COL 362 & COL 632

Relational

13 Jan 2023

Functional Dependencies

- There are usually a variety of constraints (rules) on the data in the real world.
- **Legal instance of a relation:** an instance that satisfies all real-world constraints
  - A legal instance of a database = all relations are legal
- Require that the **value for a certain set of attributes** determines **uniquely the value for another set of attributes**.
  - A functional dependency is a generalization of the notion of a key.

Functional Dependencies

- Let R be a relation schema

$$\alpha \subseteq R \quad and \quad \beta \subseteq R$$

- The **functional dependency**

$\alpha \rightarrow \beta$ **holds on** R **if and only if**

for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β.

That is, $t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$

$$R(A_1, A_2, A_3, B_1, B_2, B_3)$$

$$A_1 A_2 A_3 \rightarrow B_1$$

$$A_1 A_2 A_3 \rightarrow B_2$$

$$A_1 A_2 A_3 \rightarrow B_1 B_2 B_3$$

Functional Dependencies

- Example (let's figure out the right FDs)

Actor (Name, DOB, Address)

Movie (Title, Year, Language, StudioName)

PlaysIn (Name, DOB, Title, Year)

Name DOB → Address

Title Year → Language StudioName

Title → Language

Language → Title Year

Name DOB → Title Year

Actor (Name, DOB, Address, MovieTitle, Year, Language)

Name DOB → Address

Name DOB Address → Language

Name DOB MovieTitle Year → Address

MovieTitle Year DOB → Name

DOB Address MovieTitle → Name

DOB Address MovieTitle Year → Name

Can you figure out the functional dependencies from the data?

Functional Dependecies from Data

Name	DOB	Address	MovieTitle	Year	Language
Ayushmaan Khurrana	1984	Mumbai	DoctorG	2022	Hindi
Ayushmaan Khurrana	1984	Mumbai	Andhadhun	2018	Hindi
Henry Cavill	1983	Beverley Hills	Man of Steel	2013	English
Leonardo DiCaprio	1974	Beverley Hills	The Revenant	2015	English

Address → Language

Name DOB → Address ✓

Trivial and non-trivial FDs

- Trivial: In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$
- Non-trivial: partial overlap of attributes
- Completely non-trivial: no overlap of attributes

$\alpha \cap \beta \neq \emptyset$
$\alpha = \beta$

Actor (Name, DOB, Address)

Movie (Title, Year, Language)

PlaysIn (Name, DOB, Title, Year)

Name DOB → Address — Completely non-trivial
Name DOB → Name — Trivial
Name DOB → Name Address — Non-trivial

Keys and superkeys

Movie (Title, Year, Language, Length)

{Title, Year}

Key
- Functionally determines *all* other attributes
- Minimal

Keys need not be unique

{Title, Year, Language}

Superkey
- Functionally determines *all* other attributes
- Not necessarily *minimal*

Terminology

- Key = Candidate Key
- Superkey
- Primary key
- Prime attribute

Inferring FDs

- Given a set of FDs, which other FDs follow from it?
- Example:
  - Given: Name DOB → Address
    Address → City
  - Inferred: Name, DOB → City

Inferred through *transitivity* of FDs

Armstrong's Axioms

- Reflexivity

  If B is a subset of A, then A → B

- Augmentation

  If A → B, then AC → BC

- Transitivity

  If A → B and B → C, then A → C

Closure of FDs

- **Given**: *F*, the set of FDs
- **Output**: F^+, the ***closure of F***, containing all FDs derivable from F

- Example: R (A, B, C)

AB → C

C → ED

Basis (i.e., F)

Is this set F^+

AB → ED

AB → E

ABC → ED

C → E

C → D