

¹ PyCPD: Pure NumPy Implementation of the Coherent Point Drift Algorithm

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⁶ Background

⁷ Point cloud registration is a common problem in many areas of computer science, particularly
⁸ computer vision. Point clouds come from many types of data such as LIDAR commonly used
⁹ for self-driving vehicles, and other sorts of 3D scanners (e.g., structured light) are commonly
¹⁰ used to map the surface of physical objects. Point clouds are also used to represent the surface
¹¹ of an anatomical structure extracted from a medical image. Point cloud registration finds a
¹² transformation from one point cloud to another. Point cloud registration has use cases in many
¹³ fields from self-driving vehicles to medical imaging and virtual reality. Typically, point cloud
¹⁴ registration is classified into rigid (only rotations or translations), affine (rigid + shearing and
¹⁵ scaling) and non-rigid also called deformable registration (non-linear deformation).

¹⁶ Point cloud registration typically requires 2 point clouds. The first point cloud is the “fixed” or
¹⁷ “target” point cloud and the second is the “moving” or “source” point cloud. We try to find
¹⁸ the transformation that will best align the moving (or source) point cloud with the fixed point
¹⁹ cloud. One of the most well known rigid point cloud registration algorithms is the Iterative
²⁰ Closest Point (ICP) algorithm ([Besl & McKay, 1992](#); [Chen & Medioni, 1992](#)). ICP is an
²¹ iterative algorithm where the following steps are iterated:

- ²² (1) for every point on the moving point cloud find the closest point on the fixed point cloud
- ²³ (2) use a least squares approach to find the optimal transformation matrix (rotation, transla-
- ²⁴ tion, scaling, shear) to align the point correspondences found in (1)
- ²⁵ (3) apply the transformation from (2) to the moving point cloud

²⁶ These steps are repeated until the root mean squared point-to-point distances from (1)
²⁷ converges.

²⁸ The coherent point drift (CPD) algorithm was created by Myronenko and Song ([Myronenko & Song, 2010](#)) to overcome many of the limitations of ICP and other previous registration
²⁹ methods. Namely, these other methods didn't necessarily generalize to greater than 3 dimensions
³⁰ and they were prone to errors such as noise, outliers, or missing points. The CPD algorithm is
³¹ a probabilistic multidimensional algorithm that is robust and works for both rigid and non-rigid
³² registration. In CPD the moving point cloud is modelled as a Gaussian Mixture Model (GMM)
³³ and the fixed point cloud is treated as observations from the GMM. The optimal transformation
³⁴ parameters maximize the Maximum Likelihood / Maximum A Posteriori (MAP) estimation
³⁵ that the observed point cloud is drawn from the GMM. A key point of the CPD algorithm
³⁶ is that it forces the points to move coherently by preserving topological structure. The CPD
³⁷ algorithm is also an iterative algorithm that iterates between an expectation (E) step and
³⁸ a maximization (M) step until convergence is achieved. The E-step estimates the posterior
³⁹ probability distributions of the GMM centroids (moving points) given the data (fixed points)
⁴⁰ then the M-step updates the transformation to maximize the posterior probability that the
⁴¹ data belong to the GMM distributions. The E- and M-steps are iterated until convergence.
⁴²

⁴³ Statement of need

⁴⁴ Due to the robustness and the broad array of uses for the CPD algorithm the original CPD
⁴⁵ paper has currently (March 2022) been referenced >2000 times. The CPD algorithm is
⁴⁶ available in Matlab. However, to the best of our knowledge, no open-source python version
⁴⁷ previously existed. In this paper we present a pure NumPy([Harris et al., 2020](#)) version of the
⁴⁸ CPD algorithm to enable general use of CPD for the Python community. Furthermore, the
⁴⁹ full implementation in Numpy makes the algorithm accessible for others to learn from. To
⁵⁰ help in learning, a blog post that coincides with this library has previously been [published]
⁵¹ (<http://siavashk.github.io/2017/05/14/coherent-point-drift/>)([Khallaghi, 2017](#)).

⁵² Summary

⁵³ The PyCPD package implements the CPD algorithm in NumPy. The library itself includes a
⁵⁴ module to implement the Expectation Maximization (EM) algorithm. Sub-modules inherent
⁵⁵ the EM functionality and implement rigid, affine, and deformable registration using EM. CPD
⁵⁶ registration using affine, rigid, and deformable methods all allow for the transformation learned
⁵⁷ from CPD to be applied to any point cloud. Thus, it is possible to learn the transformation
⁵⁸ on a subset of the points and then apply it to the whole point cloud to reduce computation
⁵⁹ time. Finally, the low-rank approximation for deformable registration that was described by
⁶⁰ Myronenko and Song ([Myronenko & Song, 2010](#)) was implemented. A low rank approximation
⁶¹ of the Gaussian kernel is used to reduce computation time and has the added benefit of
⁶² regularizing the non-rigid deformation.

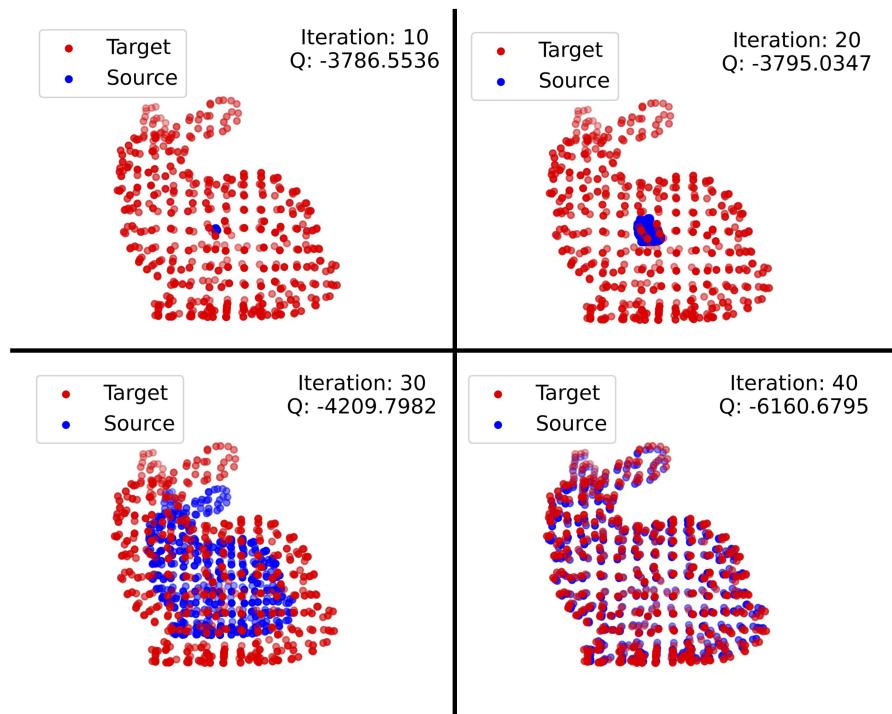


Figure 1: Visualization of the 3D rigid registration from the examples included in the library. Each panel represents a different iteration in the registration process.

⁶³ Examples of the PyCPD algorithm are included (**Figure 1**). Examples are available for 2D and

64 3D versions of all registration methods (rigid, affine, deformable). Examples of how to use the
65 low-rank approximation as well as how to use a sub-set of the points for registration are also
66 included in the examples.

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74 References

- 75 Besl, P. J., & McKay, N. D. (1992). A method for registration of 3-d shapes. *IEEE Transactions
76 on Pattern Analysis and Machine Intelligence*, 14(2), 239–256. <https://doi.org/10.1109/34.121791>
- 78 Chen, Y., & Medioni, G. (1992). Object modelling by registration of multiple range images.
79 *Image and Vision Computing*, 10(3), 145–155. [https://doi.org/https://doi.org/10.1016/0262-8856\(92\)90066-C](https://doi.org/https://doi.org/10.1016/0262-8856(92)90066-C)
- 81 Harris, C. R., Millman, K. J., Walt, S. J. van der, Gommers, R., Virtanen, P., Cournapeau, D.,
82 Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., Kerkwijk,
83 M. H. van, Brett, M., Haldane, A., Río, J. F. del, Wiebe, M., Peterson, P., ... Oliphant,
84 T. E. (2020). Array programming with NumPy. *Nature*, 585(7825), 357–362. <https://doi.org/10.1038/s41586-020-2649-2>
- 86 Khallaghi, S. (2017). *PyCPD: Tutorial on the coherent point drift algorithm*. <http://siavashk.github.io/2017/05/14/coherent-point-drift/>
- 88 Myronenko, A., & Song, X. (2010). Point set registration: Coherent point drift. *IEEE
89 Transactions on Pattern Analysis and Machine Intelligence*, 32(12), 2262–2275. <https://doi.org/10.1109/TPAMI.2010.46>