for the specification of data transformation operators. Such work focuses on defining powerful extensions to SQL and algorithms that enable users to express data cleaning specifications efficiently.

As we discover more about the data, it is important to keep updating the metadata to reflect this knowledge. This will help speed up data cleaning on future versions of the same data store.

## 2.4 Data Integration and Transformation

Data mining often requires data integration—the merging of data from multiple data stores. The data may also need to be transformed into forms appropriate for mining. This section describes both data integration and data transformation.

### 2.4.1 Data Integration

It is likely that your data analysis task will involve *data integration*, which combines data from multiple sources into a coherent data store, as in data warehousing. These sources may include multiple databases, data cubes, or flat files.

There are a number of issues to consider during data integration. *Schema integration* and *object matching* can be tricky. How can equivalent real-world entities from multiple data sources be matched up? This is referred to as the **entity identification problem**. For example, how can the data analyst or the computer be sure that *customer\_id* in one database and *cust\_number* in another refer to the same attribute? Examples of metadata for each attribute include the name, meaning, data type, and range of values permitted for the attribute, and null rules for handling blank, zero, or null values (Section 2.3). Such metadata can be used to help avoid errors in schema integration. The metadata may also be used to help transform the data (e.g., where data codes for *pay\_type* in one database may be “H” and “S”, and 1 and 2 in another). Hence, this step also relates to data cleaning, as described earlier.

*Redundancy* is another important issue. An attribute (such as *annual revenue*, for instance) may be redundant if it can be “derived” from another attribute or set of attributes. Inconsistencies in attribute or dimension naming can also cause redundancies in the resulting data set.

Some redundancies can be detected by **correlation analysis**. Given two attributes, such analysis can measure how strongly one attribute implies the other, based on the available data. For numerical attributes, we can evaluate the correlation between two attributes, *A* and *B*, by computing the **correlation coefficient** (also known as *Pearson’s product moment coefficient*, named after its inventor, Karl Pearson). This is

$$r_{A,B} = \frac{\sum_{i=1}^N (a_i - \bar{A})(b_i - \bar{B})}{N\sigma_A\sigma_B} = \frac{\sum_{i=1}^N (a_i b_i) - N\bar{A}\bar{B}}{N\sigma_A\sigma_B}, \quad (2.8)$$

where  $N$  is the number of tuples,  $a_i$  and  $b_i$  are the respective values of  $A$  and  $B$  in tuple  $i$ ,  $\bar{A}$  and  $\bar{B}$  are the respective mean values of  $A$  and  $B$ ,  $\sigma_A$  and  $\sigma_B$  are the respective standard deviations of  $A$  and  $B$  (as defined in Section 2.2.2), and  $\sum(a_i b_i)$  is the sum of the  $AB$  cross-product (that is, for each tuple, the value for  $A$  is multiplied by the value for  $B$  in that tuple). Note that  $-1 \leq r_{A,B} \leq +1$ . If  $r_{A,B}$  is greater than 0, then  $A$  and  $B$  are positively correlated, meaning that the values of  $A$  increase as the values of  $B$  increase. The higher the value, the stronger the correlation (i.e., the more each attribute implies the other). Hence, a higher value may indicate that  $A$  (or  $B$ ) may be removed as a redundancy. If the resulting value is equal to 0, then  $A$  and  $B$  are independent and there is no correlation between them. If the resulting value is less than 0, then  $A$  and  $B$  are negatively correlated, where the values of one attribute increase as the values of the other attribute decrease. This means that each attribute discourages the other. Scatter plots can also be used to view correlations between attributes (Section 2.2.3).

Note that correlation does not imply causality. That is, if  $A$  and  $B$  are correlated, this does not necessarily imply that  $A$  causes  $B$  or that  $B$  causes  $A$ . For example, in analyzing a demographic database, we may find that attributes representing the number of hospitals and the number of car thefts in a region are correlated. This does not mean that one causes the other. Both are actually causally linked to a third attribute, namely, *population*.

For categorical (discrete) data, a correlation relationship between two attributes,  $A$  and  $B$ , can be discovered by a  $\chi^2$  (chi-square) test. Suppose  $A$  has  $c$  distinct values, namely  $a_1, a_2, \dots, a_c$ .  $B$  has  $r$  distinct values, namely  $b_1, b_2, \dots, b_r$ . The data tuples described by  $A$  and  $B$  can be shown as a **contingency table**, with the  $c$  values of  $A$  making up the columns and the  $r$  values of  $B$  making up the rows. Let  $(A_i, B_j)$  denote the event that attribute  $A$  takes on value  $a_i$  and attribute  $B$  takes on value  $b_j$ , that is, where  $(A = a_i, B = b_j)$ . Each and every possible  $(A_i, B_j)$  joint event has its own cell (or slot) in the table. The  $\chi^2$  value (also known as the *Pearson  $\chi^2$  statistic*) is computed as:

$$\chi^2 = \sum_{i=1}^c \sum_{j=1}^r \frac{(o_{ij} - e_{ij})^2}{e_{ij}}, \quad (2.9)$$

where  $o_{ij}$  is the *observed frequency* (i.e., actual count) of the joint event  $(A_i, B_j)$  and  $e_{ij}$  is the *expected frequency* of  $(A_i, B_j)$ , which can be computed as

$$e_{ij} = \frac{\text{count}(A = a_i) \times \text{count}(B = b_j)}{N}, \quad (2.10)$$

where  $N$  is the number of data tuples,  $\text{count}(A = a_i)$  is the number of tuples having value  $a_i$  for  $A$ , and  $\text{count}(B = b_j)$  is the number of tuples having value  $b_j$  for  $B$ . The sum in Equation (2.9) is computed over all of the  $r \times c$  cells. Note that the cells that contribute the most to the  $\chi^2$  value are those whose actual count is very different from that expected.

**Table 2.2** A  $2 \times 2$  contingency table for the data of Example 2.1. Are *gender* and *preferred\_Reading* correlated?

	<i>male</i>	<i>female</i>	Total
<i>fiction</i>	250 (90)	200 (360)	450
<i>non-fiction</i>	50 (210)	1000 (840)	1050
Total	300	1200	1500

The  $\chi^2$  statistic tests the hypothesis that  $A$  and  $B$  are independent. The test is based on a significance level, with  $(r - 1) \times (c - 1)$  degrees of freedom. We will illustrate the use of this statistic in an example below. If the hypothesis can be rejected, then we say that  $A$  and  $B$  are statistically related or associated.

Let's look at a concrete example.

**Example 2.1** Correlation analysis of categorical attributes using  $\chi^2$ . Suppose that a group of 1,500 people was surveyed. The gender of each person was noted. Each person was polled as to whether their preferred type of reading material was fiction or nonfiction. Thus, we have two attributes, *gender* and *preferred\_reading*. The observed frequency (or count) of each possible joint event is summarized in the contingency table shown in Table 2.2, where the numbers in parentheses are the expected frequencies (calculated based on the data distribution for both attributes using Equation (2.10)).

Using Equation (2.10), we can verify the expected frequencies for each cell. For example, the expected frequency for the cell (male, fiction) is

$$e_{11} = \frac{\text{count}(\text{male}) \times \text{count}(\text{fiction})}{N} = \frac{300 \times 450}{1500} = 90,$$

and so on. Notice that in any row, the sum of the expected frequencies must equal the total observed frequency for that row, and the sum of the expected frequencies in any column must also equal the total observed frequency for that column. Using Equation (2.9) for  $\chi^2$  computation, we get

$$\begin{aligned} \chi^2 &= \frac{(250 - 90)^2}{90} + \frac{(50 - 210)^2}{210} + \frac{(200 - 360)^2}{360} + \frac{(1000 - 840)^2}{840} \\ &= 284.44 + 121.90 + 71.11 + 30.48 = 507.93. \end{aligned}$$

For this  $2 \times 2$  table, the degrees of freedom are  $(2 - 1)(2 - 1) = 1$ . For 1 degree of freedom, the  $\chi^2$  value needed to reject the hypothesis at the 0.001 significance level is 10.828 (taken from the table of upper percentage points of the  $\chi^2$  distribution, typically available from any textbook on statistics). Since our computed value is above this, we can reject the hypothesis that *gender* and *preferred\_reading* are independent and conclude that the two attributes are (strongly) correlated for the given group of people. ■

In addition to detecting redundancies between attributes, duplication should also be detected at the tuple level (e.g., where there are two or more identical tuples for a

given unique data entry case). The use of denormalized tables (often done to improve performance by avoiding joins) is another source of data redundancy. Inconsistencies often arise between various duplicates, due to inaccurate data entry or updating some but not all of the occurrences of the data. For example, if a purchase order database contains attributes for the purchaser's name and address instead of a key to this information in a purchaser database, discrepancies can occur, such as the same purchaser's name appearing with different addresses within the purchase order database.

A third important issue in data integration is the *detection and resolution of data value conflicts*. For example, for the same real-world entity, attribute values from different sources may differ. This may be due to differences in representation, scaling, or encoding. For instance, a *weight* attribute may be stored in metric units in one system and British imperial units in another. For a hotel chain, the *price* of rooms in different cities may involve not only different currencies but also different services (such as free breakfast) and taxes. An attribute in one system may be recorded at a lower level of abstraction than the "same" attribute in another. For example, the *total.sales* in one database may refer to one branch of *All\_Electronics*, while an attribute of the same name in another database may refer to the total sales for *All\_Electronics* stores in a given region.

When matching attributes from one database to another during integration, special attention must be paid to the *structure* of the data. This is to ensure that any attribute functional dependencies and referential constraints in the source system match those in the target system. For example, in one system, a *discount* may be applied to the order, whereas in another system it is applied to each individual line item within the order. If this is not caught before integration, items in the target system may be improperly discounted.

The semantic heterogeneity and structure of data pose great challenges in data integration. Careful integration of the data from multiple sources can help reduce and avoid redundancies and inconsistencies in the resulting data set. This can help improve the accuracy and speed of the subsequent mining process.

## 2.4.2 Data Transformation

In *data transformation*, the data are transformed or consolidated into forms appropriate for mining. Data transformation can involve the following:

- **Smoothing**, which works to remove noise from the data. Such techniques include binning, regression, and clustering.
- **Aggregation**, where summary or aggregation operations are applied to the data. For example, the daily sales data may be aggregated so as to compute monthly and annual total amounts. This step is typically used in constructing a data cube for analysis of the data at multiple granularities.
- **Generalization** of the data, where low-level or "primitive" (raw) data are replaced by higher-level concepts through the use of concept hierarchies. For example, categorical

attributes, like *street*, can be generalized to higher-level concepts, like *city* or *country*. Similarly, values for numerical attributes, like *age*, may be mapped to higher-level concepts, like *youth*, *middle-aged*, and *senior*.

- **Normalization**, where the attribute data are scaled so as to fall within a small specified range, such as  $-1.0$  to  $1.0$ , or  $0.0$  to  $1.0$ .
- **Attribute construction** (or *feature construction*), where new attributes are constructed and added from the given set of attributes to help the mining process.

Smoothing is a form of data cleaning and was addressed in Section 2.3.2. Section 2.3.3 on the data cleaning process also discussed ETL tools, where users specify transformations to correct data inconsistencies. Aggregation and generalization serve as forms of data reduction and are discussed in Sections 2.5 and 2.6, respectively. In this section, we therefore discuss normalization and attribute construction.

An attribute is normalized by scaling its values so that they fall within a small specified range, such as  $0.0$  to  $1.0$ . Normalization is particularly useful for classification algorithms involving neural networks, or distance measurements such as nearest-neighbor classification and clustering. If using the neural network backpropagation algorithm for classification mining (Chapter 6), normalizing the input values for each attribute measured in the training tuples will help speed up the learning phase. For distance-based methods, normalization helps prevent attributes with initially large ranges (e.g., *income*) from outweighing attributes with initially smaller ranges (e.g., binary attributes). There are many methods for data normalization. We study three: *min-max normalization*, *z-score normalization*, and *normalization by decimal scaling*.

**Min-max normalization** performs a linear transformation on the original data. Suppose that  $min_A$  and  $max_A$  are the minimum and maximum values of an attribute,  $A$ . Min-max normalization maps a value,  $v$ , of  $A$  to  $v'$  in the range  $[new\_min_A, new\_max_A]$  by computing

$$v' = \frac{v - min_A}{max_A - min_A} (new\_max_A - new\_min_A) + new\_min_A. \quad (2.11)$$

Min-max normalization preserves the relationships among the original data values. It will encounter an “out-of-bounds” error if a future input case for normalization falls outside of the original data range for  $A$ .

**Example 2.2 Min-max normalization.** Suppose that the minimum and maximum values for the attribute *income* are \$12,000 and \$98,000, respectively. We would like to map *income* to the range  $[0.0, 1.0]$ . By min-max normalization, a value of \$73,600 for *income* is transformed to  $\frac{73,600 - 12,000}{98,000 - 12,000} (1.0 - 0) + 0 = 0.716$ . ■

In **z-score normalization** (or *zero-mean normalization*), the values for an attribute,  $A$ , are normalized based on the mean and standard deviation of  $A$ . A value,  $v$ , of  $A$  is normalized to  $v'$  by computing

$$v' = \frac{v - \bar{A}}{\sigma_A}, \quad (2.12)$$

where  $\bar{A}$  and  $\sigma_A$  are the mean and standard deviation, respectively, of attribute  $A$ . This method of normalization is useful when the actual minimum and maximum of attribute  $A$  are unknown, or when there are outliers that dominate the min-max normalization.

**Example 2.3 z-score normalization** Suppose that the mean and standard deviation of the values for the attribute *income* are \$54,000 and \$16,000, respectively. With z-score normalization, a value of \$73,600 for *income* is transformed to  $\frac{73,600 - 54,000}{16,000} = 1.225$ . ■

**Normalization by decimal scaling** normalizes by moving the decimal point of values of attribute  $A$ . The number of decimal points moved depends on the maximum absolute value of  $A$ . A value,  $v$ , of  $A$  is normalized to  $v'$  by computing

$$v' = \frac{v}{10^j}, \quad (2.13)$$

where  $j$  is the smallest integer such that  $\text{Max}(|v'|) < 1$ .

**Example 2.4 Decimal scaling.** Suppose that the recorded values of  $A$  range from  $-986$  to  $917$ . The maximum absolute value of  $A$  is  $986$ . To normalize by decimal scaling, we therefore divide each value by  $1,000$  (i.e.,  $j = 3$ ) so that  $-986$  normalizes to  $-0.986$  and  $917$  normalizes to  $0.917$ . ■

Note that normalization can change the original data quite a bit, especially the latter two methods shown above. It is also necessary to save the normalization parameters (such as the mean and standard deviation if using z-score normalization) so that future data can be normalized in a uniform manner.

In **attribute construction**,<sup>5</sup> new attributes are constructed from the given attributes and added in order to help improve the accuracy and understanding of structure in high-dimensional data. For example, we may wish to add the attribute *area* based on the attributes *height* and *width*. By combining attributes, attribute construction can discover missing information about the relationships between data attributes that can be useful for knowledge discovery.

## 2.5 Data Reduction

Imagine that you have selected data from the *AllElectronics* data warehouse for analysis. The data set will likely be huge! Complex data analysis and mining on huge amounts of data can take a long time, making such analysis impractical or infeasible.

<sup>5</sup>In the machine learning literature, attribute construction is known as *feature construction*.

**Data reduction** techniques can be applied to obtain a reduced representation of the data set that is much smaller in volume, yet closely maintains the integrity of the original data. That is, mining on the reduced data set should be more efficient yet produce the same (or almost the same) analytical results.

Strategies for data reduction include the following:

1. **Data cube aggregation**, where aggregation operations are applied to the data in the construction of a data cube.
2. **Attribute subset selection**, where irrelevant, weakly relevant, or redundant attributes or dimensions may be detected and removed.
3. **Dimensionality reduction**, where encoding mechanisms are used to reduce the data set size.
4. **Numerosity reduction**, where the data are replaced or estimated by alternative, smaller data representations such as parametric models (which need store only the model parameters instead of the actual data) or nonparametric methods such as clustering, sampling, and the use of histograms.
5. **Discretization and concept hierarchy generation**, where raw data values for attributes are replaced by ranges or higher conceptual levels. Data discretization is a form of numerosity reduction that is very useful for the automatic generation of concept hierarchies. Discretization and concept hierarchy generation are powerful tools for data mining, in that they allow the mining of data at multiple levels of abstraction. We therefore defer the discussion of discretization and concept hierarchy generation to Section 2.6, which is devoted entirely to this topic.

Strategies 1 to 4 above are discussed in the remainder of this section. The computational time spent on data reduction should not outweigh or “erase” the time saved by mining on a reduced data set size.

## 2.5.1 Data Cube Aggregation

Imagine that you have collected the data for your analysis. These data consist of the *AllElectronics* sales per quarter, for the years 2002 to 2004. You are, however, interested in the annual sales (total per year), rather than the total per quarter. Thus the data can be *aggregated* so that the resulting data summarize the total sales per year instead of per quarter. This aggregation is illustrated in Figure 2.13. The resulting data set is smaller in volume, without loss of information necessary for the analysis task.

Data cubes are discussed in detail in Chapter 3 on data warehousing. We briefly introduce some concepts here. Data cubes store multidimensional aggregated information. For example, Figure 2.14 shows a data cube for multidimensional analysis of sales data with respect to annual sales per item type for each *AllElectronics* branch. Each cell holds an aggregate data value, corresponding to the data point in multidimensional space. (For readability, only some cell values are shown.) Concept