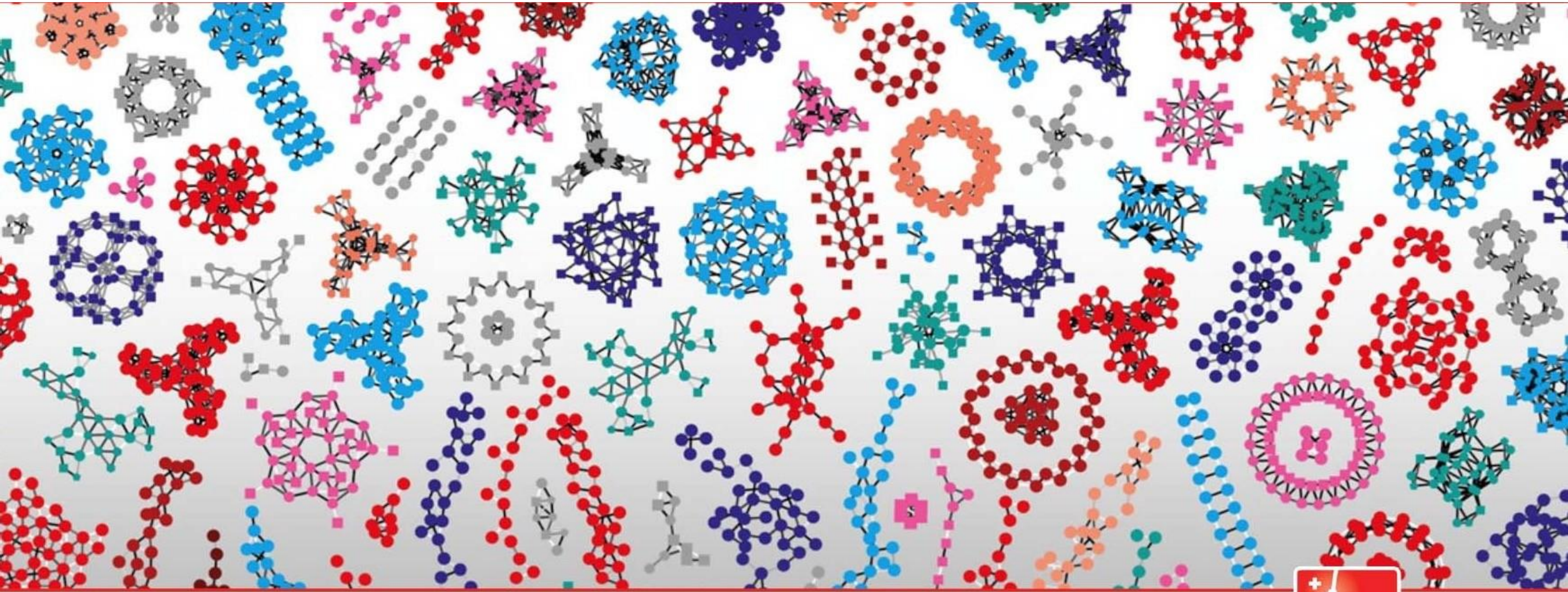

Swiss Institute of Bioinformatics

# Introduction to Statistics

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# Parametric and non parametric tests

# *T-tests: summary*

T-test in general
Used to compare means

One-sample t-test
Compare the mean of a sample to a given number

Two-sample t-test
Compare the means of two samples

Paired t-test
Compare the difference between pairs of related data points

## *Assumptions for the t-test*

- This test is one of the most widely used tests. However, it can be used only if the background assumptions are satisfied.
  - Data values must be **independent**.
  - Data in each group must be obtained via a **random sample** from the population.
  - The variances for the two independent groups are **equal**.
  - Data in each group are **normally** distributed.
  - Data values are **continuous**.

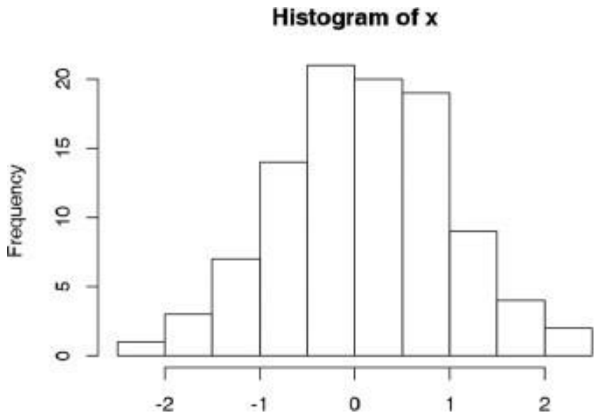
## *T-test in practice*

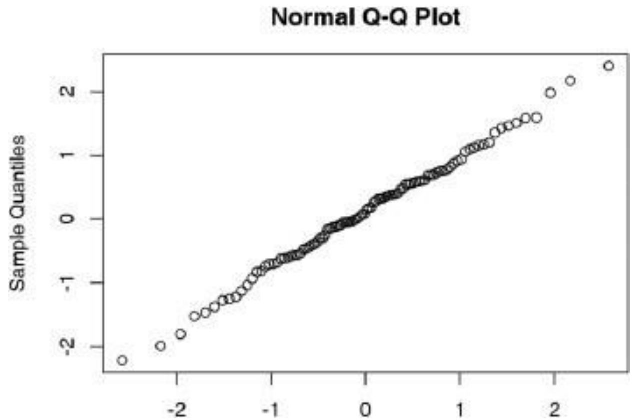
- Analysts generally don't use the standard (Student) t-test.
- The Welch t-test takes into account different sample variances.
- If the variances are actually the same, it provides the same results as the standard Student t-test.

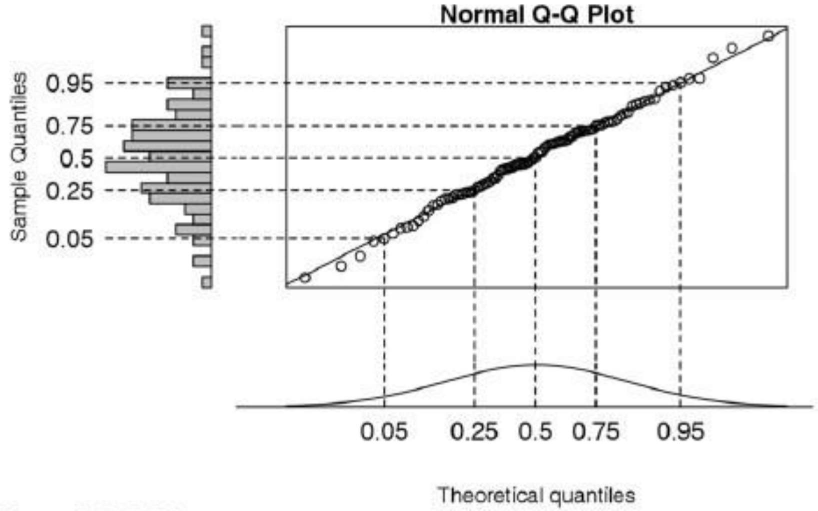
```
> t.test(KO_WT$weight ~ KO_WT$genotype)

                    Welch Two Sample t-test

data:  KO_WT$weight by KO_WT$genotype
t = -1.4261, df = 18.905, p-value = 0.1702
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.0078465  0.5705536
sample estimates:
mean in group KO mean in group WT 
        30.32366         31.54231
```

# *Graphical tools to assess normality*

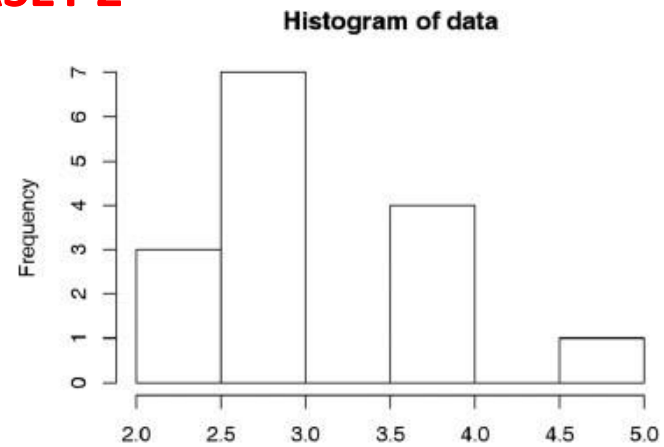
**DATASET 1**

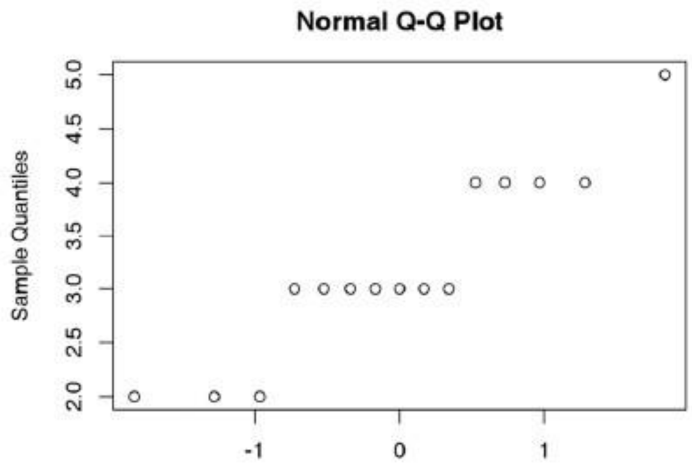



**DATASET 2**



## *Normality tests*

- Examples: Shapiro-Wilk, Kolmogorov-Smirnov.

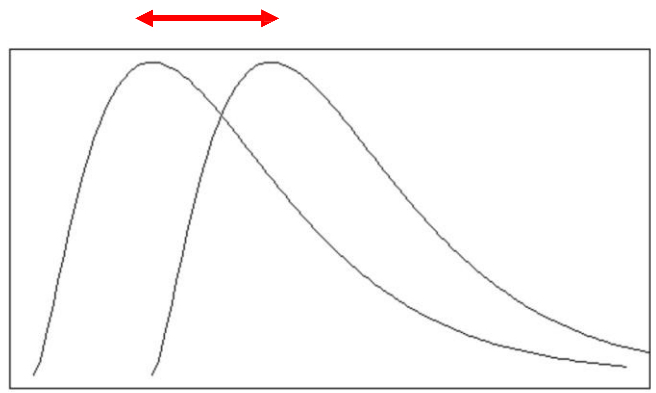
```
> shapiro_test(weight$weight)
# A tibble:  1 x 3
  variable        statistic p.value
  <chr>               <dbl>   <dbl>
1 weight$weight       0.902   0.166
```

- Normality tests are statistical tests
  - They test ***against*** normality (and can not show/prove normality)
  - They may not have enough power
  - Conversely, if *n* is large, even a very small departure from normality may be significant.

## *What happens if the data is not normally distributed ?*

- Transform the data using the log function
  - Examples of data that should be logged:
    - Highly asymetrical data
    - Data spanning several orders of magnitude
    - Data originating from ratios

- Another possible solution: non-parametric tests
  - Wilcoxon rank-sum test (WRS)
  - Mann–Whitney U test
  - Mann–Whitney–Wilcoxon (MWW)



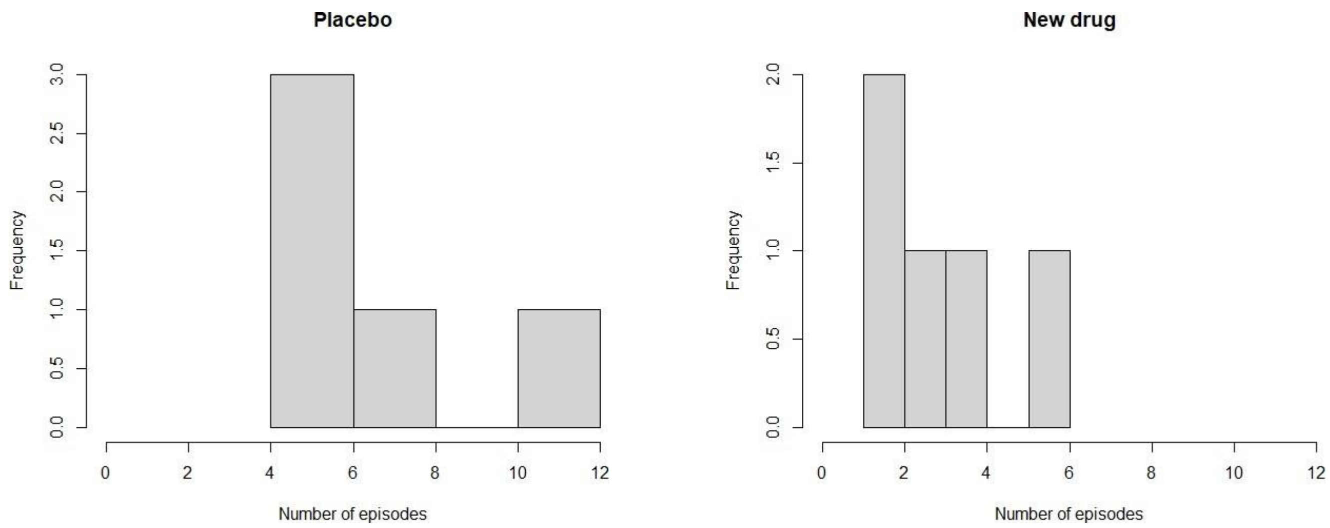
# *Mann–Whitney U test*

- General formulation:
  - All the observations from both groups are independent of each other
  - The variable is ordinal or numerical
  - $H_0$: the distributions of both populations are equal
  - $H_1$: the distributions are not equal

# *Mann–Whitney U test: an example*

Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of either the new drug or a placebo. Is the difference significant ?

| Placebo | 7 | 5 | 6 | 4 | 12 |
|---|---|---|---|---|---|
| New drug | 3 | 6 | 4 | 2 | 1 |



# *Mann–Whitney U test: an example*

| | | Total sample (ordered) | | Ranks | |
|---|---|---|---|---|---|
| Placebo | New drug | Placebo | New drug | Placebo | New drug |
| 7 | 3 | | 1 | | 1 |
| 5 | 6 | | 2 | | 2 |
| 6 | 4 | | 3 | | 3 |
| 4 | 2 | 4 | 4 | 4.5 | 4.5 |
| 12 | 1 | 5 | | 6 | |
| | | 6 | 6 | 7.5 | 7.5 |
| | | 7 | | 9 | |
| | | 12 | | 10 | |

R1 = 4.5+6+7.5+9+10 = 37

R2 = 1+2+3+4.5+7.5= 18

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 5(5) + \frac{5(6)}{2} - 37 = 3$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 = 5(5) + \frac{5(6)}{2} - 18 = 22$$

Test statistic for the Mann Whitney U Test is denoted **U** =min($U_1$,$U_2$)

# *Mann–Whitney U test: an example*

**Alpha = .05 (two-tailed)**

| n1\n2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | | | | | | | 0 | 0 | 0 |
| 3 | | | | 0 | 1 | 1 | 2 | 2 | 3 |
| 4 | | | 0 | 1 | 2 | 3 | 4 | 4 | 5 |
| 5 | | 0 | 1 | $\boxed{2}$ | 3 | 5 | 6 | 7 | 8 |
| 6 | | 1 | 2 | 3 | 5 | 6 | 8 | 10 | 11 |
| 7 | | 1 | 3 | 5 | 6 | 8 | 10 | 12 | 14 |
| 8 | 0 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 |
| 9 | 0 | 2 | 4 | 7 | 10 | 12 | 15 | 17 | 20 |
| 10 | 0 | 3 | 5 | 8 | 11 | 14 | 17 | 20 | 23 |

**Decision for the test: if U < $U_{crit}$ we reject H0**

We do not reject H0 because 3 > 2.

We do not have statistically significant evidence at α =0.05, to show that the two populations of numbers of episodes of shortness of breath are not equal.

Low power because of small sample sizes ?

```
> t.test(placebo,newdrug)

       Welch Two Sample t-test

data:  placebo and newdrug

t = 2.199, df = 6.6639, p-value = 0.06574

alternative hypothesis: true difference in means is not equal to 0
```

# *Take home message*

- How to handle (non-)normality
  - Start by making sure that your data is of the right type
  - If the data spans several orders of magnitude, is made of ratios or is very asymmetrical, consider taking the log.
  - If n is large (>20 or 30), then you should not have to worry.
  - As long as your data is not "too" far from normality (and you are careful with borderline p-values), the t-test will return a meaning full value.

- How to handle (non-)normality: alternatively
  - Perform both a t-test and a Wilcoxon rank-sum test
  - If they return a coherent result, it is stronger that if you performed only one test.
  - If the results are incoherent, look at the data to find out what produced this.