Fields and Galois Theory

PMATH 348

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Preface

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Introduction

One way to motivate Galois theory is to consider polynomial equations over real numbers. The simplest polynomial equation that we can solve is linear equation: $ax + b = 0$ with $a, b \in \mathbb{R}$ and $a \neq 0$. The solution to this equation is $x = -\frac{b}{a}$.

We can also solve the quadratic equations: $ax^2 + bx + c = 0$ with $a, b, c \in \mathbb{R}$ and $a \neq 0$. It solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

When $b^2 - 4ac < 0$, we need the notion of complex numbers to such an equation.

We notice that the solutions of linear and quadratic equations only use addition, subtraction, multiplication, division and the nth root. This motivates the definition of radical expression.

radical expression

An expression involving only $+, -, *, \div, \sqrt[n]{}$ is called **radical**.

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