```
import numpy as np
import pandas as pd
from scipy.stats import norm
import matplotlib.pyplot as plt
```

Q1:

```
# Load the dataset (change the file path to your dataset)
data = pd.read_csv('StudentsPerformance.csv')

# Print the first few rows of the dataset to understand its structure
display(data)
```

	gender	race/ethnicity	parental level of education	lunch	test preparation course	math score	reading score	writing score
0	female	group B	bachelor's degree	standard	none	72	72	74
1	female	group C	some college	standard	completed	69	90	88
2	female	group B	master's degree	standard	none	90	95	93
3	male	group A	associate's degree	free/reduced	none	47	57	44
4	male	group C	some college	standard	none	76	78	75
...	...	...	...	...	...	...	...	...
995	female	group E	master's degree	standard	completed	88	99	95
996	male	group C	high school	free/reduced	none	62	55	55
997	female	group C	high school	free/reduced	completed	59	71	65
998	female	group D	some college	standard	completed	68	78	77
999	female	group D	some college	free/reduced	none	77	86	86

1000 rows × 8 columns

```
math_scores = data['math score']
reading_scores = data['reading score']
writing_scores = data['writing score']
```

```
def likelihood(params, data):
    mu, sigma = params
    return -np.sum(norm.logpdf(data, loc=mu, scale=sigma))
```

```
from scipy.optimize import minimize

# Initial parameter guess
initial_guess = [math_scores.mean(), math_scores.std()]

# Perform MLE for math scores
result = minimize(lambda params: likelihood(params, math_scores), initial_guess)
mu_mle, sigma_mle = result.x

print(f"MLE estimate of mu for math scores: {mu_mle}")
print(f"MLE estimate of sigma for math scores: {sigma_mle}")
```

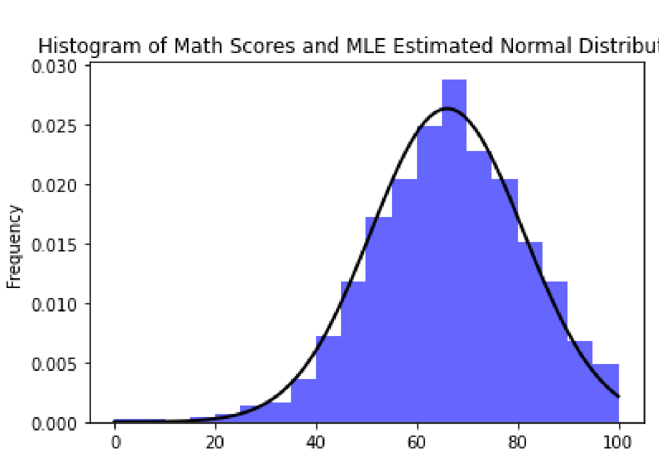
```
MLE estimate of mu for math scores: 66.089
MLE estimate of sigma for math scores: 15.155496761094891
```

```
# Create a histogram of math scores
plt.hist(math_scores, bins=20, density=True, alpha=0.6, color='b')

# Create a PDF of the estimated normal distribution for math scores
x = np.linspace(math_scores.min(), math_scores.max(), 100)
pdf = norm.pdf(x, loc=mu_mle, scale=sigma_mle)
plt.plot(x, pdf, 'k-', lw=2)

# Label the plot for math scores
plt.xlabel('Math Scores')
plt.ylabel('Frequency')
plt.title('Histogram of Math Scores and MLE Estimated Normal Distribution')

# Show the plot
plt.show()
```



Q2:

```
from scipy.optimize import minimize
import numpy as np

# Define the sequence of coin flips (H for heads, T for tails)
coin_flips = ['H', 'T', 'H', 'H', 'T', 'H', 'T', 'H', 'H', 'H']

# Define the negative log-likelihood function for a binomial distribution
def neg_log_likelihood(p):
 # Calculate the likelihood of the observed sequence
 likelihood = np.prod([p if flip == 'H' else 1 - p for flip in coin_flips])
 # Take the negative natural logarithm of the likelihood
 return -np.log(likelihood)

# Use a numerical optimization method to find the MLE estimate for p
initial_guess = 0.5 # Initial guess for the probability of heads
result = minimize(neg_log_likelihood, initial_guess, method='Nelder-Mead')

# The MLE estimate for the probability of heads (p) is in the result
mle_estimate = result.x[0]

# Print the MLE estimate for p
print("Maximum Likelihood Estimate (MLE) for the probability of heads (p):", mle_estimate)
```

```
Maximum Likelihood Estimate (MLE) for the probability of heads (p): 0.7000000000000005
```

```
# Create a range of p values for visualization
p_values = np.linspace(0, 1, 100)

# Calculate the negative log-likelihood for each p value
negative_log_likelihoods = [-neg_log_likelihood(p) for p in p_values]

# Visualize the likelihood as a function of p
plt.plot(p_values, negative_log_likelihoods)
plt.axvline(x=mle_estimate, color='r', linestyle='--', label=f'MLE Estimate for p: {mle_estimate:.3f}')
plt.xlabel('Probability of Getting Heads (p)')
plt.ylabel('Negative Log-Likelihood')
plt.title('MLE for Probability of Getting Heads')
plt.legend()
plt.grid(True)
plt.show()
```

```
<ipython-input-15-987cae3ef542>:12: RuntimeWarning: divide by zero encountered in log
  return -np.log(likelihood)
```