Partial order relation

A partial order relation is a homogeneous relation that is transitive and antisymmetric.[4] There are two common sub-definitions for a partial order relation, for reflexive and irreflexive partial order relations, also called "non-strict" and "strict" respectively. The two definitions can be put into a one-to-one correspondence, so for every strict partial order there is a unique corresponding non-strict partial order, and vice versa. The term **partial order** typically refers to a non-strict partial order relation.

Non-strict partial order

A **reflexive**, **weak**, or **non-strict partial order**[5] is a homogeneous relation ≤ on a set that is reflexive, antisymmetric, and transitive. That is, for all it must satisfy:

1. Reflexivity: $a \leq a$ i.e. every element is related to itself.
2. Antisymmetry: if $a \leq b$ and $b \leq a$ then, $a=b$ i.e. no two distinct elements precede each other.
3. Transitivity: if $a \leq b$ and $b \leq c$ then $a \leq c$.

A non-strict partial order is also known as an antisymmetric preorder.

Strict partial order

An **irreflexive**, **strong**, or **strict partial order** is a homogeneous relation < on a set that

is irreflexive, asymmetric, and transitive; that is, it satisfies the following conditions for all

1. Irreflexivity: not $a < a$ i.e. no element is related to itself (also called anti-reflexive).

2. Asymmetry: if $a < b$ then not $b < a$.

3. Transitivity: if $a < b$ and $b < c$ then $a < c$.

Irreflexivity and transitivity together imply asymmetry. Also, asymmetry implies irreflexivity. In other words, a transitive relation is asymmetric if and only if it is irreflexive.[6] So the definition is the same if it omits either irreflexivity or asymmetry (but not both).

A strict partial order is also known as a strict preorder.

A relation R on a set A is called a partial order relation if it satisfies the following three properties:

1. Relation R is Reflexive, i.e. $aRa \ \forall \ a \in A$.
2. Relation R is Antisymmetric, i.e., aRb and $bRa \Longrightarrow a = b$.
3. Relation R is transitive, i.e., aRb and $bRc \Longrightarrow aRc$.

Equivalence Relation

A relation R on a set A is said to be an **equivalence relation** if and only if the relation R is reflexive, symmetric and transitive. The equivalence relation is a relationship on the set which is generally represented by the symbol "∼".

Reflexive: A relation is said to be reflexive, if (a, a) ∈ R, for every a ∈ A.

Symmetric: A relation is said to be symmetric, if (a, b) ∈ R, then (b, a) ∈ R.

Transitive: A relation is said to be transitive if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.

In terms of equivalence relation notation, it is defined as follows:

A binary relation ∼ on a set A is said to be an equivalence relation, if and only if it is reflexive, symmetric and transitive.

(i.e) For all x, y, z in set A,

- x ∼ x (Reflexivity)
- x ∼ y if and only if y ∼ x (Symmetry)
- If x∼y and y∼z, then x∼z (Transitivity)

Equivalence relations can be explained in terms of the following examples:

- The sign of 'is equal to (=)' on a set of numbers; for example, 1/3 = 3/9.
- For a given set of triangles, the relation of 'is similar to (~)' and 'is congruent to (≅)' shows equivalence.
- For a given set of integers, the relation of 'congruence modulo n (≡)' shows equivalence.
- The image and domain are the same under a function, which shows the relation of equivalence.
- For a set of all angles, 'has the same cosine'.
- For a set of all real numbers,' has the same absolute value'.

Equivalence Class

We have indicated that an equivalence relation on a set is a relation with a certain combination of properties (reflexive, symmetric, and transitive) that allow us to sort the elements of the set into certain classes. We saw this happen in the preview activities. We can now illustrate specifically what this means. For example,

C[0]C[1]C[2]==={a∈Z | a≡0 (mod 3)}, {a∈Z | a≡1 (mod 3)}, and{a∈Z | a≡2 (mod 3)}.(7.3.1)(7.3.1) C[0]={a∈Z | a≡0 (mod 3)},C[1]={a∈Z | a≡1 (mod 3)}, andC[2]={a∈Z | a≡2 (mod 3)}.

The main results that we want to use now are Theorem 3.31 and Corollary 3.32 on page 150. This corollary tells us that for any $a \in Z a \in Z$, $a a$ is congruent to precisely one of the integers 0, 1, or 2. Consequently, the integer $a a$ must be congruent to 0, 1, or 2, and it cannot be congruent to two of these numbers. Thus

1. For each $a \in Z a \in Z$, $a \in C[0] a \in C[0]$, $a \in C[1] a \in C[1]$, or $a \in C[2] a \in C[2]$; and
2. $C[0] \cap C[1] = \emptyset C[0] \cap C[1] = \emptyset$, $C[0] \cap C[2] = \emptyset C[0] \cap C[2] = \emptyset$, and $C[1] \cap C[2] = \emptyset C[1] \cap C[2] = \emptyset$.

Composition of Relations

Let A, B, and C be sets, and let R be a relation from A to B and let S be a relation from B to C. That is, R is a subset of A × B and S is a subset of B × C. Then R and S give rise to a relation from A to C indicated by R∘S and defined by:

1. a (R∘S)c **if for** some b ∈ B we have aRb and bSc. is,
2. R ∘ S = {(a, c)| there exists b ∈ B **for** which (a, b) ∈ R and (b, c) ∈ S}

The relation R∘S is known the composition of R and S; it is sometimes denoted simply by RS.

Let R is a relation on a set A, that is, R is a relation from a set A to itself. Then R∘R, the composition of R with itself, is always represented. Also, R∘R is sometimes denoted by R^2. Similarly, $R^3 = R^2 \circ R = R \circ R \circ R$, and so on. Thus R^n is defined for all positive n.