DISCRETE MATHEMATICS

Unit - I

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PROPOSITIONAL CALCULUS —

① A proposition is a sentence which is either true or false but not both.

→ We use letters like P, Q, R etc. to denote a proposition.

For eg:- P : 2+3=5
Q : Delhi is capital of India.
R : The sum of two odd integers is an odd integer.

LOGICAL OPERATORS —

Logical operators are used to join two more proposition to form a new proposition.

① **Negation (~)**

The negation of a proposition p is denoted by ~P (not P)

P : I go to market
~P : I do not go to market.

Truth Table (Negation)

P	~P
T	F
F	T

② **Disjunction (OR / v)**

The disjunction of two proposition P & Q is denoted by P v Q.

P : I go to market
Q : I play football

P v Q : I go to market OR I play football.

Truth Table (Disjunction)

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

③ **Conjuction (AND / ∧)**

The conjuction of two proposition P & Q is denoted by P∧Q

P: I go to market
Q: I play football

P∧Q : I go to market and I play football.

Truth Table (Conjuction)

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

Examples to practice →

P : 2+3=5 T
Q : Delhi is the capital of India. T
R : Five is an even number F

1) P∧Q = T
2) P∨Q = T
3) P∨R = T∨F = T
4) P∧~P = T∧(~F) = T∧(T) = T∧T = T
5) ~Q∧~R = ~T∧~F = F∧T = F

Conditional & Biconditional Statement →

CONDITIONAL If p and q are given statements then the statement 'if p then q' denoted as $p \to q$ is called conditional statement or implication.

p → called → antecedent or hypothesis
q → called → consequent or conclusion.

$p \to q$ read as "p implies q" or "p only if q"

Eg: p: I am hungry q: I will eat

$p \to q$: If I am hungry then I will eat

Truth Table of Conditional Statement →

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

BICONDITIONAL If p and q are given statements then the statement "p if and only if q" written as $p \leftrightarrow q$ is called as biconditional statement or equivalence.

Truth Table of biconditional statement →

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Notes → the symbols $\vee$, $\wedge$, $\sim$, $\rightarrow$, $\leftrightarrow$ defined above are called connectives.

Well-formed formulas →

1) A statement is replaced by the variable

P : Today is a sunday

2) A statement formula is an expression which is a string consisting of string of variable, parenthesis and connective symbols.

$P \wedge Q$, $P \vee Q$, $P \rightarrow Q$

3) Every string of these ☝ symbols is a statement formula.

4) If A is a well-formed formula, then $\sim A$ is a well-formed formula.

Examples →

1) $\sim(P \wedge Q)$ —— well formed formula
2) $\sim(P \vee Q)$ —— well formed formula
3) $(P \rightarrow (P \vee Q))$ —— WFF
4) $(P \rightarrow (Q \rightarrow R))$ —— WFF
5) $(((P \rightarrow Q) \wedge (Q \rightarrow R)) \leftrightarrows (P \rightarrow R))$ —— WFF
6) $\sim P \wedge Q$ —— not well formed formula.

It should be like this $(\sim P \wedge Q)$ or $\sim(P \wedge Q)$

$\sim P \wedge Q$ does not contain any parenthesis that's why it is not well formed formula.

7) $(P \rightarrow Q) \rightarrow (\wedge Q)$ —— not well Formed formula

Some variable should be their before 'or' operator like $(P \wedge Q)$

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Tautologies & Contradiction →

1) Tautologies → truth values are anytime true.
2) Contradiction → truth values are false for any variable
3) Contigency → Some truth value are true & some are false.

Q. 1) $P \vee Q$ (contigency) 2) $P \wedge \sim P$ (Contradiction)

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

P	~P	P∧~P
T	F	F
F	T	F

3) $\sim(P \wedge q)$ (contigency) 4) $P \vee \sim P$ (Tautologies)

P	q	P∧q	~(P∧q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

P	~P	P∨~P
T	F	T
F	T	T

Duality Law → the formula A and A* are said to be duals of each other if either one can be obtained from another by replacing ∧ by ∨ and ∨ by ∧.

If a formula A contains special variable T or F then A*, its dual is obtained by replacing T by F and F by T

Eg: 1) $(P \vee Q) \wedge R$
$(P \wedge Q) \vee R$

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Eg 2) $(P \wedge Q) \vee T$
$(P \vee Q) \wedge F$
3) $\sim(P \vee Q)$
$\sim(P \wedge Q)$

Logical equivalence →

Definitions → the compound propositions P & Q are said to be logically equivalent if $P \leftrightarrow Q$ is a tautology. & Logical equivalence is denoted by $\equiv$ or $\Leftrightarrow$

Eg: $P \wedge T \equiv P$

P	T	$P \wedge T$
T	T	T
F	T	F

Famous logical equivalence →

1) Identity laws :

(a) $P \wedge T \equiv P$ (b) $P \vee F \equiv P$

2) Domination laws :

(a) $P \vee T \equiv T$ (b) $P \wedge F \equiv F$

3) Idempotent laws :

(a) $P \vee P \equiv P$ (b) $P \wedge P \equiv P$

4) Double Negation laws :

a) $\neg(\neg P) \equiv P$

5) Commutative laws :

(a) $p \vee q \equiv q \vee p$

(b) $p \wedge q \equiv q \wedge p$

6) Associative laws :

(a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7) Distributive laws :

(a) $p \vee (q \wedge r) \equiv (q \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \equiv (q \wedge q) \vee (p \wedge r)$

8) De Morgan's laws :

(a) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(b) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

9) Absorption laws :

(a) $p \vee (p \wedge q) \equiv p$

(b) $p \wedge (p \vee q) \equiv p$

10) Negation laws :

(a) $p \vee \neg p \equiv T$

(b) $p \wedge \neg p \equiv F$

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Solved examples :-

(1) Let P, Q and R be the propositions.
P : I go to market
Q : I buy a pen
R : I watch a movie.

Write proposition for the following sentences:

(a) I do not go to market and I do not ~~buy~~ buy a pen. $(\sim P \wedge \sim Q)$

(b) If I buy a pen, then I do not watch a movie $(Q \rightarrow \sim R)$

(c) I go to market, but I do not watch a movie $(P \wedge \sim R)$

(d) If I do not go to market and do not buy a pen then I do not watch a movie. $(\sim P \wedge \sim Q)$

(e) Watching a movie is necessary for me to go market and buy a pen. $(P \wedge Q) \rightarrow R$

(f) Going to market is sufficient for me to buy a pen and watch a movie. $P \rightarrow (Q \wedge R)$

(g) Neither I go to market nor I buy a pen $(\sim P \wedge \sim Q)$

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Example of biconditional statement →

1) Let P be a proposition "You get promoted" and let q be a proposition "you have connection.

P: You get promoted
q: You have connection.

then $P \leftrightarrow q$ is the statement : "You get promoted if and only if you have connection"

Q. Show that $\sim p \vee q$ and $p \rightarrow q$ are logically equivalent

p	q	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\boxed{\sim p \vee q \equiv p \rightarrow q}$

Q Show that $\sim(p \vee q)$ & $\sim p \wedge \sim q$ are logically equivalen

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$\boxed{\sim(p \vee q) \equiv \sim p \wedge \sim q}$