

1) (22, 1, 42, 10) and (20, 0, 36, 8)

a) $\sqrt{(22 - 20)^2 + (1 - 0)^2 + (42 - 36)^2 + (10 - 8)^2}$

b) $|22 - 20| + |1 - 0| + |42 - 36| + |10 - 8| =$

c) $\sqrt[3]{(22 - 20)^3 + (1 - 0)^3 + (42 - 36)^3 + (10 - 8)^3}$

d) $\max(|22 - 20|, |1 - 0|, |42 - 36|, |10 - 8|) =$

2) (a)

#Euclidean distance:

$$D(x1,x) = \sqrt{(1.5 - 1.4)^2 + (1.7 - 1.6)^2} = 0.1414$$

$$D(x2,x) = \sqrt{(2 - 1.4)^2 + (1.9 - 1.6)^2} = 0.6708$$

$$D(x3,x) = \sqrt{(1.6 - 1.4)^2 + (1.8 - 1.6)^2} = 0.2828$$

$$D(x4,x) = \sqrt{(1.2 - 1.4)^2 + (1.5 - 1.6)^2} = 0.2236$$

$$D(x5,x) = \sqrt{(1.5 - 1.4)^2 + (1 - 1.6)^2} = 0.6082$$

$$\text{Ranking} = x1, x4, x3, x5, x2$$

#Manhattan distance:

$$D(x1,x) = |1.5-1.4| + |1.7 - 1.6| = 0.1 + 0.1 = 0.2$$

$$D(x2,x) = |2-1.4| + |1.9-1.6| = 0.6 + 0.3 = 0.9$$

$$D(x3,x) = |1.6-1.4| + |1.8-1.6| = 0.2 + 0.2 = 0.4$$

$$D(x4,x) = |1.2-1.4| + |1.5-1.6| = 0.2 + 0.1 = 0.3$$

$$D(x_5, x) = |1.5 - 1.4| + |1 - 1.6| = 0.1 + 0.6 = 0.7$$

$$\text{Ranking} = x_1, x_4, x_3, x_5, x_2$$

#Supremum distance:

$$D(x_1, x) = \max(|1.5 - 1.4|, |1.7 - 1.6|) = \max(0.1, 0.1) = 0.1$$

$$D(x_2, x) = \max(|2 - 1.4|, |1.9 - 1.6|) = \max(0.6, 0.3) = 0.6$$

$$D(x_3, x) = \max(|1.6 - 1.4|, |1.8 - 1.6|) = \max(0.2, 0.2) = 0.2$$

$$D(x_4, x) = \max(|1.2 - 1.4|, |1.5 - 1.6|) = \max(0.2, 0.1) = 0.2$$

$$D(x_5, x) = \max(|1.5 - 1.4|, |1 - 1.6|) = \max(0.1, 0.6) = 0.6$$

$$\text{Ranking} = x_1, x_3, x_4, x_2, x_5$$

#cosine similarity:

$$d1.d = (1.5 * 1.4) + (1.7 * 1.6) = 2.1 + 2.72 = 4.82$$

$$\|d1\| = \sqrt{1.5 * 1.5 + 1.7 * 1.7} = 2.2671$$

$$\|d\| = \sqrt{1.4 * 1.4 + 1.6 * 1.6} = 2.1260$$

$$\cos(x_1, x) = \frac{d1.d6}{\|d1\| \|d6\|} = 1.000030$$

$$d2.d = (2 * 1.4) + (1.9 * 1.6) = 2.8 + 3.04 = 5.84$$

$$\|d2\| = \sqrt{2 * 2 + 1.4 * 1.4} = 2.4413$$

$$\|d\| = 2.1260$$

$$\cos(x_2, x) = \frac{d_2 \cdot d_6}{\|d_2\| \|d_6\|} = 1.125196$$

$$d_3 \cdot d = (1.6 * 1.4) + (1.8 * 1.6) = 2.24 + 2.88 = 5.12$$

$$\|d_3\| = \sqrt{1.6 * 1.6 + 1.8 * 1.8} = 2.4083$$

$$\|d\| = 2.1260$$

$$\cos(x_3, x) = \frac{d_3 \cdot d_6}{\|d_3\| * \|d_6\|} = 0.999991$$

$$d_4 \cdot d = (1.2 * 1.4) + (1.5 * 1.6) = 1.68 + 2.4 = 4.08$$

$$\|d_4\| = \sqrt{1.2 * 1.2 + 1.5 * 1.5} = 1.9209$$

$$\|d\| = 2.1260$$

$$\cos(x_4, x) = \frac{d_4 \cdot d_6}{\|d_4\| \|d_6\|} = 1.000817$$

$$d_5 \cdot d = (1.5 * 1.4) + (1 * 1.6) = 2.1 + 1.6 = 3.7$$

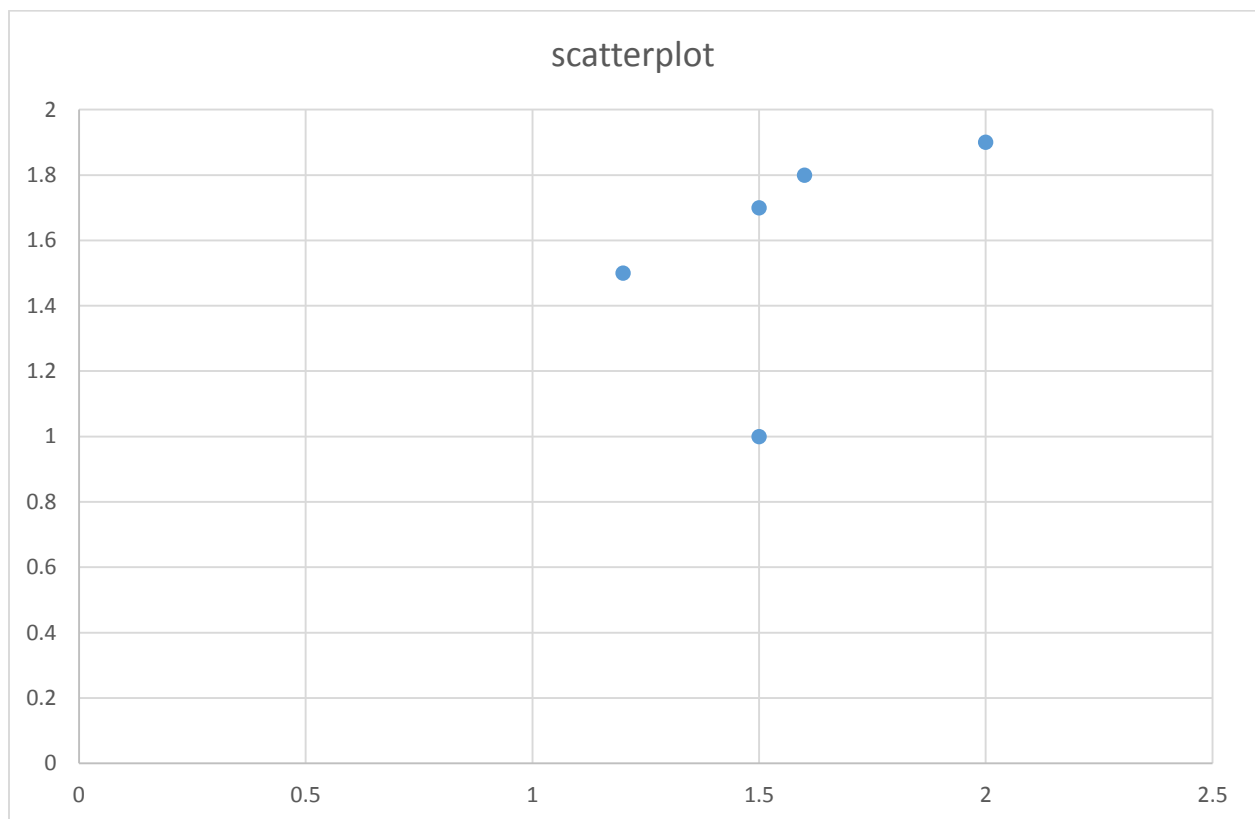
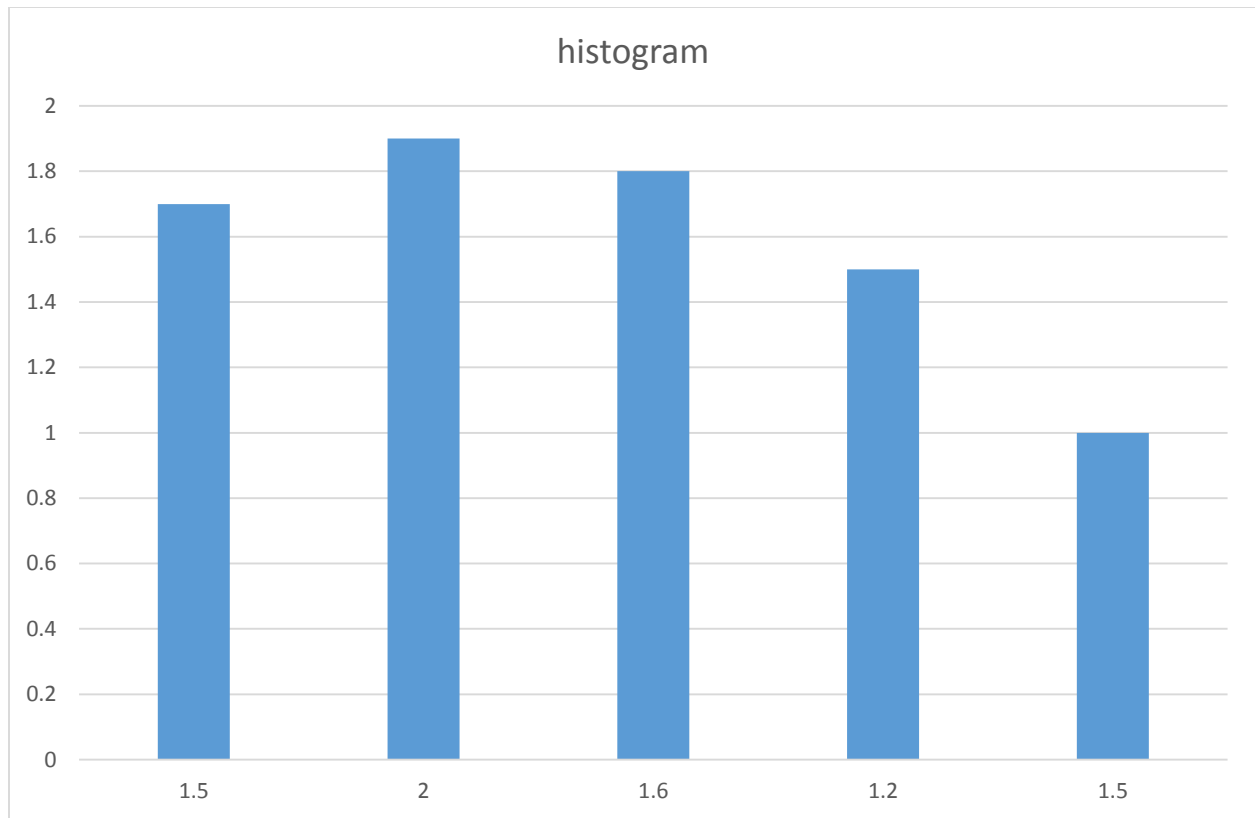
$$\|d_5\| = \sqrt{(1.5 * 1.5) + 1} = 1.8027$$

$$\|d\| = 2.1260$$

$$\cos(x_5, x) = \frac{d_5 \cdot d_6}{\|d_5\| \|d_6\|} = 0.965417$$

Ranking = x_5, x_3, x_1, x_4, x_2

c)



3)

There are three methods to handle ratio-scaled variables for computing the dissimilarity between objects.

1 # treat ratio-scaled variables like interval-scaled variables.

This, however, is not usually a good choice since it is likely that the scale may be distorted.

2 # apply logarithmic transformation to a ratio-scaled variable f having value x_{if} for object i by using the formula $y_{if} = \log(x_{if})$. The y_{if} values can be treated as interval valued. Notice that for some ratio-scaled variables, log-log or other transformation may be applied, depending on the variable's definition and the application.

3 # treat x_{if} as continuous ordinal data and treat their ranks as interval-valued.