SPECTRAL CLUSTERING

AND

CRYPTO CURRENCY NETWORK STRUCTURE

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MSCF ML II Group Project

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2. Spectral Clustering
3. Applications to Crypto
4. Further Research

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Paper Replication

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Further Research

Analysis of a network structure of the foreign currency exchange market

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Received: date / Accepted: date

Abstract We analyze structure of the world foreign currency exchange (FX) market viewed as a network of interacting currencies. We analyze daily time series of FX data for a set of 63 currencies, including gold, silver and platinum. We group together all the exchange rates with a common base currency and study each group separately. By applying the methods of filtered correlation matrix we identify clusters of closely related currencies. The clusters are formed typically according to the economical and geographical factors. We also study topology of weighted minimal spanning trees for different network representations (i.e., for different base currencies) and find that in a majority of representations the network has a hierarchical scale-free structure. In addition, we analyze the temporal evolution of the network and detect that its structure is not stable over time. A medium-term trend can be identified which affects the USD node by decreasing its centrality. Our analysis shows also an increasing role of euro in the world's currency market.

Keywords Foreign exchange market · Correlation matrix · Networks · Minimal Spanning Tree

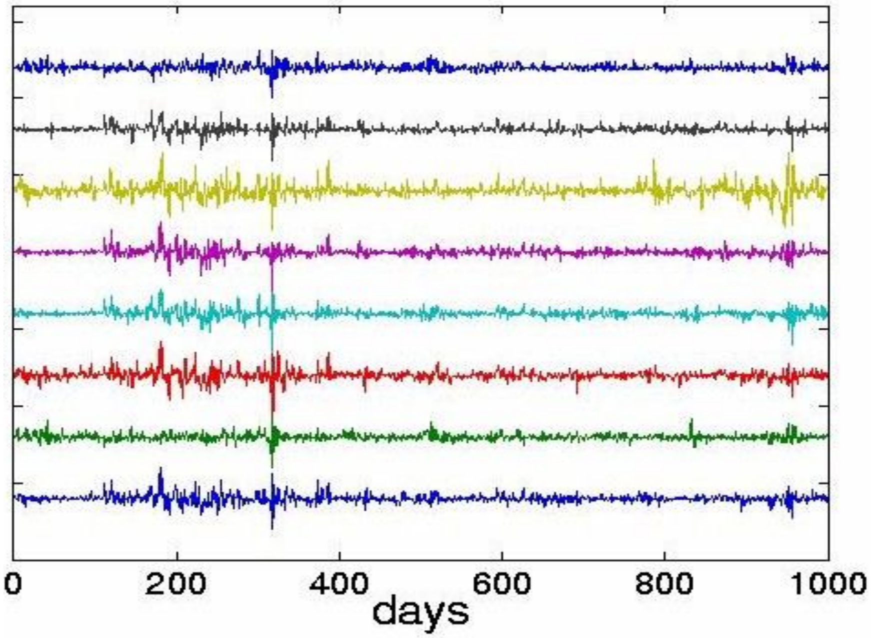
- The FX market can be viewed as a network of interacting exchange rates.
- Interactions are assumed to be strongly nonlinear.
- There can be no independent frame of reference.

DATA

- 60 most traded currencies
- 3 precious metals (XAG, XAU, PTC)
- From/To: Jan 1999 - Jun 2008 (9.5 yr)
- Daily data: 5 days a week
- 2,394 observations

EXCHANGE RATE EIGENSPECTRA

1) Select base currency. NxN matrix X



2) Compute correlation matrix.

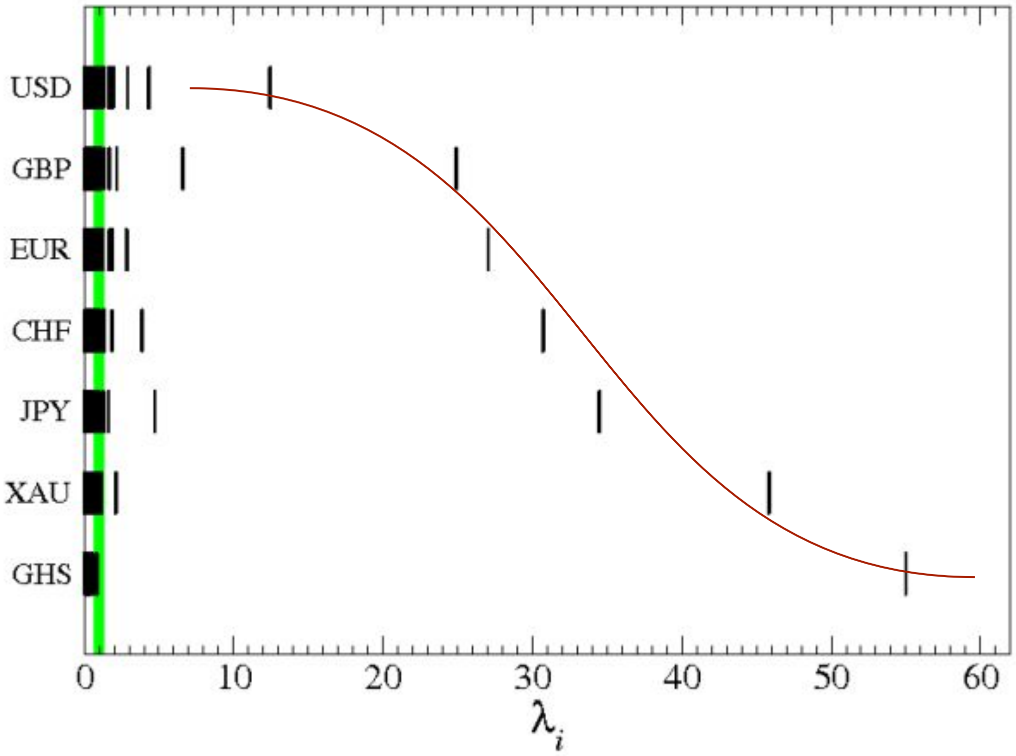
3) Compute eigenvalues.

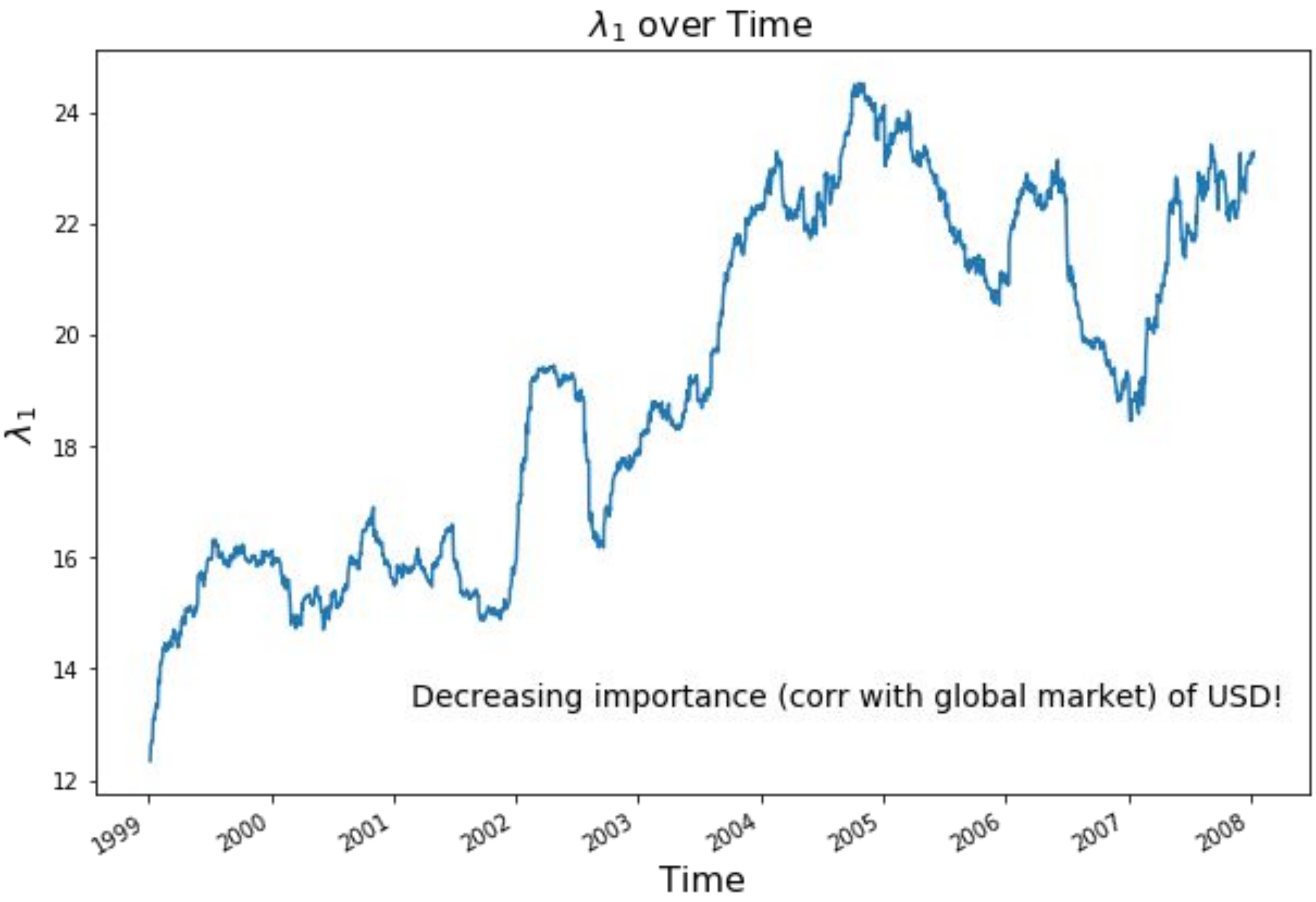
3) Inspect largest eigenvalue.

EXCHANGE RATE EIGENSPECTRA

$$0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$

- Small max eigenvalue: Central currency.
- Large max eigenvalue: Peripheral currency.



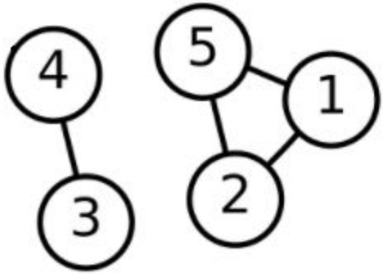

λ_1 over Time
24
22
20
18
16
14
12
λ_1
Decreasing importance (corr with global market) of USD!
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
Time

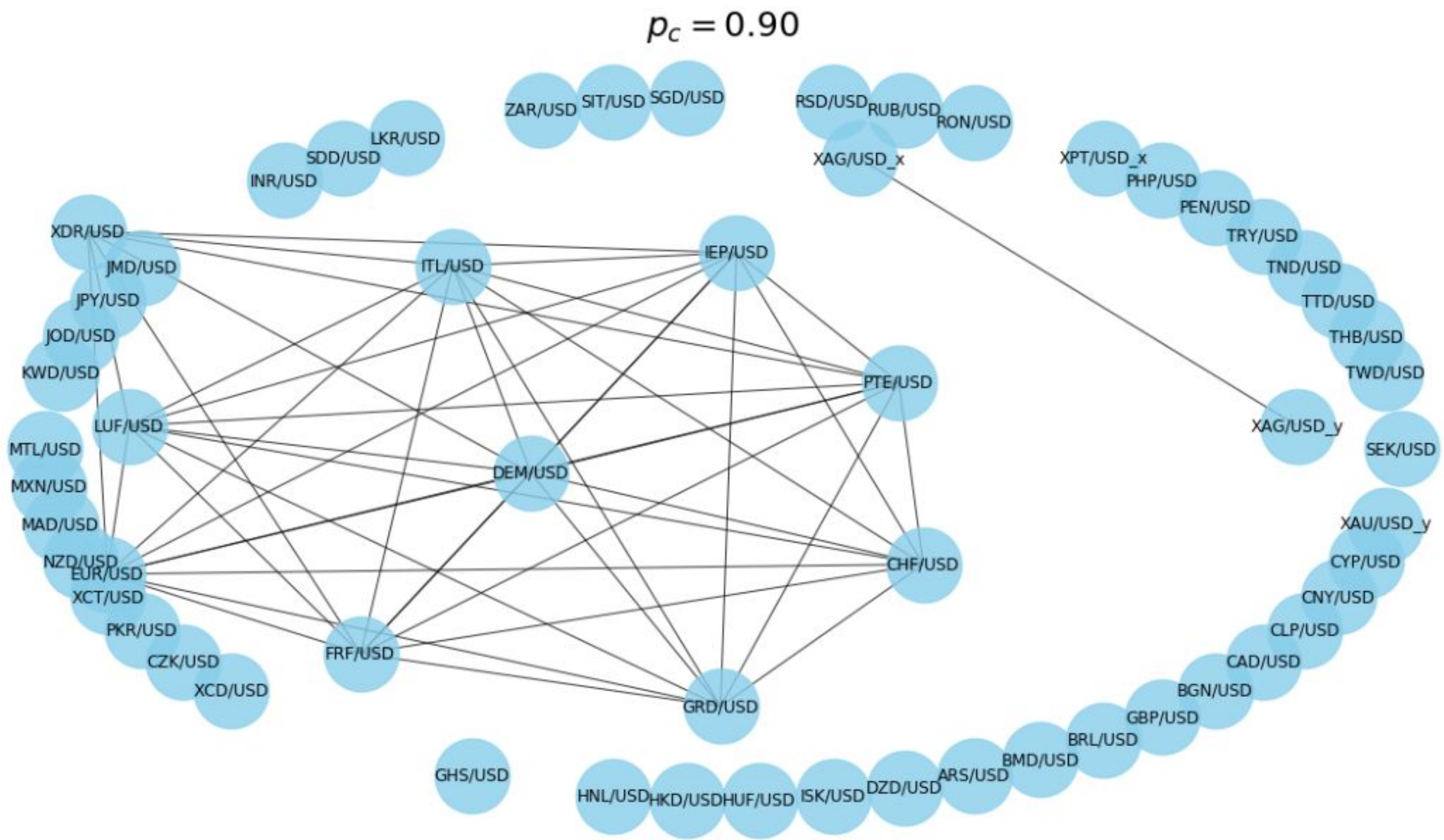
CLUSTER STRUCTURE

1) Correlation matrix.
2) Threshold by ρ.
3) Draw links.
4) Try different bases.

$$C = \begin{pmatrix} 1 & 0.6 & 0.2 & 0.2 & 0.8 \\ 0.6 & 1 & 0.3 & 0.1 & 0.6 \\ 0.2 & 0.3 & 1 & 0.7 & 0 \\ 0.2 & 0.1 & 0.7 & 1 & 0.4 \\ 0.8 & 0.6 & 0 & 0.4 & 1 \end{pmatrix}$$

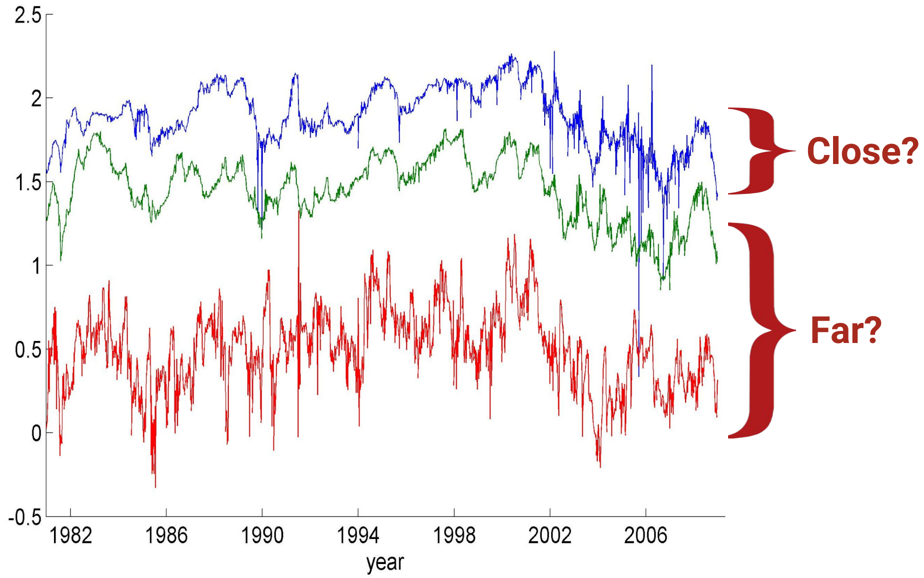
$$T = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$



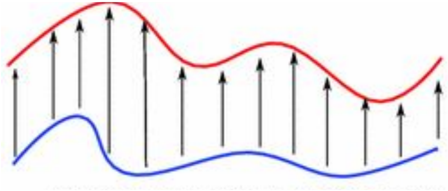


$p_c = 0.90$
ZAR/USD
SIT/USD
SGD/USD
RSD/USD
RUB/USD
RON/USD
LKR/USD
SDD/USD
INR/USD
XAG/USD_x
XPT/USD_x
PHP/USD
PEN/USD
TRY/USD
TND/USD
TTD/USD
THB/USD
TWD/USD
XDR/USD
JMD/USD
JPY/USD
JOD/USD
KWD/USD
ITL/USD
IEP/USD
PTE/USD
LUF/USD
DEM/USD
XAG/USD_y
SEK/USD
MTL/USD
MXN/USD
MAD/USD
NZD/USD
EUR/USD
XCT/USD
PKR/USD
CZK/USD
XCD/USD
FRF/USD
CHF/USD
GRD/USD
XAU/USD_y
CYP/USD
CNY/USD
CLP/USD
CAD/USD
BGN/USD
GBP/USD
BRL/USD
BMD/USD
ARS/USD
DZD/USD
ISK/USD
HUF/USD
HKD/USD
HNL/USD
GHS/USD

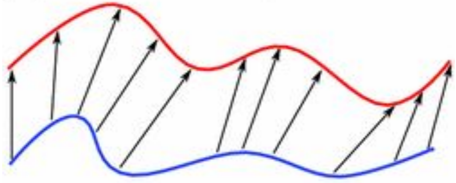
DISSIMILARITY OF EXCHANGE RATES



- Euclidean distance (bad).



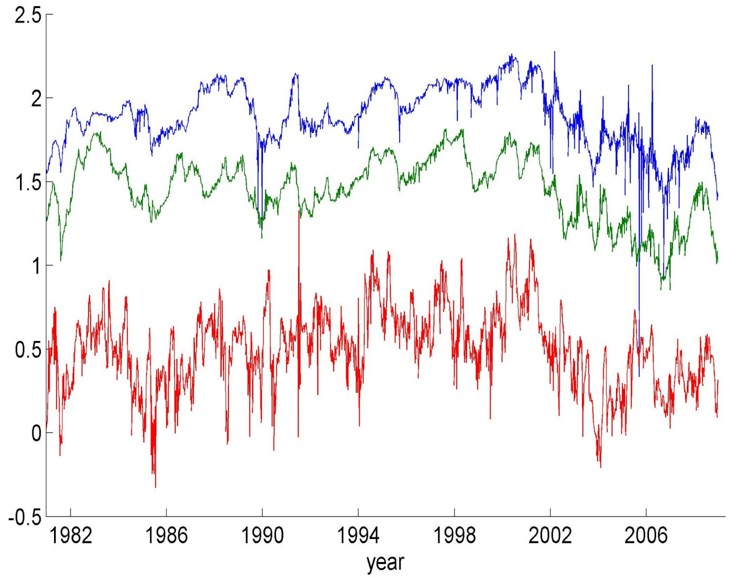
- DTW/MJC (okay).



- **Pearson dissimilarity** (good).

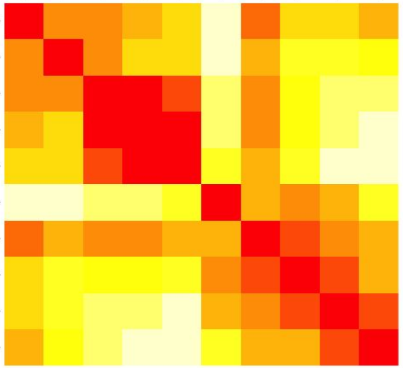
$$d^{B}_{\mathrm{X,Y}} = \sqrt{2(1 - C^{\mathrm{B}}_{\mathrm{X,Y}})}.$$

DISSIMILARITY OF EXCHANGE RATES



$$d^{B}_{\mathrm{X,Y}} = \sqrt{2(1 - C^{\mathrm{B}}_{\mathrm{X,Y}})}.$$

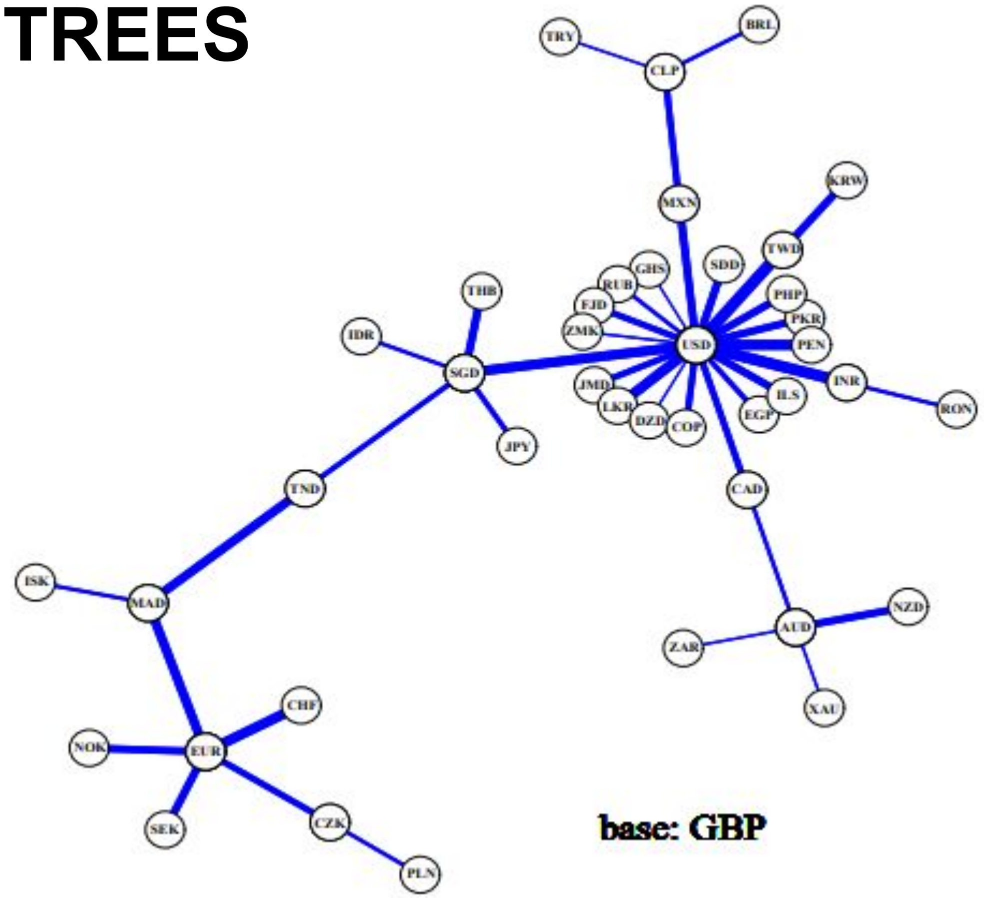
Dissimilarity matrix



MINIMAL SPANNING TREES

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \ldots & \ldots & 0 \end{bmatrix}$$

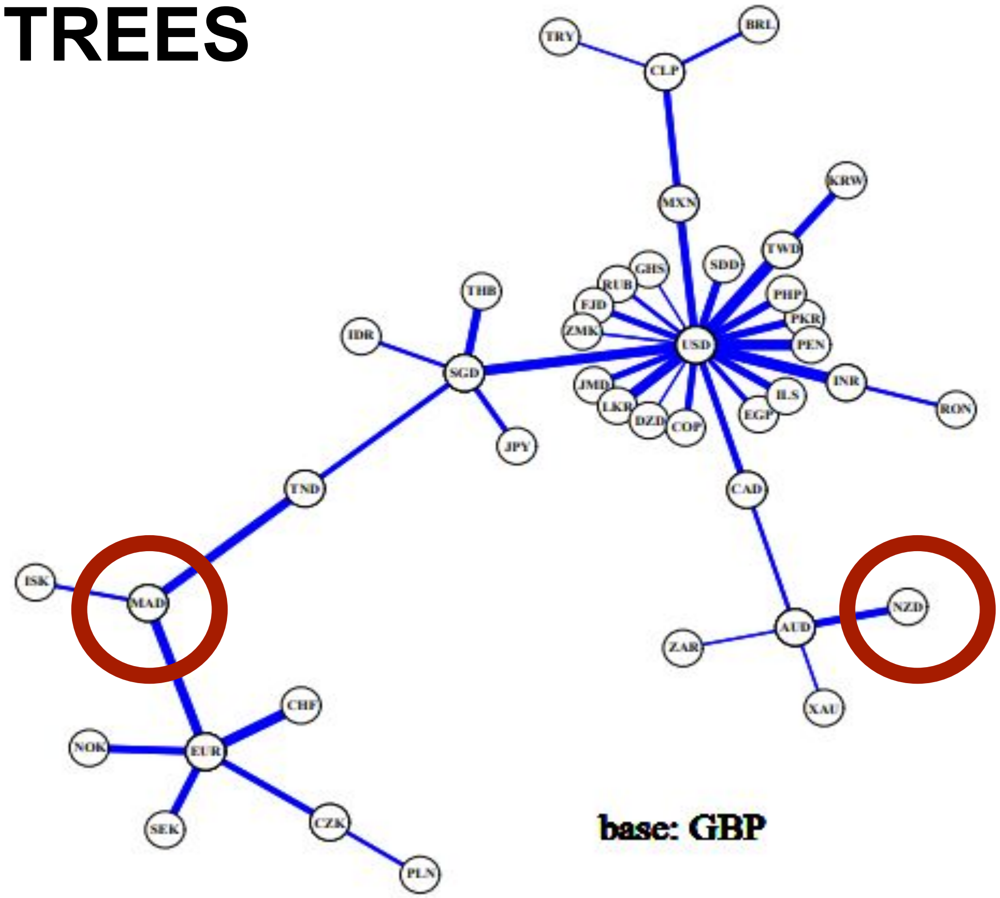
$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ \text{[redacted]} & \text{[redacted]} & 0 & & \\ \text{[redacted]} & \text{[redacted]} & \vdots & & \\ \text{[redacted]} & d(n,2) & \text{[redacted]} & \ldots & 0 \end{bmatrix}$$



MINIMAL SPANNING TREES

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ \text{[redacted]}) & d\text{[redacted]}) & 0 & & \\ \text{[redacted]} & \text{[redacted]} & \vdots & & \\ \text{[redacted]}) & d(n,2) & \text{[redacted]} & \dots & 0 \end{bmatrix}$$

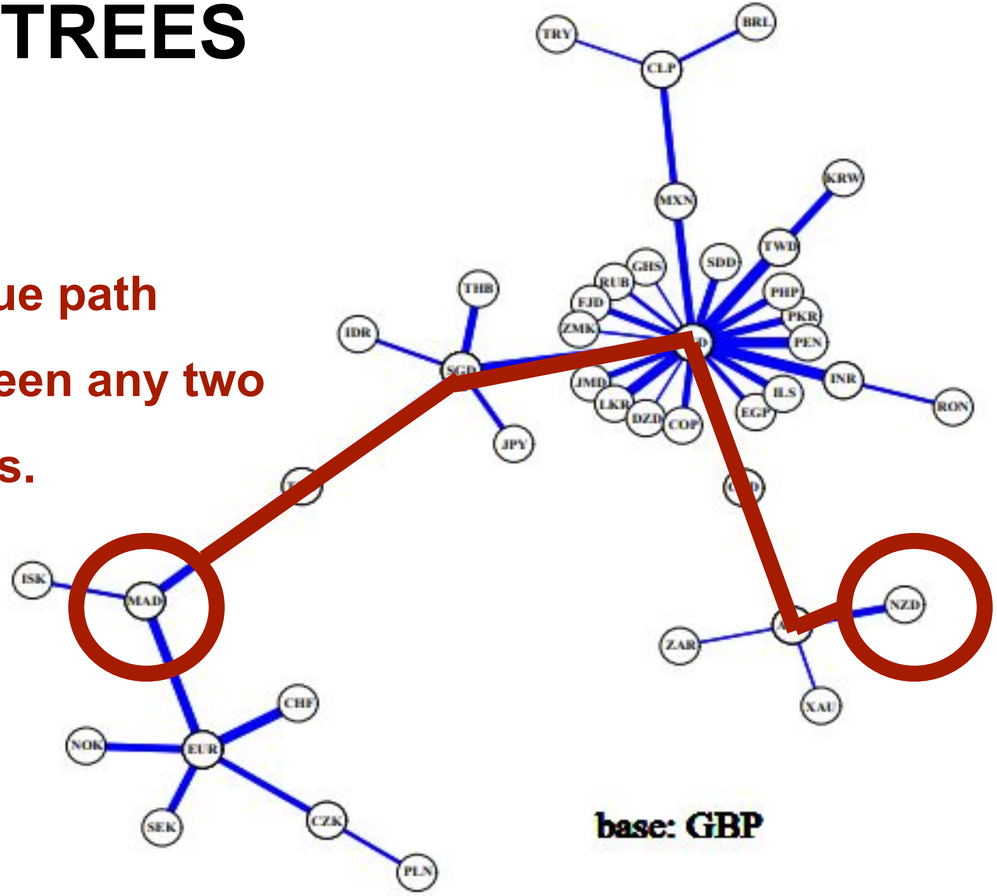


MINIMAL SPANNING TREES

$$
\begin{bmatrix}
0 & & & & \\
d(2,1) & 0 & & & \\
d(3,1) & d(3,2) & 0 & & \\
\vdots & \vdots & \vdots & & \\
d(n,1) & d(n,2) & \dots & \dots & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 & & & & \\
d(2,1) & 0 & & & \\
[redacted]) & d[redacted]) & 0 & & \\
[redacted] & [redacted] & \vdots & & \\
d[redacted]1) & d(n,2) & [redacted] & \dots & 0
\end{bmatrix}
$$

Unique path between any two nodes.



OBSERVATIONS

- Thresholding ‘C’ cluster analysis indicated relationships among:
  - Precious metals
  - European legacy currencies and the Euro
- Spectral clustering indicated relationships among:
  - Jordan Dinar and Kuwaiti Dinar given pegged to USD relationship (2003-2007)
  - Kuwaiti Dinar and a basket of global currencies (2007-Present)

1

2

3

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Paper Replication

Spectral Clustering

Introduction

Laplacian

Interpretation

Applications to Crypto

Further Research

INTRO TO SPECTRAL CLUSTERING

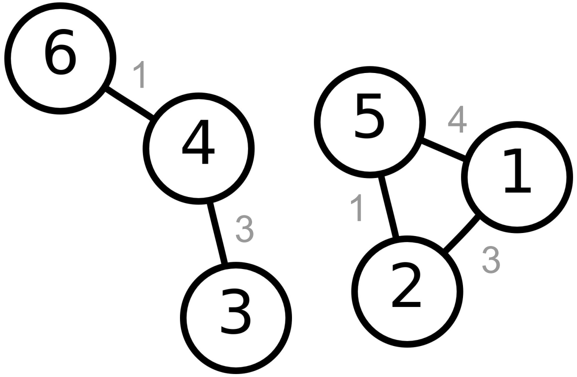
- Most traditional clustering methods
  - Perform poorly on non-convex data: “non-blobs”
  - Have difficulty with high dimensional data
- Time series data has both of these qualities
- Spectral clustering fixes these issues!
- Steals from well developed graph theory ideas

K-means

Mean Shift

BUILDING BLOCKS

G



$$W = \begin{pmatrix} 0 & 3 & 0 & 0 & 4 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

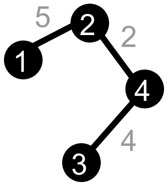
Note: Each subgraph making up *G* is a so called *connected component*

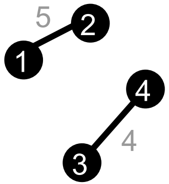
PROPERTIES OF LAPLACIAN MATRIX

$$L = D - W$$

1. *L* is symmetric and positive semi-definite
2. Row and columns sum to 0
3. *N* non-negative eigenvalues $0 = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_N$
4. $f'Lf = \frac{1}{2}\sum_{i,j=1}^{N} w_{ij}(f_i - f_j)^2 \quad \forall f \in \mathbb{R}^N$
5. Multiplicity of eigenvalue 0 equals the **number of connected components**

WHY IT WORKS





$$L_f = \begin{pmatrix} 5 & -5 & 0 & 0 \\ -5 & 6 & 0 & -1 \\ 0 & 0 & 4 & -4 \\ 0 & -1 & -4 & 5 \end{pmatrix} \qquad L_d = \begin{pmatrix} 5 & -5 & 0 & 0 \\ -5 & 5 & 0 & 0 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & -4 & 4 \end{pmatrix}$$

$$\color{red}{0 = \lambda_1^f} < \lambda_2^f \leq ... \leq \lambda_N^f \qquad \color{red}{0 = \lambda_1^d = \lambda_2^d} < \lambda_3^d \leq ... \leq \lambda_N^d$$

$$v_1^f = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad v_1^d = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2^d = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

ALGORITHM

Algorithm 1 Spectral Clustering

Input: Weighted adjacency matrix $W \in \mathbb{R}^{N \times N}$, number of clusters k

Compute degree matrix D

$L \leftarrow D - W$

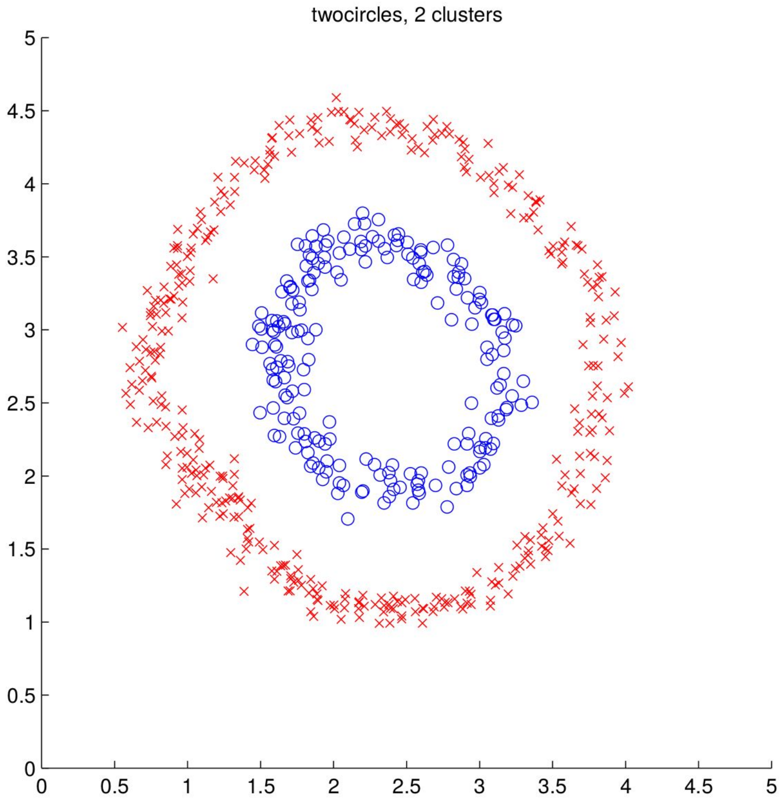
Compute first k eigenvectors $v_1, ..., v_k$ of L and stack them into $X \in \mathbb{R}^{N \times k}$

$y_i \leftarrow$ normalized ith row of X (note: $y_i \in \mathbb{R}^k$)

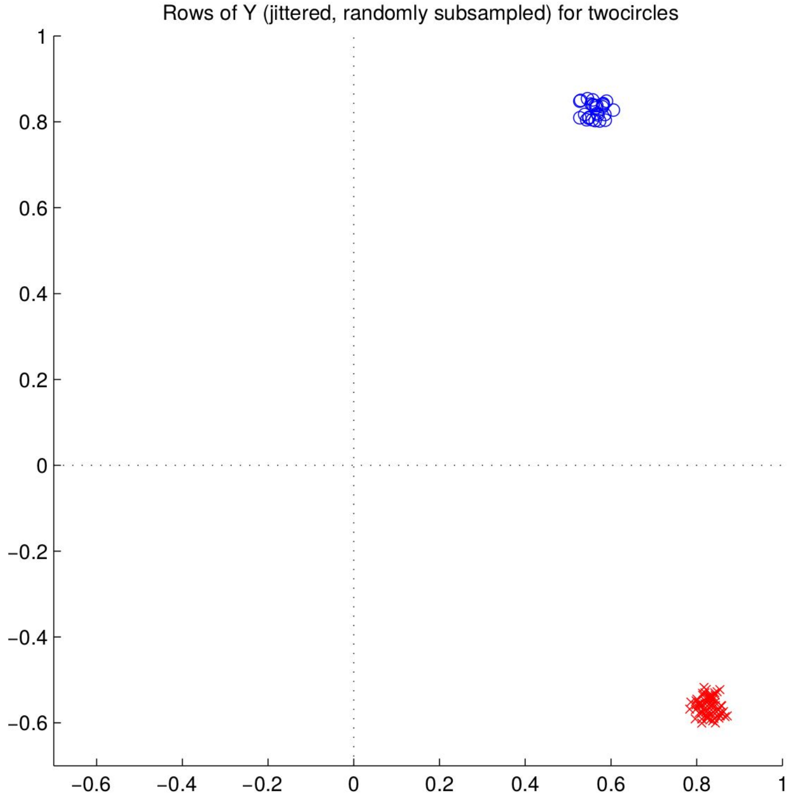
Cluster $(y_i)_{i=1,...,n}$ into clusters $C_1, ..., C_k$ using k-means

Output: Clusters $A_1, ..., A_k$ where $A_i = \{j | y_j \in C_i\}$

HOW IT WORKS



twocircles, 2 clusters
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Rows of Y (jittered, randomly subsampled) for twocircles
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

1

2

3

4

Paper Replication

Spectral Clustering

Applications to Crypto

Data

Analysis

Observations

Conclusion

Further Research

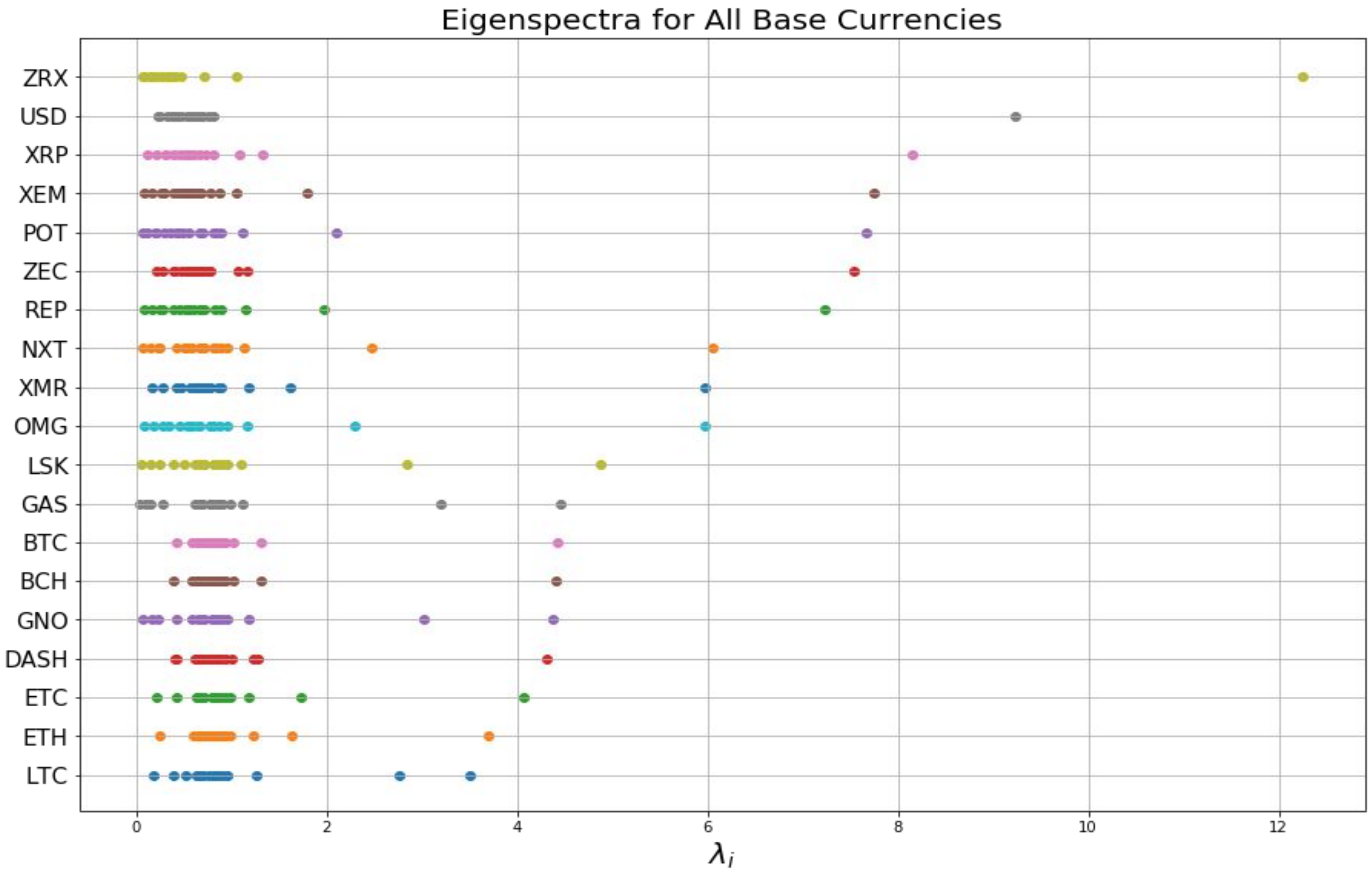
DATA

- 18 crypto currencies
- USD
- One exchange: Poloniex
- From/To: Jul 2017 - Apr 2018 (10 mo.)
- Hourly data 24/7
- 7,167 observations

BITCOIN DASH RIPPLE NEO
OMISEGO IOTA ETHEREUM ZCASH
LITECOIN DOGECOIN STRATIS MONERO
ETHEREUM CLASSIC NXT STEEM ARK

ANALYSIS & CONCLUSIONS

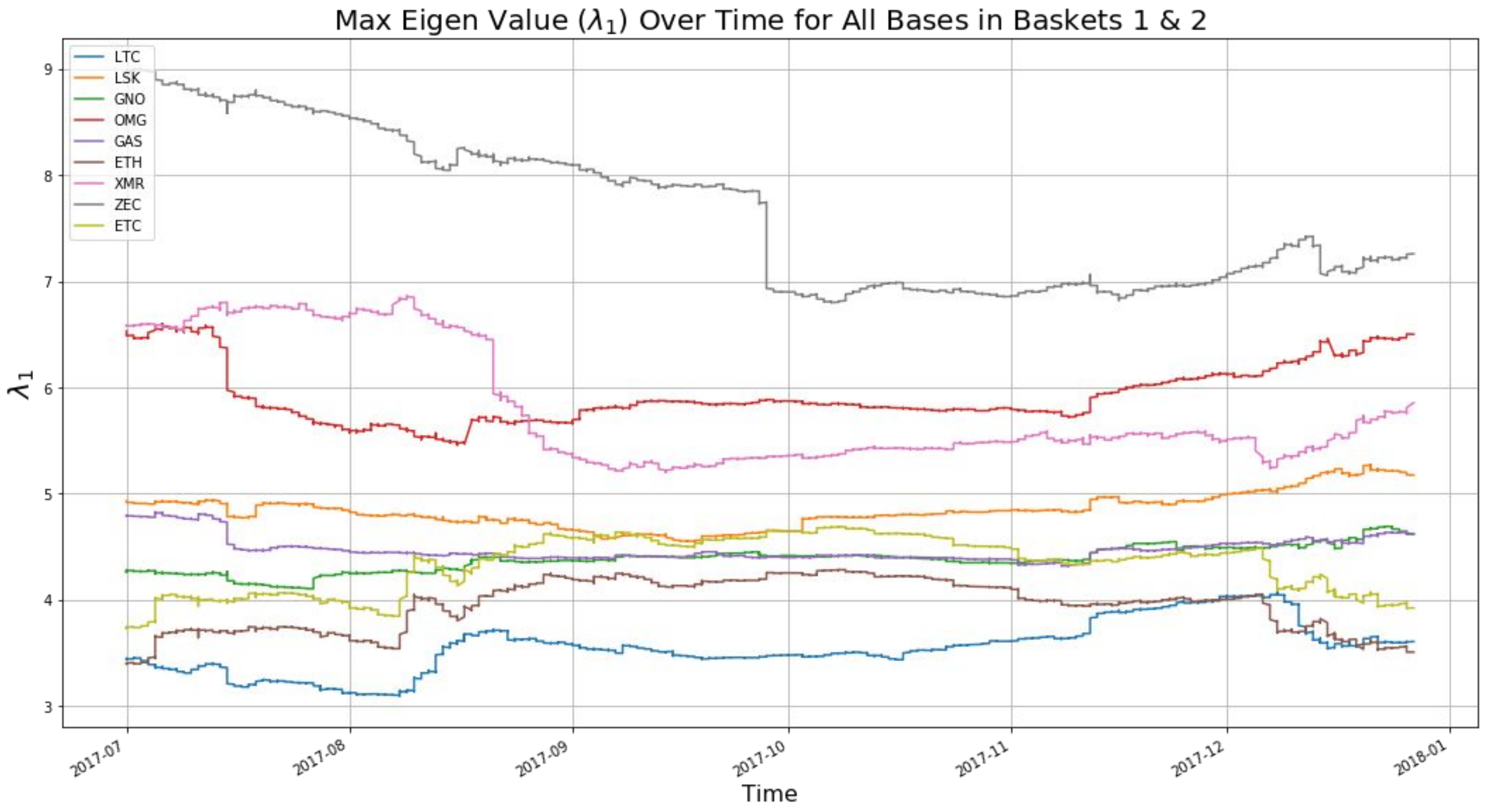
- Important dynamics shaping crypto markets:
  - “Bridge Currencies” and “Niche Currencies”
  - Differentiated Investment and Speculative Investment
- The eigenspectra analysis highlighted:
  - Relatively large max eigenvalues for USD and XRP
  - Relative small max eigenvalues for Bridge Currencies
    - BTC, LTC, ETH, BCH

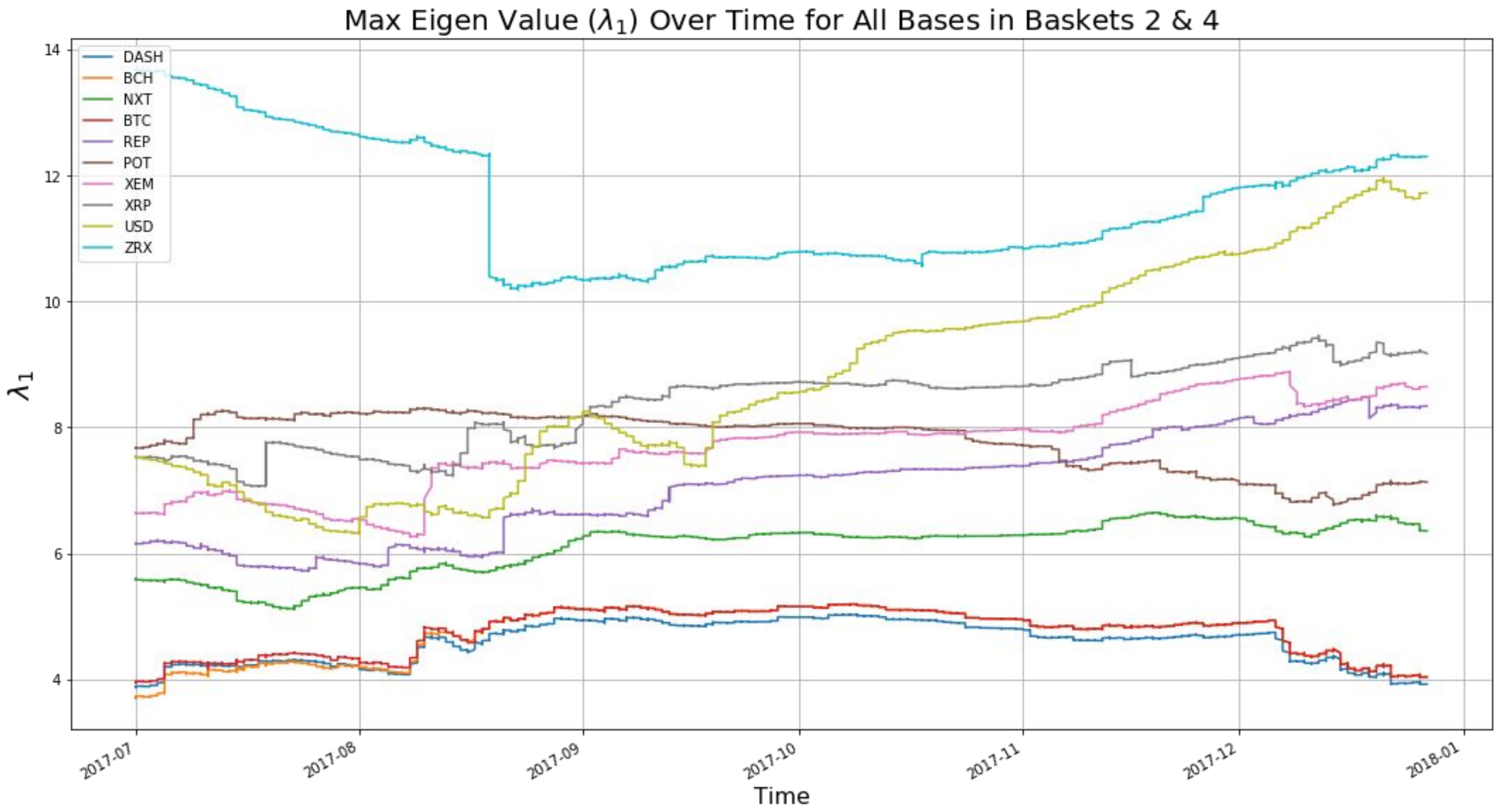


Eigenspectra for All Base Currencies
ZRX
USD
XRP
XEM
POT
ZEC
REP
NXT
XMR
OMG
LSK
GAS
BTC
BCH
GNO
DASH
ETC
ETH
LTC
0
2
4
6
8
10
12
λ_i

ANALYSIS & CONCLUSIONS

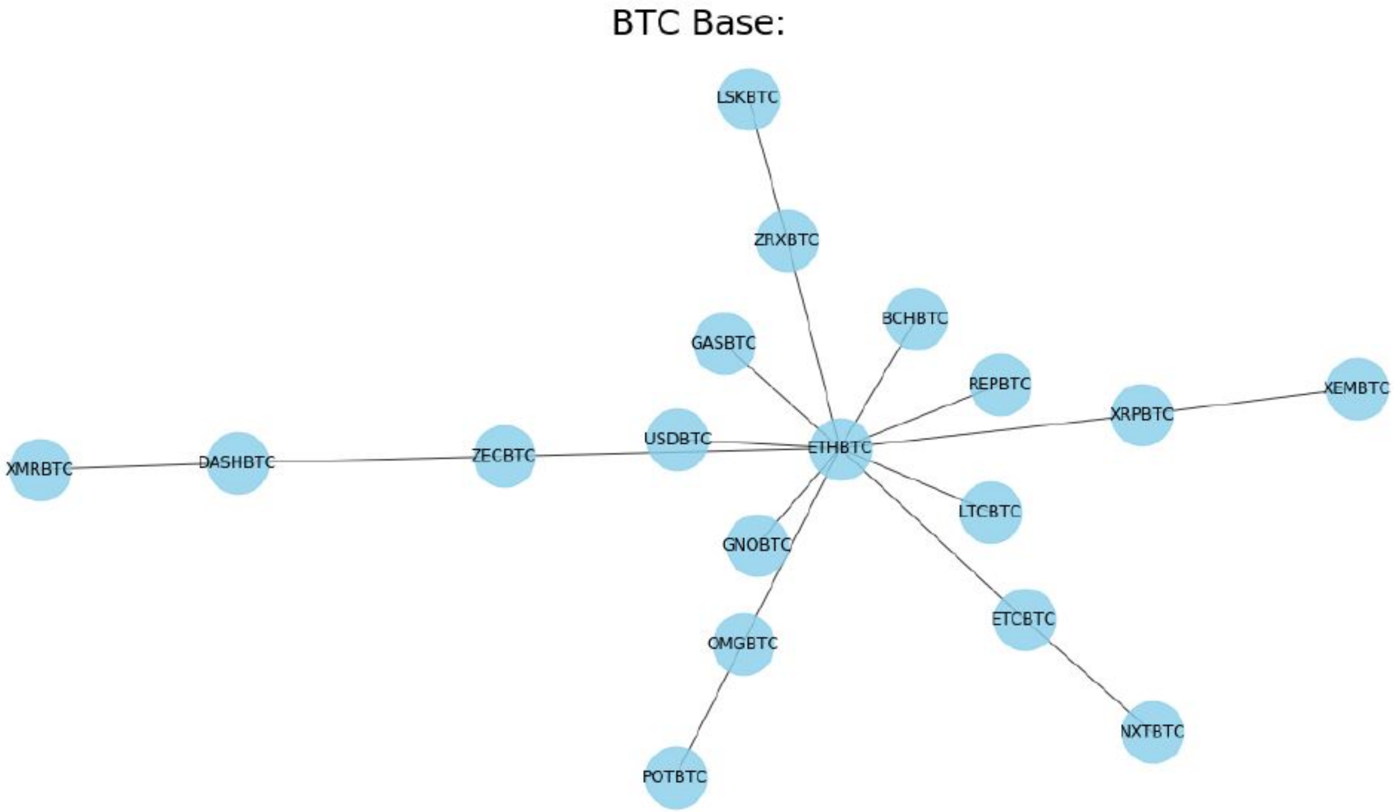
- The eigenspectra converged over time due to the influx of speculative capital
  - Sharp decreases in max eigenvalue associated with shock events in cryptocurrency price
- Spectral clustering and minimal spanning tree analysis indicate:
  - Unique relationships with ETH base
  - Unexpected relationship between BTC and ZEC


Max Eigen Value (λ_1) Over Time for All Bases in Baskets 1 & 2
LTC
LSK
GNO
OMG
GAS
ETH
XMR
ZEC
ETC
λ_1
9
8
7
6
5
4
3
2017-07
2017-08
2017-09
2017-10
2017-11
2017-12
2018-01
Time

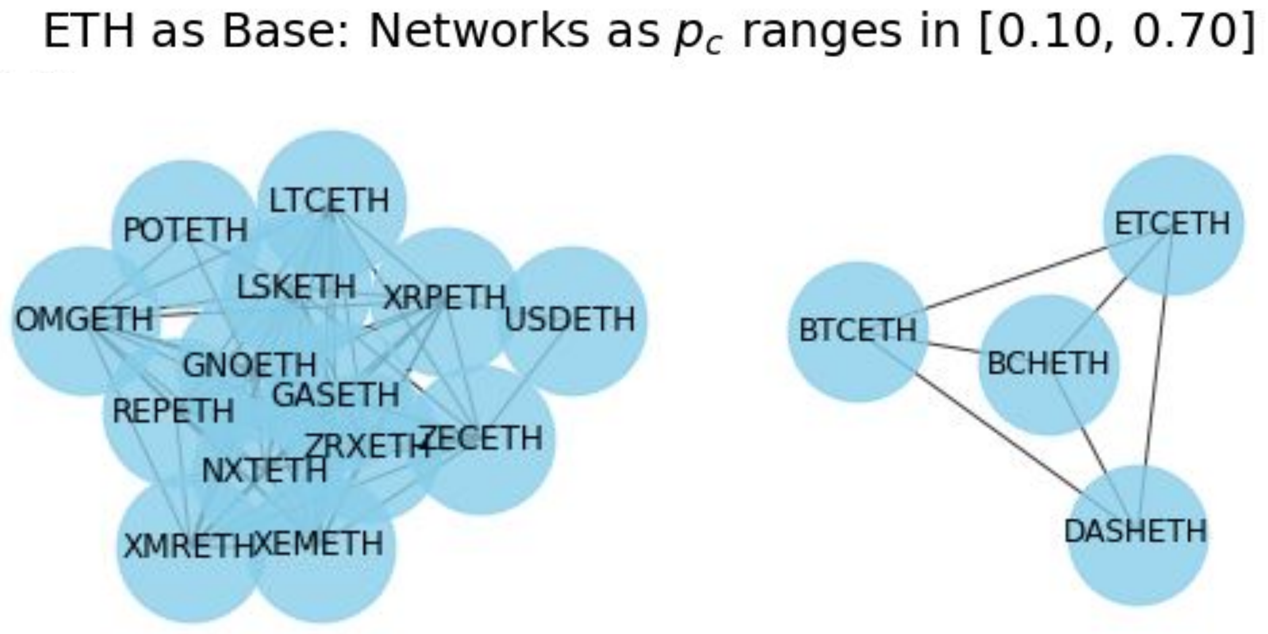


Max Eigen Value (λ_1) Over Time for All Bases in Baskets 2 & 4
DASH
BCH
NXT
BTC
REP
POT
XEM
XRP
USD
ZRX
λ_1
Time
14
12
10
8
6
4
2017-07
2017-08
2017-09
2017-10
2017-11
2017-12
2018-01

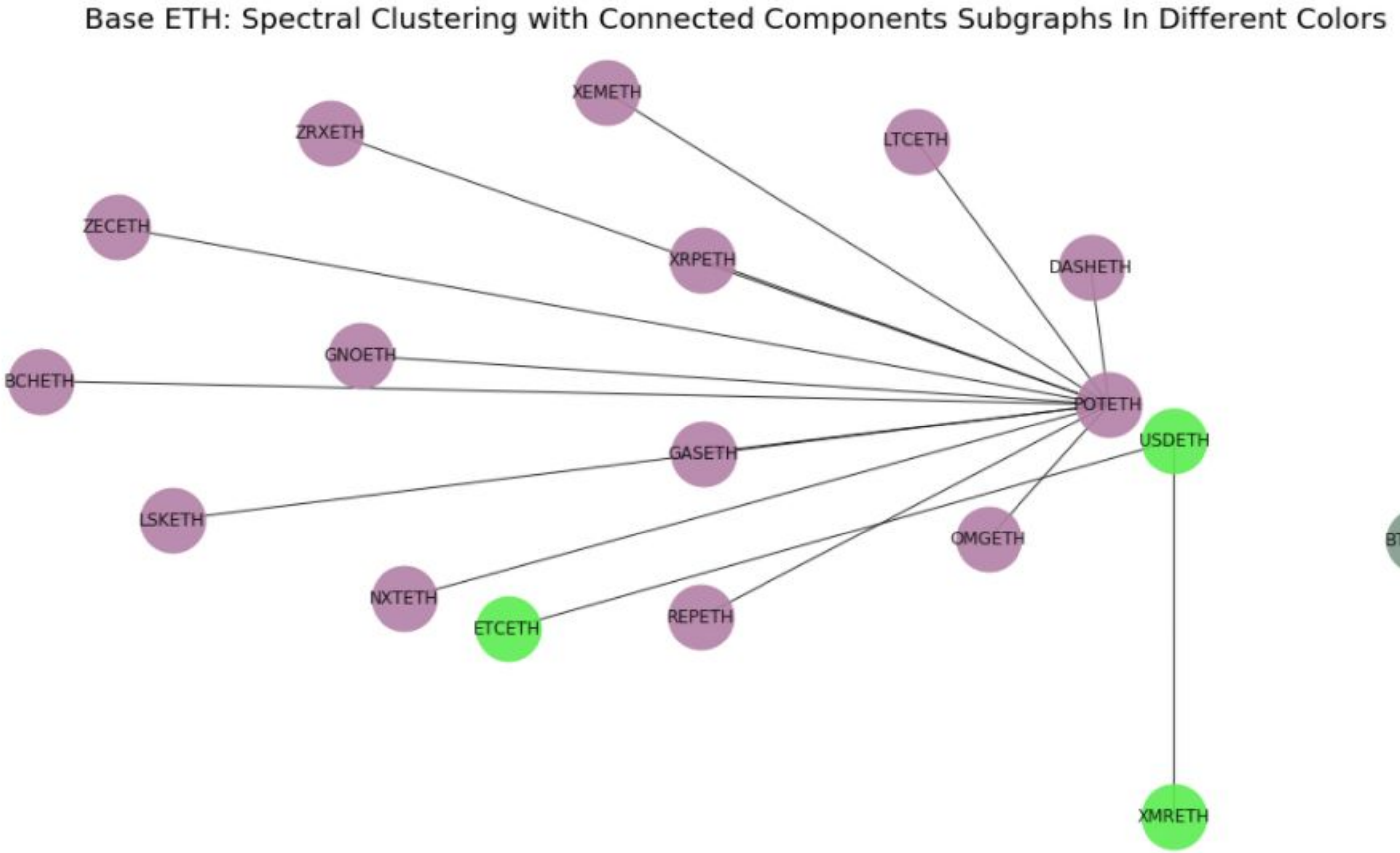
MINIMAL SPANNING TREES



THRESHOLDING *C* METHOD



SPECTRAL CLUSTERING



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Further Research

Cointegration.

Network Evolution.

Triangular arbitrage.

FURTHER RESEARCH

Methodology

Visualization

TRIANGULAR ARBITRAGE

THANK YOU!

For a copy, please contact:

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Nick Eterovic,

Eloy Lanau-Rosello

Spectral Clustering and Network Analysis of the Cryptocurrency Market

Replication of Kwapień et al.[1]

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