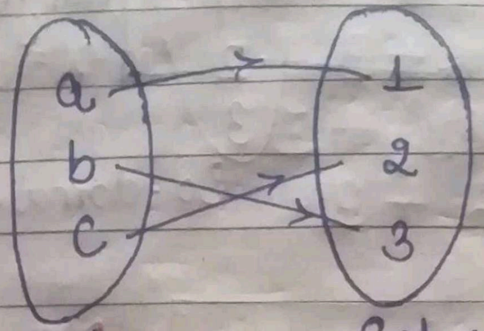
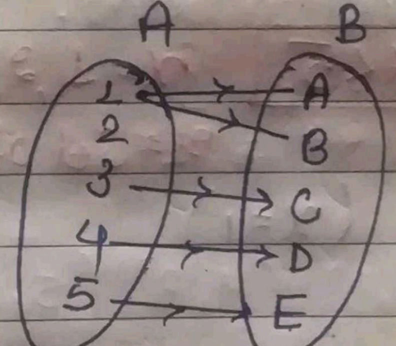
Chapter - 5

Relation & Function



Set x Set y

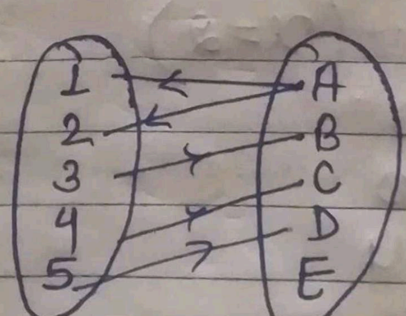
$R = \{(a,1)\ (b,3)\ (c,2)\}$



← It is not a function

SON MOTHER

One Son doesn't have two mother

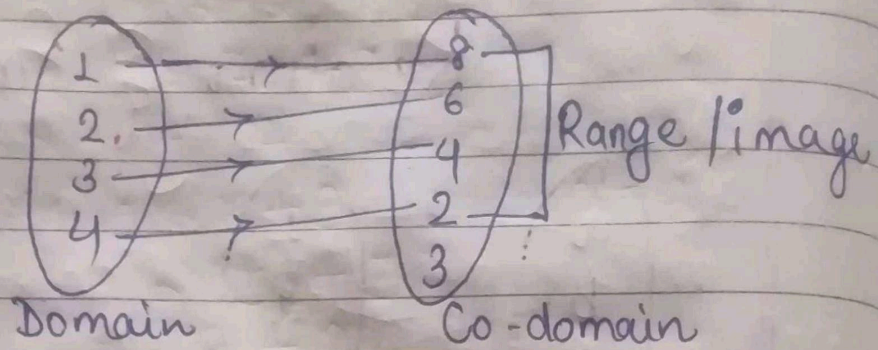


← It is a function

SON MOTHER

One mother can have two son.

Co-domain, Domain, Range →



Q1. Given $\boxed{2x+y=10}$ Find domain, co-domain ; Range:

$\boxed{x,y \in N}$ $\boxed{x<5}$ $x = 1, 2, 3, 4 \ldots$

x = set of natural no.

Solun

$2x+y=10 \quad (x=1)$

$2(1)+y=10$

$2+y=10$

$y=10-2$

$\boxed{y=8}$

$2x+y=10 \quad (x=2)$

$2(2)+y=10$

$4+y=10$

$y=10-4$

$\boxed{y=6}$

$2x + y = 10 \qquad (x = 3)$

$2(3) + y = 10$

$6 + y = 10$

$y = 10 - 6$

$\boxed{y = 4}$

$2x + y = 10 \qquad (x = 4)$

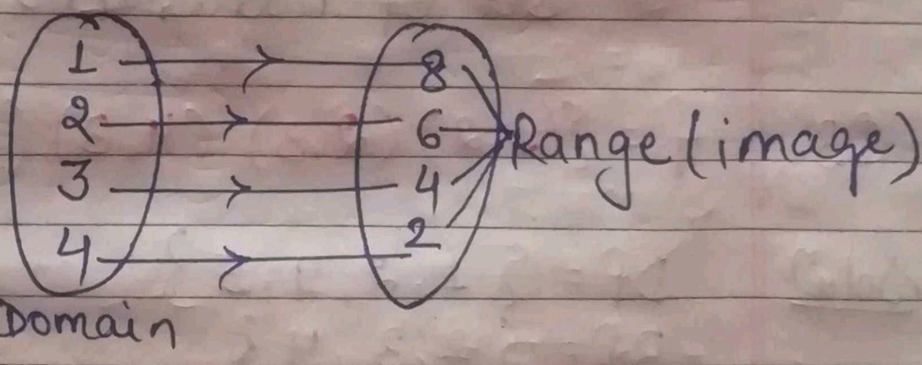
$2(4) + y = 10$

$8 + y = 10$

$y = 10 - 8$

$\boxed{y = 2}$

$R = \{(1,8)\ (2,6)\ (3,4)\ (4,2)\}$



Q2. Explain $f: N \rightarrow N$ defined by $f(x) = 2x + 3$ as set of ordered pair and find domain, range & co-domain?

Solu^n $f(x) = 2x + 3 \qquad (x = 1)$

$= 2(1) + 3$

$= 2 + 3$

$= 5$

$f(x) = 2x + 3 \quad (x = 2)$
$= 2(2) + 3$
$= 4 + 3$
$= 7$

$f(x) = 2x + 3 \quad (x = 3)$
$= 2(3) + 3$
$= 6 + 3$
$= 9$

functions $= \{(1,5)(2,7)(3,4) \ldots\}$
$Rf = \{5, 7, 9 \ldots\}$
$Df = \{1, 2, 3 \ldots\}$
= Set of natural no.
$CDf = \{5, 7, 9 \ldots\}$
= Set of natural no.

Q3. $f = \{(1,2)(2,3)(4,3)(5,4)\}$

Solu^n $Df = \{1, 2, 4, 5\}$
$Rf = \{2, 3, 4\}$

Equivalence Relation

① Reflexive $(A \times A)$

$A = \{1, 2, 3\}$ → $(1,1)$, $(2,2)$, $(3,3)$

$R = \{(a,a) \ \forall \ a \in A\}$

② Not Reflexive

$\exists \ a \in A$

$(a,a) \notin R$

③ Symmetric

For $a, b \in A$

$\Rightarrow (a,b) \in R$

$\Rightarrow (b,a) \in R$

④ Not Symmetric

$a, b \in A$

$(a,b) \in R$

But $(b,a) \notin R$

⑤ Transitive

For $a, b, c \in A$

(a,b) & $(b,c) \in R$

$(a,c) \in R$

⑥ Not Transitive

$a, b, c \in A$

(a,b) $(b,c) \in R$

but $(a,c) \notin R$

Q1. Check whether the relation R defined on the set $A = (1, 2, 3, 4, 5, 6)$ as $R = \{(a, b) : b = a+1\}$ is reflexive, symmetric, transitive.

<u>Solⁿ</u> $(a, b) \in R$ if $b = a+1$

(i) Reflexive

No it is not reflexive

$1 \in A$ but $(1, 1) \notin R$

$a = 1$

$b = a+1 \Rightarrow 1+1 = 2$

$b = 2$

(ii) Symmetric

No it is not symmetric

$\therefore (1, 2) \in A$

but $(2, 1) \notin R$

$\therefore a = 1 \quad b = a+1$

$= 1+1$

$b = 2$

$(1, 2) \in R$

but $a = 2 \quad b = 2+1$

$b = 3$

$(2, 1) \notin R$

(iii) Transitive

No it is not transitive

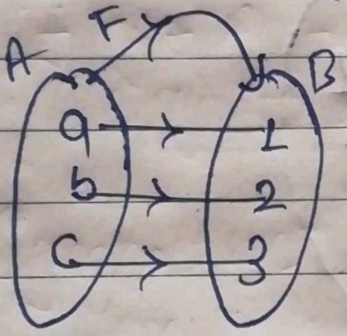
$(1,2) \in R$ $[a=1 \quad b=2]$

$(2,3) \in R$ $[a=2 \quad b=3]$

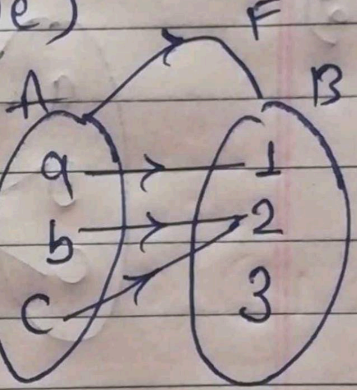
But $(1,3) \notin R$ $(a=1 \ \forall \ b=3)$

Types of function

① One-One (injective) & Many-one

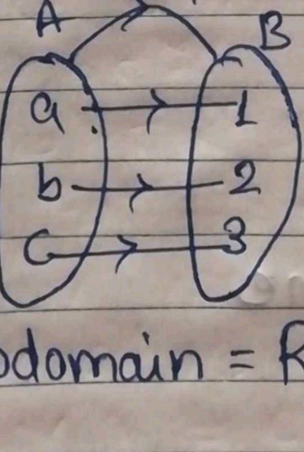


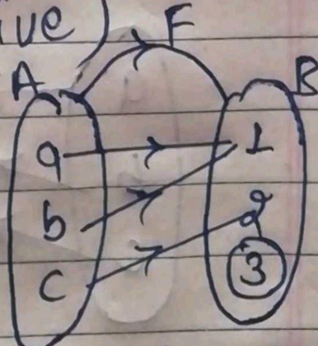
one-one



Many-one

② Onto & into (Subjective)

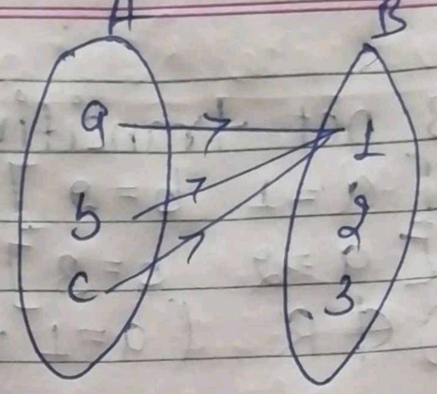




co-domain ≠ Range (into)

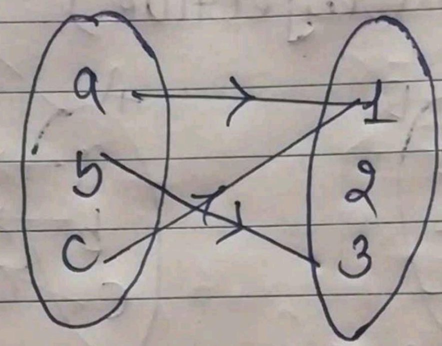
codomain = Range (onto)

eg: 1)



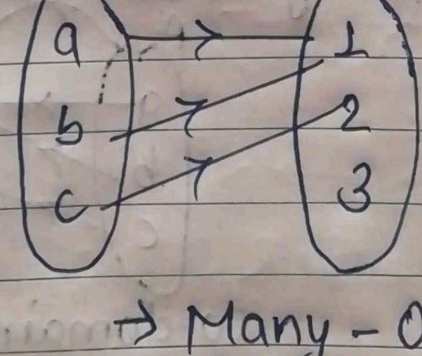
→ Many-one
→ into

2)



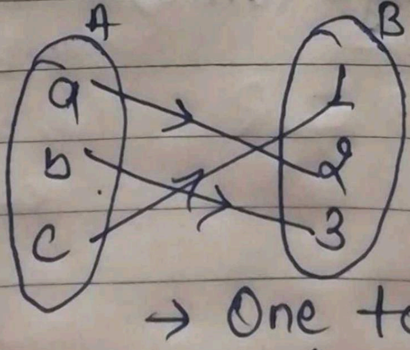
→ Many-One
→ into

3)



→ Many-One
→ into

4)



→ One to one
→ onto

⇒ Solved Questions on Reflexive, Symmetric and transitive

Q2. Relation R in a Set $A = \{1, 2, 3 \ldots 14\}$ defined by $R = \{(x, y) : 3x - y = 0\}$

<u>Solu^n</u>

$$3x - y = 0$$

$$3x = y \qquad x, y \in A$$

$$R = \{(1,3)\ (2,6)\ (3,4)\ (4,12)\}$$

$$3x = y \qquad (x = 1)$$

$$3(1) = y$$

$$3 = y$$

$$\boxed{y = 3}$$

$$3x = y \qquad (x = 2)$$

$$3(2) = y$$

$$6 = y$$

$$\boxed{y = 6}$$

$$3x = y \qquad (x = 3)$$

$$3(3) = y$$

$$9 = y$$

$$\boxed{y = 9}$$

$$3x = y \qquad (x = 4)$$

$$3(4) = y$$

$$12 = y$$

$$\boxed{y = 12}$$

① Reflexive (It is not Reflexive)

$1 \in A \quad (1,1) \notin R$

$2 \in A \quad (2,2) \notin R$

$3 \in A \quad (3,3) \notin R$

② Symmetric (It is not Symmetric)

$(1,3) \in R$ but $(3,1) \notin R$

$(2,6) \in R$ but $(6,2) \notin R$

③ Transitive (It is not Transitive)

$(\overset{a}{1},\overset{b}{3}) \in R \qquad (\overset{b}{3},\overset{c}{9}) \in R$

but $(1,9) \notin R$

Q3. Relation R in a set $A = \{1,2,4,5,6\}$ defined by $R = \{(x,y) : y$ is divisible by $x\}$

Solu^n^ y is divisible by x

$(x, y \in A)$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,2)\ (2,4)\ (2,6)\ (3,3)\ (3,6)\ (4,4)$

$(5,5)\ (6,6)$

(11)

Dt.

Pg.

① Reflexive (It is Reflexive)

$(1,1)(2,2)(3,3)(4,4)(5,5)(6,6) \in R$

② Symmetric (It is not Symmetric)

$(1,2) \in R$ but $(2,1) \notin R$

③ Transitive : (It is transitive)

$(1,1) \in R \quad (1,2) \in R$

$(1,2) \in R$

$(3,6) \in R \quad (6,6) \in R$

$(3,6) \in R$

Algebra of Set :-

① Idempotent laws →

(a) $A \cup A = A$
(b) $A \cap A = A$

② Associative laws →

(a) $(A \cup B) \cup C = A \cup (B \cup C)$
(b) $(A \cap B) \cap C = A \cap (B \cap C)$

③ Commutative laws →

(a) $A \cup B = B \cup A$
(b) $A \cap B = B \cap A$

④ Distributative laws →

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

⑤ Demorgan's laws →

(a) $(A \cup B)' = A' \cap B'$
(b) $(A \cap B)' = A' \cup B'$

⑥ Identity laws →

(a) $A \cup \phi = A$
(b) $A \cap U = A$
(c) $A \cup U = U$
(d) $A \cap \phi = \phi$

(7) Involution laws →

(a) $(A')' = A$

(8) Complement laws →

(a) $A \cup A' = U$
(b) $A \cap A' = \phi$
(c) $U' = \phi$
(d) $\phi' = U$

Composition of Relations

Let A, B and C be sets.
Let R is the relation from A to B ie. $R \subseteq A \times B$
S is the relation from B to C ie. $S \subseteq B \times C$

The composition of R and S, denoted by $R \circ S$

Eg: Let $A = \{1, 2, 3\}$
$B = \{p, q, r\}$
$C = \{x, y, z\}$

$R = \{(1,p), (1,r), (2,p), (2,q)\}$

$S = \{(p,y) (q,x) (q,y) (r,z)\}$

Compute $R \circ S$

Solu^n Given $R = \{(1,p)\ (1,r)\ (2,p)\ (2,q)\}$

$S = \{(p,y)\ (q,x)\ (q,y)\ (r,z)\}$

$RoS = \{(1,y)\ (1,z)\ (2,y)\ (2,x)\}$

Q2. Let $R = \{(1,2)\ (3,4)\ (2,2)\}$
$S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

Find RoS, SoR, $Ro(SoR)$, RoR, SoS

Given $R = \{(1,2)\ (3,4)\ (2,2)\}$
$S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

(i) $RoS = \{(1,5)\ (3,2)\ (2,5)\}$

(ii) $SoR = \{(4,2)\ (3,2)\ (1,4)\}$

(iii) $Ro(SoR)$

$R = \{(1,2)\ (3,4)\ (2,2)\}$
$SoR = \{(4,2)\ (3,2)\ (1,4)\}$

$Ro(SoR) = \{(3,2)\}$

(iv) RoR
$R = \{(1,2)\ (3,4)\ (2,2)\}$
$R = \{(1,2)\ (3,4)\ (2,2)\}$

$RoR = R^2 = \{(1,2)\ (2,2)\}$

(v) $SoS = S^2 = \{(4,5)\ (3,3)\ (1,1)\}$

POSET (Partial Ordering Relation)

A Relation R is said to be a partial order Relation if R is reflexive, Antisymmetric and transitive. ~~It i~~

$A = \{1, 2, 3\}$
$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$
$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (1,3)\}$

Relation R_1 is Reflexive, Antisymmetric and transitive.

Note: In Antisymmetric if (a,b) is present in relation then (b,a) should not present in that relation

$$(a,b) \in R_1 \text{ then } (b,a) \notin R_1$$

but (a,a) (b,b) (c,c) are allowed in antisymmet

Relation R_2 is also Reflexive, Antisymmetric and transitive

Page No.:

Binary Relation

Let 'A' and 'B' are the two non-empty sets then the subset "R" of $A \times B$ is called Binary Relation from 'A' to 'B'.

Eg: $A = \{1, 2\}$ $B = \{3, 4\}$

$A \times B = \{ \underbrace{(1,3)}_{\text{1st Element}} \underbrace{(1,4)}_{\text{2nd Element}} \underbrace{(2,3)}_{\text{3rd Element}} \underbrace{(2,4)}_{\text{4th element}} \}$

Subset $= 2^n = 2^4 = 16$

$R_1 = \{ \}$ $R_2 = \{1,3\}$ $R_3 = \{1,4\}$ $R_4 = \{2,3\}$ $R_5 = \{2,4\}$

$R_6 = \{(1,3)(1,4)\}$ $R_7 = \{(1,3)(2,3)\}$

$R_8 = \{(1,3)(2,4)\}$ $R_9 = \{(1,4)(2,3)\}$

$R_{10} = \{(1,4)(2,4)\}$ $R_{11} = \{(2,3)(2,4)\}$

$R_{12} = \{(1,3),(1,4)(2,3)\}$ $R_{13} = \{(1,3)(2,3)(2,4)\}$

$R_{14} = \{(1,4)(2,3)(2,4)\}$ $R_{15} = \{(1,3)(1,4)(2,4)\}$

$R_{16} = \{(1,3)(1,4)(2,3)(2,4)\}$

Page No.:

Eg: ② $A = \{1, 2, 3\}$ $B = \{4, 5\}$

$A \times B = \{(1,4)(1,5)(2,4)(2,5)(3,4)(3,5)\}$

$R = \{(1,4)(2,5)\}$

$2^n = 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Binary Relation $= 64$

Q1 If a set has n elements, then how many relations are there from A to A?

Solu^n If Set A contain n elements then $A \times A$ contain n^2 elements

Total no. of possible subsets of any set $= 2^n$

$\therefore$ No. of possible subset of any set contain n^2 elements $= 2^{n^2}$

Hence Total no. of possible relation from A to A $= 2^{n^2}$

Q2. If A has m elements and B has n element. How many relations are there from A to B and vice-versa?

Solu^n Total no. of elements in $A \times B = m \times n$

Total no. of possible subset $= 2^{m \times n}$

$\therefore$ Total no. of possible relation from A to B $= 2^{m \times n}$

Graph Representation of Relation:-

→ A graph consist set of vertices and set of edges.

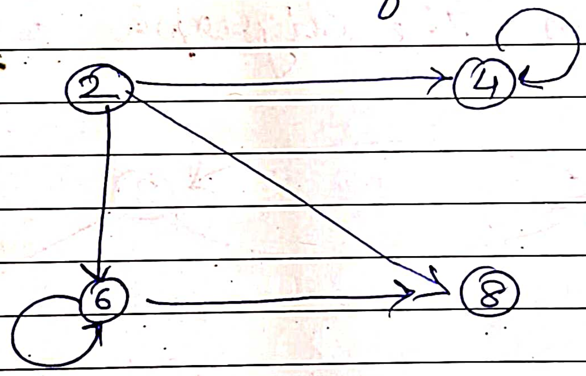
→ Each element of the set is represented by a point. These points are called nodes or vertices.

Eg:- Let $A = \{2, 4, 6\}$. $\qquad R \subseteq A \times B$

$B = \{4, 6, 8\}$.

$R = \{(2,4)\ (2,6)\ (2,8)\ (4,4)\ (6,6)\ (6,8)\}$

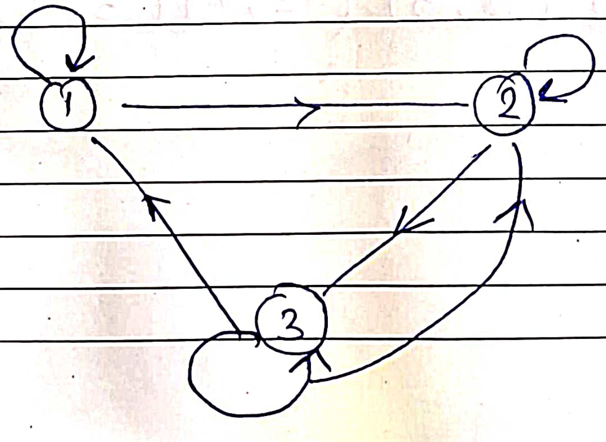
graph representation of R.



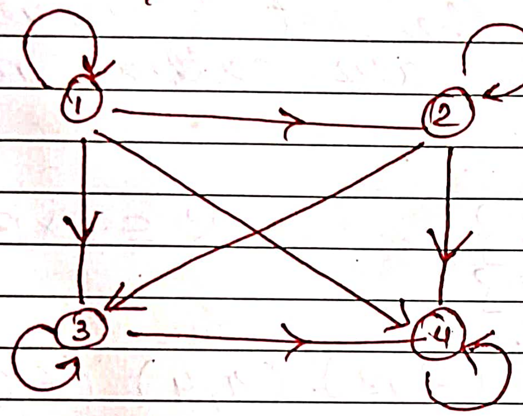
Q1. Draw the directed graph that represents the relation :-

$R = \{(1,1)\ (2,2)\ (1,2)\ (2,3)\ (3,2)\ (3,1)\ (3,3)\}$

on $A = \{1, 2, 3\}$

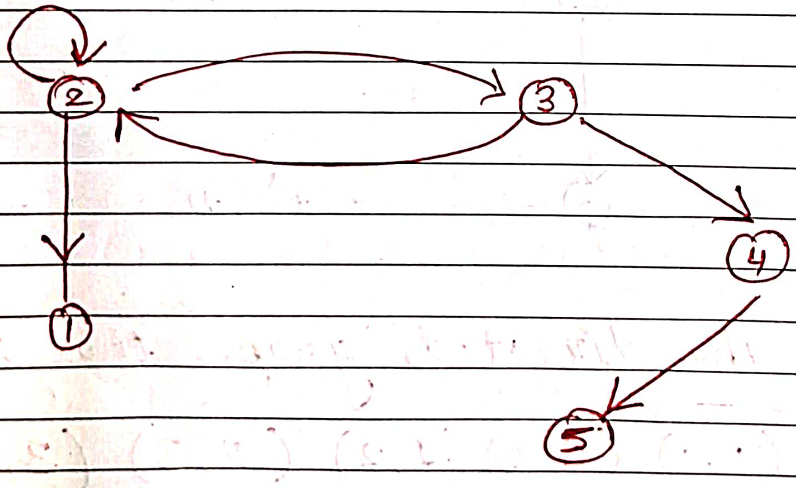


Q2. Determine whether the relation for the directed graph shown in figure are reflexive, symmetric, antisymmetric and/or transitive.



The given graph is
1) Reflexive
2) Not Symmetric
3) Antisymmetric
4) Transitive

Q3. Write the relation as a set of ordered pair from the digraph as shown in figure.



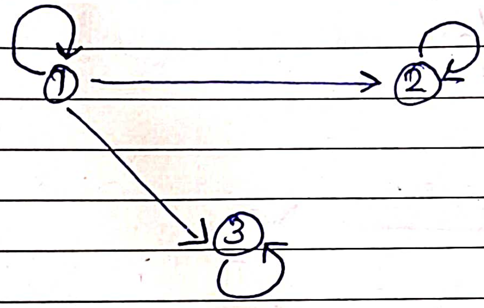
$R = \{(2,2)\ (2,3)\ (2,1)\ (3,2)\ (3,4)\ (4,5)\}$

Graph Representation of properties —

① **Reflexive**

Let $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (1,3)\}$

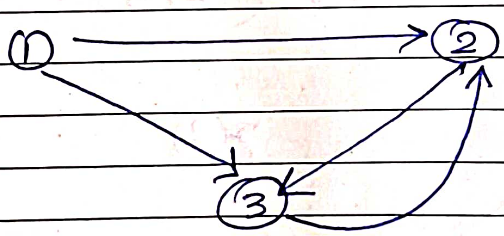


Self loop on every vertic

② **Irreflexive**

Let $A = \{1, 2, 3\}$

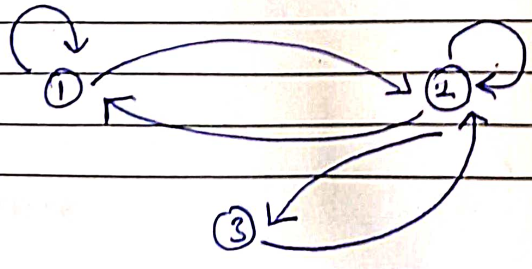
$R = \{(1,2)\ (2,3)\ (1,3)\ (3,2)\}$



No Self loop

③ **Symmetric**

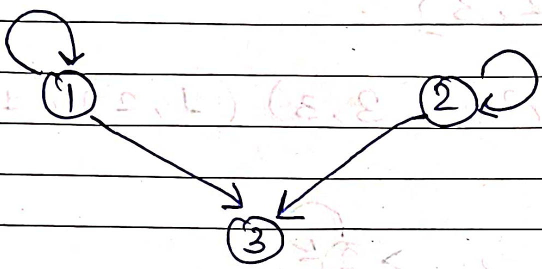
Let $A = \{(1,1)\ (1,2)\ (2,1)\ (2,3)\ (3,2)$
$(2,2)\}$



closed Loop between two vertex

④ Antisymmetric

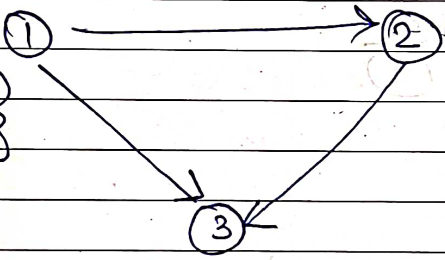
Let $A = \{1, 2, 3\}$
$R = \{(1,1)\ (2,2)\ (2,3)\ (1,3)\}$



No closed loop
b/w two vertex

⑤ Asymmetric

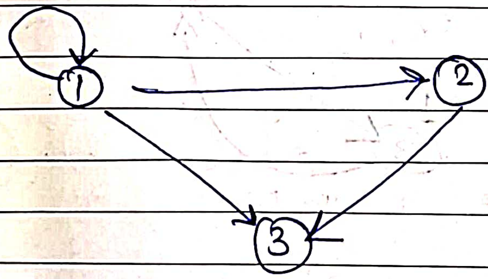
Let $A = \{1, 2, 3\}$
$R = \{(1,2)(2,3)$
$(1,3)\}$



No closed loop
No ~~stet~~ self loop

⑥ Transitive

Let $A = \{1, 2, 3\}$
$R = \{(1,1)\ (1,2)\ (2,3)\ (1,3)\}$



Matrix Representation of Relations →

Eg: ① Let $A = \{1, 3, 4\}$
$R = \{(1,1)\ (1,3)\ (3,3)\ (4,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Eg ② Let $A = \{1, 2, 3, 4, 8\}$ $B = \{1, 4, 6, 9\}$

aRb iff $a|b$ ie. a divides b
$a \in A$
$b \in B$ Find the relation matrix.

Solu^n^ $R = \{(1,1)\ (1,4)\ (1,6)\ (1,9)\ (2,4)\ (2,6)$
$(3,6)\ (3,9)\ (4,4)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 6 & 9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$