TRANSFORMASI LAPLACE

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- **Transformasi fungsi dari ranah waktu ke ranah frekwensi**
- Metode operasional yang dapat digunakan secara mudah untuk menyelesaikan persamaan diferensial linier.
- Kelebihan:
  a) memungkinkan penggunaan teknik grafis untuk meramal kinerja sistem tanpa menyelesaikan persamaan diferensial sistem.
  b) diperolehnya penyelesaian secara serentak baik komponen transien maupun komponen keadaan tunak.

PERJANJIAN NOTASI

f(t) → fungsi di domain waktu

F(s) → fungsi di domain frekwensi

Pada materi berikutnya notasi menyesuaikan perjanjian tersebut

ILUSTRASI

ELEMEN RANGKAIAN LISTRIK DALAM DOMAIN-S

Untuk dapat mentransformasi suatu rangkaian listrik ke dalam transformasi Laplace, maka perlu didefinisikan elemen-elemen di dalam rangkaian tersebut ke dalam domain-s.

Adapun transformasi elemen-elemen rangkaian listrik ke dalam domain-s didefinisikan sebagai berikut.(John Bird, 2007: 640).

1. Resistor (R)

Dalam domain waktu (t), resistor didefinisikan oleh hukum Ohm, yaitu:

$$v_R(t) = Ri(t).$$

Transformasi Laplace dari persamaan ini yaitu,

$$\mathcal{L}\{v_R(t)\} = \mathcal{L}\{Ri(t)\} = RI(s).$$

Diperoleh v_R di dalam domain-s,

$$V_R(s) = RI(s).$$

2. Kapasitor (C)

Sebuah kapasitor dalam domain waktu *(t)* didefinisikaan sebagai,

$$i(t) = C\frac{dv_c(t)}{dt} \text{ atau } v_c(t) = \frac{1}{C}\int i(t)\,dt.$$

Transformasi Laplace dari persamaan ini yaitu,

$$\mathcal{L}\{v_c(t)\} = \mathcal{L}\left\{\frac{1}{C}\int i(t)\,dt\right\} = \frac{1}{C}\frac{I(s)}{s}.$$

Diperoleh impedansi kapasitor dalam domain-s,

$$V_c(s) = \frac{1}{sC}I(s).$$

3. Induktor (L)

Sebuah induktor dalam domain waktu *(t)* didefinisikaan sebagai,

$$v_L(t) = L\frac{di(t)}{dt}.$$

Transformasi Laplace dari persamaan ini yaitu,

$$\mathcal{L}\{v_L(t)\} = \mathcal{L}\left\{L\frac{di}{dt}\right\} = sLI(s) - Li(0).$$

Impedansi Induktor dalam domain-s didefinisikan oleh,

$$V_L(s) = L[sI(s) - i(0)].$$

APLIKASI TRANSFORMASI LAPLACE PADA RANGKAIAN LISTRIK

Jika diberikan suatu rangkaian listrik, maka prosedur/langkah-langkah untuk mencari penyelesaiannya dengan menggunakan transformasi Laplace yaitu, (John Bird, 2007:642):

1. Gunakan hukum yang berlaku pada rangkaian tersebut untuk menentukan persamaan diferensialnya (Hukum Kirchoff dan hukum Ohm).
2. Ambil transformasi Laplace pada kedua ruas persamaan yang terbentuk.
3. Masukkan nilai awal yang diberikan dan susun persamaan pembantu.
4. Gunakan invers transformasi Laplace untuk menentukan penyelesaiannya.

DEFINITION OF THE LAPLACE TRANSFORM

Let $F(t)$ be a function of t specified for $t > 0$. Then the *Laplace transform* of $F(t)$, denoted by $\mathcal{L}\{F(t)\}$, is defined by

$$\mathcal{L}\{F(t)\} \quad = \quad f(s) \quad = \quad \int_0^\infty e^{-st} F(t)\, dt \qquad (1)$$

where we assume at present that the parameter s is real. Later it will be found useful to consider s complex.

The Laplace transform of $F(t)$ is said to *exist* if the integral (*1*) *converges* for some value of s; otherwise it does not exist. For sufficient conditions under which the Laplace transform does exist, see Page 2.

NOTATION

If a function of t is indicated in terms of a capital letter, such as $F(t)$, $G(t)$, $Y(t)$, etc., the Laplace transform of the function is denoted by the corresponding lower case letter, i.e. $f(s)$, $g(s)$, $y(s)$, etc. In other cases, a tilde ($\sim$) can be used to denote the Laplace transform. Thus, for example, the Laplace transform of $u(t)$ is $\tilde{u}(s)$.

LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

The adjacent table shows Laplace transforms of various elementary functions. For details of evaluation using definition (1), see Problems 1 and 2. For a more extensive table see Appendix B, Pages 245 to 254.

	$F(t)$	$\mathcal{L}\{F(t)\} = f(s)$
1.	1	$\frac{1}{s}$ $\quad s > 0$
2.	t	$\frac{1}{s^2}$ $\quad s > 0$
3.	t^n $n = 0, 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$ $\quad s > 0$ *Note.* Factorial $n = n! = 1 \cdot 2 \cdots n$ Also, by definition $0! = 1$.
4.	e^{at}	$\frac{1}{s-a}$ $\quad s > a$
5.	$\sin at$	$\frac{a}{s^2+a^2}$ $\quad s > 0$
6.	$\cos at$	$\frac{s}{s^2+a^2}$ $\quad s > 0$
7.	$\sinh at$	$\frac{a}{s^2-a^2}$ $\quad s > \lvert a\rvert$
8.	$\cosh at$	$\frac{s}{s^2-a^2}$ $\quad s > \lvert a\rvert$

SECTIONAL OR PIECEWISE CONTINUITY

A function is called *sectionally continuous* or *piecewise continuous* in an interval $\alpha \leqq t \leqq \beta$ if the interval can be subdivided into a finite number of intervals in each of which the function is continuous and has finite right and left hand limits.

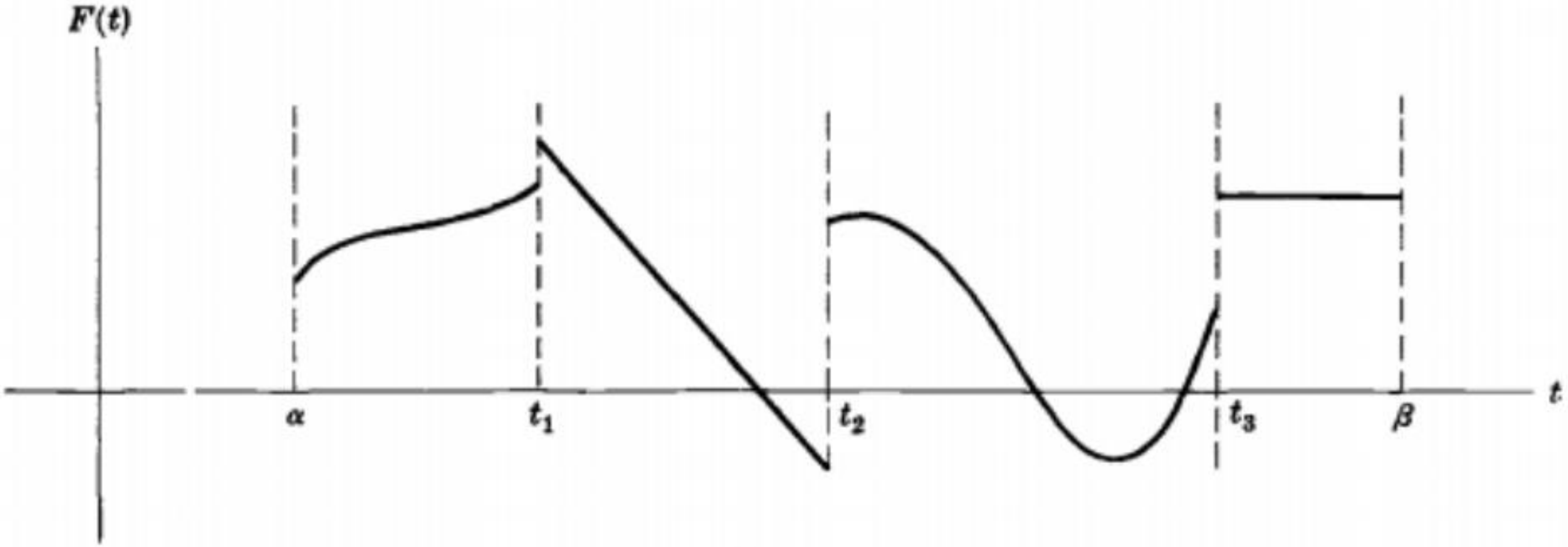

Fig. 1-1

An example of a function which is sectionally continuous is shown graphically in Fig. 1-1 above. This function has discontinuities at t_1, t_2 and t_3. Note that the right and left hand limits at t_2, for example, are represented by $\lim_{\epsilon \to 0} F(t_2 + \epsilon) = F(t_2 + 0) = F(t_2+)$ and $\lim_{\epsilon \to 0} F(t_2 - \epsilon) = F(t_2 - 0) = F(t_2-)$ respectively, where ϵ is positive.

FUNCTIONS OF EXPONENTIAL ORDER

If real constants $M > 0$ and γ exist such that for all $t > N$

$$|e^{-\gamma t} F(t)| < M \qquad \text{or} \qquad |F(t)| < Me^{\gamma t}$$

we say that $F(t)$ is a *function of exponential order* γ *as* $t \to \infty$ or, briefly, is of *exponential order*.

Example 1. $F(t) = t^2$ is of exponential order 3 (for example), since $|t^2| = t^2 < e^{3t}$ for all $t > 0$.

Example 2. $F(t) = e^{t^3}$ is not of exponential order since $|e^{-\gamma t} e^{t^3}| = e^{t^3 - \gamma t}$ can be made larger than any given constant by increasing t.

Intuitively, functions of exponential order cannot "grow" in absolute value more rapidly than $Me^{\gamma t}$ as t increases. In practice, however, this is no restriction since M and γ can be as large as desired.

Bounded functions, such as $\sin at$ or $\cos at$, are of exponential order.

SUFFICIENT CONDITIONS FOR EXISTENCE OF LAPLACE TRANSFORMS

Theorem 1-1. If $F(t)$ is sectionally continuous in every finite interval $0 \leq t \leq N$ and of exponential order γ for $t > N$, then its Laplace transform $f(s)$ exists for all $s > \gamma$.

For a proof of this see Problem 47. It must be emphasized that the stated conditions are *sufficient* to guarantee the existence of the Laplace transform. If the conditions are not satisfied, however, the Laplace transform may or may not exist [see Problem 32]. Thus the conditions are not *necessary* for the existence of the Laplace transform.

For other sufficient conditions, see Problem 145.

SOME IMPORTANT PROPERTIES OF LAPLACE TRANSFORMS

In the following list of theorems we assume, unless otherwise stated, that all functions satisfy the conditions of *Theorem 1-1* so that their Laplace transforms exist.

1. Linearity property.

Theorem 1-2. If c_1 and c_2 are any constants while $F_1(t)$ and $F_2(t)$ are functions with Laplace transforms $f_1(s)$ and $f_2(s)$ respectively, then

$$\mathcal{L}\{c_1F_1(t) + c_2F_2(t)\} = c_1\mathcal{L}\{F_1(t)\} + c_2\mathcal{L}\{F_2(t)\} = c_1f_1(s) + c_2f_2(s) \qquad (2)$$

The result is easily extended to more than two functions.

Example.

$$\begin{aligned}\mathcal{L}\{4t^2 - 3\cos 2t + 5e^{-t}\} &= 4\mathcal{L}\{t^2\} - 3\mathcal{L}\{\cos 2t\} + 5\mathcal{L}\{e^{-t}\} \\ &= 4\left(\frac{2!}{s^3}\right) - 3\left(\frac{s}{s^2+4}\right) + 5\left(\frac{1}{s+1}\right) \\ &= \frac{8}{s^3} - \frac{3s}{s^2+4} + \frac{5}{s+1}\end{aligned}$$

The symbol $\mathcal{L}$, which transforms $F(t)$ into $f(s)$, is often called the *Laplace transformation operator*. Because of the property of $\mathcal{L}$ expressed in this theorem, we say that $\mathcal{L}$ is a *linear operator* or that it has the *linearity property*.

2. First translation or shifting property.

Theorem 1-3. If $\mathcal{L}\{F(t)\} = f(s)$ then

$$\mathcal{L}\{e^{at}F(t)\} = f(s-a) \tag{3}$$

Example. Since $\mathcal{L}\{\cos 2t\} = \frac{s}{s^2+4}$, we have

$$\mathcal{L}\{e^{-t}\cos 2t\} = \frac{s+1}{(s+1)^2+4} = \frac{s+1}{s^2+2s+5}$$

3. Second translation or shifting property.

Theorem 1-4. If $\mathcal{L}\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$, then

$$\mathcal{L}\{G(t)\} = e^{-as}f(s) \tag{4}$$

Example. Since $\mathcal{L}\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$, the Laplace transform of the function

$$G(t) = \begin{cases} (t-2)^3 & t > 2 \\ 0 & t < 2 \end{cases}$$

is $6e^{-2s}/s^4$.

4. Change of scale property.

Theorem 1-5. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right) \tag{5}$$

Example. Since $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$, we have

$$\mathcal{L}\{\sin 3t\} = \frac{1}{3}\frac{1}{(s/3)^2+1} = \frac{3}{s^2+9}$$

5. Laplace transform of derivatives.

Theorem 1-6. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\{F'(t)\} \quad = \quad s\,f(s) \; - \; F(0) \tag{6}$$

if $F(t)$ is continuous for $0 \leqq t \leqq N$ and of exponential order for $t > N$ while $F'(t)$ is sectionally continuous for $0 \leqq t \leqq N$.

Example. If $F(t) = \cos 3t$, then $\mathcal{L}\{F(t)\} = \dfrac{s}{s^2+9}$ and we have

$$\mathcal{L}\{F'(t)\} \quad = \quad \mathcal{L}\{-3 \sin 3t\} \quad = \quad s\left(\frac{s}{s^2+9}\right) - 1 \quad = \quad \frac{-9}{s^2+9}$$

The method is useful in finding Laplace transforms without integration [see Problem 15].

Theorem 1-7. If in *Theorem 1-6*, $F(t)$ fails to be continuous at $t = 0$ but $\lim\limits_{t \to 0} F(t) = F(0+)$ exists [but is not equal to $F(0)$, which may or may not exist], then

$$\mathcal{L}\{F'(t)\} \quad = \quad s\,f(s) \; - \; F(0+) \tag{7}$$

Theorem 1-8. If in *Theorem 1-6*, $F(t)$ fails to be continuous at $t = a$, then

$$\mathcal{L}\{F'(t)\} = s\,f(s) - F(0) - e^{-as}\{F(a+) - F(a-)\} \tag{8}$$

where $F(a+) - F(a-)$ is sometimes called the *jump* at the discontinuity $t = a$. For more than one discontinuity, appropriate modifications can be made.

Theorem 1-9. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\{F''(t)\} = s^2 f(s) - s\,F(0) - F'(0) \tag{9}$$

if $F(t)$ and $F'(t)$ are continuous for $0 \leq t \leq N$ and of exponential order for $t > N$ while $F''(t)$ is sectionally continuous for $0 \leq t \leq N$.

If $F(t)$ and $F'(t)$ have discontinuities, appropriate modification of (*9*) can be made as in *Theorems 1-7* and *1-8*.

Theorem 1-10. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\{F^{(n)}(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) - \cdots - s\,F^{(n-2)}(0) - F^{(n-1)}(0) \tag{10}$$

if $F(t), F'(t), \ldots, F^{(n-1)}(t)$ are continuous for $0 \leq t \leq N$ and of exponential order for $t > N$ while $F^{(n)}(t)$ is sectionally continuous for $0 \leq t \leq N$.

6. Laplace transform of integrals.

Theorem 1-11. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\left\{\int_0^t F(u)\,du\right\} = \frac{f(s)}{s} \tag{11}$$

Example. Since $\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$, we have

$$\mathcal{L}\left\{\int_0^t \sin 2u\,du\right\} = \frac{2}{s(s^2+4)}$$

as can be verified directly.

7. Multiplication by t^n.

Theorem 1-12. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^{(n)}(s) \tag{12}$$

Example. Since $\mathcal{L}\{e^{2t}\} = \dfrac{1}{s-2}$, we have

$$\mathcal{L}\{te^{2t}\} = -\frac{d}{ds}\left(\frac{1}{s-2}\right) = \frac{1}{(s-2)^2}$$

$$\mathcal{L}\{t^2e^{2t}\} = \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right) = \frac{2}{(s-2)^3}$$

8. Division by t.

Theorem 1-13. If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(u)\,du \tag{13}$$

provided $\lim\limits_{t\to 0} F(t)/t$ exists.

Example. Since $\mathcal{L}\{\sin t\} = \dfrac{1}{s^2+1}$ and $\lim\limits_{t\to 0}\dfrac{\sin t}{t} = 1$, we have

$$\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{du}{u^2+1} = \tan^{-1}(1/s)$$

9. Periodic functions.

Theorem 1-14. Let $F(t)$ have period $T > 0$ so that $F(t+T) = F(t)$ [see Fig. 1-2].

Then
$$\mathcal{L}\{F(t)\} = \frac{\int_0^T e^{-st} F(t)\, dt}{1 - e^{-sT}} \tag{14}$$

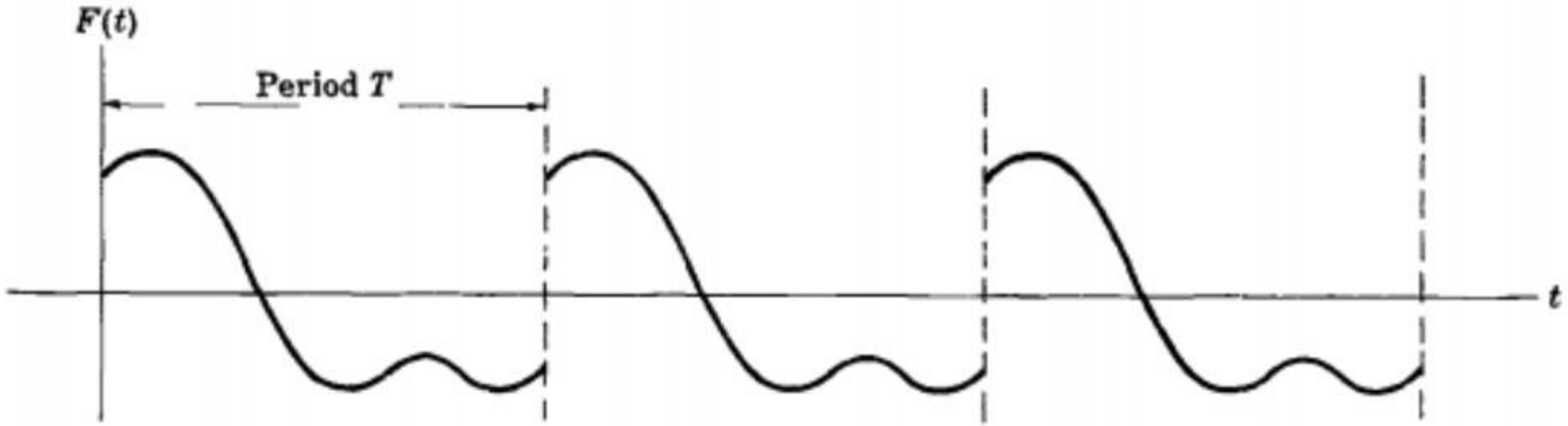

Fig. 1-2

Supplementary Problems

LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

51. Find the Laplace transforms of each of the following functions. In each case specify the values of s for which the Laplace transform exists.

(a) $2e^{4t}$	*Ans.* (a) $2/(s-4)$,	$s > 4$
(b) $3e^{-2t}$	(b) $3/(s+2)$,	$s > -2$
(c) $5t - 3$	(c) $(5-3s)/s^2$,	$s > 0$
(d) $2t^2 - e^{-t}$	(d) $(4+4s-s^3)/s^3(s+1)$,	$s > 0$
(e) $3\cos 5t$	(e) $3s/(s^2+25)$,	$s > 0$
(f) $10\sin 6t$	(f) $60/(s^2+36)$,	$s > 0$
(g) $6\sin 2t - 5\cos 2t$	(g) $(12-5s)/(s^2+4)$,	$s > 0$
(h) $(t^2+1)^2$	(h) $(s^4+4s^2+24)/s^5$,	$s > 0$
(i) $(\sin t - \cos t)^2$	(i) $(s^2-2s+4)/s(s^2+4)$,	$s > 0$
(j) $3\cosh 5t - 4\sinh 5t$	(j) $(3s-20)/(s^2-25)$,	$s > 5$

52. Evaluate (a) $\mathcal{L}\{(5e^{2t}-3)^2\}$, (b) $\mathcal{L}\{4\cos^2 2t\}$.

Ans. (a) $\dfrac{25}{s-4}-\dfrac{30}{s-2}+\dfrac{9}{s},\ s>4$ (b) $\dfrac{2}{s}+\dfrac{2s}{s^2+16},\ s>0$

53. Find $\mathcal{L}\{\cosh^2 4t\}$. *Ans.* $\dfrac{s^2-32}{s(s^2-64)}$

54. Find $\mathcal{L}\{F(t)\}$ if (a) $F(t)=\begin{cases}0 & 0<t<2\\ 4 & t>2\end{cases}$, (b) $F(t)=\begin{cases}2t & 0\leqq t\leqq 5\\ 1 & t>5\end{cases}$

Ans. (a) $4e^{-2s}/s$ (b) $\dfrac{2}{s^2}(1-e^{-5s})-\dfrac{9}{s}e^{-5s}$

55. Prove that $\mathcal{L}\{t^n\}=\dfrac{n!}{s^{n+1}}$, $n=1,2,3,\ldots$.

56. Investigate the existence of the Laplace transform of each of the following functions.

(a) $1/(t+1)$, (b) e^{t^2-t}, (c) $\cos t^2$ *Ans.* (a) exists, (b) does not exist, (c) exists

58. Evaluate each of the following.

(a) $\mathcal{L}\{t^3e^{-3t}\}$ Ans. (a) $6/(s+3)^4$

(b) $\mathcal{L}\{e^{-t}\cos 2t\}$ (b) $(s+1)/(s^2+2s+5)$

(c) $\mathcal{L}\{2e^{3t}\sin 4t\}$ (c) $8/(s^2-6s+25)$

(d) $\mathcal{L}\{(t+2)^2e^t\}$ (d) $(4s^2-4s+2)/(s-1)^3$

(e) $\mathcal{L}\{e^{2t}(3\sin 4t - 4\cos 4t)\}$ (e) $(20-4s)/(s^2-4s+20)$

(f) $\mathcal{L}\{e^{-4t}\cosh 2t\}$ (f) $(s+4)/(s^2+8s+12)$

(g) $\mathcal{L}\{e^{-t}(3\sinh 2t - 5\cosh 2t)\}$ (g) $(1-5s)/(s^2+2s-3)$

59. Find (a) $\mathcal{L}\{e^{-t}\sin^2 t\}$, (b) $\mathcal{L}\{(1+te^{-t})^3\}$.

Ans. (a) $\dfrac{2}{(s+1)(s^2+2s+5)}$ (b) $\dfrac{1}{s}+\dfrac{3}{(s+1)^2}+\dfrac{6}{(s+2)^3}+\dfrac{6}{(s+3)^4}$

60. Find $\mathcal{L}\{F(t)\}$ if $F(t) = \begin{cases} (t-1)^2 & t > 1 \\ 0 & 0 < t < 1 \end{cases}$. Ans. $2e^{-s}/s^3$

61. If $F_1(t), F_2(t), \ldots, F_n(t)$ have Laplace transforms $f_1(s), f_2(s), \ldots, f_n(s)$ respectively and $c_1, c_2, \ldots, c_n$ are any constants, prove that

$$\mathcal{L}\{c_1F_1(t) + c_2F_2(t) + \cdots + c_nF_n(t)\} = c_1f_1(s) + c_2f_2(s) + \cdots + c_nf_n(s)$$

62. If $\mathcal{L}\{F(t)\} = \dfrac{s^2 - s + 1}{(2s+1)^2(s-1)}$, find $\mathcal{L}\{F(2t)\}$. *Ans.* $(s^2 - 2s + 4)/4(s+1)^2(s-2)$

63. If $\mathcal{L}\{F(t)\} = \dfrac{e^{-1/s}}{s}$, find $\mathcal{L}\{e^{-t}F(3t)\}$. *Ans.* $\dfrac{e^{-3/(s+1)}}{s+1}$

64. If $f(s) = \mathcal{L}\{F(t)\}$, prove that for $r > 0$,

$$\mathcal{L}\{r^t F(at)\} = \frac{1}{s - \ln r} f\left(\frac{s - \ln r}{a}\right)$$

LAPLACE TRANSFORMS OF DERIVATIVES

65. (a) If $\mathcal{L}\{F(t)\} = f(s)$, prove that

$$\mathcal{L}\{F'''(t)\} = s^3 f(s) - s^2 F(0) - s F'(0) - F''(0)$$

stating appropriate conditions on $F(t)$.

(b) Generalize the result of (a) and prove by use of mathematical induction.

66. Given $F(t) = \begin{cases} 2t & 0 \leqq t \leqq 1 \\ t & t > 1 \end{cases}$. (a) Find $\mathcal{L}\{F(t)\}$. (b) Find $\mathcal{L}\{F'(t)\}$. (c) Does the result $\mathcal{L}\{F'(t)\} = s\mathcal{L}\{F(t)\} - F(0)$ hold for this case? Explain.

Ans. (a) $\dfrac{2}{s^2} - \dfrac{e^{-s}}{s} - \dfrac{e^{-s}}{s^2}$, (b) $\dfrac{2}{s} - \dfrac{e^{-s}}{s}$

67. (a) If $F(t) = \begin{cases} t^2 & 0 < t \leqq 1 \\ 0 & t > 1 \end{cases}$, find $\mathcal{L}\{F''(t)\}$.

LAPLACE TRANSFORMS OF INTEGRALS

69. Verify directly that $\mathcal{L}\left\{\int_0^t (u^2 - u + e^{-u})\,du\right\} = \frac{1}{s}\mathcal{L}\{t^2 - t + e^{-t}\}$.

70. If $f(s) = \mathcal{L}\{F(t)\}$, show that $\mathcal{L}\left\{\int_0^t dt_1 \int_0^{t_1} F(u)\,du\right\} = \frac{f(s)}{s^2}$.

$\left[\text{The double integral is sometimes briefly written as } \int_0^t \int_0^t F(t)\,dt^2.\right]$

71. Generalize the result of Problem 70.

72. Show that $\mathcal{L}\left\{\int_0^t \frac{1 - e^{-u}}{u}\,du\right\} = \frac{1}{s}\ln\left(1 + \frac{1}{s}\right)$.

73. Show that $\int_{t=0}^{\infty} \int_{u=0}^{t} \frac{e^{-t}\sin u}{u}\,du\,dt = \frac{\pi}{4}$.

MULTIPLICATION BY POWERS OF t

74. Prove that (a) $\mathcal{L}\{t\cos at\} = \dfrac{s^2-a^2}{(s^2+a^2)^2}$

(b) $\mathcal{L}\{t\sin at\} = \dfrac{2as}{(s^2+a^2)^2}$

75. Find $\mathcal{L}\{t(3\sin 2t - 2\cos 2t)\}$. *Ans.* $\dfrac{8+12s-2s^2}{(s^2+4)^2}$

76. Show that $\mathcal{L}\{t^2\sin t\} = \dfrac{6s^2-2}{(s^2+1)^3}$.

77. Evaluate (a) $\mathcal{L}\{t\cosh 3t\}$, (b) $\mathcal{L}\{t\sinh 2t\}$. *Ans.* (a) $(s^2+9)/(s^2-9)^2$, (b) $4s/(s^2-4)^2$

78. Find (a) $\mathcal{L}\{t^2\cos t\}$, (b) $\mathcal{L}\{(t^2-3t+2)\sin 3t\}$.

Ans. (a) $(2s^3-6s)/(s^2+1)^3$, (b) $\dfrac{6s^4-18s^3+126s^2-162s+432}{(s^2+9)^3}$

79. Find $\mathcal{L}\{t^3\cos t\}$. *Ans.* $\dfrac{6s^4-36s^2+6}{(s^2+1)^4}$

DIVISION BY t

81. Show that $\mathcal{L}\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\} = \ln\left(\dfrac{s+b}{s+a}\right)$.

82. Show that $\mathcal{L}\left\{\dfrac{\cos at - \cos bt}{t}\right\} = \dfrac{1}{2}\ln\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$.

83. Find $\mathcal{L}\left\{\dfrac{\sinh t}{t}\right\}$. *Ans.* $\dfrac{1}{2}\ln\left(\dfrac{s+1}{s-1}\right)$

84. Show that $\displaystyle\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}\,dt = \ln 2$.

[*Hint.* Use Problem 81.]

85. Evaluate $\displaystyle\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt$. *Ans.* $\ln(3/2)$

86. Show that $\displaystyle\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \frac{\pi}{2}$.

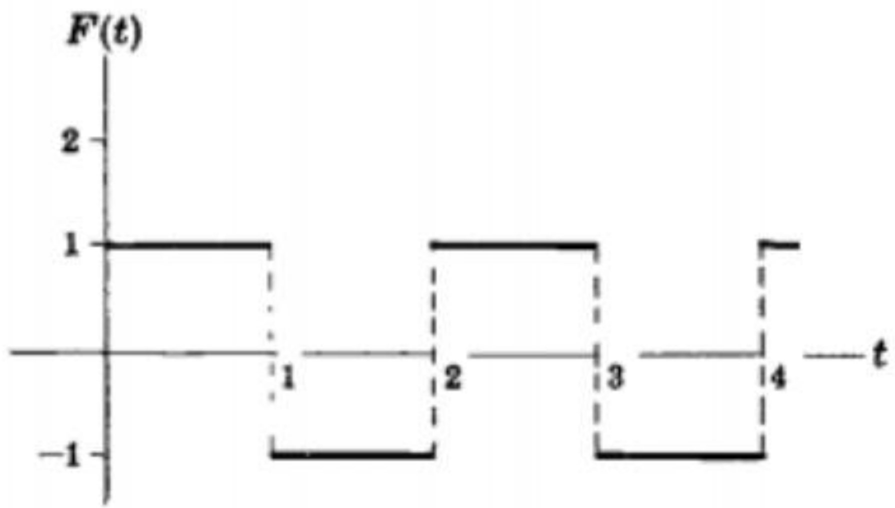

Fig. 1-7

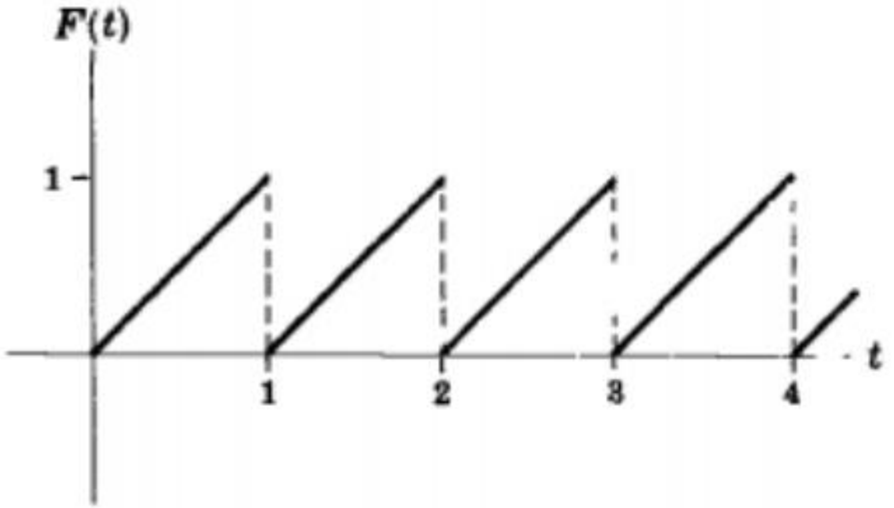

Fig. 1-8

88. Find $\mathcal{L}\{F(t)\}$ where $F(t)$ is the periodic function shown graphically in Fig. 1-8 above.

Ans. $\dfrac{1}{s^2} - \dfrac{e^{-s}}{s(1-e^{-s})}$

89. Let $F(t) = \begin{cases} 3t & 0 < t < 2 \\ 6 & 2 < t < 4 \end{cases}$ where $F(t)$ has period 4. (*a*) Graph $F(t)$. (*b*) Find $\mathcal{L}\{F(t)\}$.

Ans. (*b*) $\dfrac{3 - 3e^{-2s} - 6se^{-4s}}{s^2(1-e^{-4s})}$