

# <sup>1</sup> PyCPD: Pure NumPy Implementation of the Coherent Point Drift Algorithm

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## Software

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## <sup>6</sup> Background

<sup>7</sup> Point cloud registration is a common problem in many areas of computer science, particularly  
<sup>8</sup> computer vision. Point clouds come from many types of data such as LIDAR commonly used  
<sup>9</sup> for self-driving vehicles, and other sorts of 3D scanners (e.g., structured light) are commonly  
<sup>10</sup> used to map the surface of physical objects. Point clouds are also used to represent the surface  
<sup>11</sup> of an anatomical structure extracted from a medical image. Point cloud registration finds a  
<sup>12</sup> transformation from one point cloud to another. Point cloud registration has use cases in many  
<sup>13</sup> fields from self-driving vehicles to medical imaging and virtual reality. Typically, point cloud  
<sup>14</sup> registration is classified into rigid (only rotations or translations), affine (rigid + shearing and  
<sup>15</sup> scaling) and non-rigid also called deformable registration (non-linear deformation).

<sup>16</sup> Point cloud registration typically requires 2 point clouds. The first point cloud is the “fixed” or  
<sup>17</sup> “target” point cloud and the second is the “moving” or “source” point cloud. We try to find  
<sup>18</sup> the transformation that will best align the moving (or source) point cloud with the fixed point  
<sup>19</sup> cloud. One of the most well known rigid point cloud registration algorithms is the Iterative  
<sup>20</sup> Closest Point (ICP) algorithm ([Besl & McKay, 1992](#); [Chen & Medioni, 1992](#)). ICP is an  
<sup>21</sup> iterative algorithm where the following steps are iterated:

- <sup>22</sup> (1) for every point on the moving point cloud find the closest point on the fixed point cloud
- <sup>23</sup> (2) use a least squares approach to find the optimal transformation matrix (rotation, transla-
- <sup>24</sup> tion, scaling, shear) to align the point correspondences found in (1)
- <sup>25</sup> (3) apply the transformation from (2) to the moving point cloud

<sup>26</sup> These steps are repeated until the root mean squared point-to-point distances from (1)  
<sup>27</sup> converges.

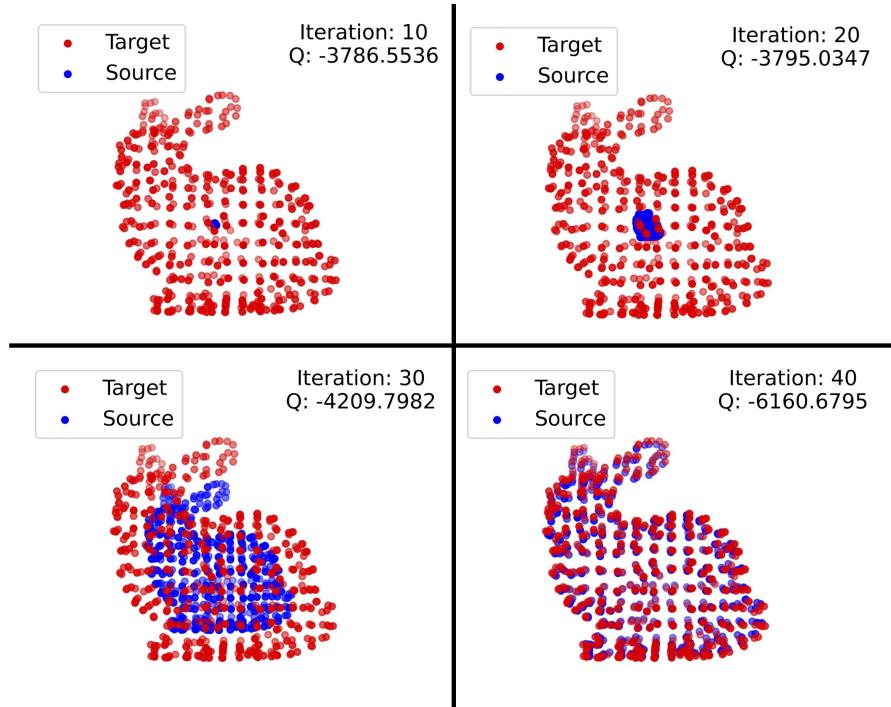
<sup>28</sup> The coherent point drift (CPD) algorithm was created by Myronenko and Song ([Myronenko & Song, 2010](#)) to overcome many of the limitations of ICP and other previous registration  
<sup>29</sup> methods. Namely, these other methods didn't necessarily generalize to greater than 3 dimensions  
<sup>30</sup> and they were prone to errors such as noise, outliers, or missing points. The CPD algorithm is  
<sup>31</sup> a probabilistic multidimensional algorithm that is robust and works for both rigid and non-rigid  
<sup>32</sup> registration. In CPD the moving point cloud is modelled as a Gaussian Mixture Model (GMM)  
<sup>33</sup> and the fixed point cloud is treated as observations from the GMM. The optimal transformation  
<sup>34</sup> parameters maximize the Maximum Likelihood / Maximum A Posteriori (MAP) estimation  
<sup>35</sup> that the observed point cloud is drawn from the GMM. A key point of the CPD algorithm  
<sup>36</sup> is that it forces the points to move coherently by preserving topological structure. The CPD  
<sup>37</sup> algorithm is also an iterative algorithm that iterates between an expectation (E) step and  
<sup>38</sup> a maximization (M) step until convergence is achieved. The E-step estimates the posterior  
<sup>39</sup> probability distributions of the GMM centroids (moving points) given the data (fixed points)  
<sup>40</sup> then the M-step updates the transformation to maximize the posterior probability that the  
<sup>41</sup> data belong to the GMM distributions. The E- and M-steps are iterated until convergence.  
<sup>42</sup>

## <sup>43</sup> Statement of need

<sup>44</sup> Due to the robustness and the broad array of uses for the CPD algorithm the original CPD  
<sup>45</sup> paper has currently (March 2022) been referenced >2000 times. The CPD algorithm is  
<sup>46</sup> available in Matlab. However, no open-source python version previously existed. In this paper  
<sup>47</sup> we present a pure NumPy([Harris et al., 2020](#)) version of the CPD algorithm to enable general  
<sup>48</sup> use of CPD for the Python community. Furthermore, the full implementation in Numpy makes  
<sup>49</sup> the algorithm accessible for others to learn from. To help in learning, a blog post that coincides  
<sup>50</sup> with this library has previously been published ([Khallaghi, 2017](#)).

## <sup>51</sup> Summary

<sup>52</sup> The PyCPD package implements the CPD algorithm in NumPy. The library itself includes a  
<sup>53</sup> module to implement the Expectation Maximization (EM) algorithm. Sub-modules inherent  
<sup>54</sup> the EM functionality and implement rigid, affine, and deformable registration using EM. CPD  
<sup>55</sup> registration using affine, rigid, and deformable methods all allow for the transformation learned  
<sup>56</sup> from CPD to be applied to any point cloud. Thus, it is possible to learn the transformation  
<sup>57</sup> on a subset of the points and then apply it to the whole point cloud to reduce computation  
<sup>58</sup> time. Finally, the low-rank approximation for deformable registration that was described by  
<sup>59</sup> Myronenko and Song ([Myronenko & Song, 2010](#)) was implemented. A low rank approximation  
<sup>60</sup> of the Gaussian kernel is used to reduce computation time and has the added benefit of  
<sup>61</sup> regularizing the non-rigid deformation.



**Figure 1:** Visualization of the 3D rigid registration from the examples included in the library. Each panel represents a different iteration in the registration process.

<sup>62</sup> Examples of the PyCPD algorithm are included (**Figure 1**). Examples are available for 2D and  
<sup>63</sup> 3D versions of all registration methods (rigid, affine, deformable). Examples of how to use the

64 low-rank approximation as well as how to use a sub-set of the points for registration are also  
65 included in the examples.

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71 finding a bug when transforming a point cloud using rigid registration parameters. - sandyhsia  
72 for finding a bug when updating the variance during deformable registration.

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