In [1]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split
from sklearn.svm import SVC
from sklearn.metrics import classification_report, confusion_matrix
import warnings
from sklearn.exceptions import UndefinedMetricWarning
```

In [2]:
```python
df = pd.read_csv('Auto.csv')
```

In [3]:
```python
df.head()
```

Out[3]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin	name
0	18.0	8	307.0	130	3504	12.0	70	1	chevrolet chevelle malibu
1	15.0	8	350.0	165	3693	11.5	70	1	buick skylark 320
2	18.0	8	318.0	150	3436	11.0	70	1	plymouth satellite
3	16.0	8	304.0	150	3433	12.0	70	1	amc rebel sst
4	17.0	8	302.0	140	3449	10.5	70	1	ford torino

1. Create a binary variable that takes on a value of 1 for cars with gas mileage above the median and 0 for cars with gas mileage below the median.

In [4]:
```python
median_mpg = df['mpg'].median()

# Create a new column 'mpg_indicator' based on the median
df['mpg_indicator'] = np.where(df['mpg'] > median_mpg, 0, 1)

df.head()
df['mpg_indicator'].value_counts()
```

Out[4]:
```
mpg_indicator
1    196
0    196
Name: count, dtype: int64
```

2. Fit a support vector classifier to the data using a linear kernel. Use a seed of 123, if needed. Experiment with different values of C to predict whether a car gets high or low gas mileage.

In [5]:
```
X = df[['cylinders', 'displacement', 'horsepower', 'weight', 'acceleration','year','origin']]
y = df['mpg_indicator']

# Split data into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=123)
```

In [6]:
```
C_values = [0.001, 0.01, 0.1, 1, 5, 10, 100]

q2results = []

for C in C_values:
    svc_model = SVC(kernel='linear', C=C)
    svc_model.fit(X_train, y_train)

    y_pred = svc_model.predict(X_test)

    print(f"Results for C = {C}:")
    print(classification_report(y_test, y_pred))

    crosstab_df = pd.crosstab(y_test, y_pred, rownames=['Actual'], colnames=['Predicted'])
    print(crosstab_df)
    print("-" * 60)

    q2report = classification_report(y_test, y_pred, output_dict=True)
    accuracy = q2report['accuracy']
    error_rate = 1 - accuracy

    q2results.append({
        'C Value': C,
        'Accuracy': accuracy,
```

```
        'Error Rate': error_rate
         })
```

```
Results for C = 0.001:
              precision    recall  f1-score   support

           0       0.82      0.93      0.87        55
           1       0.93      0.83      0.87        63

    accuracy                           0.87       118
   macro avg       0.88      0.88      0.87       118
weighted avg       0.88      0.87      0.87       118

Predicted   0   1
Actual
0          51   4
1          11  52
------------------------------------------------------------
Results for C = 0.01:
              precision    recall  f1-score   support

           0       0.86      0.91      0.88        55
           1       0.92      0.87      0.89        63

    accuracy                           0.89       118
   macro avg       0.89      0.89      0.89       118
weighted avg       0.89      0.89      0.89       118

Predicted   0   1
Actual
0          50   5
1           8  55
------------------------------------------------------------
Results for C = 0.1:
              precision    recall  f1-score   support

           0       0.85      0.91      0.88        55
           1       0.92      0.86      0.89        63

    accuracy                           0.88       118
   macro avg       0.88      0.88      0.88       118
weighted avg       0.88      0.88      0.88       118

Predicted   0   1
```

```
Actual
0           50   5
1            9  54
------------------------------------------------------------
Results for C = 1:
              precision    recall  f1-score   support

           0       0.84      0.87      0.86        55
           1       0.89      0.86      0.87        63

    accuracy                           0.86       118
   macro avg       0.86      0.86      0.86       118
weighted avg       0.87      0.86      0.86       118

Predicted   0   1
Actual
0           48   7
1            9  54
------------------------------------------------------------
Results for C = 5:
              precision    recall  f1-score   support

           0       0.84      0.87      0.86        55
           1       0.89      0.86      0.87        63

    accuracy                           0.86       118
   macro avg       0.86      0.86      0.86       118
weighted avg       0.87      0.86      0.86       118

Predicted   0   1
Actual
0           48   7
1            9  54
------------------------------------------------------------
Results for C = 10:
              precision    recall  f1-score   support

           0       0.84      0.87      0.86        55
           1       0.89      0.86      0.87        63

    accuracy                           0.86       118
```

```
   macro avg       0.86      0.86      0.86       118
weighted avg       0.87      0.86      0.86       118

Predicted   0   1
Actual
0          48   7
1           9  54
------------------------------------------------------------
Results for C = 100:
              precision    recall  f1-score   support

           0       0.86      0.87      0.86        55
           1       0.89      0.87      0.88        63

    accuracy                           0.87       118
   macro avg       0.87      0.87      0.87       118
weighted avg       0.87      0.87      0.87       118

Predicted   0   1
Actual
0          48   7
1           8  55
------------------------------------------------------------
```

3. Report the cross-validation errors associated with different values of this parameter. Discuss the empirical implications of those results.

In [7]:

```
q2results_df = pd.DataFrame(q2results)
q2results_df
```

Out[7]:

	C Value	Accuracy	Error Rate
0	0.001	0.872881	0.127119
1	0.010	0.889831	0.110169
2	0.100	0.881356	0.118644
3	1.000	0.864407	0.135593
4	5.000	0.864407	0.135593
5	10.000	0.864407	0.135593
6	100.000	0.872881	0.127119

Outcomes:

C = 0.010: Error Rate = 11% (Lowest Error Rate - Best!)

C = (1,5,10): Error Rate = 13.6% (Highest Error Rate - Worst!)

The Implications:

Finding the Balance: The 'C' value controls how complex the model tries to be.

1. Very Small C (like 0.001): Makes the model very simple. It did pretty well, but not the absolute best.
2. Medium C (like 0.01): This seems to be the sweet spot for this data. It's complex enough to find the pattern but simple enough not to get confused by noise.
3. Large C (1, 5, 10): Making the model even more flexible didn't help. The mistake rate stayed higher than the best setting, suggesting overfitting was still happening.

Choosing the Right Model:

Based on these tests, we should choose C = 0.01 for our final model because it's expected to make the fewest mistakes on new, unseen data.

4. Now repeat Q2, this time using SVMs with radial and polynomial basis kernels, with different values of gamma and degree and C. Discuss the empirical implications of those results.

In [8]:

```
warnings.filterwarnings("ignore", category=UndefinedMetricWarning)

kernels = ['rbf', 'poly']
gamma_values = [0.001, 0.01, 0.1, 1]
degree_values = [2, 3, 4]
C_values = [0.001, 0.01, 0.1, 1, 5, 10, 100]

q4results = []

for kernel in kernels:
    for C in C_values:
        if kernel == 'rbf':
            for gamma in gamma_values:
                svc_model = SVC(kernel=kernel, C=C, gamma=gamma)
                svc_model.fit(X_train, y_train)
                y_pred = svc_model.predict(X_test)

                q4report = classification_report(y_test, y_pred, output_dict=True, zero_division=0)
                accuracy = q4report['accuracy']
                error_rate = 1 - accuracy

                q4results.append({
                    'Kernel': kernel,
                    'C': C,
                    'Gamma': gamma,
                    'Accuracy': accuracy,
                    'Error Rate': error_rate
                })

        elif kernel == 'poly':
            for degree in degree_values:
                svc_model = SVC(kernel=kernel, C=C, degree=degree)
                svc_model.fit(X_train, y_train)
                y_pred = svc_model.predict(X_test)

                q4report = classification_report(y_test, y_pred, output_dict=True, zero_division=0)
```

```
                accuracy = q4report['accuracy']
                error_rate = 1 - accuracy

                q4results.append({
                    'Kernel': kernel,
                    'C': C,
                    'Degree': degree,
                    'Accuracy': accuracy,
                    'Error Rate': error_rate
                })

q4results_df = pd.DataFrame(q4results)
q4results_df
```

Out[8]:

	Kernel	C	Gamma	Accuracy	Error Rate	Degree
0	rbf	0.001	0.001	0.466102	0.533898	NaN
1	rbf	0.001	0.010	0.466102	0.533898	NaN
2	rbf	0.001	0.100	0.466102	0.533898	NaN
3	rbf	0.001	1.000	0.466102	0.533898	NaN
4	rbf	0.010	0.001	0.466102	0.533898	NaN
5	rbf	0.010	0.010	0.466102	0.533898	NaN
6	rbf	0.010	0.100	0.466102	0.533898	NaN
7	rbf	0.010	1.000	0.466102	0.533898	NaN
8	rbf	0.100	0.001	0.550847	0.449153	NaN
9	rbf	0.100	0.010	0.466102	0.533898	NaN
10	rbf	0.100	0.100	0.466102	0.533898	NaN
11	rbf	0.100	1.000	0.466102	0.533898	NaN
12	rbf	1.000	0.001	0.898305	0.101695	NaN
13	rbf	1.000	0.010	0.737288	0.262712	NaN
14	rbf	1.000	0.100	0.466102	0.533898	NaN
15	rbf	1.000	1.000	0.466102	0.533898	NaN
16	rbf	5.000	0.001	0.864407	0.135593	NaN
17	rbf	5.000	0.010	0.754237	0.245763	NaN
18	rbf	5.000	0.100	0.466102	0.533898	NaN
19	rbf	5.000	1.000	0.466102	0.533898	NaN
20	rbf	10.000	0.001	0.872881	0.127119	NaN
21	rbf	10.000	0.010	0.754237	0.245763	NaN

	Kernel	C	Gamma	Accuracy	Error Rate	Degree
22	rbf	10.000	0.100	0.466102	0.533898	NaN
23	rbf	10.000	1.000	0.466102	0.533898	NaN
24	rbf	100.000	0.001	0.864407	0.135593	NaN
25	rbf	100.000	0.010	0.754237	0.245763	NaN
26	rbf	100.000	0.100	0.466102	0.533898	NaN
27	rbf	100.000	1.000	0.466102	0.533898	NaN
28	poly	0.001	NaN	0.466102	0.533898	2.0
29	poly	0.001	NaN	0.508475	0.491525	3.0
30	poly	0.001	NaN	0.644068	0.355932	4.0
31	poly	0.010	NaN	0.737288	0.262712	2.0
32	poly	0.010	NaN	0.796610	0.203390	3.0
33	poly	0.010	NaN	0.822034	0.177966	4.0
34	poly	0.100	NaN	0.830508	0.169492	2.0
35	poly	0.100	NaN	0.838983	0.161017	3.0
36	poly	0.100	NaN	0.830508	0.169492	4.0
37	poly	1.000	NaN	0.872881	0.127119	2.0
38	poly	1.000	NaN	0.855932	0.144068	3.0
39	poly	1.000	NaN	0.847458	0.152542	4.0
40	poly	5.000	NaN	0.864407	0.135593	2.0
41	poly	5.000	NaN	0.855932	0.144068	3.0
42	poly	5.000	NaN	0.847458	0.152542	4.0
43	poly	10.000	NaN	0.855932	0.144068	2.0

	Kernel	C	Gamma	Accuracy	Error Rate	Degree
44	poly	10.000	NaN	0.864407	0.135593	3.0
45	poly	10.000	NaN	0.855932	0.144068	4.0
46	poly	100.000	NaN	0.855932	0.144068	2.0
47	poly	100.000	NaN	0.855932	0.144068	3.0
48	poly	100.000	NaN	0.847458	0.152542	4.0

Overall Best Result:

The RBF kernel with C = 1.0 and Gamma = 0.001 was the clear winner.

Performance: It had the highest accuracy (making correct predictions 89.8% of the time) and the lowest error rate (0.10).

Comparing Kernels (RBF vs. Polynomial):

RBF was very sensitive to the Gamma setting.

- If Gamma is too high (C ≥ 0.1), RBF performed poorly, sometimes worse than just guessing!
- It needed a small Gamma (C ≤ 0.1) combined with a moderate-to-high C (like 1, 5, or 10) to work well.

The Polynomial kernel gave decent results (best was 87.3% accuracy with C = 1, Degree = 2).

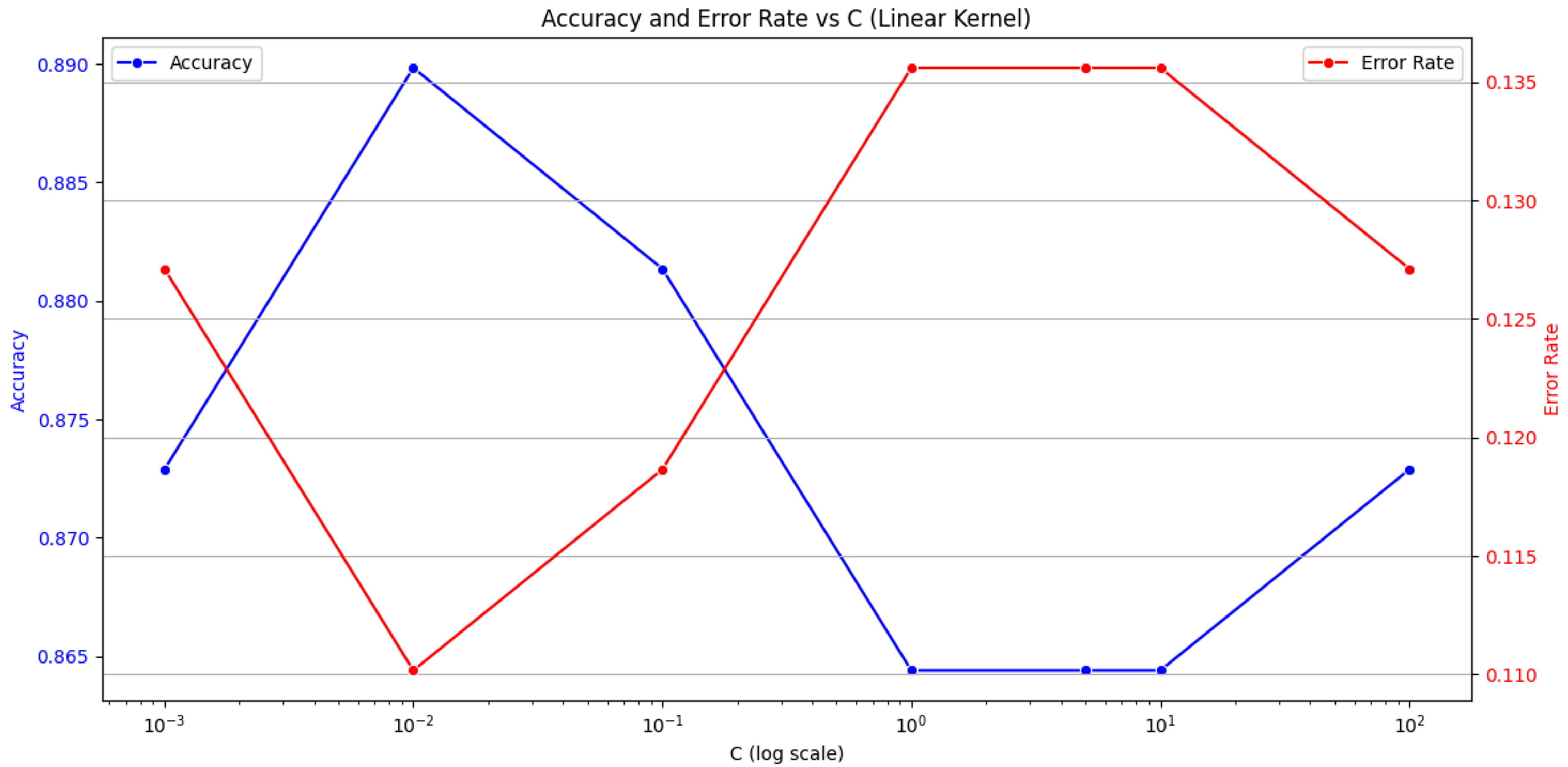
- Polynomial results were generally more stable across different Degree settings, especially once C was 0.1 or higher.
- It didn't have one setting that caused a total performance collapse like RBF did with high Gamma.

For this data, RBF seems better.

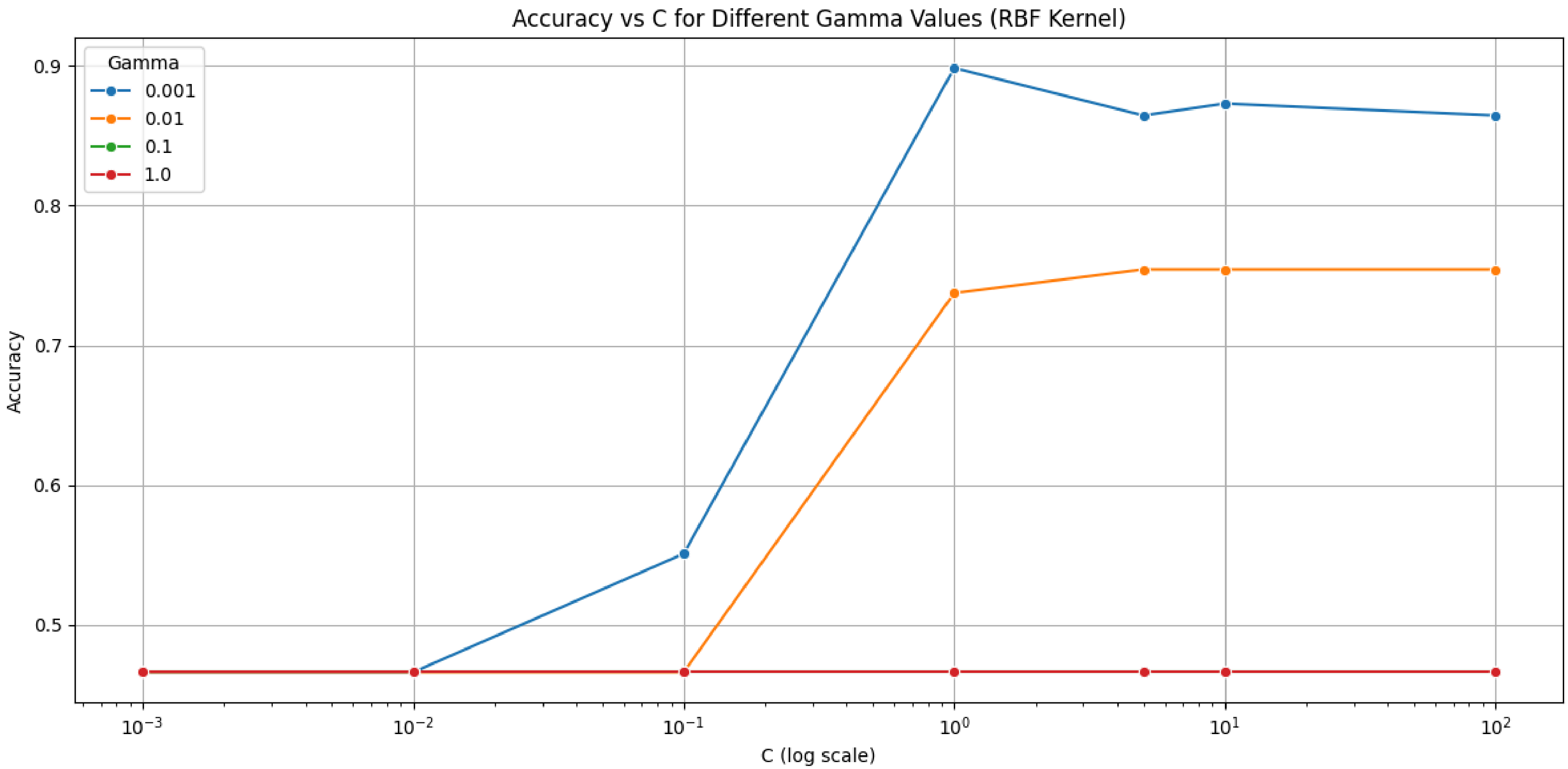
5. Generate relevant plots to support your findings in Q2 and Q4.

In [9]:
```
# Plot with Accuracy on the left y-axis
plt.figure(figsize=(12, 6))
sns.lineplot(data=q2results_df, x='C Value', y='Accuracy', marker='o', label='Accuracy', color='blue')
plt.xscale('log')
```

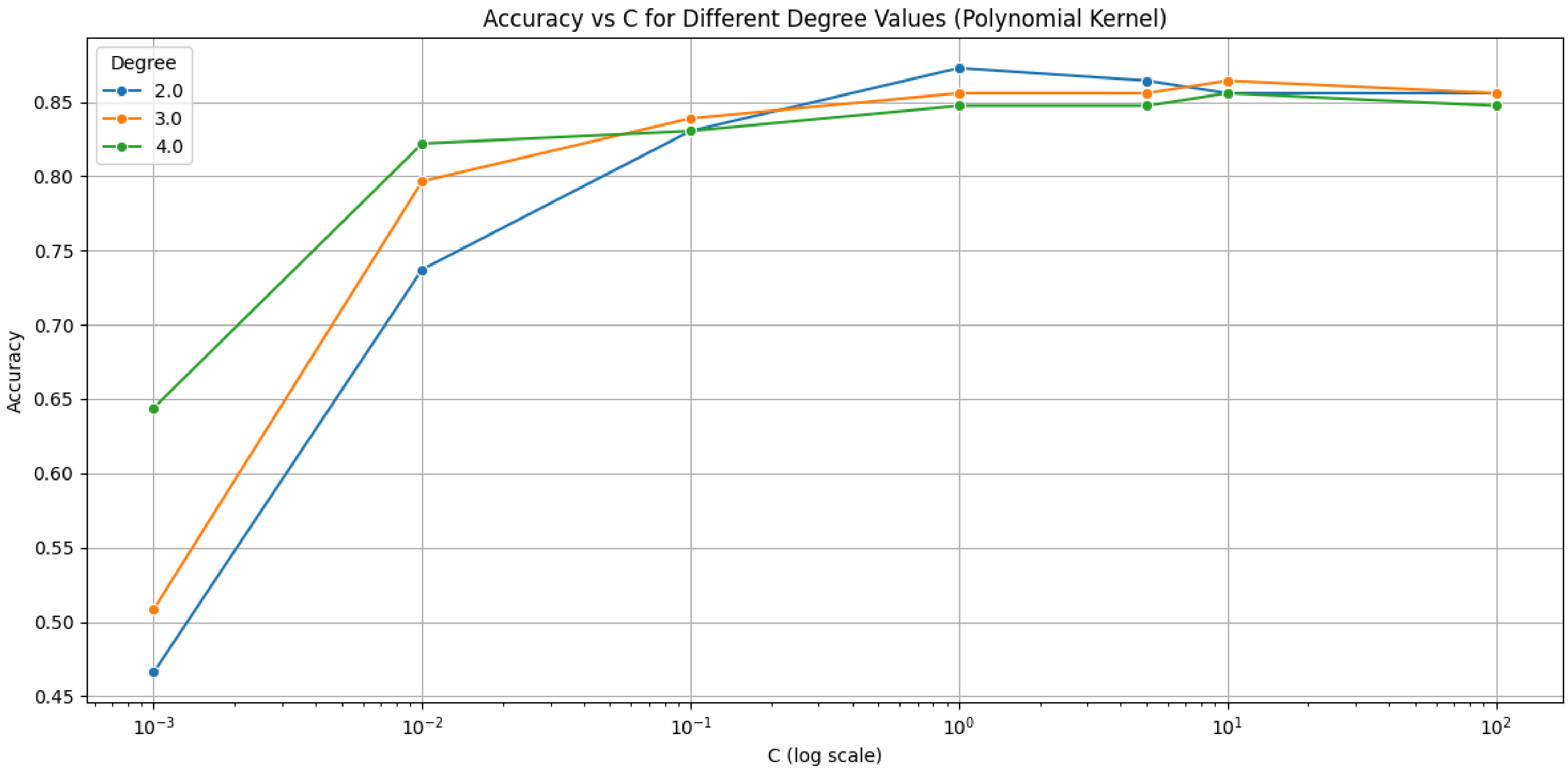
```python
plt.xlabel('C (log scale)')
plt.ylabel('Accuracy', color='blue')
plt.tick_params(axis='y', labelcolor='blue')
plt.legend(loc='upper left')
ax2 = plt.gca().twinx()
sns.lineplot(data=q2results_df, x='C Value', y='Error Rate', marker='o', label='Error Rate', color='red', ax=ax2)
ax2.set_ylabel('Error Rate', color='red')
ax2.tick_params(axis='y', labelcolor='red')
plt.title('Accuracy and Error Rate vs C (Linear Kernel)')
plt.grid(True)
plt.tight_layout()
plt.show()

#######################################################

rbf_df = q4results_df[q4results_df['Kernel'] == 'rbf']
plt.figure(figsize=(12, 6))
sns.lineplot(data=rbf_df, x='C', y='Accuracy', hue='Gamma', marker='o', palette='tab10')
plt.xscale('log')
plt.title('Accuracy vs C for Different Gamma Values (RBF Kernel)')
plt.xlabel('C (log scale)')
plt.ylabel('Accuracy')
plt.legend(title='Gamma')
plt.grid(True)
plt.tight_layout()
plt.show()

#######################################################

poly_df = q4results_df[q4results_df['Kernel'] == 'poly']
plt.figure(figsize=(12, 6))
sns.lineplot(data=poly_df, x='C', y='Accuracy', hue='Degree', marker='o', palette='tab10')
plt.xscale('log')
plt.title('Accuracy vs C for Different Degree Values (Polynomial Kernel)')
plt.xlabel('C (log scale)')
plt.ylabel('Accuracy')
plt.legend(title='Degree')
plt.grid(True)
plt.tight_layout()
plt.show()
```



Accuracy and Error Rate vs C (Linear Kernel)
Accuracy
Error Rate
0.890
0.885
0.880
0.875
0.870
0.865
0.135
0.130
0.125
0.120
0.115
0.110
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
C (log scale)



Accuracy vs C for Different Gamma Values (RBF Kernel)
Gamma
0.001
0.01
0.1
1.0
0.9
0.8
0.7
0.6
0.5
Accuracy
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
C (log scale)



Accuracy vs C for Different Degree Values (Polynomial Kernel)
Degree
2.0
3.0
4.0
Accuracy
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
C (log scale)

In [ ]: