

Unit-II

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Types of Boolean Function \rightarrow

① SOP Form (sum of product form)

$$(x_1 \cdot x_2 \cdot x_3) + (x_1 \cdot x_2 \cdot x_3) + (x_1 \cdot x_2 \cdot x_3)$$

OR

$$(x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$$

Disjunction Normal Form (D.N.F)

A Boolean expression is said to be D.N.F if it is join of min terms.

② POS form (product of sum form)

$$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + x_3)$$

$$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3)$$

Conjunctive normal form (C.N.F)

A Boolean expression is said to be C.N.F if it is meet of maxterms.

Solving Questions Using Disjunction Normal form. :-

$$[p \rightarrow q \equiv \sim p \vee q]$$

i) $p \wedge (p \rightarrow q)$

$$\Rightarrow p \wedge (\sim p \vee q)$$

$$\Rightarrow (p \wedge \sim p) \vee (p \wedge q)$$

required DNF Form.

2) $p \vee \{ \sim p \rightarrow (q \vee (q \rightarrow \sim r)) \}$

$\Rightarrow p \vee \{ \sim p \rightarrow (q \vee (\sim q \vee \sim r)) \}$

$\Rightarrow p \vee \{ \sim \sim p \vee (q \vee (\sim q \vee \sim r)) \}$

$\Rightarrow p \vee \{ p \vee (q \vee (\sim q \vee \sim r)) \}$

$\Rightarrow p \vee \{ p \vee (q \vee \sim q) \vee (q \vee \sim r) \}$

$\Rightarrow p \vee p \vee (q \vee \sim q) \vee (q \vee \sim r)$ required DNF form

Solving Question using Conjunctive Normal forms :-

$[p \rightarrow q = \sim p \vee q]$

1) $\sim(p \rightarrow (q \wedge r))$

$\Rightarrow (\sim p \rightarrow \sim(q \wedge r))$

$\Rightarrow (\sim p \rightarrow \sim q) \wedge (\sim p \rightarrow \sim r)$

$\Rightarrow (\sim \sim p \vee \sim q) \wedge (\sim \sim p \vee \sim r)$

$\Rightarrow (p \vee \sim q) \wedge (p \vee \sim r)$ required CNF forms.

2) $(p \rightarrow (q \wedge r)) \wedge (\sim p \rightarrow (\sim q \wedge \sim r))$

$\Rightarrow ((p \rightarrow q) \wedge (p \rightarrow r)) \wedge ((\sim p \rightarrow \sim q) \wedge (\sim p \rightarrow \sim r))$

$\Rightarrow ((\sim p \vee q) \wedge (\sim p \vee r)) \wedge (\sim \sim p \vee \sim q) \wedge (\sim \sim p \vee \sim r)$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee r) \wedge (p \vee \sim q) \wedge (p \vee \sim r)$

required CNF form

3) $(q \vee (p \wedge r)) \wedge \sim((p \vee r) \wedge q)$

$\Rightarrow ((q \vee p) \wedge (q \vee r)) \wedge (\sim(p \vee r) \vee \sim q)$

$[\therefore \sim(p \vee q) = \sim p \wedge \sim q]$
demorgan's law.



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$$\Rightarrow (q \vee p) \wedge (q \vee r) \wedge ((\sim p \wedge \sim r) \vee \sim q)$$

$$\Rightarrow (q \vee p) \wedge (q \vee r) \wedge ((\sim p \vee \sim q) \wedge (\sim p \vee \sim q))$$

$$\Rightarrow (q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$$

required CNF form

$$4) \sim \{ (p \vee \sim q) \wedge \sim r \}$$

$$\Rightarrow \{ \sim p (p \vee \sim q) \vee \sim \sim r \}$$

$$\Rightarrow \{ (\sim p \wedge \sim \sim q) \vee r \}$$

$$\Rightarrow \{ (\sim p \wedge q) \vee r \}$$

$$\Rightarrow (\sim p \vee r) \wedge (q \vee r) \text{ required CNF form}$$

Principal Disjunctive Normal form (PDNF)

For a given formula, an equivalent formula consisting of disjunction of minterms only is known as its "principal disjunctive Normal form"



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① By using Truth table

Ex-1 obtain the PDNF of $P \rightarrow Q$

P	Q	$P \rightarrow Q$	Minterm
T	T	T	$P \wedge Q$ ✓
T	F	F	$P \wedge \sim Q$
F	T	T	$\sim P \wedge Q$ ✓
F	F	T	$\sim P \wedge \sim Q$ ✓

we will consider minterm of only True values -

∴ PDNF of $P \rightarrow Q$ is $(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

Principal Conjunction Normal form (PCNF)

For a given formula, an equivalent formula consisting of conjunctions of the maxterm only is known as its "Principal Conjunctive Normal form".

→ PCNF is also called the "product-of-sums Canonical form".

Methods to obtain PCNF of a given formula :-

- 1) Using truth table
- 2) without using Truth table

① By using truth table →

Ex ① Obtain the PCNF of $P \leftrightarrow Q$

Given formula : $P \leftrightarrow Q$

Number of variables in the given formula is $n=2$

Number of rows in the truth table 2^n
 $= 2^2$
 $= 4$

Truth table :-

P	Q	$P \leftrightarrow Q$	Maxterm
T	T	T	
T	F	F	$\sim P \vee Q$
F	T	F	$P \vee \sim Q$
F	F	T	

∴ The PCNF of $P \leftrightarrow Q$ is $(\sim P \vee Q) \wedge (P \vee \sim Q)$