Applied Cryptography

CO 487

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LaTeXed by Sibelius Peng

Preface

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Sibelius Peng

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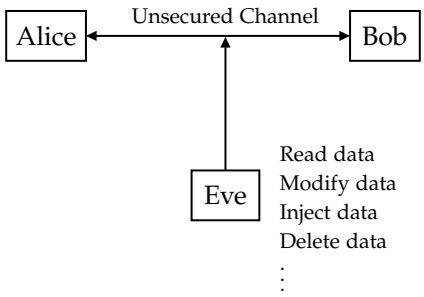
Introduction

Cryptography is about securing communications in the presence of *malicious* adversaries.



Note that even if we call him "Eve", he can do more than eavesdropping... The adversary is malicious, powerful and unpredictable.

Fundamental Goals of Cryptography

1. Confidentiality: Keeping data secret from all but those authorized to see it.
2. Data integrity: Ensuring data has not been altered by unauthorized means.
3. Data origin authentication: Corroborating the source of data.
4. Non-repudiation: Preventing an entity from denying previous commitments or actions.

Some examples of unsecured channel:

- Secure Browsing: The Internet
- Online Shopping: The Internet
- Automatic Software Upgrades: The Internet
- Cell Phone Service: Wireless
- Wi-Fi: Wireless
- Bluetooth: Wireless
- Messaging: Wired/Wireless

Communicating Parties

Alice and Bob are two communicating devices.

Alice	Bob	Communication channel
person	person	telephone cable
person	person	cellular network
person	web site	internet
iPhone	wireless	router wireless
iPhone	headphones	wireless
iPhone	service provider	cellular network
your car's brakes	another car	wireless
smart card	bank machine	financial network
smart meter	energy provider	wireless
military commander	satellite	space

1.1 Secure Web Transactions

Transport Layer Security (TLS): The cryptographic protocol used by web browsers for secure web transactions for secure access to amazon, gmail, hotmail, facebook etc.

TLS is used to assure an individual user (called a client) of the authenticity of the web site (called the server) he or she is visiting, and to establish a secure communications channel for the remainder of the session.

Symmetric-key cryptography: The client and server a priori share some secret information k, called a key.

They can subsequently engage in secure communications by encrypting their messages with AES and authenticating the resulting ciphertexts with HMAC.

How do they establish the shared secret key k?

Public-key cryptography: Communicating parties a priori share some authenticated (but non-secret) information.

To establish a secret key, the client selects the secret session key k, and encrypts it with the server's RSA public key. Then only the server can decrypt the resulting ciphertext with its RSA private key to recover k.

How does the client obtain an authentic copy of the server's RSA public key?

Signature scheme: The server's RSA public key is signed by a Certifying Authority using the RSA signature scheme.

The client can verify the signature using the Certifying Authority's RSA public verification key which is embedded in its browser. In this way, the client obtains an authentic copy of the server's RSA public key.

1.2 The TLS Protocol

1. When a client first visits a secured web page, the server transmits its certificate to the client.
    - The certificate contains the server's identifying information (e.g., web site name and URL) and RSA public key, and the RSA signature of a certifying authority.

- The certifying authority (e.g., Verisign) is trusted to carefully verify the server's identity before issuing the certificate.

2. Upon receipt of the certificate, the client verifies the signature using the certifying authority's public key, which is embedded in the browser. A successful verification confirms the authenticity of the server and of its RSA public key.
3. The client selects a random session key k, encrypts it with the server's RSA public key, and transmits the resulting ciphertext to the server.
4. The server decrypts the ciphertext to obtain the session key, which is then used with symmetric-key schemes to encrypt (e.g. with AES) and authenticate (e.g. with HMAC) all sensitive data exchanged for the remainder of the session.
5. The establishment of a secure link is indicated by a closed padlock in the browser. Clicking on this icon reveals the server's certificate and information about the certifying authority.

TLS is one of the most successful security technologies ever deployed. But is TLS really secure?

There are many potential security vulnerabilities:

1. The crypto is weak (e.g., AES, HMAC, RSA).
2. Quantum attacks on the underlying cryptography.
3. Weak random number generation.
4. Issuance of fraudulent certificates
   - In 2001, Verisign erroneously issued two Class 3 code-signing certificates to a person masquerading as a Microsoft representative.
   - Mistake due to human error.
5. Software bugs (both inadvertent and malicious).
6. Phishing attacks.
7. TLS only protects data during transit. It does not protect your data when it is collected at the server.

   Many servers store large amounts of credit card data and other personal information.
8. The National Security Agency (NSA)

1.3 Cryptography in Context

Cybersecurity is comprised of the concepts, technical measures, and administrative measures used to protect networks, computers, programs and data from deliberate or inadvertent unauthorized access, disclosure, manipulation, loss or use. Also known as information security. Cybersecurity includes the study of computer security, network security and software security.

Note that Cryptography $\neq$ Cybersecurity.

- Cryptography provides some mathematical tools that can assist with the provision of cybersecurity services. It is a small, albeit an indispensable, part of a complete security solution.
- Security is a chain
  - Weak links become targets; one flaw is all it takes.

- Cryptography is usually not the weakest link. However, when the crypto fails the damage can be catastrophic.

2

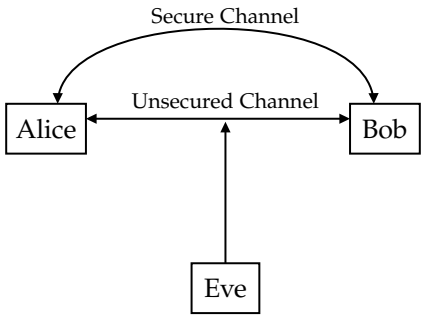
Symmetric-Key Cryptography

2.1 Basic concepts

symmetric-key encryption scheme

A **symmetric-key encryption scheme (SKES)** consists of:

- M - the plaintext space,
- C - the ciphertext space,
- K - the key space,
- a family of encryption functions: $E_k : M \to C$, $\forall k \in K$,
- a family of decryption functions: $D_k : M \to C$, $\forall k \in K$, such that $D_k(E_k(m)) = m$ for all $m \in M, k \in K$.



1. Alice and Bob agree on a secret key $k \in K$ by communicating over the secure channel.
2. Alice computes $c = E_k(m)$ and sends the ciphertext c to Bob over the unsecured channel.
3. Bob retrieves the plaintext by computing $m = D_k(c)$.

Some examples:

- WWII: Enigma Machine, Alan Turing,

  https://en.wikipedia.org/wiki/Cryptanalysis_of_the_Enigma

- WWII: Lorenz Machine, Bill Tutte, Colossus

http://tinyurl.com/COBillTutte, http://billtuttememorial.org.uk/

2.1.1 The Simple Substitution Cipher

- M = all English msgs.
- C = all encrypted msgs.
- K = all permutations of the English alphabet.
- $E_k(m)$: Apply permutation k to m, one letter at a time.
- $D_k(c)$: Apply inverse permutations k^{-1} to c, one letter at a time.

Example:

$$k = \begin{array}{cccccccccccccccccccccccc} a & b & c & d & e & f & g & h & i & j & k & l & m & n & o & p & q & r & s & t & u & v & w & x & y & z \\ D & N & X & E & S & K & O & J & T & A & F & P & Y & I & Q & U & B & R & Z & G & V & C & H & M & W & L \end{array}$$

Encryption: $m = the\ big\ dog$, $c = E_k(the\ big\ dog) = GJS\ NTO\ EQO$.

Decryption: $c = GJS\ NTO\ EQO$, $m = E_k^{-1}(GJS\ NTO\ EQO) = the\ big\ dog$.

Is the simple substitution cipher a secure SKES? but wait, what does it mean for a SKES to be secure? We need a **security definition**.

1. What are the computational powers of the adversary?
2. How does the adversary interact with the two communicating parties?
3. What is the adversary's goal?

Security model: Defines the computational abilities of the adversary, and how she interacts with the communicating parties.

Basic assumption: The adversary knows everything about the SKES, except the particular key k chosen by Alice and Bob. (Avoid security by obscurity!!) Security should only depend on the secrecy of the secret keying material, not on the secrecy of the algorithms that describe the cryptographic scheme.

Let's now consider the computational power of the adversary.

- **Information-theoretic security**: Eve has infinite computational resources.
- **Complexity-theoretic security**: Eve is a 'polynomial-time Turing machine'.
- **Computational security**: Eve has 36,768 Intel E5-2683 V4 cores running at 2.1 GHz at her disposal. See: Graham in the basement of MC

  We say: Eve is "*computationally bounded*".

In this course, we will be concerned with computational security.

Next consider how the adversary can interact with Alice and Bob.

- Passive attacks:
  - **Ciphertext-only attack**: The adversary knows some ciphertext (that was generated by Alice or Bob).
  - **Known-plaintext attack**: The adversary also knows some plaintext and the corresponding ciphertext. (stronger because more info than before)
- Active attacks:

  - **Chosen-plaintext attack**: The adversary can also choose some plaintext and obtains the corresponding ciphertext. This is at least as strong as known-plaintext attack because the attacker gets to choose the plain text for which it is given the corresponding ciphertext.

- Other attacks (not considered in this course):
  - Clandestine attacks: bribery, blackmail, etc.
  - Side-channel attacks: monitor the encryption and decryption equipment (timing attacks, power analysis attacks, electromagnetic-radiation analysis, etc.)

Finally, we will consider the adversary's goal (from strongest to weakest).

1. Recover the secret key.
2. Systematically recover plaintext from ciphertext (without necessarily learning the secret key).
3. Learn some partial information about the plaintext from the ciphertext (other than its length).

If the adversary can achieve 1 or 2, the SKES is said to be **totally insecure** (or **totally broken**).

If the adversary cannot learn any partial information about the plaintext from the ciphertext (except possibly its length), the SKES is said to be **semantically secure**. So the ciphertext hides all the meaning about the corresponding plaintext.

security of SKES

A symmetric-key encryption scheme is said to be **secure** if it is semantically secure against chosen-plaintext attack by a computationally bounded adversary.

Note that the definitions captures the computational abilities of the adversary, namely some concrete upper bound on its computational resources. It captures how the adversary interacts with Alice and Bob, namely by mounting a chosen-plaintext attack. And finally, it captures the goal of the adversary, namely breaking semantic security. Note also that in the definition the attacker's capabilities are *as strong as possible*, namely a chosen-plaintext attack and its goal is *as weak as possible* namely, breaking semantic security

From the definition, we can also deduce what it means to break an encryption scheme.

1. The adversary is given a challenge ciphertext c (generated by Alice or Bob using their secret key k).
2. During its computation, the adversary can select plaintexts and obtains (from Alice or Bob) the corresponding ciphertexts.
3. After a feasible amount of computation, the adversary obtains some information about the plaintext m corresponding to the challenge ciphertext c (other than the length of m).

If the attacker can win this game specified in these three steps, then we'll say that the encryption scheme is in secure.

Desirable Properties of a SKES

1. Efficient algorithms should be known for computing E_k and D_k (i.e., for encryption and decryption).
2. The secret key should be small (but large enough to render exhaustive key search infeasible).
3. The scheme should be secure.

4. The scheme should be secure even against the designer of the system.

The last point is to ensure that the designer hasn't somehow inserted a backdoor into the encryption scheme whereby the adversary can learn a secret keying material by looking at ciphertext, while this method would be hard to find by other people. This fourth property can sometimes be very difficult to obtain in practice.

Now we can see whether the simple substitution cipher is secure: Totally insecure against a chosen-plaintext attack. This is because the adversary can simply give the plaintext message that's comprised of the English alphabet from a to z to Alice for encryption. Alice returns to the adversary the ciphertext, which it turns out is the secret key itself. And so by simply giving Alice a short plaintext and getting back the corresponding ciphertext the adversary has learnt Alice's secret key.

What about security under a ciphertext-only attack? Is exhaustive key search possible?

- Given sufficient amounts of ciphertext c, decrypt c using each possible key until c decrypts to a plaintext message which "makes sense".
- In principle, 30 characters of ciphertext are sufficient on average to yield a unique plaintext that is a sensible English message. In practice, a few hundred characters are needed.

Exhaustive search:

- Number of keys to try is $26! \approx 2^{88}$
- If the adversary uses 10^6 computers, each capable of trying 10^9 keys per second, then exhaustive key search takes about 10^4 years.
- So, exhaustive key search is infeasible.

In this course,

- 2^{40} operations is considered very easy.
- 2^{56} operations is considered easy.
- 2^{64} operations is considered feasible.
- 2^{80} operations is considered barely feasible.
- 2^{128} operations is considered infeasible.

The Bitcoin network is presently performing hash operations at the rate of $2^{66.4}$ per second (or $2^{91.3}$ per year).

The Landauer limit from thermodynamics suggests that exhaustively trying 2^{128} symmetric keys would require $\gg$ 3000 gigawatts of power for one year (which is $\gg$ 100% of the world's energy production). http://en.wikipedia.org/wiki/Brute-force_attack

security level

A cryptographic scheme is said to have a **security level** of ℓ bits if the fasted known attack on the scheme takes approximately 2^{ℓ} operations.

As of the year 2021, a security level of 128 bits is desirable in practice.

Of course, simple frequency analysis of ciphertext letters can be used to recover the key. One would simply find the most frequently occurring ciphertext letter; this would most likely correspond to the English letter e which is the most frequently occurring letter in the English alphabet. Hence, the

simple substitution cipher is totally insecure even against a ciphertext-only attack.

2.1.2 Polyalphabetic Ciphers

Basic idea: Use several permutations, so a plaintext letter is encrypted to one of several possible ciphertext letters.

Example: Vigenère cipher

Secret key is an English word having no repeated letters. E.g., $k = CRYPTO$.

Example of encryption:

	m	=	t	h	i	s	i	s	a	m	e	s	s	a	g	e
+	k	=	C	R	Y	P	T	O	C	R	Y	P	T	O	C	R
	c	=	V	Y	G	H	B	G	C	D	C	H	L	O	I	V

Here, $A = 0, B = 1, \ldots, Z = 25$; addition of letters is mod 26.

Decryption is subtraction modulo 26: $m = c - k$.

Frequency distribution of ciphertext letters is flatter (than for a simple substitution cipher).

The Vigenère cipher is totally insecure against a chosen-plaintext attack. Not unexpectedly, the Vigenère cipher is also totally insecure against a ciphertext-only attack.

2.2 The One-Time Pad

The one-time pad is a symmetric-key encryption scheme invented by Vernam in 1917 for the telegraph system. The key is a random[1] string of letters. Example of encryption:

	m	=	t	h	i	s	i	s	a	m	e	s	s	a	g	e
+	k	=	Z	F	K	W	O	G	P	S	M	F	J	D	L	G
	c	=	S	M	S	P	W	Y	P	F	Q	X	C	D	R	K

Note that the key is as long as the plaintext. One major disadvantage of the one-time pad is that the secret key is of the same length as the plaintext. This is especially inconvenient if Alice and Bob are exchanging long messages over a period of time, and so it might be tempting in practice to reuse the secret key. The key should *not* be re-used: If $c_1 = m_1 + k$ and $c_2 = m_2 + k$, then $c_1 - c_2 = m_1 - m_2$. Thus $c_1 - c_2$ depends only on the plaintext (and not on the key) and hence can leak information about the plaintext. In particular, if m_1 is known, then m_2 can be easily computed.

Convention: From now on, unless otherwise stated, messages and keys will be assumed to be binary/bit strings.

$\oplus$

$\oplus$ is bitwise exclusive-or (XOR) i.e., bitwise addition modulo 2. For example:

$$1011001010 \oplus 1001001001 = 0010000011.$$

Note that $x \oplus x = 0$ and $x \oplus y = y \oplus x$. Hence if $x = y \oplus z$, then $x \oplus y = z$.

So, for the one-time pad:

Encryption: $c = m \oplus k$.

[1] independently and uniformly at random

Decryption: $m = c \oplus k$.

https://cryptosmith.com/2008/05/31/stream-reuse/ is an example which reuses the same key...

Security of One-Time Pad

Perfect secrecy: The one-time pad is semantically secure against ciphertext-only attack by an adversary with infinite computational resources. This can be proven formally using concepts from information theory [Shannon 1949].

The bad news: Shannon (1949) proved that if plaintexts are m-bit strings, then any symmetric-key encryption scheme with perfect secrecy must have $|K| \geq 2^m$. So, perfect secrecy (and the one-time pad) is fairly useless in practice. However, all is not lost. One can use one-time pads to inspire the construction of so-called stream ciphers.

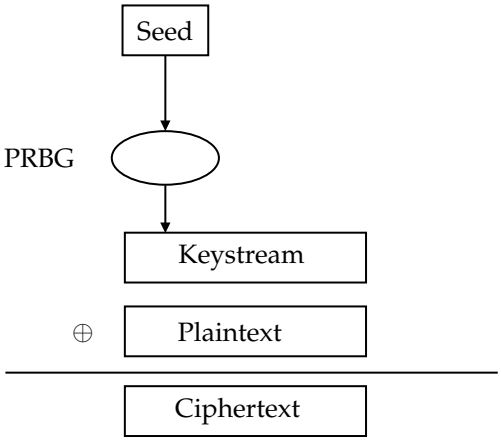
2.3 Stream Ciphers

Basic idea: Instead of using a random key in the one-time pad, use a "pseudorandom" key.

A **pseudorandom bit generator** (PRBG) is a deterministic algorithm that takes as input a (relatively small random) seed, and outputs a longer "pseudorandom" sequence called the keystream.

Using a PRBG for encryption (a stream cipher):

The seed is the secret key shared by Alice and Bob.



The XOR portion of the stream cipher emulates a one-time pad. Note though that we no longer have perfect secrecy because the keystream is no longer purely random but pseudo-random. Thus security depends on the quality of the PRBG.

Security Requirements for the PRBG

The keystream should be "indistinguishable" from a random sequence (the **indistinguishability requirement**).

If an adversary knows a portion c_1 of ciphertext and the corresponding plaintext m_1, then she can easily find the corresponding portion $k_1 = c_1 \oplus m_1$ of the keystream. Thus, given portions of the keystream, it should be infeasible to learn any information about the rest of the keystream (the **unpredictability requirement**).

Aside: Don't use UNIX random number generators (`rand` and `srand`) for cryptography!

- $X_0 = \text{seed}$, $X_{i+1} = aX_i + b \mod n$, $i \geq 0$.

- a, b and n are fixed and public constants.

And so the UNIX routines `rand` and `srand` do not meet the unpredictability requirement.

One difficulty with using stream ciphers in practice is that we still have the requirement as we did with the one-time pad that keystream should never be reused.

2.4 The RC4 Stream Cipher

Designed by Ron Rivest in 1987.

Until recently, was widely used in commercial products including Adobe Acrobat, Windows password encryption, Oracle secure SQL, TLS, etc.

- *Pros*: Extremely simple; extremely fast; variable key length. No catastrophic weakness has been found since 1987.
- *Cons*: Design criteria are proprietary; not much public scrutiny until the year 2001.

In the past 10 years, many weaknesses have been found in RC4 and so its use has rapidly declined in practice.

RC4 has two components: (i) a *key scheduling algorithm*, and (ii) a *keystream generator*.

In the following, $K[i], \overline{K}[i]$ and $S[i]$ are 8-bit integers (bytes).

Algorithm 1: RC4 Key Scheduling Algorithm

Input: Secret key $K[0], K[1], \ldots, K[d-1]$. (Keylength is $8d$ bits.)
Output: 256-long array: $S[0], S[1], \ldots, S[255]$.

for $i = 0, \ldots, 255$ **do**
  $S[i] \leftarrow i$
  $\overline{K}[i] \leftarrow K[i \mod d]$
$j \leftarrow 0$
for $i = 0, \ldots, 255$ **do**
  $j \leftarrow (\overline{K}[i] + S[i] + j) \mod 256$
  Swap$(S[i], S[j])$

Idea: S is a "random-looking" permutation of $\{0, 1, 2, \ldots, 255\}$ that is generated from the secret key.

Algorithm 2: RC4 Keystream Generator

Input: 256-long byte array: $S[0], S[1], \ldots, S[255]$ produced by the RC4 Key Scheduling Algorithm
Output: Keystream.

$i \leftarrow 0; j \leftarrow 0$
while *keystream bytes are required* **do**
  $i \leftarrow (i+1) \mod 256$
  $j \leftarrow (S[i] + j) \mod 256$
  Swap$(S[i], S[j])$
  $t \leftarrow (S[i] + S[j]) \mod 256$
  Output$(S[t])$

ENCRYPTION: The keystream bytes are stored with the plaintext bytes to produce ciphertext bytes.

2.4.1 Wired Equivalent Privacy

Wireless Security

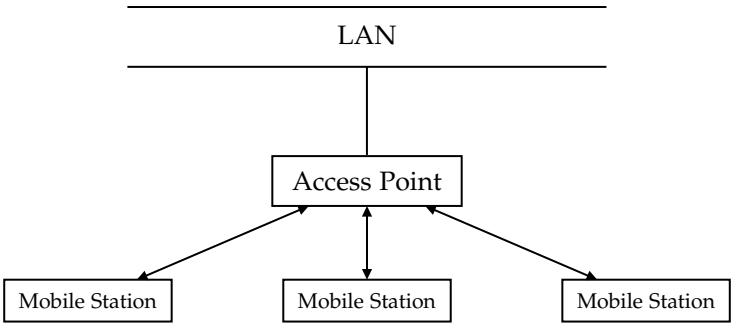
Wireless networks have become prevalent. Popular standards for wireless networks: IEEE 802.11 (longer range, higher speeds, commonly used for wireless LANs) and Bluetooth (short range, low speed).

New security concerns:

- More attack opportunities (no need for physical access).
- Attack from a distance ($>$ 1 km with good antennae).
- No physical evidence of attack.

Case Study: IEEE 802.11 Security

IEEE 802.11 standard for wireless LAN communications includes a protocol called **Wired Equivalent Privacy** (WEP). This standard was ratified in September 1999. This was a very exciting time because this was the first time that wi-fi was available to the consumer market. Multiple amendments: 802.11a (1999), 802.11b (1999), 802.11g (2003), 802.11i (2004), 802.11n (2009)... WEP's goal is (only) to protect link-level data during wireless transmission between mobile stations and access points. A mobile station might be your cell phone or your laptop, and an access point is a wi-fi router somewhere in the building.



Main Security Goals of WEP

1. *Confidentiality*: Prevent casual eavesdropping.

   To achieve this, RC4 is used for encryption.

2. *Data Integrity*: Prevent tampering with transmitted messages.

   To achieve this, an 'integrity checksum' is used for WEP.

3. *Access Control*: Protect access to a wireless network infrastructure.

   This was achieved by discarding all packets that are not properly encrypted using WEP.

Description of WEP Protocol

Mobile stations share a secret key k with access point. Here k is either 40 bits or 104 bits in length. The IEEE standard does not specify how the key is to be distributed. But in practice, one shared key per LAN is common; this key is manually injected into each access point and mobile station; the key is not changed very frequently.

Plaintext messages are divided into packets of some fixed length (e.g., 1500 bytes). These packets were

encrypted using RC4. Since a mobile station or access point might encrypt many packets in a given period of time, WEP had to be careful not to reuse RC4 keystreams. So WEP uses a per-packet 24-bit initialization vector (IV) v to process each packet. WEP does not specify how the IVs are managed. In practice: either a random IV is generated for each packet; or the IV is set to 0 and incremented by 1 for each use.

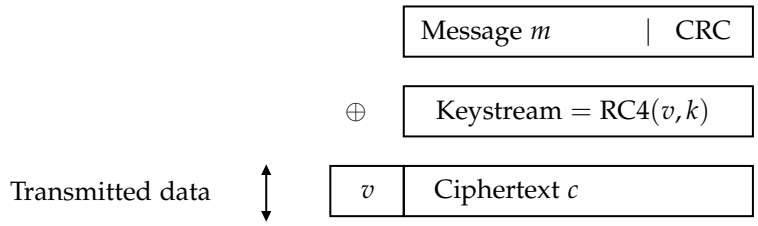
To send a packet m, an entity does the following:

1. Select a 24-bit IV v.
2. Compute a 32-bit checksum: $S = \text{CRC}(m)$.

   802.11 specifies that a CRC-32 checksum be used. CRC-32 is linear. That is, for any two messages m_1 and m_2 of the same bitlength,

   $$\text{CRC}(m_1 \oplus m_2) = \text{CRC}(m_1) + \text{CRC}(m_2).$$

   (The details of CRC-32 are not important to us)
3. Compute $c = (m \parallel S) \oplus \text{RC4}(v \parallel k)$.
   - $\parallel$ (and also a comma) denotes concatenation
   - $(v \parallel k)$ is the key used the RC4 stream cipher.
4. Send (v, c) over the wireless channel.



The receiver of (v, c) does the following:

1. Compute $(m \parallel s) = c \oplus \text{RC4}(v \parallel k)$.
2. Compute $S' = \text{CRC}(m)$; reject the packet if $S' \neq s$.

Are confidentiality, data integrity, and access control achieved? Intuitively the answer is yes. No (Borisov, Goldberg & Wagner; 2001),

Problem 1: IV Collision

Suppose that two packets (v, c) and (v, c') use the same IV v. Let m, m' be the corresponding plaintexts. Then $c \oplus c' = (m \parallel S) \oplus (m' \parallel S')$. Thus, the eavesdropper can compute $m \oplus m'$. If m is known, them m' is immediately available. If m is not known, then one may be able to use the expected distribution of m and m' to discover information about them. (Some contents of network traffic is predictable.)

Since there are only 2^{24} choices for the IV, collisions are guaranteed after enough time - a few days on a busy network (5 Mbps). If IVs are randomly selected, then one can expect a collision after about 2^{12} packets. This is due to the birthday paradox.

Birthday paradox

Suppose that an urn contains n numbered balls. Suppose that balls are drawn from the urn, one at a time, with replacement. The expected number of draws before a ball is selected for a second time (called a collision) is approximately $\sqrt{\pi n/2} \approx \sqrt{n}$.

Collisions are more likely if keys k are long-lived and the same key is used for multiple mobile stations in a network.

Conclusion: WEP does not provide a high degree of confidentiality.

Problem 2: Checksum is Linear

CRC-32 is used to check integrity. This is fine for random errors, but not for deliberate ones.

It is easy to make controlled changes to (encrypted) packets: Suppose (v, c) is an encrypted packet. Let $c = \text{RC4}(c \parallel k) \oplus (m \parallel S)$, where k, m, S are unknown to the adversary. Let $m' = m \oplus \Delta$, where Δ is a bit string. (The 1's in Δ correspond to the bits of m an attacker wishes to change.) Let $c' = c \oplus (\Delta \parallel \text{CRC}(\Delta))$. Then (v, c') is a valid encrypted packet for m'.

$$\begin{aligned} c' &= c \oplus (\Delta \parallel \text{CRC}(\Delta)) \\ &= \text{RC4}(v \parallel k) \oplus (m \parallel S) \oplus (\Delta \parallel \text{CRC}(\Delta)) \\ &= \text{RC4}(v \parallel k) \oplus (m \oplus \Delta \parallel \text{CRC}(m) \oplus \text{CRC}(\Delta)) \\ &= \text{RC4}(c \parallel k) \oplus (m' \parallel \text{CRC}(m \oplus \Delta)) \\ &= \text{RC4}(v \parallel k) \oplus (m' \parallel \text{CRC}(m')) \end{aligned}$$

In this way, the adversary has successfully made a controlled change to the plaintext m without actually knowing the plaintext.

Conclusion: WEP does not provide data integrity.

Problem 3: Integrity Function is Unkeyed

Suppose that an attacker learns the plaintext m corresponding to a single encrypted packet (v, c). Then, the attacker can compute the RC4 keystream $\text{RC4}(v \parallel k) = c \oplus (m \parallel \text{CRC}(m))$. Henceforth, the attacker can compute a valid encrypted packet for *any* plaintext m' of her choice: (v, c'), where $c' = \text{RC4}(v \parallel k) = c \oplus (m' \parallel \text{CRC}(m'))$.

Conclusion: WEP does not provide access control.

Optional Reading: Sections 1, 2, 3, 4.1, 4.2, 6 of "Intercepting mobile communications: The insecurity of 802.11", by N. Borisov, I. Goldberg and D. Wagner.

2.4.2 Fluhrer-Mantin-Shamir Attack

Shortly after these attacks on WEP were disclosed, a *more devastating attack* was discovered. This was due to Fluhrer, Mantin & Shamir, 2001. Shamir is the "S" in RSA.

The attack makes the following three assumptions:

1. The same 104-bit key k is used for a long period of time. [Most products do this.]
2. The IV is incremented for each packet, or a random IV is selected for each packet. [Most products do this.]

3. The first plaintext byte of each packet (i.e. the first byte of each m) is known to the attacker. [Most wireless protocols prepend the plaintext with some header bytes which are non-secret.]

In the attack, a passive adversary who can collect about 5,000,000 encrypted packets can very easily recover k (and thus totally break the system). [Details not covered in this course.]

The attack can be easily mounted in practice: Can buy a $100 wireless card and hack drivers to capture (encrypted) packets. On a busy wireless network (5Mbps), 5 million packets can be captured in a few hours, and then k can be immediately computed.

Implementation details: A. Stubbefield, J. Ionnidis, A. Rubin, "Using the Fluhrer, Mantin and Shamir attack to break WEP", AT&T Technical Report, August 2001.

If you want to mount the WEP attack today, all you have to do is find a wireless network that still uses WEP and then go to one of these two websites: Aircrack-ng, WEPCrack. The latest enhancement of the Fleur-Mantin-Shamir attack is aircrack ptw. This can break WEP in under 60 seconds (only about 40,000 packets are needed).

WEP was blamed for the 2007 theft of 45 million credit-card numbers from T.J. Maxx. (T.J. Maxx is an American department store chain) A subsequent class action lawsuit settled for $40,900,000.

See: http://tinyurl.com/WEP-TJMaxx

IEEE 802.11 Update

http://en.wikipedia.org/wiki/Wi-Fi_Protected_Access

WPA2 (Wi-Fi Protected Access)

- Implements the new IEEE 802.11i standard (2004).
- Uses the AES block cipher instead of RC4.
- Deployed ubiquitously in Wi-fi networks.
  - But deployments of WEP are still out there :-(
  - See https://wigle.net/stats.
- KRACK attack disclosed in October 2017.

WPA3

- Adopted in January 2018.
- Further improvements to WPA2.
- Dragonblood attack disclosed in April 2019.

The Fluhrer-Mantin-Shamir attack exploits known biases in the first few bytes of the keystream. The attack can be defeated by discarding the first few bytes (e.g., 768 bytes) of the keystream. [Details omitted]

As of 2013, approximately 50% of all TLS traffic was secured using RC4. 2013-2021: Because of several new weaknesses discovered, RC4 has been deprecated in applications such as TLS. (As of October 2019, $\approx$ 11.6% of websites have RC4 enabled, and only $\approx$ 1.1% actually use it.)

Conclusions: Don't use RC4. Instead use ChaCha20 or AES-CTR (more on these later).

Lessons Learned

1. Details matter: RC4 was considered to be a secure stream cipher. However, it was improperly used in WEP and the result was a highly insecure communications protocol. Do not assume that "obvious" ways of using cryptographic functions are secure.
2. Attacks only get better; they never get worse.
   (a) Moore's law (1965): computers get twice as fast every two years.
   (b) Known attacks are constantly being tweaked and improved.
   (c) New attacks are constantly being invented.
3. Designing security is hard:
   - Designing cryptographic protocols is complicated and difficult.
   - Clearly state your security objectives.
   - Hire cryptography experts to design your security. Programming or engineering experts are not good enough.
   - Make your protocols available for public scrutiny.
4. There is a big demand in industry for "security engineers": People who have a deep understanding of applied cryptography and have excellent programming skills.

2.5 ChaCha20 Stream Cipher

Designed by Dan Bernstein in 2008.

The ChaCha20 stream cipher is conceptually simple, word-oriented, and uses only simple arithmetic operations (integer addition modulo 2^{32}, xor, and left rotations). It is extremely fast in software, and does not require any special hardware. To date, no security weaknesses have been found. ChaCha20 is widely deployed in practice, including in TLS.

Notation:

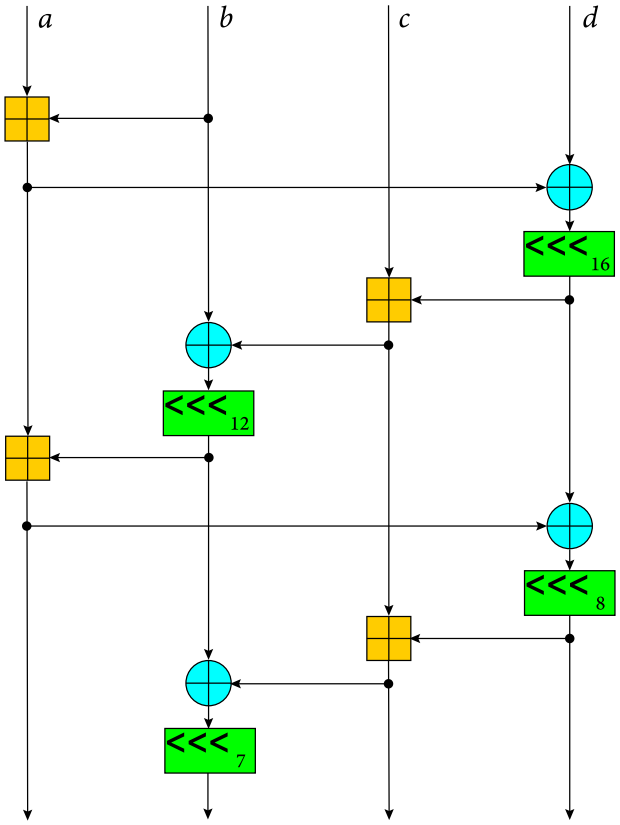
- 256-bit key: $k = (k_1, k_2, \ldots, k_8)$
- 96-bit nonce: $n = (n_1, n_2, n_3)$
- 128-bit constant: $f = (f_1, f_2, f_3, f_4)$
- 32-bit counter: c
- The initial state is:

$$\begin{array}{|c|c|c|c|} \hline f_1 & f_2 & f_3 & f_4 \\ \hline k_1 & k_2 & k_3 & k_4 \\ \hline k_5 & k_6 & k_7 & k_8 \\ \hline c & n_1 & n_2 & n_3 \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline S_1 & S_2 & S_3 & S_4 \\ \hline S_5 & S_6 & S_7 & S_8 \\ \hline S_9 & S_{10} & S_{11} & S_{12} \\ \hline S_{13} & S_{14} & S_{15} & S_{16} \\ \hline \end{array}$$

A hexadecimal digit is a 4-bit number.

A nonce (or IV) is a non-repeating quantity (either a counter, or a randomly-generated string).

2.5.1 ChaCha20 Quarter Round Function



picture from wiki.

Notation:

- $\oplus$: XOR
- $\boxplus$: integer addition modulo 2^{32}
- $\lll t$: left-rotate by t bit positions.

QR: Quarter Round Function

Input Four 32-bit words a, b, c, d

$$
\begin{array}{lll}
a \leftarrow a \boxplus b, & d \leftarrow d \oplus a, & d \leftarrow d \lll 16 \\
c \leftarrow c \boxplus d, & b \leftarrow b \oplus c, & b \leftarrow b \lll 12 \\
a \leftarrow a \boxplus b, & d \leftarrow d \oplus a, & d \leftarrow d \lll 8 \\
c \leftarrow c \boxplus d, & b \leftarrow b \oplus c, & b \leftarrow b \lll 7
\end{array}
$$

Output a, b, c, d

ENCRYPTION: The keystream bytes are xored with the plaintext bytes to produce ciphertext bytes. The nonce is included in the ciphertext.

Algorithm 3: ChaCha20 Keystream Generator

1 Select a nonce n and initialize the counter c.
2 **while** *keystream bytes are required* **do**
3 Create the initial state S.
4 Make a copy S' of S.
5 Update S by repeating the following 10 times:

$$\begin{array}{ll} QR(S_1, S_5, S_9, S_{13}) & QR(S_2, S_6, S_{10}, S_{14}) \\ QR(S_3, S_7, S_{11}, S_{15}) & QR(S_4, S_8, S_{12}, S_{16}) \\ QR(S_1, S_6, S_{11}, S_{16}) & QR(S_2, S_7, S_{12}, S_{13}) \\ QR(S_3, S_8, S_9, S_{14}) & QR(S_4, S_5, S_{10}, S_{15}) \end{array}$$

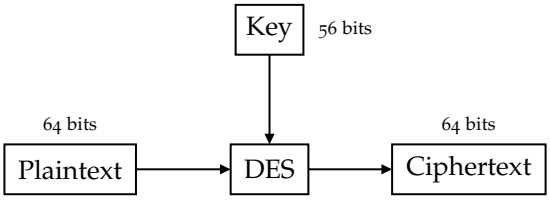
6 Output $S \oplus S'$ (64 keystream bytes).
7 Increment the counter.

2.6 Block Ciphers

A block cipher is a SKES that breaks up the plaintext into blocks of a fixed length (e.g. 128 bits), and encrypts the blocks one at a time.

In contrast, a stream cipher encrypts the plaintext one character (usually a bit) at a time.

Historically important example of a block cipher: The Data Encryption Standard (DES)



Key length: 56 bits; Size of key space 2^{56}; Block length: 64 bits.

2.6.1 Brief History of Block Ciphers

Late 1960's: Feistel network and LUCIFER designed at IBM.

1972: NBS (now NIST: National Institute of Standards and Technology) solicits proposals for encryption algorithms for the protection of computer data.

1973-1974: IBM develops DES.

1975: NSA (National Security Agency) "fixes" DES. Reduces the key length from 64 bits to 56 bits. (NSA tried for 48 bits; compromised at 56 bits.)

The National Security Agency: Founded in 1952. Budget: Classified (estimated: $10.8 billion/year). Number of employees: Classified (30,000–40,000). Signals Intelligence (SIGINT): produce foreign intelligence information. Information Assurance (IA): protects all classified and sensitive information that is stored or sent through US government equipment. Very influential in setting US government export policy for cryptographic products (especially encryption). Canadian counterpart: Communications Security Establishment (CSE)

NSA Shenanigans

Edward Snowden. CIA/NSA contractor who disclosed in 2013 up to 200,000 NSA classified documents to the press. Currently in Russia under temporary asylum. Documents revealed the enormous capabilities of NSA and its partners (including CSE and GCHQ): "Groundbreaking cryptanalytic capabilities" Collects vast amounts of internet communications and automatically analyzes the data Directly attacks routers, switches, firewalls, etc. Installs malware on individual computers Breaks into individual computers Bruce Schneier: "The NSA is subverting the Internet and turning it into a massive surveillance tool."

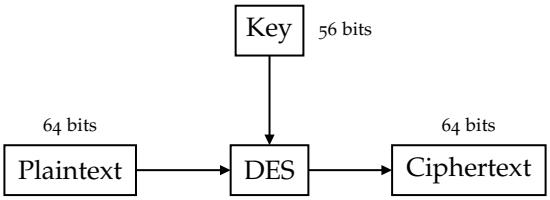
1977: DES adopted as US Federal Information Processing Standard (FIPS 46). 1981: DES adopted as a US banking standard (ANSI X3.92). 1997: NIST begins the AES (Advanced Encryption Standard) competition. 1999: 5 finalists for AES announced. 2001: Rijndael adopted for AES (FIPS 197). AES has three key lengths: 128, 192 and 256 bits. 2021: No significant weaknesses found with AES (as yet). AES uptake is rapidly increasing. However, (Triple-)DES is still deployed.

2.6.2 Some Desirable Properties of Block Ciphers

Design principles described by Claude Shannon in 1949:

- Security:
  - *Diffusion*: each ciphertext bit should depend on all plaintext bits.
  - *Confusion*: the relationship between key and ciphertext bits should be complicated.
  - *Key length*: should be small, but large enough to preclude exhaustive key search.
- Efficiency:
  - Simplicity (easier to implement and analyze).
  - High encryption and decryption rate.
  - Suitability for hardware or software.

2.6.3 The Data Encryption Standard (DES)



Key length: 56 bits; Size of key space 2^{56}; Block length: 64 bits.

The design principles of DES are still classified, making analysis difficult. Hundreds of research papers written on the security of DES. (Triple-)DES is still deployed is practice, although its use is rapidly decreasing.

DES Problem 1: Small Key Size

Exhaustive search on key space takes 256 steps and can be easily parallelized. DES challenges from RSA Security (3 known PT/CT pairs):

The unkn	own mess	age is:	????????

June 1997: Broken in 3 months by Internet search. 1999: Broken in 56 hours by DeepCrack machine

(1800 chips; $250,000). 2006 Broken in 153 hours by COPACOBANA machine ($10,000). 2012 Broken in 11.5 hours by crack.sh ($200). RC5 64-bit challenge: July 2002: Broken in 1757 days. Participation by 331,252 individuals.

DES Problem 2: Small Block Size

If plaintext blocks are distributed "uniformly at random", then the expected number of ciphertext blocks observed before a collision occurs is $\approx 2^{32}$ (by the birthday paradox). Hence the ciphertext reveals some information about the plaintext.

Small block length is also damaging to some authentication applications (more on this later).

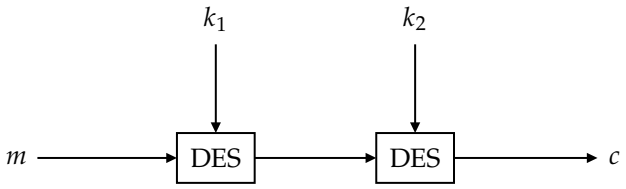
The only (substantial) weaknesses known in DES are the obvious ones: small key length and small block length.

Question: How can one construct a more secure block cipher from DES? (i.e. without changing the internals of DES.) Multiple encryption: Re-encrypt the ciphertext one or more times using independent keys. Hope that this increases the effective key length. Multiple encryption does not always result in increased security. Example: If E_π denotes the encryption function for the simple substitution cipher with key π, then is $E_{\pi_2} \circ E_{\pi_1}$ any more secure than E_π? No, since $E_{\pi_2} \circ E_{\pi_1} = E_{\pi_2 \circ \pi_1}$.

2.6.4 Double-DES

Key is $k = (k_1, k_2)$, $k_1, k_2 \in_R \{0,1\}^{56}$. Here $k \in_R K$ means that k is chosen uniformly and independently at random from K.

Encryption: $c = E_{k_2}(E_{k_1}(m))$. (E = DES encryption, E^{-1} = DES decryption)



Decryption: $m = E_{k_1}^{-1}(E_{k_2}^{-1}(c))$.

The Double-DES key length is $\ell = 112$, so exhaustive key search takes 2^{112} steps (infeasible). Note: Block length is unchanged.

Main idea: $c = E_{k_2}(E_{k_1}(m))$ if and only if $E_{k_2}^{-1}(c) = E_{k_1}(m)$.

Algorithm 4: Meet-In-The-Middle Attack on Double-DES

Input: 3 known PT/CT pairs $(m_1, c_1), (m_2, c_2), (m_3, c_3)$
Output: The secret key (k_1, k_2)

foreach $h_2 \in \{0,1\}^{56}$ **do**
  Compute $E_{h_2}^{-1}(c_1)$, and store $[E_{h_2}^{-1}(c_1), h_2]$ in a table sorted by first component.
foreach $h_1 \in \{0,1\}^{56}$ **do**
  Compute $E_{h_1}(m_1)$.
  Search for $E_{h_1}(m_1)$ in the table. (we say that $E_{h_1}(m_1)$ matches table entry $[E_{h_2}^{-1}(c_1), h_2]$ if $E_{h_2}^{-1}(c_1) = E_{h_1}(m_1)$.)
  foreach *match* $[E_{h_2}^{-1}(c_1), h_2]$ *in the table* **do**
    if $E_{h_2}(E_{h_1}(m_2)) = c_2$ **then**
      if $E_{h_2}(E_{h_1}(m_3)) = c_3$ **then**
        Output (h_1, h_2) and STOP.

Number of known plaintext/ciphertext pairs needed for unique key determination:

Question: Let E be a block cipher with key space $K = \{0,1\}^\ell$, and plaintext and ciphertext space $\{0,1\}^L$.

Let $k' \in K$ be the secret key chosen by Alice and Bob, and let $(m_i, c_i), 1 \le i \le t$, be known plaintext/-ciphertext pairs, where the plaintext m_i are distinct. (Note that $c_i = E_{k'}(m_i)$ for all $1 \le i \le t$)

Then how large should t be to ensure (with probability very close to 1) that there is only one key $k \in K$ such that $E_k(m_i) = c_i$ for all $1 \le i \le t$?

Answer: Select t so that $FK \approx 0$.

For each $k \in K$, then encryption function $E_k : \{0,1\}^L \to \{0,1\}^L$ is a permutation.

We make the heuristic assumption that for each $k \in K, E_k$ is a random function (i.e., a randomly selected function). This assumption is certainly false since E_k is not random, and because a random function is almost certainly not a permutation. Nonetheless, it turns out that the assumption is good enough for our analysis.

Now, fix $k \in K, k \ne k'$. The probability that $E_k(m_i) = c_i$ for all $1 \le i \le t$ is

$$\underbrace{\frac{1}{2^L} \cdot \frac{1}{2^L} \cdots \frac{1}{2^L}}_{t} = \frac{1}{2^{Lt}}.$$

Thus the expected number of false keys $k \in K$ (not including k') for which $E_k(m_i) = c_i$ for all $1 \le i \le t$ is

$$FK = \frac{2^\ell - 1}{2^{Lt}}.$$

Meet-In-The-Middle Attack on Double DES

Let E be the DES encryption function, so Double-DES encryption is $c = E_{k_2}(E_{k_1}(m))$.

1. If $\ell = 112, L = 64, t = 3$, then $FK \approx 1/2^{80} \approx 0$. Thus if a Double-DES key (h_1, h_2) is found for which $E_{h_2}(E_{h_1}(m_i)) = c_i$ for $i = 1, 2, 3$, then with very high probability we have $(h_1, h_2) = (k_1, k_2)$.
2. If $\ell = 112, L = 64, t = 1$, then $FK \approx 2^{48}$. Thus the expected number of Double-DES keys (h_1, h_2) for which $E_{h_2}(E_{h_1}(m_1)) = c_1$ is $\approx 2^{48}$.
3. If $\ell = 112, L = 64, t = 2$, then $FK \approx 1/2^{16}$. Thus the expected number of Double-DES keys (h_1, h_2) for which $E_{h_2}(E_{h_1}(m_1)) = c_1$ and $E_{h_2}(E_{h_1}(m_2)) = c_2$ is $\approx 1/2^{16}$.

Analysis:

- Number of DES operations is $\approx 2^{56} + 2^{56} + 2 \cdot 2^{48} \approx 2^{57}$. (We are not counting the time to do the sorting and searching)
- Space requirements: $2^{56}(64 + 56)$ bits $\approx 1,080,863$ Tbytes.

Time-memory tradeoff. [Exercise] The attack can be modified to decrease the storage requirements at the expense of time: Time: 2^{56+s} steps; memory: 2^{56-s} units, $1 \le s \le 55$.

Conclusions: Double-DES has the same effective key length as DES. Double-DES is not much more secure than DES.

Storage Units

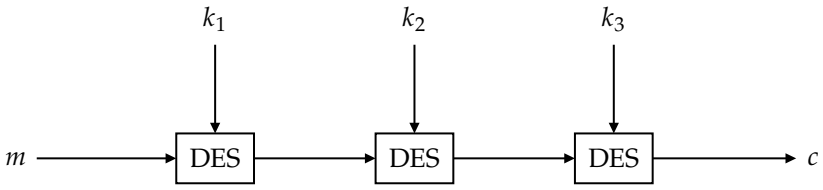
- 10^3 bytes = 1 Kbyte (kilo) $\approx 2^{10}$ bytes
- 10^3 Kbytes = 1 Mbyte (mega) $\approx 2^{20}$ bytes

- 10^3 Mbytes = 1 Gbyte (giga) $\approx 2^{30}$ bytes
- 10^3 Gbytes = 1 Tbyte (tera) $\approx 2^{40}$ bytes
- 10^3 Tbytes = 1 Pbyte (peta) $\approx 2^{50}$ bytes
- 10^3 Pbytes = 1 Ebyte (exa) $\approx 2^{60}$ bytes

2.6.5 Triple-DES

Triple-DES. Key is $k = (k_1, k_2, k_3)$, $k_1, k_2, k_3 \in_R \{0,1\}^{56}$.

Encryption: $c = E_{k_3}(E_{k_2}(E_{k_1}(m)))$ where E = DES encryption, E^{-1} = DES decryption



Decryption: $m = E_{k_1}^{-1}(E_{k_2}^{-1}(E_{k_3}^{-1}(c)))$.

Key length of Triple-DES is $\ell = 168$, so exhaustive key search takes 2^{168} steps (infeasible)

Meet-in-the-middle attack takes $\approx 2^{112}$ steps. [Exercise]

So, the effective key length of Triple-DES against exhaustive key search is ≈ 112 bits.

No proof that Triple-DES is more secure than DES.

Note: Block length is unchanged.

Triple-DES is still deployed: especially by some financial institutions, where its use can be expected to continue until 2030.

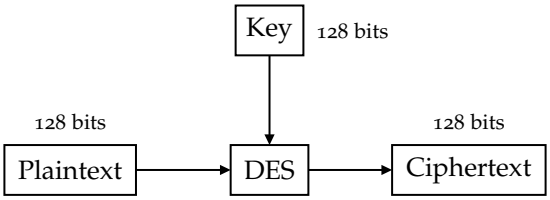
2.6.6 The Advanced Encryption Standard (AES)

http://www.nist.gov/aes

1997: Call issued for AES candidate algorithms.

Requirements:

- Key lengths: 128, 192 and 256 bits.
- Block length: 128 bits.
- Efficient on both hardware and software platforms.
- Availability on a worldwide, non-exclusive, royalty-free basis.



The AES Process: 1998: 15 submissions in Round 1. 1999: 5 finalists selected by NIST: MARS, RC6, Rijndael, Serpent, Twofish. 1999: NSA performed a hardware efficiency comparison. 2000: Rijndael was selected. 2001: The AES standard is officially adopted (FIPS 197). Rijndael is an example of a

substitution-permutation network. 2021: No attacks have been found on AES that are (significantly) faster than exhaustive key search.

2.6.7 Substitution-Permutation Networks

A substitution-permutation network (SPN) is an iterated block cipher where a round consists of a substitution operation followed by a permutation operation.

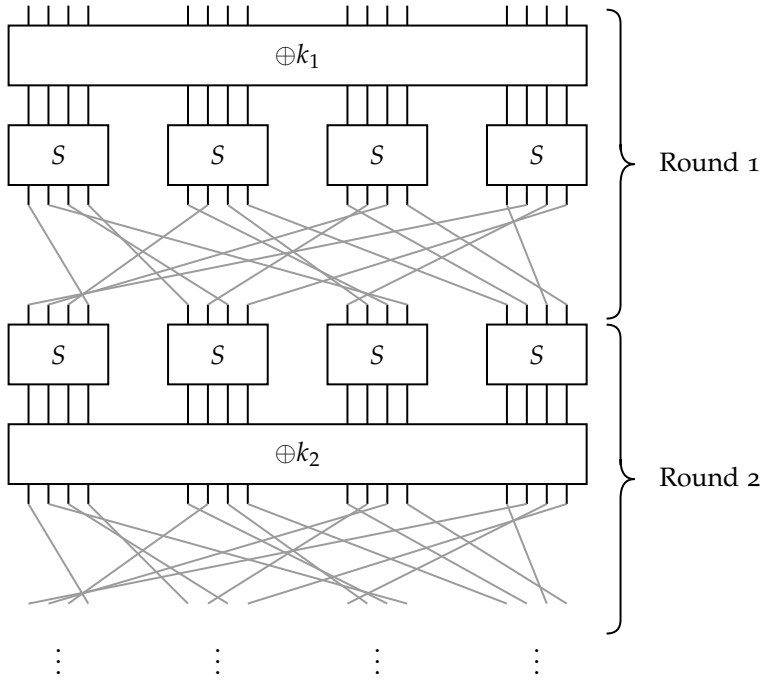
- n: the block length.
- ℓ: the key length.
- h: the number of rounds.
- A fixed invertible function $S : \{0,1\}^b \to \{0,1\}^b$, where b is a divisor of n.
- A fixed permutation P on $\{1, 2, \ldots, n\}$.
- A key scheduling algorithm that determines subkeys $k_1, k_2, \ldots, k_h, k_{h+1}$ from a key k.

Note: $n, \ell, h, S, P$and the key scheduling algorithm are public. The only secret in AES is the key k that is selected.

Here is a description of encryption:

Algorithm 5: Encryption of Substitution-Permutation Networks

```
A ← plaintext
for i = 1..h do
    A ← A ⊕ k_i // XOR
    A ← S(A) // Substitution
    A ← P(A) // Permutation
A ← A ⊕ k_{h+1}
ciphertext ← A
```



2.6.8 AES

AES is an SPN, where the permutation operation is replaced by two invertible linear transformations. All operations are byte oriented (e.g., $b = 8$ so the S-box maps 8-bits to 8-bits). This allows AES to be efficiently implemented on software platforms. The block length of AES is $n = 128$ bits. Each subkey is 128 bits. AES accepts three different key lengths. The number of rounds h depends on the key length:

cipher	key length ℓ	h
AES-128	128	10
AES-192	192	12
AES-256	256	14

AES Round Operations

Each round updates a variable called State which consists of a 4×4 array of bytes (note: $4 \cdot 4 \cdot 8 = 128$, the block length). State is initialized with the plaintext:

$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$
$a_{1,0}$	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$
$a_{2,0}$	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$
$a_{3,0}$	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$

$\leftarrow$ plaintext

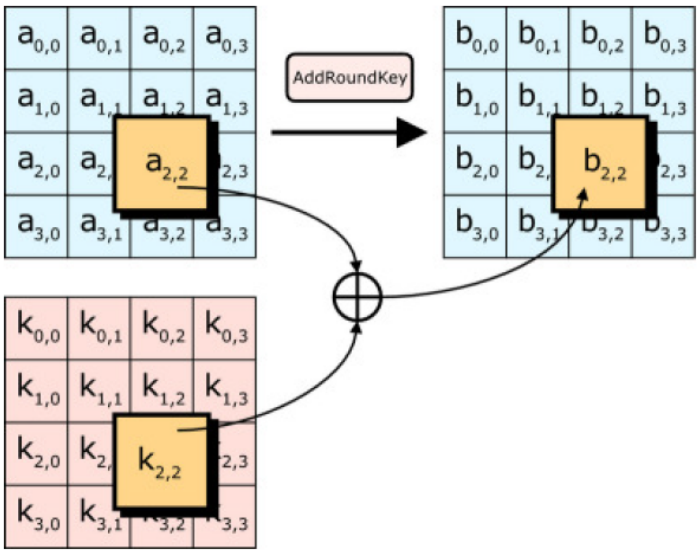
After h rounds are completed, a final subkey is XOR-ed with State, the result being the ciphertext.

The AES round function uses four invertible operations:

1. `AddRoundKey` (key mixing)
2. `SubBytes` (S-box)
3. `ShiftRows` (permutation)
4. `MixColumns` (linear transformation).

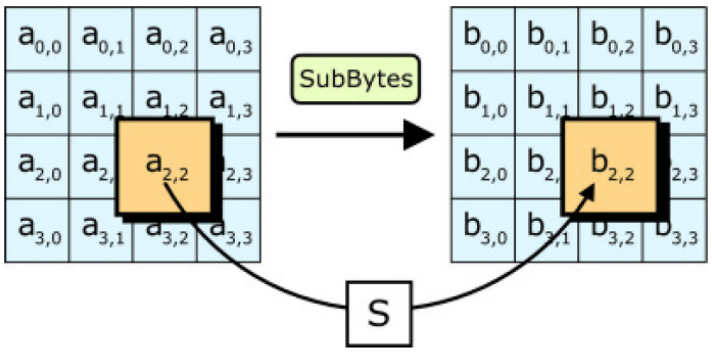
Add Round Key

Bitwise-XOR each byte of State with the corresponding byte of the subkey.



Substitute Bytes

Take each byte in State and replace it with the output of the S-box.



$S : \{0,1\}^8 \to \{0,1\}^8$ is a fixed, public, invertible function. The S-box is a non-linear function.

The Finite Field $GF(2^8)$

The elements of the finite field $GF(2^8)$ are the polynomials of degree at most 7 in $\mathbb{Z}_2[y]$, with addition and multiplication performed modulo the irreducible polynomial $f(y) = y^8 + y^4 + y^3 + y + 1$.

Interpret an 8-bit string $a = a_7a_6a_5a_4a_3a_2a_1a_0$ as coefficients of the polynomial $a(y) = a_7y^7 + a_6y^6 + a_5y^5 + \cdots + a_1y + a_0$ and vice versa.

Example:

Let $a = 11101100 = \texttt{ec}$ and $b = 00111011 = \texttt{3b}$, so $a(y) = y^7 + y^6 + y^5 + y^3 + y^2$ and $b(y) = y^5 + y^4 + y^3 + y + 1$.

1. Addition: $a(y) + b(y) = y^7 + y^6 + y^t + y^2 + y + 1$, so $\texttt{ec} + \texttt{3b} = \texttt{d7}$.
2. Multiplication: $a(y) \cdot b(y) = y^{12} + y^{10} + y^8 + y^4 + y^2$, which leaves a remainder of $r(y) = y^7 + y^6 + y^3$ upon division by $f(y)$. Hence $a \cdot b = 11001000$ in $GF(2^8)$, or $\texttt{ec} \cdot \texttt{3b} = \texttt{c8}$.
3. Inversion: $\texttt{ec}^{-1} = 5f$ since $\texttt{ec} \cdot \texttt{5d} = \texttt{01}$.

S-box Definition

Let $p \in \{0,1\}^8$, and consider p as an element of $GF(2^8)$.

1. Let $q = p^{-1}$ if $p \neq 0$, and $q = p$ if $p = 0$.
2. Define $q = (q_7q_6q_5q_4q_3q_2q_1q_0)$.
3. Compute

$$\begin{pmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

Then $S(p) = r = (r_7r_6r_5r_4r_3r_2r_1r_0)$.

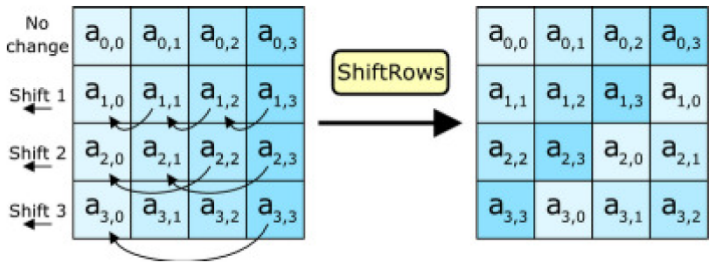
The AES S-box

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

For example, $S(\texttt{a8}) = \texttt{c2}$.

Shift Rows

Permute the bytes of State by applying a cyclic shift to each row.



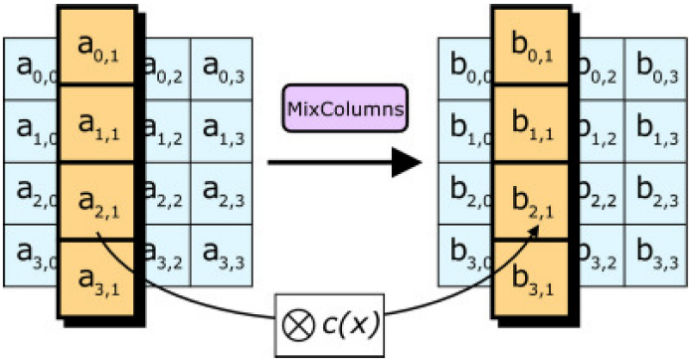
Mix Columns

1. Read column i of `State` as a polynomial:

$$(a_{0,i}, a_{1,i}, a_{2,i}, a_{3,i}) = a_{0,i} + a_{1,i}x + a_{2,i}x^2 + a_{3,i}x^3$$

   (interpret the coefficients as elements of the finite field $GF(2^8)$).

2. Multiply this polynomial with the constant polynomial $c(x) = \texttt{03} \cdot x^3 + \texttt{01} \cdot x^2 + \texttt{01} \cdot x + \texttt{02}$ and reduce modulo $x^4 - 1$. This gives a new polynomial: $b_{0,i} + b_{1,i}x + b_{2,i}x^2 + b_{3,i}x^3$



The $\otimes c(x)$ Operation

- Let $a(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, where each $a_i \in GF(2^8)$.
- Let $c(x) = \texttt{02} + \texttt{01}x + \texttt{01}x^2 + \texttt{03}x^3$, where `01`, `02`, `03` are elements in $GF(2^8)$ (written in hexadecimal).
- To compute $a(x) \otimes c(x)$:
  - Compute $d(x) = a(x) \times c(x)$ (polynomial multiplication, where coefficient arithmetic is in $GF(2^8)$).
  - Divide by $x^4 - 1$ to find the remainder polynomial $r(x)$ (equivalently, replace x^4 by 1, x^5 by x, and x^6 by x^2).
  - Then $a(x) \otimes c(x) = r(x)$.

For example, let $a(x) = \texttt{d0f112bb} = \texttt{d0} + \texttt{f1}x + \texttt{12}x2 + \texttt{bb}x^3$. Then $a(x) \otimes c(x) = \texttt{1a} + \texttt{a4}x + \texttt{d3}x^2 + \texttt{e5}x^3 = \texttt{1aa4d3e5}$.

AES Encryption

From the key k derive $h + 1$ subkeys $k_0, k_1, \ldots, k_h$.

Here is a description of encryption:

Algorithm 6: AES Encryption

```
State ← plaintext
State ← State ⊕ k_0
for i = 1..h − 1 do
    State ← SubBytes(State)
    State ← ShiftRows(State)
    State ← MixColumns(State)
    State ← State ⊕ k_i
State ← SubBytes(State)
State ← ShiftRows(State)
State ← State ⊕ k_h
ciphertext ← State
```

Note that in the final round, `MixColumns` is not applied. This helps make decryption more similar to encryption, and thus is useful when implementing AES. [Details omitted]

AES Decryption

From the key k derive $h+1$ subkeys $k_0, k_1, \ldots, k_h$.

Here is a description of decryption:

Algorithm 7: AES Decryption

```
State ← ciphertext
State ← State ⊕ k_h
State ← InvShiftRows(State)
State ← InvSubBytes(State)
for i = h − 1...1 do
    State ← State ⊕ k_i
    State ← InvMixColumns(State)
    State ← InvShiftRows(State)
    State ← InvSubBytes(State)
State ← State ⊕ k_0
plaintext ← State
```

`InvMixColumns` is multiplication by $d(x) = \texttt{0e} + \texttt{09}x + \texttt{0d}x^2 + \texttt{0b}x^3$ modulo $x^4 - 1$.

AES Key Schedule (for 128-bit keys)

- For 128-bit keys, AES has 10 rounds, so we need 11 subkeys.
- The first subkey $k_0 = (r_0, r_1, r_2, r_3)$ is the actual AES key.
- The second subkey is $k_1 = (r_4, r_5, r_6, r_7)$.
- The third subkey is $k_2 = (r_8, r_9, r_9, r_{10})$.
- ...
- The eleventh subkey is $k_{10} = (r_{40}, r_{41}, r_{42}, r_{43})$.

AES Key Schedule Functions f_i

The functions $f_i : \{0,1\}^{32} \to \{0,1\}^{32}$ are defined as follows:

1. The input is divided into 4 bytes: (a, b, c, d).
2. Left-rotate the bytes: (b, c, d, a).
3. Apply the AES S-box to each byte: $(S(b), S(c), S(d), S(a))$.
4. XOR the leftmost byte with the constant ℓ_i, and output the result: $(S(b) \oplus \ell_i, S(c), S(d), S(a))$.

The constants ℓ_i:

i	ℓ_i	i	ℓ_i	i	ℓ_i	i	ℓ_i	i	ℓ_i
1	0x01	2	0x02	3	0x04	4	0x08	5	0x10
6	0x20	7	0x40	8	0x80	9	0x1b	10	0x36

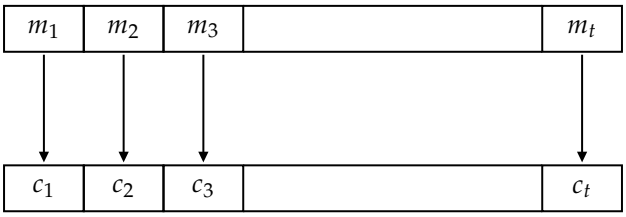
2.6.9 Block Cipher Modes of Operation

In practice, one might need to encrypt a large quantity of data. Plaintext message is $m = m_1, m_2, \ldots, m_t$, where each m_i is an L-bit block.

Question: How should we use a block cipher $E_k : \{0,1\}^L \to \{0,1\}^L$ to encrypt m?

Electronic Codebook (ECB) Mode

Encrypt blocks independently, one at a time: $c = c_1, c_2, \ldots, c_t$, where $c_i = E_k(m_i)$.

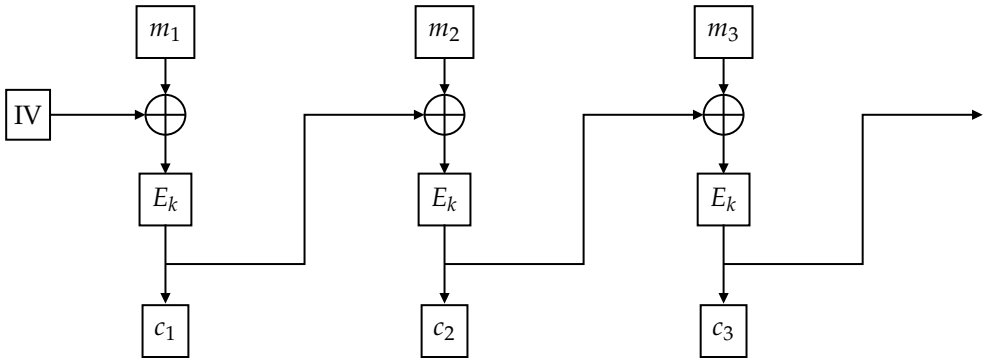


Decryption: $m_i = E_k^{-1}(c_i)$, $i = 1, 2, \ldots, t$.

Drawback: Identical plaintexts result in identical ciphertexts (under the same key), and thus ECB encryption is not (semantically) secure against chosen-plaintext attacks. [Why?]

Cipher Block Chaining (CBC) Mode

Encryption: Select $c_0 \in_R \{0,1\}^L$ (c_0 is a random non-secret IV). Then compute $c_i = E_k(m_i \oplus c_{i-1})$, $i = 1, 2, \ldots, t$.



Ciphertext is $(c_0, c_1, c_2, \ldots, c_t)$.

Decryption: $m_i = E_k^{-1}(c_i) \oplus c_{i-1}$, $i = 1, 2, \ldots, t$.

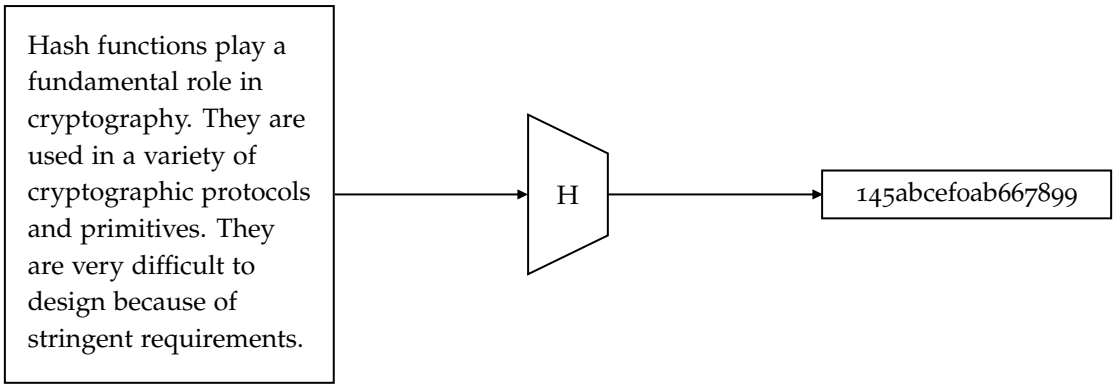
Identical plaintexts with different IVs result in different ciphertexts and for this reason CBC encryption is (semantically) secure against chosen-plaintext attacks (for a well chosen block cipher E).

3

Hash Functions

Hash functions play a fundamental role in cryptography. They are used in a variety of cryptographic primitives and protocols. They are very difficult to design because of very stringent security and performance requirements. The main candidates available today are: SHA-1; SHA-2 family: SHA-224, *SHA-256*, SHA-384, SHA-512

What is a Hash Function?



See

- http://www.xorbin.com/tools/md5-hash-calculator
- http://www.xorbin.com/tools/sha1-hash-calculator
- http://www.xorbin.com/tools/sha256-hash-calculator

hash function

A **hash function** is a mapping H such that:

1. H maps binary messages of arbitrary lengths $\leq L$ to outputs of a fixed length n: $H : \{0,1\}^{\leq L} \rightarrow \{0,1\}^n$. ($L$ is large, e.g., $L = 2^{64}$; n is small, e.g., $n = 256$)
2. $H(x)$ can be efficiently computed for all $x \in \{0,1\}^{\leq L}$.

H is called an n-bit hash function. $H(x)$ is called the hash or message digest of x.

Note:
The description of a hash function is public. There are no secret keys.

For simplicity, we will usually write $\{0,1\}^*$ instead of $\{0,1\}^{\leq L}$.

Toy Hash Function

$H : \{0,1\}^{\leq 4} \to \{0,1\}^2$

x	$H(x)$	x	$H(x)$	x	$H(x)$	x	$H(x)$
0	00	1	01				
00	11	01	01	10	01	11	00
000	00	001	10	010	11	011	11
100	11	101	01	110	01	111	10
0000	00	0001	11	0010	11	0011	00
0100	01	0101	10	0110	10	0111	01
1000	11	1001	01	1010	00	1011	01
1100	10	1101	00	1110	00	1111	11

$(00, 1000)$ is a collision.

1001 is a preimage of 01.

10 is a second preimage of 1011.

Example: SHA-256

SHA-256: $\{0,1\}^* \to \{0,1\}^{256}$

SHA-256("Hello there") =
`0x4e47826698bb4630fb4451010062fadbf85d61427cbdfaed7ad0f23f239bed89`

SHA-256("Hello There") =
`0xabf5dacd019d2229174f1daa9e62852554ab1b955fe6ae6bbbb214bab611f6f5`

SHA-256("Welcome to CO 487") =
`0x819ba4b1e568e516738b48d15b568952d4a35ea73f801c873907d3ae1f5546fb`

SHA-256("Welcome to CO 687") =
`0x404fb0ee527b8f9f01c337e915e8beb6e03983cfd9544296b8cf0e09c9d8753d`

Davies-Meyer hash function. Let E_k be an m-bit block cipher with n-bit key k. Let IV be a fixed m-bit initializing value.

To compute $H(x)$, do:

- Break up $x \parallel 1$ into n-bit blocks: $\overline{x} = x_1, x_2, \ldots, x_t$, padding out the last block with 0 bits if necessary.
- Define $H_0 = IV$.
- Compute $H_i = E_{x_i}(H_{i-1}) \oplus H_{i-1}$ for $i = 1, \ldots, t$.
- Define $H(x) = H_t$.

Hash functions are the Swiss Army knife of cryptography. Hash functions are used for all kinds of applications they were not designed for. One reason for this widespread use of hash functions is speed.

Preimage Resistance (PR)

$H : \{0,1\}^* \to \{0,1\}^n$.

Preimage resistance hash function

Given a hash value $y \in_R \{0,1\}^n$, it is computationally infeasible to find (with non-negligible probability of success) any $x \in \{0,1\}^*$ such that $H(x) = y$. x is called a preimage of y.

Note: $x \in_R S$ means that x is chosen independently and uniformly at random from S.

Password protection on a multi-user computer system:

- Server stores (userid, H(password)) in a password file. Thus, if an attacker gets a copy of the password file, she does not learn any passwords.
- Requires preimage-resistance.

2nd Preimage Resistance (2PR)

$H : \{0,1\}^* \to \{0,1\}^n$.

Second-preimage resistance hash function

Given an input $x \in_R \{0,1\}^*$, it is computationally infeasible to find (with non-negligible probability of success) a second input $x' \in \{0,1\}^*$, $x' \neq x$, such that $H(x') = H(x)$.

Modification Detection Codes (MDCs).

- To ensure that a message m is not modified by unauthorized means, one computes $H(m)$ and protects $H(m)$ from unauthorized modification.
- e.g. virus protection. Requires 2nd preimage resistance.

Other Applications of Hash Functions

1. Message Authentication Codes (HMAC). [More on this later]
2. Pseudorandom bit generation: Distilling random bits s from several "pseudorandom" sources $x_1, \ldots, x_t$. Output $s = H(x_1, \ldots, x_t)$.
3. Key derivation function (KDF): Deriving a cryptographic key from a shared secret. [More on this later]
4. Proof-of-work in cryptocurrencies (Bitcoin): [More on this later]
5. Quantum-safe signature schemes: [More on this later]

Typical Cryptographic Requirements

$H : \{0,1\}^* \to \{0,1\}^n$

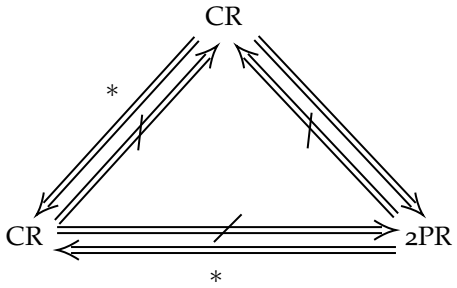
Preimage resistance, 2nd preimage resistance.

Collision resistance

It is computationally infeasible to find (with non-negligible probability of success) two distinct inputs $x, x' \in \{0,1\}^*$ such that $H(x) = H(x')$.

Relationships between PR, 2PR, CR

Let $H : \{0,1\}^* \to \{0,1\}^n$ be a hash function.



$*$ for somewhat uniform hash functions

1. CR implies 2PR

Proof:
Suppose that H is not 2PR.

Select $x \in_R \{0,1\}^*$. Since H is not 2PR, we can efficiently find $x' \in \{0,1\}^*$, $x' \neq x$, with $H(x') = H(x)$. Thus we have efficiently found a collision (x, x') for H. Hence H is not CR. □

Note that this proof establishes the contrapositive statement.

2. 2PR does not guarantee CR

Proof:
Suppose that $H : \{0,1\}^* \to \{0,1\}^n$ is 2PR.

Consider $\overline{H} : \{0,1\}^* \to \{0,1\}^n$ defined by:

$$\overline{H}(x) = \begin{cases} H(0), & \text{if } x = 1, \quad \overline{H}(1) = H(0) = \overline{H}(0) \\ H(x), & \text{if } x \neq 1. \end{cases}$$

Suppose that $\overline{H}$ is not 2PR. So, given $x \in_R \{0,1\}^*$, we can efficiently find $x' \in \{0,1\}^*$, $x' \neq x$, with $\overline{H}(x') = \overline{H}(x)$. Now, if $x' \neq 1$, then $\overline{H}(x') = \overline{H}(x') = H(x)$; while if $x' \neq 1$, then $\overline{H}(x') = \overline{H}(1) = H(0) = H(x)$. In either case, we have found a second preimage for x with respect to H. So, $\overline{H}$ must be 2PR.

Also, $\overline{H}$ is not CR, since $(0,1)$ is a collision for $\overline{H}$. □

3. CR does not guarantee PR

Proof:
Suppose that $H : \{0,1\}^* \to \{0,1\}^n$ is CR.

Consider $\overline{H} : \{0,1\}^* \to \{0,1\}^{n+1}$ defined by:

$$\overline{H}(x) = \begin{cases} 1 \parallel x, & \text{if } x \in \{0,1\}^n, \\ 0 \parallel H(x), & \text{if } x \notin \{0,1\}^n. \end{cases}$$

Then $\overline{H}$ is CR (since H is).

And, $\overline{H}$ is not PR since preimages can be efficiently found for at least half of all $y \in \{0,1\}^{n+1}$. □

However, if H is "somewhat uniform" (i.e., all hash values have roughly the same number of preimages), then CR does not imply PR.

Proof:
Suppose H is not PR. Select $x \in_R \{0,1\}^*$ and compute $y = H(x)$. Since H is not PR, we can efficiently find a preimage x' of y. Since H is somewhat uniform, we expect that y has many preimages, and thus $x' \neq x$ with very high probability. Thus, (x, x') is a collision for H that we have efficiently found, so H is not CR. □

Note that we shall henceforth assume that hash functions are somewhat uniform.

4. PR does not guarantee 2PR

Proof:
Suppose $H : \{0,1\}^* \to \{0,1\}^n$ is PR.

Define $\overline{H} : \{0,1\}^* \to \{0,1\}^n$ by

$$\overline{H}(x_1, x_2, \ldots, x_t) = H(0, x_2, \ldots, x_t).$$

Then $\overline{H}$ is PR, but $\overline{H}$ is not 2PR. □

5. 2PR implies PR (for somewhat uniform H)

Proof:
Exercise. □

3.1 Generic Attacks

generic attack

A **generic attack** on a hash function $H : \{0,1\}^* \to \{0,1\}^n$ does not exploit any properties the specific hash function may have.

In the analysis of a generic attack, we view H as a random function in the sense that for each $x \in \{0,1\}^*$, we assume that the value $y = H(x)$ was chosen by selecting $y \in_R \{0,1\}^n$.

From a security point of view, a random function is an ideal hash function. However, random functions are not suitable for practical applications because they cannot be compactly stored.

Generic Attack for Finding Preimages

$H : \{0,1\}^* \to \{0,1\}^n$.

Given $y \in_R \{0,1\}^n$, repeatedly select arbitrary $x \in \{0,1\}^*$ until $H(x) = y$.

Expected number of steps is $\approx 2^n$. (Here, a 'step' is a hash function evaluation.)

This attack is infeasible if $n \geq 128$.

Note:
It has been proven that this generic attack for finding preimages is optimal, i.e., no faster generic attack exists.

Of course, for a specific hash function, there might exist a preimage finding algorithm that is faster than the generic attack.

Generic Attack for Finding Collisions

Select arbitrary $x \in \{0,1\}^*$ and store $(H(x), x)$ in a table sorted by first entry. Continue until a collision is found.

Expected number of steps: $\sqrt{\pi 2^n/2} \approx \sqrt{2^n}$ (by birthday paradox). (Here, a "step" is a hash function evaluation.)

This attack is infeasible if $n \geq 256$.

It has been proven that this generic attack for finding collisions is optimal in terms of the number of hash function evaluations.

Expected space required: $\sqrt{\pi 2^n/2} \approx \sqrt{2^n}$.

If $n = 128$:

- Expected running time: 2^{64} steps. [feasible]
- Expected space required: 7×10^8 Tbytes. [infeasible]

VW Parallel Collision Search

VW: Van Oorschot & Wiener (1993)

Expected number of steps: $\sqrt{2^n}$

Expected space required: negligible.

Easy to parallelize: m-fold speedup with m processors.

The collision-finding algorithm can easily be modified to find "meaningful" collisions. [see optional readings]

Conclusion: If collision resistance is desired, then use an n-bit hash function with $n \geq 256$

Parallel collision search (VW Method)

Problem: Find a collision for $H : \{0,1\}^* \to \{0,1\}^n$.

Assumption: H is a random function.

VW: Paul van Oorschot, Michael Wiener (1993)

Notation: Let $N = 2^n$. Define a sequence $\{x_i\}_{i \geq 0}$ by $x_0 \in_R \{0,1\}^n$, $x_i = H(x_{i-1})$ for $i \geq 1$. Let j be the smallest index for which $x_j = x_i$ for some $i < j$; such a j must exist. Then $x_{j+\ell} = x_{i+\ell}$ for all $\ell \geq 1$. By the birthday paradox, $E[j] \approx \sqrt{\frac{\pi N}{2}} \approx \sqrt{N}$. In fact, $E[i] \approx \frac{1}{2}\sqrt{N}$ and $E[j-1] \approx \frac{1}{2}\sqrt{N}$.

Now, $i \neq 0$ with overwhelming probability; in that event, (x_{i-1}, x_{j-1}) is a collision for H.

How to find (x_{i-1}, x_{j-1}) without using much storage?

Answer: Store only distinguished points.

Select an easily testable distinguishing property for elements of $\{0,1\}^n$ (e.g., leading 32 bits are all 0). Let θ be the proportion of elements of $\{0,1\}^n$ that are distinguished.

VW algorithm Compute $x_0, x_1, x_2, \ldots$ and only store the points in the sequence that are distinguished.

Algorithm 8: VW collision finding

```
// Stage 1 (detecting a collision)
Select x_0 ∈_R {0,1}^n
Store (x_0, 0, −) in a sorted table
LP ← x_0 // LP = last point stored
for d = 1, 2, 3, 4, ... do
    Compute x_d = H(x_{d−1})
    if x_d is distinguished then
        If x_d is already in table, say x_d = x_b where b < d then go to stage 2.
        Store (x_d, d, LP) in a table.
        LP ← x_d
// Stage 2 (finding a collision)
Set ℓ_1 ← b − d, ℓ_2 ← d − c.
Suppose ℓ_1 ≥ ℓ_2; set k ← ℓ_1 − ℓ_2
Compute x_{a+1}, x_{a+2}, ..., x_{a+R}.
for m = 1, 2, 3, ... do
    Compute (x_{a+k+m}, x_{c+m})
Until x_{a+k+m} = x_{c+m}
The collision is (x_{a+k+m−1}, x_{c+m−1})
```

Analysis

- Stage 1: Expected # of H-evaluations is

$$\underbrace{\sqrt{\frac{\pi N}{2}}}_{\text{collision occurs}} + \underbrace{\frac{1}{\theta}}_{\text{collision is detected}} \approx \sqrt{N} + \frac{1}{\theta}$$

- Stage 2: Expected # of H-evaluations is $\leq \frac{3}{\theta}$ (see optional readings)

Overall expected running time: $\sqrt{N} + \frac{4}{\theta}$

Storage: $\approx \theta\sqrt{N}(3N)$ bits (each table entry has bitlength $3n$)

> Example:
>
> Let $n = 128$. take $\theta = \frac{1}{2^{32}}$. Then the expected time of VW collision search is 2^{64} H-evaluations (feasible), and the expected storage is 241 Gbytes (free).

Parallelizing VW collision search (m processors)

Run a copy of VW on each of the m processors. Report distinguished points to a central server.

Analysis:

- Expected time $\approx \sqrt{N}/m + 4/\theta$
- Expected storage $\approx \theta\sqrt{N}(3n)$ bits

> **Note:**
> Factor−m speedup.
>
> No communication between processors.

- Occasional communication with central server

3.2 Iterated Hash Functions (Merkle Meta-Method)

Components:

- Fixed initializing value $IV \in \{0,1\}^n$,
- Compression function $f : \{0,1\}^{n+r} \to \{0,1\}^n$ (efficiently computable).

To compute $H(x)$ where x has bitlength $b < 2^r$ do:

- Break up x into r-bit blocks: $\overline{x} = x_1, x_2, \ldots, x_t$, padding out the last block with 0 bits if necessary.
- Define x_{t+1}, the length-block, to hold the right-justified binary representation of b.
- Define $H_0 = IV$.
- Compute $H_i = f(H_{i-1}, x_i)$ for $i = 1, 2, \ldots, t+1$.
- Define $H(x) = H_{t+1}$.

The H_i"s are called chaining variables.

3.2.1 Collision Resistance of Iterated Hash Functions

Theorem 3.1: Merkle

If the compression function f is collision resistant, then the hash function H is also collision resistant.

Merkle's theorem reduces the problem of designing collision-resistant hash functions to that of designing collision-resistant compression functions.

3.3 Provable Security

A major theme of cryptographic research is to formulate precise security definitions and assumptions, and then prove that a cryptographic protocol is secure. A proof of security is certainly desirable since it rules out the possibility of attacks being discovered in the future. However, it isn't always easy to assess the practical security assurances (if any) that a security proof provides.

- The assumptions might be unrealistic, or false, or circular.
- The security proof might be fallacious.
- The security model might not account for certain kinds of realistic attacks.
- The security proof might be asymptotic.
- The security proof might have a large tightness gap.

Optional reading: http://anotherlook.ca

Proof:
Suppose that H is not collision resistant. Then we can efficiently find $x, x' \in \{0,1\}^*$, $x \neq x'$, with $H(x) = H(x')$.

Let

$$\overline{x} = x_1, x_2, \ldots, x_t, \quad b = \texttt{bitlength}(x), \quad x_{t+1} = \text{length block}$$
$$\overline{x}' = x'_1, x'_2, \ldots, x'_t, \quad b' = \texttt{bitlength}(x'), \; x'_{t'+1} = \text{length block}.$$

We efficiently compute:

$$\begin{aligned} H_0 &= IV \\ H_1 &= f(H_0, x_1) \\ H_2 &= f(H_1, x_2) \\ &\vdots \\ H_{t-1} &= f(H_{t-2}, x_{t-1}) \\ H_t &= f(H_{t-1}, x_t) \\ H_{t+1} &= f(H_t, x_{t+1}) \end{aligned}$$

and

$$\begin{aligned} H_0 &= IV \\ H'_1 &= f(H_0, x'_1) \\ H'_2 &= f(H'_1, x'_2) \\ &\vdots \\ H'_{t'-1} &= f(H'_{t'-2}, x'_{t'-1}) \\ H'_{t'} &= f(H_{t'-1}, x'_{t'}) \\ H_{t'+1} &= f(H'_{t'}, x'_{t'+1}) \end{aligned}$$

Since $H(x) = H(x')$, we have $H_{t+1} = H'_{t'+1}$.

Now if $b \neq b'$, then $x_{t+1} \neq x'_{t'+1}$. Thus $(H_t, x_{t+1}), (H'_{t'}, x'_{t'+1})$ is a collision for f that we have efficiently found. Hence f is not collision resistant.

Suppose $b = b'$. Then $t = t'$ and $x_{t+1} = x'_{t'+1}$. Let i be the largest index, $0 \leq i \leq t$, for which

$$(H_i, x_{i+1}) \neq (H'_i, x'_{i+1}).$$

Such an i must exist since $x \neq x'$. Then

$$H_{i+1} = f(H_i, x_{i+1}) = f(H'_i, x'_{i+1}) = H'_{i+1},$$

so $(H_i, x_{i+1}), (H'_i, x'_{i+1})$ is a collision for f that we have efficiently found. Hence f is not collision resistant. □

3.4 MDx-Family of Hash Functions

MDx is a family of iterated hash functions. MD4 was proposed by Ron Rivest in 1990. MD4 has 128-bit outputs. Wang et al. (2004) found collisions for MD4 by hand. Leurent (2008) discovered an algorithm for finding MD4 preimages in 2^{102} steps.

3.4.1 MD5 Hash Function

MD5 is a strengthened version of MD4. Designed by Ron Rivest in 1991.

MD5 has 128-bit outputs. Wang and Yu (2004) found MD5 collisions in 2^{39} steps. MD5 collisions can

now be found in 2^{24} steps (in a few seconds on a laptop computer).

Sasaki & Aoki (2009) discovered a method for finding preimages for MD5 in $2^{123.4}$ steps.

Summary MD5 should not be used if collision resistance is required, but is probably okay as a preimage-resistant hash function.

MD5 is still used today. (2006) MD5 is implemented more than 850 times in Microsoft Windows source code. (2014) Microsoft issued a patch that restricts the use of certificates with MD5 in Windows: http:tinyurl.com/MicrosoftMD5

Flame Malware

See https://en.wikipedia.org/wiki/Flame_(malware)

Discovered in May 2012. Highly sophisticated espionage tool. Targeted computers in Iran and the Middle East. Suspected to originate from the US and/or Israeli government; US government has denied all responsibility. Contains a forged Microsoft certificate for Windows code signing. Forged certificate used a new, "zero-day MD5 chosen-prefix" collision attack. Microsoft no longer allows the use of MD5 for code signing.

3.5 SHA-1

Secure Hash Algorithm (SHA) was designed by NSA and published by NIST in 1993 (FIPS 180). 160-bit iterated hash function, based on MD4. Slightly modified to SHA-1 (FIPS 180-1) in 1994 in order to fix an (undisclosed) security weakness. Wang et al. (2005) found collisions for SHA in 239 steps. Wang et al. (2005) discovered a collision-finding algorithm for SHA-1 that takes 263 steps. The first SHA-1 collision was found on February 23, 2017. No preimage or 2nd preimage attacks (that are faster than the generic attacks) are known for SHA-1.

Microsoft's SHA-1 Plan

See http://tinyurl.com/MicrosoftSHA1

As of May 9, 2017: TLS server-authentication certificates that use SHA-1 will be considered invalid.

Still unaffected:

- Code signature file hashes
- Code signing certificates
- Timestamp signature hashes
- Timestamping certificates
- etc.

3.6 SHA-2 Family

In 2001, NSA proposed variable output-length versions of SHA-1. Output lengths are 224 bits (SHA-224 and SHA-512/224), 256 bits (SHA-256 and SHA-512/256), 384 bits (SHA-384) and 512 bits (SHA-512). 2020: No weaknesses in any of these hash functions have been found. Note: The security levels of these hash functions against collision-finding attacks is the same as the security levels of Triple-DES, AES-128, AES-192 and AES-256 against exhaustive key search attacks. The SHA-2 hash functions are standardized in FIPS 180-2.

3.7 Description of SHA-256

Iterated hash function (Merkle meta-method).

$n = 256, r = 512$.

Compression function is $f : \{0,1\}^{256+512} \to \{0,1\}^{256}$.

Input: bitstring x of arbitrary bitlength $b \geq 0$.

Output: 256-bit hash value $H(x)$ of x.

SHA-256 Notation

A, B, C, D, E, F, G, H	32-bit words.
$+$	addition modulo 2^{32}.
$\overline{A}$	bitwise complement.
$A \gg s$	shift A right through s positions.
$A \hookrightarrow s$	rotate A right through s positions.
AB	bitwise AND.
$A \oplus B$	bitwise exclusive-OR.
$f(A, B, C)$	$AB \oplus \overline{A}C$.
$g(A, B, C)$	$AB \oplus AC \oplus BC$.
$r_1(A)$	$(A \hookrightarrow 2) \oplus (A \hookrightarrow 13) \oplus (A \hookrightarrow 22)$.
$r_2(A)$	$(A \hookrightarrow 6) \oplus (A \hookrightarrow 11) \oplus (A \hookrightarrow 25)$.
$r_3(A)$	$(A \hookrightarrow 7) \oplus (A \hookrightarrow 18) \oplus (A \gg 3)$
$r_4(A)$	$(A \hookrightarrow 17) \oplus (A \hookrightarrow 19) \oplus (A \gg 10)$.

3.7.1 SHA-256 Constants

32-bit initial chaining values (IVs):

$$h_1 = \texttt{0x6a09e667},$$
$$h_2 = \texttt{0xbb67ae85},$$
$$h_3 = \texttt{0x3c6ef372},$$
$$h_4 = \texttt{0xa54ff53a},$$
$$h_5 = \texttt{0x510e527f},$$
$$h_6 = \texttt{0x9b05688c},$$
$$h_7 = \texttt{0x1f83d9ab},$$
$$h_8 = \texttt{0x5be0cd19}.$$

These words were obtained by taking the first 32 bits of the fractional parts of the square roots of the first 8 prime numbers.

Per-round integer additive constants:

$$y_0 = \texttt{0x428a2f98},$$
$$y_1 = \texttt{0x71374491},$$
$$y_2 = \texttt{0xb5c0fbcf},$$
$$y_3 = \texttt{0xe9b5dba5},$$
$$\vdots$$
$$y_{62} = \texttt{0xbef9a3f7},$$
$$y_{63} = \texttt{0xc67178f2}.$$

These words were obtained by taking the first 32 bits of the fractional parts of the cube roots of the first 64 prime numbers.

Algorithm 9: SHA-256 Preprocessing

Pad x (with 1 followed by as few o's as possible) so that its bitlength is 64 less than a multiple of 512.
Append a 64-bit representation of $b \mod 2^{64}$.
The formatted input is $x_0, x_1, \ldots, x_{16m-1}$, where each x_i is a 32-bit word.
Initialize chaining variables: $(H_1, H_2, \ldots, H_6, H_7, H_8) \leftarrow (h_1, h_2, \ldots, h_6, h_7, h_8)$.
foreach *i from* 0 *to* $m-1$ **do**
    Copy the i-th block of sixteen 32-bit words into temporary storage: $X_j \leftarrow x_{16i+j}$, $0 \leq j \leq 15$.
    `// Expand the 16-word block into a 64-word block:`
    for *j from* 16 *to* 63 **do**
        $X_j \leftarrow r_4(X_{j-2}) + X_{j-7} + r_3(X_{j-15}) + X_{j-16}$.
    `// Initialize working variables:`
    $(A, B, \ldots, F, G, H) \leftarrow (H_1, H_2, \ldots, H_6, H_7, H_8)$.
    for *j from* 0 *to* 63 **do**
        $T_1 \leftarrow H + r_2(E) + f(E, F, G) + y_j + X_j$
        $T_2 \leftarrow r_1(A) + g(A, B, C)$
        $H \leftarrow G, \quad G \leftarrow F, \quad F \leftarrow E, \quad E \leftarrow D + T_1$
        $D \leftarrow C, \quad C \leftarrow B, \quad B \leftarrow A, \quad A \leftarrow T_1 + T_2$.
    Update chaining values: $(H_1, H_2, \ldots, H_7, H_8) \leftarrow (H_1 + A, H_2 + B, \ldots, H_7 + G, H_8 + H)$.

Output: SHA-256$(x) = H_1 \parallel H_2 \parallel H_3 \parallel H_4 \parallel H_5 \parallel H_5 \parallel H_6 \parallel H_7 \parallel H_8$.

Performance: Speed benchmarks (2017) for software implementations on an Intel Core i9 2.9 GHz 6-core Coffee Lake (8950HK) using OpenSSL 1.1.1d.

3.8 SHA-3

The SHA-2 design is similar to SHA-1, and thus there are lingering concerns that the SHA-1 weaknesses could eventually extend to SHA-2

SHA-3: NIST hash function competition. 64 candidates submitted by Oct 31 2008 deadline. 2012: Keecak was selected as the winner.

Keecak uses the "sponge construction" and not the Merkle iterated hash design. SHA-3 is being used in practice, but is not as widely deployed as SHA-2.

NIST's Policy on Hash Functions

August 5, 2015

See: http://csrc.nist.gov/groups/ST/hash/policy.html

Should stop using SHA-1 for digital signatures and other applications that require collision resistance. May still use SHA-1 for HMAC, KDFs, and random number generators. May use SHA-2 for all applications that employ secure hash algorithms. SHA-3 may also be used, but this is not required.

4

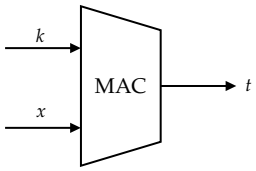
Message authentication code schemes

4.1 Definition

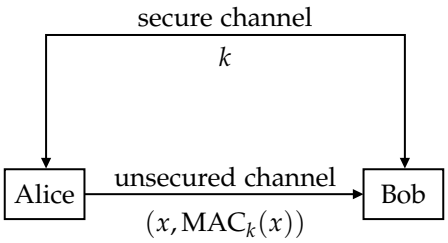
message authentication code

A **message authentication code** scheme is a family of functions $\mathrm{MAC}_k : \{0,1\}^* \to \{0,1\}^n$ parameterized by an ℓ-bit key k, where each function MAC_k can be efficiently computed.

$t = \mathrm{MAC}_k(x)$ is called the MAC or tag of x with key k.



MAC schemes are used for providing (symmetric-key) data integrity and data origin authentication.



To provide data integrity and data origin authentication:

1. Alice and Bob establish a secret key $k \in \{0,1\}^\ell$.
2. Alice computes tag $t = \mathrm{MAC}_k(x)$ and sends (x,t) to Bob.
3. Bob verifies that $t = \mathrm{MAC}_k(x)$.

Note: No confidentiality or non-repudiation.

To avoid replay, add a timestamp or sequence number.

Security Definition

Let K be the secret key shared by Alice and Bob.

The adversary does not know k, but is allowed to obtain (from Alice or Bob) tags for messages of her choosing. The adversary's goal is to obtain the tag of any new message, i.e., a message whose tag she did not already obtain from Alice or Bob.

secure MAC scheme

A MAC scheme is **secure** if given some tags $\text{MAC}_k(x_i)$ for x_i's of one's own choosing, it is computationally infeasible to compute (with non-negligible probability of success) a messagetag pair $(x, \text{MAC}_k(x))$ for any new message x.

More concisely, a MAC scheme is secure if it is existentially unforgeable against chosen-message attack.

Note: A secure MAC scheme can be used to provide data integrity and data origin authentication.

An ideal MAC scheme has the following property: For each key $k \in \{0,1\}^\ell$, the function $\text{MAC}_k : \{0,1\}^* \to \{0,1\}^n$ is a random function.

Ideal MAC schemes are useless in practice. However, when analyzing a generic attack on a MAC scheme, it is reasonable to assume that the MAC scheme is ideal.

Generic Attacks on MAC schemes

Guessing the MAC of a message $x \in \{0,1\}^*$:

- Select $y \in_R \{0,1\}^n$ and guess that $\text{MAC}_k(x) = y$.
- Assuming that MAC_k is random function, the probability of success is $1/2^n$.
- Note: Guesses cannot be directly checked.
- MAC guessing is infeasible if $n \geq 128$.

Exhaustive search on the key space:

- Given r known message-tag paris $(x_1, t_1), \ldots, (x_r, t_r)$, one can check whether a guess k of the key is correct by verifying that $\text{MAC}_k(x_i) = t_i$, for $i = 1, 2, \ldots, r$.
- Assuming that MAC_k's are random functions, the expected number of keys for which the tags verify is
$$1 + FK = 1 + (2^\ell - 1)/2^{nr}$$
Example: If $\ell = 128$, $n = 128$, $r = 2$, then $FK \approx 1/2^{128}$.
- Expected number of steps $\approx 2^\ell$.
- Exhaustive search is infeasible if $\ell \geq 128$.

MACs Based on Block Ciphers

CBC-MAC

Let E be an n-bit block cipher with key space $\{0,1\}^\ell$.

Assumption: Suppose that plaintext messages all have lengths that are multiplies of $n/$

To compute $\text{MAC}_k(x)$:

1. Divide x into n-bit blocks $x_1, x_2, \ldots, x_r$.
2. Compute $H_1 = E_k(x_1)$.
3. For $2 \leq i \leq r$, compute $H_i = E_k(H_{i-1} \oplus x_i)$.
4. Then $\text{MAC}_k(x) = H_r$.

Security of CBC-MAC

Rigorous security analysis [Bellare, Kilian & Rogaway 1994]:

Informal statement of a Theorem Suppose that E is an "ideal" encryption scheme. (That is, for each $k \in \{0,1\}^\ell$, $E_k : \{0,1\}^n \to \{0,1\}^n$ is a 'random' permutation.) Then CBC-MAC with fixed-length inputs is a secure MAC scheme.

CBC-MAC (as described before without additional measures) is not secure if variable length messages are allowed.

Here is a chosen-message attack on CBC-MAC:

1. Select an arbitrary n-bit block x_1.
2. Obtain the tag t_1 of the one-block message x_1 (so $t_1 = E_k(x_1)$).
3. Obtain the tag t_2 of the one-block message t_1 (so $t_2 = E_k(t_1)$).
4. Then t_2 is the tag of the 2-block message $(x_1, 0)$ (since $t_2 = E_k(0 \oplus E_k(x_1)) = E_k(E_k(x_1)) = E_k(t_1)$).

Encrypted CBC-MAC (EMAC)

One countermeasure for variable-length messages is Encrypted CBC-MAC:

Encrypt the last block under a second key s: $\text{EMAC}_{k,s}(x) = E_s(H)r$, where $H_r = \text{CBC-MAC}_k(x)$.

Rigorous security analysis [Petrank & Rackoff 2000]: Informal statement of a Theorem: Suppose that E is an "ideal" encryption scheme. Then EMAC is a secure MAC scheme (for inputs of any length).

MACs Based on Hash Functions

Hash functions were not originally designed for message authentication; in particular they are not "keyed" primitives.

How to use them to construct secure MACs?

- Let H be an iterated n-bit hash function (without the length-block).
- Let $n + r$ be the input blocklength of the compression function $f : \{0,1\}^{n+r} \to \{0,1\}^n$.

  Example: For SHA-256, $n = 256, r = 512$.

- Let $k \in \{0,1\}^n$.
- Let K denote k padded with $(r-n)$ o's. So K has bitlength r.

Secret Prefix Method

MAC definition: $\text{MAC}_k(x) = H(K, x)$.

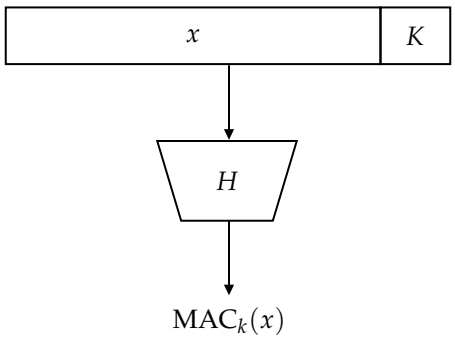
This is insecure. Here is a length extension attack:

- Suppose that $(x, \text{MAC}_k(x))$ is known.
- Suppose that the bitlength of x is a multiple of r.
- Then $\text{MAC}_k(x \parallel y)$ can be computed for any y (without knowledge of k).

Also insecure if a length block is postpended to $K \parallel x$ prior to application of H.

Secret Suffix Method

MAC definition: $\text{MAC}_k(x) = H(x, K)$.



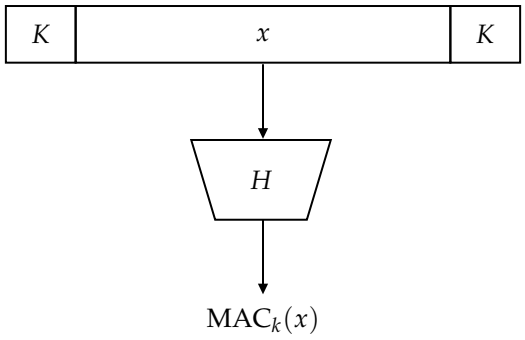
The attack on the secret prefix method does not work here.

Suppose that a collision (x_1, x_2) can be found for H (i.e., $H(x_1) = H(x_2)$). We assume that x_1 and x_2 both have bitlengths that are multiples of r.

Thus $H(x_1, K) = H(x_2, K)$, and so $\text{MAC}_k(x_1) = \text{MAC}_k(x_2)$. Then the MAC for x_1 can be requested, giving the MAc for x_2. Hence if H is not collision resistant, then the secret suffix method MAC is insecure.

Envelope Method

MAC definition: $\text{MAC}_k(x) = H(K, x, K)$.



The MAC key is used both at the start and end of the MAC computation.

The envelope method appears to be secure (i.e., no serious attacks have been found).

HMAC

"Hash-based" MAC; Bellare, Canetti & Krawczyk (1996).

Define two r-bit strings (in hexadecimal notation): ipad $=$ `0x36`, opad $=$ `0x5C`; each repeated $r/8$ times.

MAC definition: $\text{HMAC}_k(x) = H(K \oplus \text{opad}, H(K \oplus \text{ipad}, x))$.

Security analysis is rigorous:

Informal statenebt of a Theorem Suppose that the compression function f used in H is a secure MAC with fixed length messages and a secret IV as the key. Then HMAC is a secure MAC algorithm.

Recommended for practice: use SHA-256 as the hash function.

HMAC is specified in IETF RFC 2104 and FIPS 198. (IETF = Internet Engineering Task Force)

HMAC is used in IPsec (Internet Protocol Security) and TLS.

HMAC is commonly used as a key derivation function (KDF).

Suppose that Alice has a secret key k, and wishes to derive several session keys (e.g., to encrypt data in different communication sessions).

Alice computes $sk_1 = \text{HMAC}_k(1), sk_2 = \text{HMAC}_k(2), sk_3 = \text{HMAC}_k(3), \ldots$

Rationale: Without knowledge of k, an adversary is unable to learn anything about any particular session key sk_j , even though it may have learnt some other session keys.

GSM

Global standards for mobile communications:

- 2G, 2.5G: GSM (Global System for Mobile Communication)
- 3G: UMTS (Universal Mobile Telecommunications System)
- 4G: LTE (Long Term Evolution)

We will sketch the basic security mechanisms in GSM. GSM security is notable since it uses only symmetric-key primitives. UMTS and LTE security improves upon GSM security in several ways, but will not be discussed here.

GSM Security

Cryptographic ingredients:

- Enc: A symmetric-key encryption scheme.
- MAC: A symmetric-key MAC scheme.
- KDF: A key derivation function.

Setup: A SIM card manufacturer randomly selects a secret key k, and installs it in a SIM card. A copy of k is given to the cell phone service provider. When a user purchases cell phone service, she gets the SIM card which she installs in her phone.

Note: A different key k is chosen for each user.

Objectives:

1. Entity authentication: Cell phone service provider needs to be assured that entities accessing its service are legitimate subscribers.
2. Confidentiality: Users need the assurance that their cell phone communications are private.

Alice: cell phone user, Bob: cell phone service provider.

1. Alice sends an authentication request to Bob.
2. Bob selects a challenge $r \in_R \{0,1\}^{128}$, and sends r to Alice.
3. Alice's SIM card uses k to compute the response $t = \mathrm{MAC}_k(r)$. Alice sends t to Bob.
4. Bob retrieves Alice's key k from its database, and verifies that $t = \mathrm{MAC}_k(r)$.
5. Alice and Bob compute an encryption key $KE = \mathrm{KDF}_k(r)$, and thereafter use the encryption algorithm Enc_{K_E} to encrypt and decrypt messages for each other for the remainder of the session.

One drawback with using only symmetric-key crypto is that the SIM card manufacturer and the cell phone service providers have to securely maintain a large database of SIM keys k.

In 2015, the Snowden leaks revealed that NSA and GCHQ had stolen SIM keys from Gemalto, which manufactures about 2 billion SIM cards each year. See http://tinyurl.com/NSASIM

5

Authentic Encryption

Authenticated Encryption (AE)

A symmetric-key encryption scheme E provides confidentiality, e.g., $E = \text{AES}$.

A MAC scheme provides authentication (data origin authentication and data integrity), e.g., MAC = HMAC.

What if confidentiality and authentication are both required?

5.1 Encrypt-and-MAC

First Method: Encrypt-and-MAC

- Alice sends $(c, t) = (E_{k_1}(m), \text{MAC}_{k_2}(m))$ to Bob, where m is the plaintext and k_1, k_2 are secret keys she shares with Bob.
- Bob decrypts c to obtain $m = E_{k_1}^{-1}(c)$ and then verifies that $t = \text{MAC}_{k_2}(m)$.

However, this generic method might have some security vulnerabilities.

5.2 Encrypt-then-MAC

Second Method: Encrypt-then-MAC.

- Alice sends $(c, t) = (E_{k_1}(m), \text{MAC}_{k_2}(E_{k_1}(m)))$ to Bob, where m is the plaintext and k_1, k_2 are secret keys she shares with Bob.
- Bob first verifies that $t = \text{MAC}_{k_2}(c)$ and then decrypts c to obtain $m = E_{k_1}^{-1}(c)$.

This method has been deemed to be secure, provided of course that the encryption scheme E and the MAC scheme employed are secure.

5.3 Special-Purpose AE Schemes

Many specialized authenticated encryption schemes have been developed, the most popular of these being Galois/Counter Mode (GCM).

These modes can be faster than generic Encrypt-then-MAC, and also allow for the authentication (but

not encryption) of "header" data.

Example: AES-GCM

- Authenticated encryption scheme proposed by David McGrew and John Viega in 2004.
- Adopted as a NIST standard in 2007.
- Uses the CTR mode of encryption and a custom-designed MAC scheme.

CTR: CounTeR Mode of Encryption

Let $k \in_R \{0,1\}^{128}$ be the secret key. Let $M = (M_1, M_2, \ldots, M_u)$ be a plaintext message, where each M_i is a 128-bit block, $u \leq 2^{32} - 2$.

To encrypt M, Alice does the following:

1. Select $IV \in_R \{0,1\}^{96}$.
2. Let $J_0 = IV \parallel 0^{31} \parallel 1$.
3. For i from 1 to u do
   $J_i \leftarrow J_{i-1} + 1$. [increment the counter]
   Compute $C_i = \text{AES}_k(J_i) \oplus M_i$.
4. Send $(IV, C_1, C_2, \ldots, C_u)$ to Bob.

To decrypt, Bob does the following:

1. Let $J_0 = IV \parallel 0^{31} \parallel 1$.
2. For i from 1 to u do
   $J_i \leftarrow J_{i-1} + 1$. [increment the counter]
   Compute $M_i = \text{AES}_k(J_i) \oplus C_i$.

Note:

1. CTR mode of encryption can be viewed as a stream cipher.
2. As was the case with CBC encryption, identical plaintexts with different IVs result in different ciphertexts.
3. It is critical that the IV should not be repeated; this can be difficult to achieve in practice.
4. Unlike CBC encryption, CTR encryption is parallelizable.
5. Note that AES^{-1} is not used.

Multiplying Blocks

Let $a = a_0 a_1 a_2 \ldots a_{127}$ be a 128-bit block.

We associate the binary polynomial $a(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{127} x^{127} \in \mathbb{Z}_2[x]$ with a.

Let $f(x) = 1 + x + x^2 + x^7 + x^{128}$.

If a and $b$are 128-bit blocks then define $c = a \bullet b$ to be the block corresponding to the polynomial $c(x) = a(x) \cdot b(x) \mod f(x)$ in $\mathbb{Z}_2[x]$.

- That is, $c(x)$ is the remainder upon dividing $a(x) \cdot b(x)$ by $f(x)$, where coefficient arithmetic is performed modulo 2.
- This is multiplication in the Galois Field $GF(2^{128})$.

- Example with 6 bit-blocks:

  Let $f(x) = 1 + x + x^6$. Let $a = 001101$ and $b = 100111$. Then $a(x) = x^2 + x^3 + x^5$, $b(x) = 1 + x^3 + x^4 + x^5$. Thus, $c(x) = x^2 + x^5$ and $c = a \bullet b = 001001$.

AES-GCM

Input:

- Data to be authenticated (but not encrypted) $A = (A_1, A_2, \ldots, A_v)$.
- Data to be encrypted and authenticated: $M = (M_1, M_2, \ldots, M_u)$.
- Secret key $k \in_R \{0,1\}^{128}$.

Output: (IV, A, C, t), where

- IV is a 96-bit initialization vector.
- $A = (A_1, A_2, \ldots, A_v)$ is authenticated data.
- $C = (C_1, C_2, \ldots, C_u)$ is the encrypted/authenticated data.
- t is a 128-bit authentication tag.

AES-GCM Encryption/Authentication

Alice does the following:

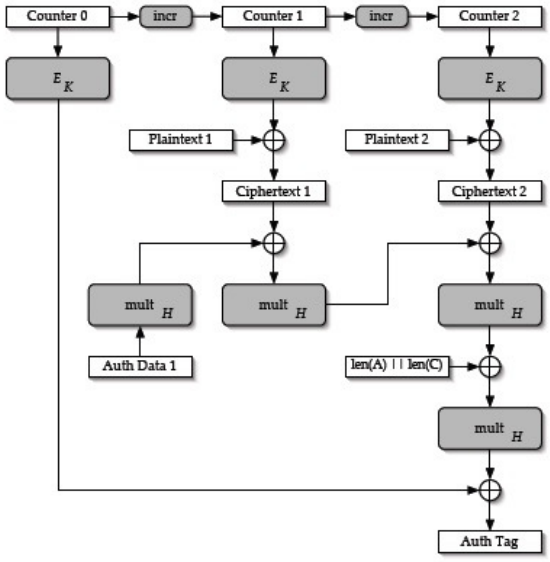
1. Let $L = L_A \parallel L_M$, where L_A, L_M are the bitlengths of A, M expressed as 64-bit integers. (L is the length block.)
2. Select $IV \in_R \{0,1\}^{96}$ and let $J_0 = IV \parallel 0^{31} \parallel 1$.
3. Encryption.

   For i from 1 to u do:
   Compute $J_i = J_{i-1} + 1$ and $C_i = \mathrm{AES}_k(J_i) \oplus M_i$.
4. Authentication.

   Let $T = 0^{128}$.
   Compute $H = \mathrm{AES}_k(0^{128})$.
   For i from 1 to v do: $T \leftarrow (T \oplus A_i) \bullet H$.
   For i from 1 to u do: $T \leftarrow (T \oplus C_i) \bullet H$.
   $T \leftarrow (T \oplus L) \bullet H$.
   Compute $t = \mathrm{AES}_k(J_0) \oplus T$.
5. Output: (IV, A, C, t).

Note: A secret key should be used to encrypt at most 2^{32} messages.

Authentication data A is one-block long ($v = 1$). Plaintext data M is two-blocks long ($u = 2$).



AES-GCM Decryption/Authentication

Upon receiving (IV, A, C, t), Bob does the following:

1. $L = L_A \parallel L_C$, where L_A, L_C are the bitlengths of A, C expressed as 64-bit integers.
2. Authentication.

   Let $T = 0^{128}$.
   Compute $H = \text{AES}_k(0^{128})$.
   For i from 1 to v do: $T \leftarrow (T + \oplus A_i) \bullet H$.
   For i from 1 to u do: $T \leftarrow (T \oplus C_i) \bullet H$.
   $T \leftarrow (T \oplus L) \bullet H$.
   Compute $t' = \text{AES}_k(J_0) \oplus T$.
   If $t' = t$ then proceed to decryption; if $t' \neq t$ then reject.
3. Decryption.

   Let $J_0 = IV \parallel 0^{31} \parallel 1$.
   For i from 1 to u do:
   Compute $J_i = J_{i-1} + 1$ and $M_i = \text{AES}_k(J_i) \oplus C_i$.
4. Accept and output (A, M).

AES-GCM: Authentication Only

- $T \leftarrow 0^{128}$, $H \leftarrow \text{AES}_k(0^{128})$
- For i from 1 to v do: $T \leftarrow (T + A_i)H$

  $(k, A, C, L, J_0) \rightarrow t$

$$\begin{aligned} T &= ((((0 + A_1)H + A_2)H + A_3)H + \cdots + A_v)H \\ &= A_1H^v + A_2H^{v-1} + A_3H^{v-2} + \cdots + A_{v-1}H^2 + A_vH \end{aligned}$$

- For i from 1 to u do: $T \leftarrow (T + C_i)H$. $T \leftarrow (T + L)H$

$$T = A_1H^{u+v+1} + A_2H^{u+v} + \cdots + A_vH^{u+2} + C_1H^{u+1} + \cdots + C_uH^2 + LH = f_{A,M}(H),$$

where

$$f_{A,M}(x) = A_1x^{u+v+1} + \cdots + A_vx^{u+2} + C_1x^{u+1} + \cdots + C_ux^2 + Lx \in GF(2^{128})[x]$$

- Hence, $t = \text{AES}_k(J_0) \oplus f_{A,M}(H)$.

Consider AES-GCM with no M, so $u = 0, L_M = 0$. The message to be authenticated is $A = (A_1, A_2, \ldots, A_v)$, where $v \leq \ell$. The tag is (IV, t), where $IV \in_R \{0,1\}^{96}$ and $t = \text{AES}_k(J_0) + f_A(H)$ (and $J_0 = IV \parallel 0^{31} \parallel 1, H = \text{AES}_k(0)$).

Attack goal: Eve has message-tag pairs (for messages of her choosing): (A^j, IV^j, t^j), $1 \leq j \leq r$. Her goal is to produce a message-tag forgery (A^*, IV^*, t^*), where $A^* \notin \{A^1, A^2, \ldots, A^r\}/$

We can assume that no two IV's in this list are the same. We can also assume that Eve does not know k or H.

Security Argument

Now, suppose that Eve outputs a forgery (A^*, IV^*, t^*).

If $IV^* \notin \{IV^1, \ldots, IV^r\}$, then $J_0^* \notin \{J_0^1, \ldots, J_0^r\}$, and so Eve doesn't know $\text{AES}_k(J_0^*)$, which serve as a one-time pad for $f_{A^*}(H)$. Thus, the probability that Eve can output a valid tag (i.e., $t^* = \text{AES}_k(J_0^*) + f_{A^*}(H)$) is only $1/2^{128}$.

Suppose that $IV^* = IV^j$, for some $1 \leq j \leq r$, so $J_0^* = J_0^j$. Then

$$t^* - t^j = \text{AES}_k(J_0^*) + f_{A^*}(H) - \text{AES}_k(J_0^j) - f_{A^j}(H) = f_{A^*}(H) - f_{A^j}(H)$$

So, Eve has produced A^*, A^j and α such that $\alpha = f_{A^*}(H) - f_{A^j}(H)$, without knowledge of H. But this can only be done with negligible probability, as the following lemma shows.

Lemma 5.1

For all distinct $A, B \in \left(\{0,1\}^{128}\right)^{\leq \ell}$ and $\alpha \in \{0,1\}^{128}$,

$$\Pr[f_A(H) - f_B(H)] = \alpha \leq (\ell + 1)/2^{128}$$

(which is negligible), where the probability is assessed over random choices of $H \in \{0,1\}^{128}$.

Proof:

Let $A, B \in \left(\{0,1\}^{128}\right)^{\leq \ell}$ with $A \neq B$, and let $\alpha \in \{0,1\}^{128}$. Suppose $A \in \{0,1\}^{128r}$ and $B \in \{0,1\}^{128w}$. Then

$$\begin{aligned} f_A(H) - f_B(H) - \alpha = &(A_1H^{v+1} + A_2H^v + \cdots + A_vH^2 + L_AH) \\ &- (B_1H^{w+1} + B_2H^w + \cdots + B_wH^2 + L_BH) - \alpha, \end{aligned}$$

which is a polynomial in H of degree $\leq \max(v, w) + 1 \leq \ell + 1$.

Since a nonzero polynomial of degree $\leq \ell + 1$ can have at most $\ell + 1$ roots, there are at most $\ell + 1$ $H \in \{0,1\}^{128}$ satisfying $f_A(H) - f_B(H) - \alpha = 0$. □

Some Features of AES-GCM

1. Performs authentication and encryption.

2. Supports authentication only (by using empty M).
3. Very fast implementations on Intel and AMD processors because of special AES-NI and PCLMULQDQ instructions for the AES and • operations.
4. Encryption and decryption can be parallelized.
5. AES-GCM can be used in streaming mode.
6. Security is justified by a security proof:

   Original McGrew-Viega security proof (2004) was wrong. The proof was fixed in 2012 by Iwata-Ohashi-Minematsu.

AES-GCM is widely used today.

5.4 NSA Surveillance

GCHQ/NSA MUSCULAR program

Disclosed by Edward Snowden on October 30, 2013.

Surveillance program conducted by GCHQ (British spy agency) in partnership with NSA.

An unnamed telecommunications operator provided GCHQ with secret access to its fibre optic cables that transported data between Google and Yahoo! data centres.

Millions of records collected each day.

In November 2013, Google and Yahoo! announced they were encrypting all traffic between their data centres.

5.5 Google Encryption

Google Data Centres

Google has 21 data centres around the world.

A data centre (DC) contains tens of thousands of servers.

Lots of physical security (cameras, biometric identification, metal detectors, vehicle barriers, etc.)

Communication between these servers and the outside world is all done via Google Front End (GFE) servers.

Servers within a data centre communicate via a LAN (Local Area Network).

Servers in different data centres communicate via a WLAN (Wide Local Area Network).

Three kinds of data to protect:

1. Data communicated between individual users (browsers) and Google (GFEs).

   Data is encrypted and authenticated using TLS. Symmetric-key enc. (e.g., AES), symmetric-key authentication (e.g., HMAC); authenticated enc. (e.g., AES-GCM); key establishment (e.g., RSA public-key enc., ECDH); public key certificates (RSA signatures). TLS is used by all web servers and browsers (not Google specific).

2. Data communicated between Google servers (perhaps in different data centres).

Data is secured using Google's version of TLS (Application Layer Transport Security, ALTS). ALTS handles roughly 1010 remote procedure calls (RPCs) per second.

3. Data stored at data centres.

Key Management Service (KMS)

All data stored within Google data centres is encrypted with AES256-GCM (GCM = Galois Counter Mode).

AES128-CTR + HMAC-SHA256 (Encrypt-then-MAC) is used in some legacy applications.

The KMS manages the many AES secret keys that are used to encrypt/decrypt data by the many storage services within data centres.

Some of the KMS requirements:

- Availability: > 99.9995% requests are served.
- Latency: 99% of requests are served in < 10 ms.
- Scalability: Handle) 107 requests/second.
- Security: Effortless and foolproof key rotation.
- Efficiency: To minimize number of machines needed.

On January 24, 2014, a KMS configuration file was truncated by error. As a result, the KMS did not know the secret keys used to decrypt stored data. Gmail, Calendar, Docs, etc. crashed for 25+ minutes.

"This got a lot of attention within Google." Subsequently, Google made many changes to its KMS. $\gg$ 99.9999% of KMS requests are served. 99.9% of requests are served in $< 200\mu s$.

Google's Key Hierarchy

1. Storage systems (millions of processes)

   Encrypts data with DEKs (Data Encryption Keys).

2. KMS (tens of thousands)

   Encrypts DEKs with KEKs (Key Encryption Keys).

3. Root KMS (hundreds)

   Encrypts KEKs with KMS Master Keys.

4. Root KMS Master Key Distributor (hundreds)

   Encrypts KMS Master Keys with the Root KMS Master Key.

5. Physical safes (two)

   The Root KMS Master Key is backed up on hardware devices.

1. Storage Systems

Suppose that a storage system wishes to encrypt some data item m. The storage system does the following.

1. Break up m into chunks, m1, . . .,m!. ! Each chunk can be up to several Gigabytes in size.

2. Generate ℓ Data Encryption Keys (DEKs), $k_1, \ldots, k_\ell$.

   Multiple sources of entropy are sampled (e.g., Intel's RDRAND instruction; inter-packet arrival times; measurement of disk seeks) The samples are then combined and hashed using a key derivation function (KDF) to produce a 256-bit secret key.

3. Encrypt with AES256-GCM: $c_1 = \text{AES}_{k_1}(m_1), \ldots, c_\ell = \text{AES}_{k_\ell}(m_\ell)$,

4. Send the DEKs $k_1, \ldots, k_\ell$ to a KMS.

5. Receive the wrapped (encrypted) keys $w_1, \ldots, w_\ell$ from the KMS.

   The KMS encrypts the DEKs with its Key Encryption Keys (KEKs).

6. Store $(c_1, w_1), \ldots, (c_\ell, w_\ell)$.

   These encrypted chunks are replicated and distributed across Google's storage systems.

To decrypt a chunk (c_j, w_j):

1. The storage system sends the wrapped key wj to the KMS.
2. The KMS decrypts wj using the appropriate KEK, and sends the DEK kj to the storage system.
3. The storage system decrypts cj using kj .

Note:

1. Each data chunk has a unique identifier.
2. The KMS maintains an Access Control List (ACL) to ensure that a data chunk can only be decrypted by the authorized storage system.
3. Note that each chunk of data is encrypted using a different DEK. This ensures that if a DEK is compromised, then only one chunk of data is potentially compromised.
4. The storage system does not store the DEKs $k_1, \ldots, k_\ell$.
5. If a chunk of data m_j is updated, it is re-encrypted with a new DEK k'_j rather than using the old DEK k_j.

2. KMS

Key Management Services (KMSs) generate the AES256-GCM Key Encryption Keys (KEKs), and maintain the Access Control Lists (one ACL list for each KEK).

The KMS encrypts/decrypts DEKs using the KEKs, in accordance with the ACL.

The KEKs never leave the KMS.

The KMS also maintains an audit trail of when a KEK was used.

KEKs are rotated (i.e., changed). The standard rotation frequency is once every 90 days.

3,4. Root KMS

The Root KMS wraps KEKs with AES256-GCM KMS Master Keys.

There are about a dozen KMS Master Keys.

The Root KMSs are run on dedicated secured machines in Google's data centres.

The KMS Master Keys are wrapped with the AES256-GCM Root KMS Master Key.

The Root KMS Master Key is stored in RAM on the Root KMS machines.

The Root KMS Master Key Distributor ensures that all Root KMSs always have the same version of the Root KMS Master Key.

5. Physical Safes

The Root KMS Master Key is backed up on two hardware devices stored in physical safes in highly secured areas in two physically separated Google locations.

Fewer than 20 Google employees have access to these safes.

The backups will be used if Google ever has to do a complete reboot.