Applied digital signal processing - Homework 1

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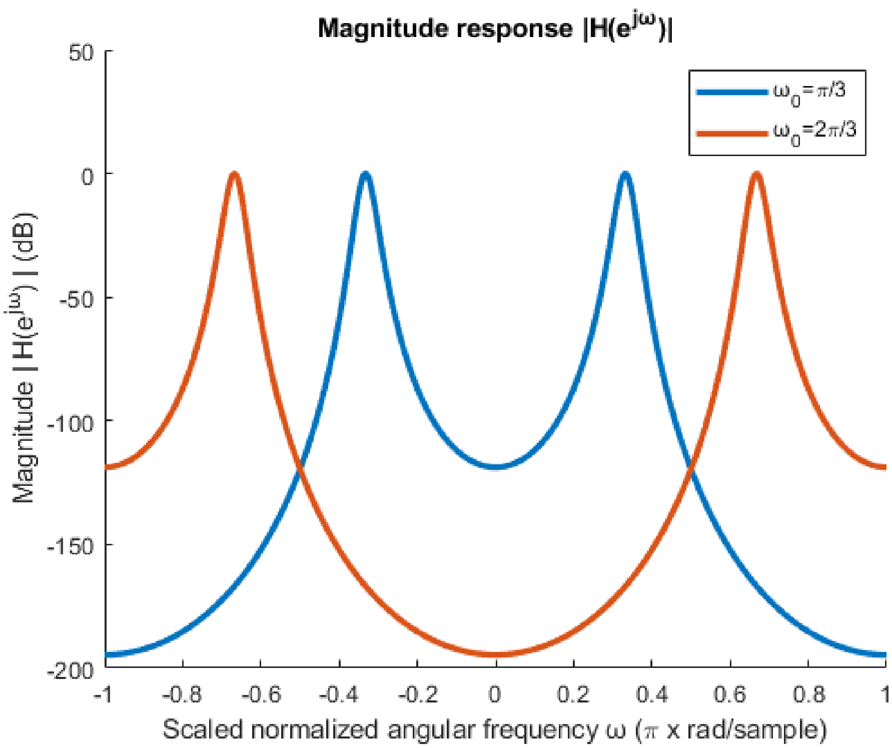
1 Magnitude response of $|H(e^{j\omega})|$

We consider the filter $H(z)$ as a cascade of K second-order filters $H_1(z) = \frac{\sqrt[K]{b_0}}{[1-2rcos(\omega_0)z^{-1}+r^2z^{-2}]} = \frac{b}{a}$ because convoluting $H_1(z)$ K times by itself means multiplying $H_1(z)$ K times by itself in the frequency domain. b and a will be used as the parameters of $freqz$ to plot the amplitude.



All of the spectrum could be summarized from $-\pi$ to π but we scale the normalized angular frequency by π to have a better intuition of what's going on. For real values of the signal the spectrum is symmetric.

This is a band-pass filter around ω_0 meaning that if you go -3 dB down from ω_0 the power is attenuated by 1/2; you are in the rejection band.
Varying ω_0 from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$ shifts the normalized frequencies by $\frac{\pi}{3}$ to the right (to the left by symmetry). The bandwith remains unchanged but the range of frequencies that are passed is not the same.

2 Autocorrelation of single echo y[n]

Starting from the definition of the autocorrelation we have

$$r_y[l] = \sum_{n=-\infty}^{+\infty} y[n]y[n-l]$$

We replace y[n] by its definition y[n] = x[n] + ax[n-D].

$$r_y[l] = \sum_{n=-\infty}^{+\infty} (x[n] + ax[n-D])(x[n-l] + ax[n-l-D])$$

$$\Leftrightarrow r_y[l] = \sum_{n=-\infty}^{+\infty} (x[n]x[n-l]+x[n]ax[n-l-D]+ax[n-D]x[n-l]+ax[n-D]ax[n-l-D])$$

$$\Leftrightarrow r_y[l] = \sum_{n=-\infty}^{+\infty} x[n]x[n-l] + \sum_{n=-\infty}^{+\infty} ax[n]x[n-(l+D)] + \sum_{n=-\infty}^{+\infty} ax[n-D]x[n-l] + \sum_{n=-\infty}^{+\infty} a^2x[n-D]x[n-l-D]$$

We can substitute n-D by n in the 2 last terms ,then we obtain

$$r_y[l] = \sum_{n=-\infty}^{+\infty} x[n]x[n-l] + \sum_{n=-\infty}^{+\infty} ax[n]x[n-(l+D)] + \sum_{n=-\infty}^{+\infty} ax[n]x[n-(l-D)] + \sum_{n=-\infty}^{+\infty} a^2x[n]x[n-l]$$

By using the definition of the autocorrelation function of the signal x[n]

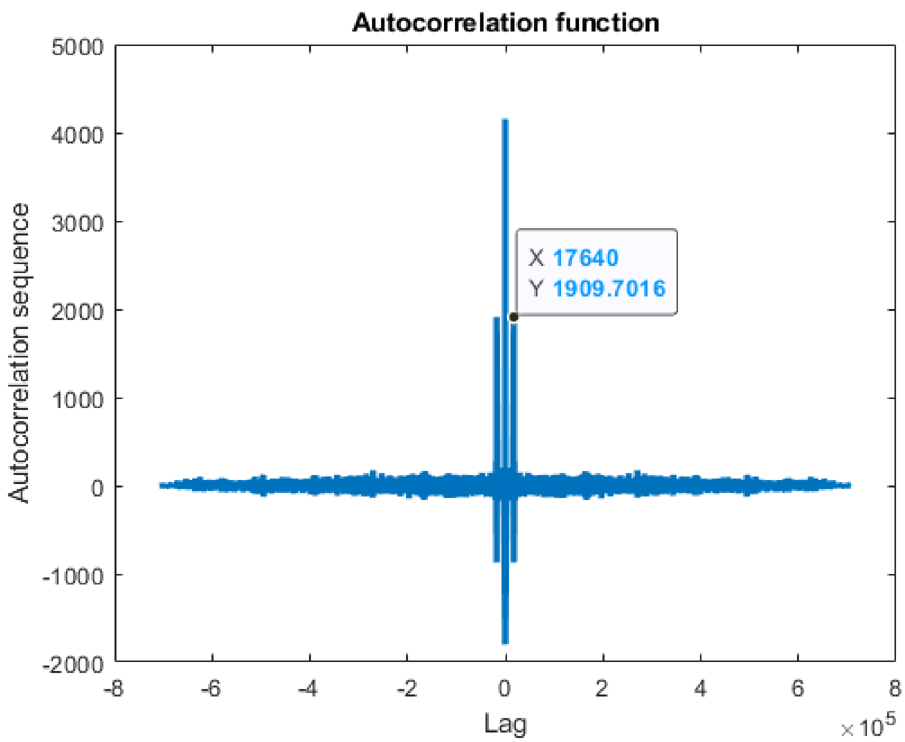
$$r_x[l] = \sum_{n=-\infty}^{+\infty} x[n]x[n-l]$$

we get

$$r_y[l] = r_x[l] + ar_x[l+D] + ar_x[l-D] + a^2r_x[l]$$
$$\Leftrightarrow r_y[l] = (1+a^2)r_x[l] + ar_x[l+D] + ar_x[l-D]$$

3 Echo cancelation

(a) Matlab functions *audioread* and *sound* allow us to read the audio then play the sound. We compute the autocorrelation function with *xcorr* then plot it. We can find the delay D by looking at the abcissa of the second peak of the autocorrelation function. We obtain D = 17640 sampling intervals. We can easily obtain the equivalent delay by using the formula $\tau = \frac{D}{F_s} \Leftrightarrow \tau = 0.4$ second where F_s is the sampling frequency returned by *audioread*.



(b) The single echo is generated using the FIR filter

$$y[n] = x[n] + 0.7x[n-d]$$

By going to the frequency domain the expression becomes

$$Y(z) = X(z) + 0.7z^{-D}X(z)$$

$$\Leftrightarrow Y(z) = X(z)(1 + 0.7z^{-D})$$

We can find the expression of the transfer function H(z) of the FIR filter

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 0.7z^{-D}$$

To get rid of the echo y[n] and get the original sound back we have to make the observed echo go through a filter that has a $H(z)^{-1}$ function transfer.
To build this filter we use the Matlab function $filter(b, a, x)$ that takes as parameters a=[1, zeros(1, D-1), α], b=1, x (the sound of the *hm1_echo.wav* that we have to filter)
where $\alpha = 0.7$ because we assume that the amplitude of the reflected sound is seventy percent of the emitted one and $D = 17640$ sampling intervals.
We can test our filter by using *sound* on the filtered sound and notice that there's no more echo.