$(\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$

$\Rightarrow \neg(p \vee q) \vee \neg(p \vee r)$ by Demorgan's law

$\Rightarrow \neg((p \vee q) \wedge (p \vee r))$ by Demorgan's law ...(2)

Using (1) and (2) in given propotion, we get

$((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$

$\Leftrightarrow ((p \vee q) \wedge (p \vee q) \wedge (p \vee r)) \vee \neg((p \vee q) \wedge (p \vee r))$

$\Leftrightarrow ((p \vee q) \wedge (p \vee r)) \vee \neg((p \vee q) \wedge (p \vee r))$

$\Leftrightarrow T$ $\because p \vee \neg p \Leftrightarrow T$

Hence given propotion is tautology.

PCNF and PDNF

Minterms using three variables:

There are $2^{(\text{no. of variables})} = 2^3 = 8$ minterms available, which are given below

$(P \wedge Q \wedge R)$, $(\neg P \wedge Q \wedge R)$, $(P \wedge \neg Q \wedge R)$, $(P \wedge Q \wedge \neg R)$, $(\neg P \wedge \neg Q \wedge R)$, $(P \wedge \neg Q \wedge \neg R)$, $(\neg P \wedge Q \wedge \neg R)$ and $(\neg P \wedge \neg Q \wedge \neg R)$.

Maxterms using three variables:

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Principal Disjunctive Normal Form (PDNF):

Sum of minterms is called PDNF.

i.e. The PDNF will be

$(\text{minterm } 1) \vee (\text{minterm } 2) \vee (\text{minterm } 3) \vee \ldots$

Principal Conjuctive Normal Form (PCNF):

Product of maxterms is called PCNF.

i.e. The PCNF will be

$(\text{maxterm } 1) \wedge (\text{maxterm } 2) \wedge (\text{maxterm } 3) \wedge \ldots$

Problem 1.6:

Obtain the PDNF and PCNF of $(P \wedge Q) \vee (\neg P \wedge R)$.

(N/D 2016)

Solution:

Let

$S \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge R)$

$\Leftrightarrow (P \wedge Q \wedge T) \vee (\neg P \wedge R \wedge T)$

$\Leftrightarrow (P \wedge Q \wedge (R \vee \neg R)) \vee (\neg P \wedge R \wedge (Q \vee \neg Q))$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

Which is sum of minterms.

$\therefore$ It represents the PDNF.

To find the PCNF, collect the remaining minterms in S, we get

$\neg S \Leftrightarrow (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge \neg R)$

$\vee (\neg P \wedge \neg Q \wedge \neg R)$

$$\neg(\neg S) \Leftrightarrow (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$$

$$S \Leftrightarrow (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee Q \vee R)$$

Which is product of maxterms.

$\therefore$ It represents the PCNF.

Problem 1.7:

Find the PCNF $(P \vee R) \wedge (P \vee \neg Q)$. Also find its PDNF, without using truth table. (A/M 2018)

Solution:

Let $S \Leftrightarrow (P \vee R) \wedge (P \vee \neg Q)$

$$\Leftrightarrow ((P \vee R) \vee F) \wedge ((P \vee \neg Q) \vee F)$$

$$\Leftrightarrow ((P \vee R) \vee (Q \wedge \neg Q)) \wedge ((P \vee \neg Q) \vee (R \wedge \neg R))$$

$$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$$

$$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$$

Which is product of maxterms.

$\therefore$ It represents the PCNF.

To find the PDNF, collect the remaining maxterms in S, we get

$$\neg S \Leftrightarrow (\neg P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R)$$

$$\neg(\neg S) \Leftrightarrow (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R)$$

$$S \Leftrightarrow (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \\ \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R)$$

Which is sum of minterms.

$\therefore$ It represents the PDNF.

Problem 1.8:

Without using truth table find the PCNF and PDNF of $P \to (Q \wedge P) \wedge (\neg P \to (\neg Q \wedge \neg R))$. (A/M 2011)

Solution:

Let $S : P \to (Q \wedge P) \wedge (\neg P \to (\neg Q \wedge \neg R))$

$\Leftrightarrow \neg P \vee (Q \wedge P) \wedge (P \vee (\neg Q \wedge \neg R))$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee P) \wedge (P \vee \neg Q) \wedge (P \vee \neg R)$

$\Leftrightarrow (\neg P \vee Q) \wedge T \wedge (P \vee \neg Q) \wedge (P \vee \neg R)$

$\Leftrightarrow (\neg P \vee Q) \wedge (P \vee \neg Q) \wedge (P \vee \neg R)$

$\Leftrightarrow (\neg P \vee Q \vee F) \wedge (P \vee \neg Q \vee F) \wedge (P \vee \neg R \vee F)$

$$\Leftrightarrow (\neg P \vee Q \vee (R \wedge \neg R)) \wedge (P \vee \neg Q \vee (R \wedge \neg R)) \\ \wedge (P \vee \neg R \vee (Q \wedge \neg Q))$$

$$\Leftrightarrow (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \\ \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee \neg R \vee Q) \wedge (P \vee \neg R \vee \neg Q)$$

$$\Leftrightarrow (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \\ \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$$

Which is product of maxterms.

$\therefore$ It represents the PCNF.

To find the PDNF, collect the remaining maxterms in S, we get

$\neg S : (P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$

Taking negation on bothsides, we get

$$\neg(\neg S) : (\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

$$S : (\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

Which is sum of minterms. $\therefore$ It represents the PDNF.

Problem 1.9:

Find the principal disjunctive normal form of the statement, $\left(q \vee (p \wedge r)\right) \wedge \neg\left((p \vee r) \wedge q\right)$. (N/D 2012)

Solution:

Let $S : \left(q \vee (p \wedge r)\right) \wedge \neg\left((p \vee r) \wedge q\right)$

$$\Leftrightarrow \left(q \vee (p \wedge r)\right) \wedge \left(\neg(p \vee r) \vee \neg q\right)$$

$$\Leftrightarrow \left(q \vee (p \wedge r)\right) \wedge \left((\neg p \wedge \neg r) \vee \neg q\right)$$

$$\Leftrightarrow (q \vee p) \wedge (q \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg r \vee \neg q)$$

$$\Leftrightarrow (p \vee q) \wedge (q \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg q \vee \neg r)$$

$$\Leftrightarrow (p \vee q \vee F) \wedge (q \vee r \vee F) \wedge (\neg p \vee \neg q \vee F) \wedge (\neg q \vee \neg r \vee F)$$

$$\Leftrightarrow (p \vee q \vee (r \wedge \neg r)) \wedge (q \vee r \vee (p \wedge \neg p)) \wedge (\neg p \vee \neg q \vee (r \wedge \neg r)) \wedge (\neg q \vee \neg r \vee (p \wedge \neg p))$$

$$\Leftrightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (q \vee r \vee p) \wedge (q \vee r \vee \neg p) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (\neg q \vee \neg r \vee p) \wedge (\neg q \vee \neg r \vee \neg p)$$

$$\Leftrightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Which is product of the maxterms.

$\therefore$ It represents the PCNF.

To find the PDNF, collect the remaining maxterms in S, we get

$$\neg S : (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r)$$

Taking negation on bothsides, we get

$\neg(\neg S): (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$

$S: (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$

Which is sum of minterms. $\therefore$ It represents the PDNF.

Problem 1.10:

Obtain the principal disjunctive normal form and principal conjunction form of the statement $p \vee (\neg p \to (q \vee (\neg q \to r)))$.

(N/D 2010)

Solution:

Let

$S: p \vee (\neg p \to (q \vee (\neg q \to r)))$

$\Leftrightarrow p \vee (\neg p \to (q \vee (q \vee r)))$

$\Leftrightarrow p \vee (p \vee (q \vee (q \vee r)))$

$\Leftrightarrow p \vee (p \vee (q \vee q) \vee r))$

$\Leftrightarrow p \vee (p \vee q \vee r)$

$\Leftrightarrow (p \vee p) \vee q \vee r$

$\Leftrightarrow p \vee q \vee r$

Which is PCNF.

To find the PDNF, collect the remaining maxterms in S, we get

$$S: (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee r)$$
$$\wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Taking negation on bothsides, we get

$$\neg(\neg S): (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee$$
$$(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$$

$S:(p\wedge\neg q\wedge\neg r)\vee(\neg p\wedge q\wedge\neg r)\vee(\neg p\wedge\neg q\wedge r)$
$\vee(p\wedge q\wedge\neg r)\vee(\neg p\wedge q\wedge r)\vee(p\wedge\neg q\wedge r)\vee(p\wedge q\wedge r)$

Which is sum of minterms.

$\therefore$ It represents the PDNF.

Problem 1.11:

Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P\to R)\wedge(Q\leftrightarrow P)$ by using equivalences. (M/J 2016),(A/M 2017)

Solution:

Let $S:(\neg P\to R)\wedge(Q\leftrightarrow P)$

$\Leftrightarrow (P\vee R)\wedge((Q\to P)\wedge(P\to Q))$

$\Leftrightarrow (P\vee R)\wedge(\neg Q\vee P)\wedge(\neg P\vee Q)$

$\Leftrightarrow (P\vee R)\wedge(P\vee\neg Q)\wedge(\neg P\vee Q)$

$\Leftrightarrow (P\vee R\vee F)\wedge(P\vee\neg Q\vee F)\wedge(\neg P\vee Q\vee F)$

$\Leftrightarrow (P\vee R\vee(Q\wedge\neg Q))\wedge(P\vee\neg Q\vee(R\wedge\neg R))\wedge(\neg P\vee Q\vee(R\wedge\neg R))$

$\Leftrightarrow (P\vee R\vee Q)\wedge(P\vee R\vee\neg Q))\wedge(P\vee\neg Q\vee R)\wedge(P\vee\neg Q\vee\neg R)$
$\wedge(\neg P\vee Q\vee R)\wedge(\neg P\vee Q\vee\neg R)$

$\Leftrightarrow (P\vee Q\vee R)\wedge(\neg P\vee Q\vee R)\wedge(P\vee\neg Q\vee R)$
$\wedge(P\vee\neg Q\vee\neg R)\wedge(\neg P\vee Q\vee\neg R)$

Which is product of maxterms.

$\therefore$ It represents the PCNF.

To find the PDNF, collect the remaining maxterms in S, we get

$\neg S:(P\vee Q\vee\neg R)\wedge(\neg P\vee\neg Q\vee R)\wedge(\neg P\vee\neg Q\vee\neg R)$

Taking negation on bothsides, we get

$\neg(\neg S):\ (\neg P\wedge\neg Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(P\wedge Q\wedge R)$

$S:(\neg P\wedge\neg Q\wedge R)\vee(P\wedge Q\wedge\neg R)\vee(P\wedge Q\wedge R)$

Which is sum of minterms.

$\therefore$ It represents the PDNF.

Problem 1.12:

Show that

$$(\neg P\rightarrow R)\wedge(Q\leftrightarrow P)=(P\vee Q\vee R)\wedge(P\vee\neg Q\vee R)\wedge$$

$$(P\vee\neg Q\vee\neg R)\wedge(\neg P\vee Q\vee R)\wedge(\neg P\vee Q\vee\neg R).$$

(M/J 2013)

Solution:

(Refer Previous Problem, the result is PCNF.)

Theory of Inference

The analysis of the validity of the formula from the given set of primises is called inference theory.

Rules:

Rule P: A given primise may be introduced at any stage in the derivation.

Rule T: A formula may be introduced in a derivation if it is tautologically implied by one or more of the preciding formulae in the derivation.

Rule CP: If we can derive S from R and a set of premises alone. In such a case R is taken as an additional premise (assumed primise).

Indirect Method of Derivation:

Whenever the assumed premise is used in a derivation then the method of derivation is called indirect method of derivation.