Algorithm Design and Data Abstraction

CS 146

Brad Lushman

Preface

Disclaimer Much of the information on this set of notes is transcribed directly/indirectly from the lectures of CS 146 during Winter 2021 as well as other related resources. I do not make any warranties about the completeness, reliability and accuracy of this set of notes. Use at your own risk.

For any questions, send me an email via https://notes.sibeliusp.com/contact.

You can find my notes for other courses on https://notes.sibeliusp.com/.



Contents

Pr	eface		1		
1	Jan 12				
	1.1	Major themes	4		
	1.2	Recursion	5		
	1.3	Impure Racket	6		
2	Jan	14	7		
	2.1	RAM	7		
	2.2	Modelling Output	7		
	2.3	Modelling input	11		
	2.4	Input in Racket	11		
3	Jan 19 12				
	3.1		12		
	3.2	TITLE TO THE TOTAL CONTROL OF THE TOTAL CONTROL OT THE TOTAL CONTROL OF	13		
	3.3		14		
	5 5	- .	14		
			15		
			15		
			15		
		3.3.5 Programs	16		
4	Jan 21				
7	4.1		17		
	4.2		18		
	4.3	Variables and input in C	19		
	4.4	Characters	20		
5	Jan	26	22		
	5.1	Improved getInt	22		
	5.2	Mutation (in Racket)	23		
			23		
	5.3		25		
6	Jan 28				
	6.1	Global variables in C	26		
	6.2		27		
	6.3		30		
	6.4	The British (D. L.)	30		

7	eb 3	33		
	termediate Mutation (Racket) cont'd	33		
	2 The same problem in C	35		
8	Feb 4			
	Advanced Mutation	39		
	2 Aliasing in C	41		
	3 Memory and vectors	42		
	4 Vectors in Racket	43		

Jan 12

1.1 Major themes

Major theme of CS 146

- side-effect ("impurity")
- programs that *do* things
- imperative programming

General outline

- impure Racket
- C
- low-level machine

Why functional programming first? Why not imperative first?

Imperative programming is harder. Side-effects are not easy things to deal with. For example, text is printed to the screen, keystrokes extracted from the keyboard, values of variables change. All these things change the state of the world. Also, the state of the world affects the program.

If we write a racket program like this one,

```
(define (f x) (+ x y))
```

That depends on the value of *y*. However, if the value of *y* can change because of the side effects, we have to add a word: it depends on *current value* of *y*.

Thus the semantics of an imperative program must take into account the current state of the world, even while changing the state of the world.

So there is then a temporal component inherent in analysis of imperative programs. It is not "what does this do?", but "what does this do at this point in time?"

Why study imperative programming at all? It seems it doesn't worth it. "The world is imperative". For example, machines work by mutating memory. Even functional programs are eventually executed imperatively.

... "or is it?" Is the world constantly mutating, or is is constantly being reinvented? When a character

appears on the screen, does that change the world or create a new one?

Either way, imperative programming matches up with real-world experience, but a functional world view may offer a unique take on side-effects.

1.2 Recursion

Recall from CS 145:

Structural recursion: the structure of the program matches the structure of data.

For example, natural numbers.

```
(define (fact n) ; A Nat is either
(if (= n 0) 1 ; 0 or
(* n (fact (- n 1))))); (+ 1 n) where n is a Nat
```

The cases in the function match the cases in the data definition. The recursive call uses arguments that either stay the same or get one step closer to the base of the data type.

Here is another example on the length of the list.

If the recursion is structural, the structure of the program matches the structure of its correctness by induction.

Claim (length *L*) produces the length of the list *L*.

Proof:

Structural induction on *L*.

Case 1 *L* is empty. Then (length *L*) produces o, which is the length of the empty list.

Case 2 *L* is (cons \times *L'*). Assume that (length *L'*) produces *n*, which is the length of *L'*. Then (length *L*) produces (+ 1 n), which is the length of (cons \times *L'*).

Correctness proof just looks like a restatement of the program itself.

Accumulative recursion one ore more extra parameters that "grow" while the other parameters "shrink".

For example,

```
(define (sum-list L)
(define (sum-list-help L acc)
(cond [(empty? L) acc]
[else (sum-list-help (rest L) (+ (first L) acc))]))
(sum-list-help L 0))
```

Proof method: induction on an invariant. For example, to prove that $(sum-list\ L)$ sums L, suffices to prove $(sum-list-help\ L\ 0)$ produces the sum of L. Let's try to prove by structural induction on L.

Case 1 L is empty. Then (sum-list-help L 0) is (sum-list-help empty 0) which gives o.

LECTURE 1. JAN 12 6

Case 2 $L = (\cos \times L')$. Assume (sum-list-help L' 0) \Rightarrow the sum of L'. Then (sum-list-help L 0) is (sum-list-help (cons $\times L'$) 0) which reduces to (sum-list-help L' (+ \times 0)) which is then equal to (sum-list-help L' \times). Then we are in trouble, because this does not match inductive hypothesis. Proof fails.

So we need a stronger statement about the relationship between L + acc that holds throughout the recursion - an invariant.

Proof:

We prove the invariant \forall L, \forall acc (sum-list-help L acc) produces acc + (sum-list L) by structural induction on L.

Case 1 L is empty. Then (sum-list-help L acc) is (sum-list-help empty acc) which gives acc, which is equal to the sum of the list + acc.

Case 2 L is (cons x L'). Assume (sum-list-help L' acc) produces the sum of L' + acc. Then (sum-list-help L acc) = (sum-list-help (cons x L') acc) \leadsto (sum-list-help L' (+ x acc)) which is equal to (sum-list L') + (x + acc) = (+ (sum-list L') x) + acc = (sum-list L) + acc

```
Then let acc = 0: (sum-list-help L 0) = (sum-list L). \Box
```

General recursion: does not follow the structure of the data. Proofs require more creativity.

How do we reason about imperative programs?

1.3 Impure Racket

```
(begin exp_1 ... exp_n)
```

evaluates all of exp_1 , ..., exp_n in left-to-right order and produces the value of exp_n . This is useless in a pure functional setting, but it is useful if exp_1 , ..., $exp_(n-1)$ are evaluated for their side-effects.

There is an implicit begin in the bodies of functions, lambdas, local, answers of cond/match. For example,

Reasoning about side-effects: for pure functional programming, we have the substitution model, so-called "stepping rules". Can the substitution model be adapted? we can have the "state of the world" an extra input & extra output at each step. So each reduction step transforms the program & also the "state of the world".

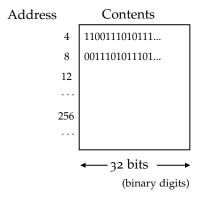
How do we model the "state of the world"? For the simple case, it is just a list of definitions. For more complex cases, we need some kind of memory model (RAM) (won't use yet).

Jan 14

2.1 RAM

For now: conceptualization of a RAM (random access machine). Memory is a sequence of "boxes", which are indexed by natural numbers ("addresses"). It contains a fixed size number (say 8 bits or 32 bits). Any box's contains can be fetched O(1) time.

For example, 32-bit RAM:



Will use in a later module, but keep it in mind.

2.2 Modelling Output

It is the simplest kind of side-effect. The "state of the world" here is the sequence of characters that have been printed to the screen. So each step of computation potentially adds characters to this sequence.

Note:

Every string is just a sequence of characters. Indeed, there is a racket function:

```
(string->list "abcd") ===> (list #\a #\b #\c #\d)
```

Substitution model $\pi_0 \Rightarrow \pi_1 \Rightarrow \pi_2 \Rightarrow \cdots \Rightarrow \pi_n$ where each π_i is a version of the program obtained by applying one reduction step to π_{i-1} .

In addition to this sequence of programs, now also: $\omega_0 \Rightarrow \omega_1 \Rightarrow \omega_1 \Rightarrow \cdots \Rightarrow \omega_n$ where each ω_i is a

version of the output sequence. Because the sequence of characters can only grow, each ω_i is a *prefix* of ω_{i+1} (can't "unprint" characters).

```
Therefore, we have a combined version: (\pi_0, \omega_0) \Rightarrow (\pi_1, \omega_1) \Rightarrow \cdots \Rightarrow (\pi_n, \omega_n).
```

Some program reductions will create definitions, (e.g., local), and these defined values will eventually change. So let's separate out the sequence of definitions δ .

So we got a triple now: $(\pi_0, \delta_0, \omega_0) \Rightarrow (\pi_1, \delta_1, \omega_1) \Rightarrow \cdots \Rightarrow (\pi_n, \delta_n, \omega_n)$ where δ_0, ω_0 , representing the beginning of the program, are empty.

If $\pi_0 = (\text{define id exp}) \dots$, then we reduce exp according to the usual CS 145 (& new CS 146) rules. This may cause characters to be sent to ω . Now exp is reduced to val. Then remove (define id val) from π and add to δ .

If $\pi_0 = \exp \ldots$, then we reduce \exp by the usual rules, which may cause characters to be sent to ω . Now \exp is reduced to val which is removed from π . So the characters that make up val added to ω .

When π is empty, then we are done. So δ , ω is the *state*, that which changes, other than the program itself. ω here is relatively harmless because changes to ω don't affect the running of the program. What about δ ? δ is not a problem yet, because variables are not yet changing. All we are doing now is adding new definitions, which is not really a change of state.

How can we affect ω ? In Racket, we can do

- (display x) which outputs the value of x with no line break
- (newline) gives the line break.
- (printf "The answer is ~a.\n" x) which is formatted print. The value of x replace ~a. And \n is the new line character. As a Racket character on its own: #\newline.

In Racket,

But then, what do display, newline, printf return? It looks that they don't return anything. We can try following:

```
1  > (define y (display "Hello"))
2  Hello
3  > y     ; y does have a value
4  > (list y) ; by a trick
5  '(#<void>)
```

They return special value #<void> which is not displayed in DrRacket. Basically, for functions, that essentially return nothing, and also the result of evaluating (void). Functions that return void are called *statements* or *commands* and that's where imperative programming gets its name.

Recall: a Racket function map. (map f (list l1 ... ln)) produces (list (f l1) ... (f ln)). It's reasonable to ask what if f is a statement? The idea: it is needed for side-effects and produces #<void>. Then (map f (list l1 ... ln)) produces (list #<void> ... #<void>) which is not useful.

Instead, now consider for-each: (for-each f (list l1 l2 ... ln)) performs (f l1), (f l2) ... (f ln) and produces #<void>. For example, we can use it as follows:

```
(define (print-with-spaces lst)
(for-each (lambda (x) (printf "~a " x)) lst))
```

This will print out each item in the list with spaces in between and will produce void at the end rather than a list of void's. Let's write for-each:

```
(define (for-each f lst)
(cond [(empty? lst) (void)]
[else (f (first lst)); implicit begin
(for-each f (rest lst))]))
```

or using if:

```
(define (for-each f lst)
(if (empty? lst)
(void)
(begin (f (first lst)) (for-each f (rest lst)))))
```

Doing nothing in one case of an if condition is common enough that there is a specialized form:

```
(define (for-each f lst)
(unless (empty? lst) (f (first lst)) (for-each f (rest lst)))); implicit begin
```

It evaluates body expressions if the test is false. Similarly, (when ...) evaluates body expressions if test is true.

Before we had output, the order of operations didn't matter (assuming no crashes/non-terminations), but now, the order of evaluation may affect the order of output. Also, before we had output, all non-terminating programs could be considered equivalent (not meaningful), but now non-terminating programs can do interesting things (e.g., print the digits of π).

Semantic model should include the possibility of non-terminating programs. What will be the meaning of the non-terminating programs be? It is what the program would produce "in the limit". Here we let Ω to denote the set of possible values of ω , which would include finite & infinite sequences of characters.

But why do we need output? We never used it in CS 145, and Racket has a REPL (Read-Eval-Print-Loop). We can just call functions and see the result. That's what Racket has, but many languages don't have this. Instead, they have compile/link/execute cycle. Under this cycle, the program is translated (by a *compiler*) to a native machine code and then executed from the command line. Then we will only see output if the program prints it. Below is an example of C program.

```
#include <stdio.h>
int main (void) {
   printf("Hello, world!\n");
   return 0;
}
```

Here we have to ask for it if we want something to show up in the screen (line 3).

What about Racket? Here is a use in Racket: tracing program.

```
(define (fact n)
(printf "fact applied to argument ~a\n" n); implicit begin
(if (= n 0) 1 (* n (fact (- n 1)))))
```

This can aid debugging.

2.3 Modelling input

Let's now talk about the input. We can imagine an infinite sequence consisting of all characters the user will ever press ι . So the model now is $(\pi, \delta, \omega, \iota)$. Every time we need to accept an input character, is the same as removing a character ι .

Here is a small problem: the sequence may *depend* on the output, so the users decide what to input *in response to* what is displayed on the screen. So a more realistic model of input would perhaps not assume all input is available at one.

The alternative: a request for input yields a function consuming one or more characters and producing the next program π , with the user's characters substituted for the read request. For example, a function (read-line), might be modeled as λ (line) line. So if user types "abc", as a result of this, we get "abc". Then the entire program reduces to a big "nesting" of input request functions, basically, one function per "prompt". If we supply user input for each prompt, it yields the final result.

Proof techniques for imperative programs will come much later.

2.4 Input in Racket

(read-line) produces a string consisting of all characters pressed until the first newline and the string we get does *not* contain the newline.

```
(read-line); pops up a little box and lets us to type
Test.
"Test."; and get back the string as the result.
(string->list (read-line)); if we type Test.
(list #\T #\e #\s #\t #\.)
```

To read a list of lines, the question then is how do we know when to stop reading? If we look carefully at the box popped up by (read-line), at the end of the box, there is a yellow button, which says "eof" (end of file). When we press that button, it also ends the search for input. "eof" means there is no more input.

```
(define (read-input)
(define nl (read-line)); nl stands for next line
(cond [(eof-object? nl) empty]
[else (cons nl (read-input))]))
```

Note that this implementation of (read-input) is not tail-recursive.

A more primitive form of input would be (read-char) which extracts one character from the input sequence.

Jan 19

3.1 More primitive input reading

read-char reads one char from the input sequence. Here is a quick demo.

peek-char examines the next char in the sequence, without removing it from the sequence. It does read the character, but does not take that from io, or the input stream.

Less primitive input: read consumes from input (and produces) an S-expression (no matter how many chars or lines it occupies)

```
property > (read); type abc
```

```
2 'abc ; symbol
3 > (read)
4 (a b c ; not closed
5 de f ghi ; racket not satisfied
6 ) ; bracket closed
7 '(a b c de f ghi)
8 > (read)
9 (a b (c d e (f)) g)
10 '(a b (c d e (f)) g)
```

3.2 Writing DrRacket

The next example is that we write DrRacket: Implementing a Racket REPL

```
(define (repl)
(define exp (read))
(cond [(eof-object? exp) (void)]
[else (display (interp (parse exp)))
(newline)
(repl)]))
```

parse figures out what that S-expression means: function/if... interp is do it.

Let's write our own version of read. Process typically happens in two steps. The first step is **Tokenization**, which converts sequence of raw characters to a sequence of *tokens* (meaningful "words"). For example, left paren, right paren, id, number... Typically, id's start with a letter, nums start with a digit. Because of that, here is a key observation: peeking at the next character tells us what kind of token we will be getting, and what to look for to complete the token. So this is asking us to build the structure: (struct token (type value)) where type is the kind of token: 'lp, 'rp, 'id, 'num; and value is the "value" of the token (numeric value, name, etc).

We gonna make a couple of helpers first:

Here is our main tokenizer:

```
i ; read-token: -> token

define (read-token)

(define fc (read-char))

(cond

[(char-whitespace? fc) (read-token)]

[(char=? fc #\() (token 'lp fc)]

[(char=? fc #\)) (token 'rp fc)]

[(char-alphabetic? fc) (token 'id (list->symbol (cons fc (read-id))))]

[(char-numeric? fc) (token 'id (list->number (cons fc (read-number))))]

[else (error "lexical error")]))
```

Note that list->symbol, list->number don't exist, but it's easy to build them.

Step 2 is **parsing**: are the tokens arranged into a sequence that has the structure of an s-exp? if so, then produce the s-exp. Let's first make a helper.

All left is to build read:

There are some good exercises:

- expand the set of token types, e.g., strings.
- handle other kinds of brackets, [], { } which have to match.

What have we lost by accepting input? We lost *referential transparency*: the same expression has the same value whenever it is evaluated. For example, (f t) always produces the same value. If we do (let ((z (f 4))) body), then every (free) z in body can be replaced by (f 4) and vice versa. "equal can be substituted for equals". It is not true anymore! because (read) doesn't produce the same value. That makes it harder to reason about programs, where simple algebraic manipulation is no longer possible.

3.3 Intro to C

C is built from expressions, statements, blocks, functions, program.

3.3.1 Expressions

Example of expressions: 1 + 2 uses infix operators. There is a notion of precedence in C unlike racket. Also a function call, f(7), the name comes first. printf("%d\n", 5) is also a function call.

Operator precedence follows usual mathematical conventions. For example, 1 + x * y, multiplication is done first. If we want plus to do first, then we do (1 + x) * y.

We can take function call in a larger expression: 3 + f(x, y, z). printf("%d\n", 5) is a function call, and C substitutes 5 in place of %d, which means display as a decimal number. It's natural to ask, what does printf produce? It produces the number of characters printed.

3.3.2 Statements

The easiest way to make a statement (command) is to take an expression and put a semicolon at the end. For example, $printf("%d\n", x)$;. Here the value produced by the expression is ignored, so expression is used only for its side-effects. Thus we could do 1 + 2;, which is legal, but useless. Also, in previous lectures, we have seen return 0;, which produces the value o as the result of this function and control returns immediately to the caller. ; is an empty statement, which does nothing. Other statement forms to come.

3.3.3 Blocks

Block is a group of statements treated as one statement.

```
1 {
2     stmt 1
3     stmt 2
4     ...
5     stmt n
6 }
```

We can think this, sort of,

```
(begin stmt 1 ... stmt n)
```

Note that the difference begin has a value, which is the value of stmt n, and this is not the case in C. Thus this is not a perfect analogy. A better analogy is that we replace begin by void, then they will get evaluated but the entire thing has void value.

3.3.4 Functions

Here is a function.

```
int f(int x, int y) {
    printf("x = %d, y = %d\n", x, y);
    return x + y;
}
```

In racket, this would be roughly equivalent to

Function call: f(4, 3) is an expression, produces 7. f(4, 3); is a statement. Thus in racket, it can be viewed as, (f 4 3) and (void (f 4 3)).

Note that contracts (type signatures) are required and enforced.

3.3.5 Programs

Program itself is a sequence of functions. The starting point is the special function, known as main, and it looks like this

For example,

```
int main() {
    f(4, 3);
    return 0;

}

// and we got our f defined before
int f(int x, int y) {
    printf("x = %d, y = %d\n", x, y);
    return x + y;
}
```

If we give it to compiler, it won't compile. Why?

Jan 21

4.1 Compile C programs

Recall from last lecture:

```
int main() {
    f(4, 3);
    return 0;
}

// and we got our f defined before
int f(int x, int y) {
    printf("x = %d, y = %d\n", x, y);
    return x + y;
}
```

won't compile. A C program compiled: there is a program called the compiler that translates the program into the binary which is the only language the computer actually speaks and the computer execute this binary code directly: not through "DrC" like in DrRacket, the program runs natively on the machine on its own.

The way to compile: gcc myfile.c -Wall -o myfile. Here -o myfile is what we want the output program to be called, name of the output. If we don't do this, the default is a.out. -Wall stands for "Warn all". To run it, ./myfile where . means the current directory. Without specifying the current directory, it won't know where to find the program to run.

Now back to our problem. The compiler will complain: main doesn't know what f is. C enforces the rule: declaration-before-use: can't use a function/variable/etc... until we tell C about it. C has this rule because C is old, and it uses one-pass compiler.

Solution 1 Put f first.

```
int f(int x, int y) {
    printf("x = %d, y = %d\n", x, y);
    return x + y;
}
int main() {
    f(4, 3);
```

```
8 return 0;
9 }
```

Ok, but... this doesn't always work. We may want a different order just for the aesthetic of the program. Moreover, reordering the programs does more than C asks.

4.2 Declaration vs. Definitions

```
int f(int x, int y) {
    // ...
}
```

is both declaration (tells C the function exists) and definition (completely constructs the function).

C only requires declaration before use. So what we can do instead is

However, this still doesn't compile. What is printf? no declaration for printf. If we knew what it was, in theory we could do

```
int printf(---???---);
int f(int x, int y);

int main() {
    f(4, 3);
    return 0;

}

int f(int x, int y) {
    // ...
}
```

Rather than declare every standard library function header before we use it, C provides "header files". So we write

```
#include <stdio.h>

int f(int x, int y);

int main() {
    f(4, 3);
    return 0;
```

```
8  }
9
10 int f(int x, int y) {
11    // ...
12  }
```

#include is not part of the C language. Rather it is a directive to the C preprocessor (which runs before
the compiler). It's sort of like macro expansion in Racket. #include <file.h> means "drop the contents
of file.h right here". stdio.h contains declarations for printf/other IO (input output) functions, and
it is located in a "standard place". For example, /usr/include directory.

Now until this point, the compiler is satisfied. However, still technically incomplete: where is the code that implements printf? printf was written once, compiled once, and put in a "standard place", for example, /usr/lib.

Code for printf must be combined with our code. This step is known as "linking", which is done by a linker, and linker runs automatically. It "knows" to link the code for printf. If we write our own modules, then we need to tell the linker about them (later).

Let's go back to main: we have the returned value o. To whom am I returning the zero? The operating system. We can type echo \$? to check the returned value. Typically, 0 usually means OK. Anything > 0 is some kind of error.

Only in the case of main, return maybe left out, and in that case, 0 is assumed.

4.3 Variables and input in C

Let's talk about variables.

```
int f(int x, int y) {
   int z = x + y;
   int w = 2;
   return z / w;
}
```

Input:

```
#include <stdio.h>
int main() {
char c = getchar();
return c;
}
```

Let's try to read in a number.

```
#include <stdio.h>
// like before, we don't care about the negatives at this point.
int getIntHelper(int acc) {
    char c = getchar();
    if (c >= '0' && c <= '9') return getIntHelper(acc * 10 + c - '0');
    else return acc; // "else" keyword is technically not needed here.
}

// An alternative way: ternary operator</pre>
```

```
int getIntHelper(int acc) {
    char c = getchar();
    return (c >= '0' && c <= '9') ? getIntHelper(acc * 10 + c - '0') : acc;
}

int getInt() {
    return getIntHelper(0);
}</pre>
```

We got boolean conditions, like c >= '0', && means "and".

```
if (test) stmt
else stmt // only needed if there is sth to do in the false case.
```

Typically here, stmt will be a block:

```
if (test) {
    stmt 1
    ...
    stmt n
}
else {
    ...
}
```

It is recommended to put curly brace for the statement(s). Consider the dangling else problem:

```
if (condition 1)
if (condition 2)
stmt 1
else // this ''else'' actually goes to the second ''if''
stmt 2
```

Don't fool by the indentation. This is actually

```
if (condition 1) {
    if (condition 2) {
        stmt 1
    }
    else { stmt 2 }
}
```

Conditional operator? : (also called the ternary operator). if else is a statement while? : creates an expression: a? b: c has value b if a is true, has value c if a is false.

Also note that there is no built-in boolean type in C. 0 means false, and non-zero (often 1) means true. We have boolean type, constants true, false in stdbool.h.

4.4 Characters

are just restricted form of integer.

int varies, but typically occupies 32 bits ($\sim 4 \times 10^9$ distinct values). char occupies always 8 bits (256 distinct values). '0' is the character 0, numerically it is 48. Similarly, '9', numerically 57.

char $c = \frac{10}{3}$; is identical to char c = 48; etc. Everything in memory is numbers, so each character must have a numerical code that represents it. The code here is known as ASCII code.

To convert a char c to its numeric value: c - '0' (c - 48). Convert a number (0 - 9) to ASCII: c + '0'.

Let's take a second look at getchar: char c = getchar(); not match the prototype: int getchar(); Why int if it's supposed to produce a char? What if there are no chars? (EOF?) If getchar returned any character in this case, there would be no way to indicate EOF (every possible returned value denotes a valid character).

If there are no chars (EOF), getchar produces an int can't possibly be a char (not in the range 0..255). The constant EOF denotes the value getchar produces an eof (often, EOF = -1).

Next question: getInt burns a character after reading an int. Does C has a function like Racket's peek-char? No, but it has ungetc which stuffs a char back into the input stream.

```
int peekchar() {
   int c = getchar();
   return c == EOF ? EOF : ungetc(c, stdin);
}
```

Here we have equality operator ==. The reason we return EOF here is because we don't want to stuff a char if we didn't receive a char. stdin is the keyboard stream (or redirected). ungetc returns the char that was stuffed.

Jan 26

5.1 Improved getInt

An improved getInt, one doesn't burn a character.

```
#include <stdio.h>
#include <ctype.h> // character predicates

int getIntHelper(int acc) {
   int c = peekchar();
   return (isdigit(c)) ? getIntHelper(10 * acc + getchar() - '0') : acc; // predicate here will be false for non-digits, EOF.
}
```

This is simpler, but not efficient because we call peekchar and getchar. To be more efficient, we don't need to call getchar twice per character:

```
int getIntHelper(int acc) {
    int c = getchar();
    return (isdigit(c)) ? getIntHelper(10 * acc + c - '0') : (ungetc(c, stdin), acc);
}
```

a, b evaluates a, then evaluates b, result is the value of b. It is equivalent (begin a b) in racket. Use is sparingly, otherwise it will affect the readability of the code.

What if there is whitespace before we reach the int? So we can write a function skip the whitespace, but we don't want it return anything.

```
void skipws() {
   int c = getchar();
   if (isspace(c)) {
       skipws();
   }
   else ungetc(c, stdin);
}
```

Here is our first example of a void function. The idea is that it returns nothing, therefore, cannot be used in an expression. For example, void x = skipws(); is illegal. There are no void variables, thus

only good for side-effects. To return from void functions, either reach the end like in skipws, or return; with no expression.

With that in place, getInt becomes simpler:

```
int getInt() {
    skipws();
    return getIntHelper(0);
}
```

5.2 Mutation (in Racket)

Basic mutation: set! which is pronounced as "set bang", instead of impolite way "set" (with the extremely high volume).

```
(define x 3)
(set! x 4); produces (void), changes delta
```

Now x is 4. Note x must have been previously defined. So we can now change the value of a variable. What can we do with that? For example,

```
1 > (lookup 'Brad)
2 false
3 > (add 'Brad 36484)
4 > (lookup 'Brad)
5 36484
```

This is not possible in pure Racket because same expression can't produces different results. How do we implement this in impure Racket:

```
(define address-book empty); global variable, and is visible throughout the entire program
(define (add name number)
(set! address-book (cons (list name number) address-book)))
```

Global data is good for defining constants to be used repeatedly. *But* not good with mutation because any part of the program could change a global variable, thus it affects the entire program. So we got hidden dependencies between different parts of the program, and therefore, it's harder to reason about the program.

5.2.1 Application: Memoization

Caching: saving the result of a computation to avoid repeating it.

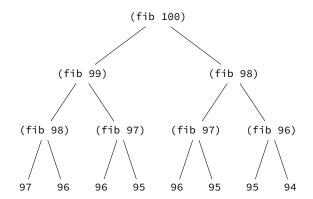
Memoization: maintaining a list or table of cached values.

Consider

```
(define (fib n)
(cond [(= n 0) 0]
[(= n 1) 1]
[else (+ (fib (- n 1))
(fib (- n 2)))]))
```

Note that this is inefficient because recursive calls are repeated.

So if want fib (100), it will be expanded as follows:



Note that (fib 98) called twice, (fib 97) called 3 times, (fib 96) called 5 times ... Thus (fib n) is $\Theta(F_n) \approx \varphi^n$ where $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$. So we can avoid repetition by keeping an association list of paris (n, F_n) .

Few notes here.

- assoc is a builtin function for association list lookup. In particular, (assoc x lst) returns the pair (x y) from lst or false if it fails.
- Any value can be used as a test. In racket, false is false, anything else is true.
- (cond [x => f]) ... if x passes (i.e., is not false), then produces (f x).

 (cond [result => second] ...) is equivalent to (cond [(list? result) (second result)] ...)

Now calls to (fib n) now happen only once. But our global variable fib-table is accessed by anyone. Can we hide it? Can we arrange that only memo-fib has access to this global variable? Here is a way to do it:

```
(define memo-fib
(local [(define fib-table empty)
(define (memo-fib n) ...)]
memo-fib))
```

Equivalently, use let:

```
(define memo-fib
(let ((fib-table empty))
(lambda (n) ...)))
```

This doesn't quite work for the address-book because we have two functions which need access to it.

5.3 Mutation in C

In C, we have an operator = performing mutation ("assignment operator"). For example,

```
int main() {
   int x = 3;
   printf("%d\n", x); // 3
   x = 4;
   printf("%d\n", x); // 4
}
```

Note that = is an operator. x = y is an expression, thus it has a value as well as an effect: its value is the value assigned. Then x = 4 sets x to 4, and has value 4. For example,

```
int main() {
   int x = 3;
   printf("%d\n", x);  // 3
   printf("%d\n", x = 4); // 4
}
```

It really has no advantages... and it has many disadvantages. Because assignment is an expression, C allows us to do x = y = z = 7; which sets all of x, y, z to y. Now consider the following,

```
int main() {
    int x = 5;
    if (x = 4) { // this assigns x to 4, and has value 4, non-zero, true.
        printf("x is 4\n"); // Thus always prints x is 4
}

x = 0;
if (x = 0) { // this assigns x to 0, and has value 0, false.
    printf("x is 0\n"); // Thus never prints x is 0
}
```

It is easy to confuse assignment with equality check: if (x == 4) ... Thus usually best to use assignment only as a statement.

One thing we can do is that we can leave variables uninitialized and assign them later. For example,

```
int main() {
int x; // uninitialized

x = 4;

...
}
```

This is actually not a good idea. Do only with a good reason. For example,

```
int x;
if (x == 0) {
    ... // will this run or not?
}
```

The answer to the question above is *we don't know*, because x's value is not known! The value of an uninitialized variable is undefined. Typically, it's whatever value was in the memory from before.

Jan 28

6.1 Global variables in C

Be careful:

```
int main() {
    printf("%d\n%d\n", f(), f());
}
```

This could produce 1 or 1 or others! Order of argument evaluation is *unspecified*.

2 0

As with the Racket fib example, we can interfere with f by mutating c.

Can we protect c from access by functions other than f? Yes by using a magic keyword: static.

```
int f() {
    static int c = 0;
    int d = c;
    c = c + 1;
    return d;
}
```

Here c is still a global variable, but it's a global variable that only f can see. In terms of variables, there are two notions which we tend to group them together because they are often the same. One is scope, one is extent or lifetime. A traditional global variable has global scope, thus everyone can see it. But more importantly here, its extent: how long it is alive, so it has a global extent. Static variable c has a local scope: only f can see it, but a global extent: it does not go away when f goes away.

6.2 Repetition

Let's say I write a function like this:

```
void sayHiNTimes(int n) {
    if (n > 0) {
        printf("Hi\n");
        sayHiNTimes(n-1);
    }
}
```

This is tail recursion: the recursion call is the last thing the function does. In C, with mutation, we can express this more idiomatically as

```
void sayHiNTimes(int n) {
    while (n > 0) {
        printf("Hi\n");
        n = n - 1;
    }
}
```

This is known as a *loop*, basically shorthand for tail-recursive computation. The body of the loop is executed repeatedly, as long as the condition remains true. In general, if we have

```
void f(int c) {

if (cont(c)) { // continuation condition

body(c);

f(update(c));

}

}
```

then it becomes

```
void f(int c) {
    while (cont(c)) {
        body(c);
        c = update(c);
    }
}
```

So in the latter version, f is not needed, i.e., the things inside may not need to be its own function anymore, if used only once. If we have accumulators, we can still do that.

```
int f(int c, int acc) {
    if (cont(c)) {
        body(c);
        return f(update1(c), update2(c, acc));
}
```

```
return g(acc);

f(acc, 0);

return g(acc);

f(acc, 0);
```

Then it becomes

```
int acc = acc0;
while (cont(c)) {
   body(c);
   acc= update2(c, acc);
   c = update1(c);
}
acc = g(acc);
```

Let's do a concrete example.

```
int getIntHelper(int acc) {
    char c = getchar();
    if (isdigit(c)) {
        return getIntHelper(10*acc+c-'0');
    }
    return acc;
}

int getInt() {
    return getIntHelper(0);
}
```

How might we change it? We can do:

```
int acc = 0;
char c = getchar();
while (isdigit(c)) {
    acc = 10 * acc + c - '0';
    c = getchar();
}
```

which is also shorter. We notice some common patterns, which we can emerge:

```
(initialize variables)
while (condition) {
    (body)
    (update variables)
}
```

It's common to forget the "update step", then we might have infinite loop. There is an alternative format which forces you to do all important things upfront, then much harder to forget them.

```
for (init; condition; update) {
    (body)
}
```

With for loop, we might do

```
int acc = 0;
char c;
for (c = getchar(); isdigit(c); c = getchar()) {
    acc = 10 * acc + c - '0';
}
```

or even

```
int acc = 0;
for (char c = getchar(); isdigit(c); c = getchar()) {
    acc = 10 * acc + c - '0';
}
```

So in the latter version, we put the initialization in the part of the loop. Is there a difference between doing these two things? And a related question to that: couldn't I also put initialization of acc in the loop as well? The answers to both questions have to do with the scope. When I declare the variable outside the loop, the scope is outside the loop. By putting the definition of c right in the loop, the scope of c is confined to the loop. Once the loop is done, there is no such c anymore. Or even, if we are inclined, we can write the loop as so:

```
int acc = 0;
for (char c = getchar(); isdigit(c); acc=10*acc+c-'0', c=getchar());
```

Note the usage of comma operator here, which makes this legible. Also, the loop body is empty, which is indicated by ;, an empty statement, or { }.

What about the peekchar version?

```
int acc = 0;
for (char c = peekchar(); isdigit(c); acc=10*acc+getchar()-'0', c=peekchar());
```

or even

```
int acc = 0;
for (char c = peekchar(); isdigit(c);c=(getchar(),peekchar())) {
    acc = 10 * acc + getchar() - '0';
}
```

Often loop is controlled by counters, then we update counters. For example, here are some very common patterns:

```
c = c + 1;

c = c - 2;

c = 10 * c;

c = c / 2;

c = c + d;
```

C has some specialized syntax, which is equivalent to above:

```
c += 1;

c -= 2;

c *= 10;

c /= 2;

c += d;
```

If we want to increment/decrement by 1, then ++c increments c; --i decrements i.

These are expressions, thus they have a value as well as an effect. ++c increments c and produces the value of c and --i similarly. Which value? the old one or the new one? We can try these out.

```
int i = 1;
printf("%d\n", ++i); // 2
```

Thus the new value. There is also a postfix versions i++, i--, which people seem to like better. These postfix versions increment/decrement i, but produce the old value of i. So it implies the old value must be remembered.

For the most cases, prefix is simpler. The one possible reason for people prefer postfix is because the name of C++, which is not called ++C. If we use increment/decrement operators, and there is no good reason for postfix, we should use prefix version.

6.3 More on Global Data

Global variables likes int i = 0;, we should avoid where possible because it creates hidden dependencies. However, Global *constants* are still useful. We can force a variable to remain constant in C. We can say

```
const int passingGrade = 50; // cannot be mutated.
```

6.4 Intermediate Mutation (Racket)

What if we want to work with multiple address books?

home is still empty, no change! Code doesn't work! Not clear how to make it work. What does substitution model say? (add-entry home "Neighbour" 34567) says substitute '() for abook in body. Then it becomes (set! '() (cons (list name number) '())). The latter part makes sense. However, (set! '() ...) doesn't makes sense: we are mutating an empty list; also based on this statement, Racket has no idea we are mutating home.

To make this work... Recall from CS 145 (??), simulation of structs using lambda. Do the same thing to create a struct with one field, called a box. A box has two operations: get the value in the box; set the value to a new value.

```
(define (make-box v)
(lambda (msg)
(cond [(equal? msg 'get) v])))
```

```
(define (get b) (b 'get))

(define b1 (make-box 7))

(get b1)

; becomes
(define b1 (lambda msg) (cond [(equal? msg 'get) 7]))

(get b1)

; becomes
(get (lambda (msg (cond [(equal? msg 'get) 7]))))

; becomes
((lambda (msg) (cond [(equal? msg 'get) 7]))) 'get)

; becomes
(cond [(equal? 'get 'get) 7])

; becomes

(cond [(equal? 'get 'get) 7])

; becomes

7
```

To support set, we can introduce a local copy of v.

```
(define (make-box v)
(define val v)
(lambda (msg)
(cond [(equal? msg 'get) val])))

(define (get b) (b 'get))

(define bl (make-box 7))
(get bl)
(j becomes
(define val_1 7)
(define bl (lambda (msg)
(cond [(equal? msg 'get) val_1])))
(j and eventually it will give us)
```

Now how do we add set? It requires an extra parameter. We can achieve this by having the box return a function.

```
; becomes
   (define val_1 7)
   (set (lambda (msg) ... val_1 ...) 4)
   ; becomes
   (define val_1 7)
   (((lambda (msg) ... val_1 ...) 'set) 4)
25
   ; becomes
   (define val_1 7)
   ((cond [(equal? 'set 'get) val_1]
          [(equal? 'set 'set) (lambda (newv) (set! val_1 newv))] 4 )
   ; becomes
31
   (define val_1 7)
   ((lambda (newv) (set! val_1 newv)) 4)
33
  ; becomes
   (define val_1 7)
   (set! val_1 4)
  ; becomes
   (define val_1 4)
  (void)
```

Feb 3

7.1 Intermediate Mutation (Racket) cont'd

Why (now) does this fix the problem?

Before we had this:

```
(define home '())
(add home ....)

reduces to
(add '() ....); value of the home substituted

reduces to
(set! '() ....); can't update home, how do we know this empty list is home not sth else?
```

Now we have

```
(define home (make-box '())); this creates a variable, local define

(define (add abook name num)
    (set abook (cons (list name num) (get abook)))); 'get' fetch the var

(add home ... ...); this is a function that can update the created variable
```

Boxes are actually built into Racket. The syntax for boxes:

Then address book example using built-in box could have been written:

```
(define home (box '(("Neighbour" 34567))))
(define work (box '(("Manager" 12345) ("Director" 23456))))
```

```
(define (add abook name num)
(set-box! abook (cons (list name num) (unbox abook))))
```

The semantics for box:

Convention: When we write an underscore before a variable name it means the variable's value is not looked up during the expression evaluation, unless (unbox __) is called on it. (Note that this is for the stepping rule, no particular meaning in Racket).

If we have $(unbox _n)$, then we want to find $(define _n v)$, then $(unbox _n)$ produces v. If we have $(set-box! _n v)$, then find $(define _n ...)$ and replace that with $(define _n v)$, then $(set-box! _n v)$ produces (void).

Let's see an example on how we step on boxes.

```
(define box1 (box 4))
(unbox box1)
(set-box! box1 true)
(unbox box1)
; becomes
(define _u1 4)
(define box1 _u1)
(unbox box1)
(set-box! box1 true)
(unbox box1)
; becomes
(define _u1 4)
(define box1 _u1)
(unbox _u1)
(set-box! box1 true)
(unbox box1)
; becomes
(define _u1 4)
(define box1 _u1)
(set-box! _u1 true)
(unbox box1)
; becomes
; (define _u1 4) no longer here
(define _u1 true)
(define box1 _u1)
(void)
(unbox box1)
```

```
34    ; becomes
35    (define _u1 true)
36    (define box1 _u1)
37    4
38    (unbox _u1)
39
40    ; becomes
41    (define _u1 true)
42    (define box1 _u1)
43    4
44    true
```

This is a bit messy, and it is one of the challenges of mutation.

7.2 The same problem in C

Suppose we want to write a function,

```
void inc(int x) {
    x = x + 1;
}

int main() {
    int x = 1;
    inc(x);
    printf("%d\n", x);
}
```

What we want is 2, but what we get is 1. Racket solution is putting the variable in a box. What is the C equivalent? one-field structure?

Structures in C:

```
struct Posn {
int x;
int y
}; // we got this cute/weird/curious semicolon here.
```

This semicolon is not optional, *needed*. The "reason" they designed this cuteness/shortcut is for the following:

```
struct Posn {
int x;
int y
pl, p2, p3;
```

where we can define the struct and declare the variables at the same time. However, we tend to use struct variables locally, and struct definitions tend to be global...

Then we can use struct as follows:

```
int main() {
    struct Posn p;
    p.x = 3;
    p.y = 4;
```

```
printf("p=(%d, %d)\n", p.x, p.y);
}
```

or we can initialize all at once:

```
int main() {
    struct Posn p = {3, 4};
    printf("p = (%d, %d)\n", p.x, p.y);
}
```

But watch out the following, which is not allowed:

```
int main() {
    struct Posn p;
    p = {3, 4};
    printf("p = (%d, %d)\n", p.x, p.y);
}
```

Let's try to write a function mutate the struct:

```
void swap (struct Posn p) {
    int temp = p.x;
    p.x = p.y;
    p.y = temp;
}

int main() {
    struct Posn p = {3, 4};
    swap(p);
    printf("p = (%d, %d)\n", p.x, p.y);
}
```

What we want is p = (4, 3), but what we get is p = (3, 4). Still doesn't work.

The problem: C (and also Racket) passes parameters by a mechanism called *call-by-value*. The idea is that the function operates on a *copy* of the argument, not the argument itself. Note that the Racket substitution model naturally implements call-by-value. For example, if we do,

```
(define x 3)
(f x) => (f 3); the value of x, not x itself
```

In C, what we have is

```
void inc(int x) {
    ++x;
}
```

Here x really does get mutated, but it's a copy of x, not the original from the caller. Therefore, the original remains the same. Similarly, in swap we wrote, the entire structure is copied into the function, thus the original structure does not change.

So there is something special about boxes. They are not equal to the value they hold, but they tell (know) you how to *find* the value: unbox/get to find the value. What does that look like in C? To find a value, we are asking where it is located: it is in memory (RAM). As we said, every value in memory has an address. If given the address, we can "find" the value. Thus addresses could function as boxes. Instead of passing a value to a function, we can pass an address. For example,

```
int main() {
   int x = 1;
   inc(&x); // & is known as ''address-of'' operator, which passes x's address, not its
   value.

printf("%d\n", x);
}

void inc(int x) { // this is wrong now. We didn't
   x = x + 1; // get an int, we get an address.
}
```

Maybe: void inc(address x) $\{...\}$ which is also wrong. We need more info than address: what type of data is stored at that address? We need to say x is the address of an int.

This is still wrong because of its body: we don't want to add 1 to the address, but we want to add 1 to the value stored at the address. So what we want is:

```
void inc(int *x) {
    *x = *x + 1; // * = dereference operator
}
```

and the dereference operator = fetch the value stored at this address (unbox in Racket). LHS of assignment: *x = expr = store the value of expr at address x (set-box! in Racket). Thus in Racket, it's equivalent to (set-box! x (+ (unbox 1) 1)).

When we see int *x, it's like x is a pointer to an int, but it's intended to be read is "*x is an int".

Alternatively we can write:

When we say *x++, we got operators on both left and right. Which of these two actually happens first? the * or the ++? In C, postfix always takes precedence over prefix. So *x++ means *(x++). The address is incremented and the old value of the address is fetched (and thrown away). Thus no change to the original variable. If we cannot give up the postfix habit, we can do:

```
void inc(int *x) {
    (*x)++;
}
```

Now consider swap. The first version didn't work. Just like inc, we can fix this by passing a pointer:

```
void swap(struct Posn *p) {
    int temp = *p.x;
    *p.x = *p.y;
    *p.y = temp;
}
```

which is wrong, and won't even compile. Same problem as before: postfix before prefix. *p.x = *p.y means *(p.x) = *(p.y) and p.x, p.y aren't pointers. We need parentheses:

```
void swap(struct Posn *p) {
    int temp = (*p).x;
    (*p).x = (*p).y;
    (*p).y = temp;
}
```

However, this is clunky. (*p).x is common enough that it has its own notation: p->x. Thus the previous code can be written as:

```
void swap(struct Posn *p) {
    int temp = p->x;
    p->x = p->y;
    p->y = temp;
}
```

Thus we have more sophisticated user input: scanf. Note that scanf("%d", x) is wrong. It should read x as a decimal integer and skip leading whitespace. Here scanf is a function which can't modify x. Instead, we can do scanf("%d", &x). We can also do scanf("%d %d", &x, &y). The space between %d's means to skip *any* amount of whitespace between the two ints (including zero). Zero space is possible if the second int is negative.

Note that scanf returns the number of arguments actually read. scanf has lots of options, very complicated.

Feb 4

8.1 Advanced Mutation

It means mutating structures and lists.

In Scheme, we can mutate parts of a cons with set-car! and set-cdr!.

In Racket, cons fields are immutable, cannot be mutated. For mutable paris, Racket provides mcons. To mutate fields, mset-car!, mset-cdr!. For structs, it is also immutable. But Racket provides the option #:mutable. It will look like this:

```
1  (struct pos (x y) #:mutable)
2  (define p (posn 3 4))
3  (set-posn-x! p 5)
4  (posn-x p); => 5
```

This has an impact on our semantics: from CS 145, we said (make-posn v1 v2) is a value. However, now, (posn v1 v2) cannot be a simple value if it is mutable. It has to behave more like a box. How would a struct behave like a box? Here are the two ways. Is a struct automatically boxed? or is a struct a box? We can find that out by some experiments.

So a struct is not automatically boxed, but it does box its contents. So we can write (posn v1 v2) as

```
(define _val1 v1) ; recall, no expansion
```

```
define _val2 v2)
(posn _val1 _val2)
```

(posn-x p) where p is (posn _val1 val2), we find the definition for _val1, fetch the value.

(set-posn-x! p v) where p is (posn _val1 val2), we find (define _val1 \dots), replace it with (define _val1 v).

So now we are ready to do some stepping. For example,

```
(define p1 (posn 3 4))
   (set-posn-x! p1 5)
   ; becomes
   (define _v1 3)
   (define _v2 4)
   (define p1 (posn _v1 _v2))
   (set-posn-x! p1 5)
   ; becomes
  (define _v1 3)
  (define _v2 4)
   (define p1 (posn _v1 _v2))
   (set-posn-x! (posn _v1 _v2) 5)
15
   ; becomes
   (define _v1 5)
  (define _v2 4)
   (define p1 (posn _v1 _v2))
   (void)
```

These rules generalize to any mutable struct, mcons.

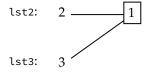
Now consider

```
(define lst1 (cons (box 1) empty)); note that we are using cons, not mutable
(define lst2 (cons 2 lst1))
(define lst3 (cons 3 lst1))
(set-box! (first (rest lst2)) 4)
(unbox (first (rest lst3))); gives 4
```

By the CS 145 understanding, this should produce 1. But in fact, it gives 4. Why?

```
lst2 = (cons 2 (cons (box1) empty))
lst3 = (cons 3 (cons (box1) empty))
```

In this case, the two (box 1)'s are actually the same object. When we define lst2 and lst3, these two lists actually share the same tail. What we actually have is



We could never tell that this was true in CS 145 because there is no way to observe a difference in

terms of whether these two lists are completely distinct lists or actually they are sharing the tail unless we can perform mutation. Thus we do need mutation to observe this.

Nevertheless, we could actually deduce they are actually sharing the tail. We can do

```
(define lst1 '(1 2 ... 100000000)); which takes O(n) time to build
(define lst2 (cons 0 (rest lst1))); O(1) time
```

If the second line is fast, it's not possible for Racket to recreate an entire list, thus it must be reusing that list. From that perspective, lst2 is sharing the tail with lst1.

Under the old substitution rules, we will get the wrong answer 1. Under the new substitution rules, boxes are rewritten as a separate define with deferred lookup. We then end up with

```
1 (define _val 1)
2 lst2 = (cons 2 (cons _val1 empty))
3 lst3 = (cons 3 (cons _val1 empty))
```

Here the shared item reflected in the rewrite.

All these force us to rethink what we mean by define. If we have

```
1  (define x 3)
2  (set! x 7)
3  X
4  ; becomes
5  (define x 3)
6  (void)
7  X
8  ; becomes
9  7
```

We cannot replace all occurrence of x with 3, otherwise we could have gotten 3 at the end. So x is not just a value, but something we can mutate, it's an entity we can access. Therefore x must denote a *location*, and the location contains the value. So we don't just have one lookup δ : var \rightarrow value, instead we have two lookups: var \rightarrow location, location \rightarrow value where the second lookup is carried out by RAM.

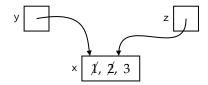
set! changes the location \rightarrow value map, but not the var \rightarrow location map (nothing changes that, at least for now). Similarly, set-box! changes the location \rightarrow value mapping. (define ...) creates a location, fills it with a value. Keep this in mind.

8.2 Aliasing in C

Does this happen in C as well? Yes. Consider

```
int main() {
   int x = 1;
   int *y = &x;
   int *z = &x; // or int *z = y;
   *y = 2;
   *z = 3;
   printf("%d %d %d\n", x, *y, *z);
}
```

The output is 3 3 3. Why? y is initialized to x's address, the y points to the location where x resides, and z is initialized to y (or &x), thus z also equals to x's address. Therefore, *y = 2 stores 2 at x's location: x = *y = *z = 2. Similarly, *z = 3 stores 3 at x's location. We can picture this visually:



Therefore, x, $\star y$, $\star z$ are three different names for the same data. This phenomenon is called *aliasing*: accessing the same data by different names. Aliasing is tricking business, and it can be subtle. Consider the following:

Hence it makes programs very difficult to understand.

8.3 Memory and vectors

Recall from the beginning of the course: Memory is a set of numbered "slots":



Each box is 8 bits (one byte), but they are usually treated in groups of 4-byte words.

Primitive data structure: the *array*, it's a "slice" of memory, and a sequence of consecutive memory locations. We will discuss at length when we return to C.

In Racket (also Scheme), it's known as the vector. It is used much like a traditional array. Unlike arrays, the slot of the vector can hold items of any size. Thus we can have unlimited integers, strings, whatever.

8.4 Vectors in Racket

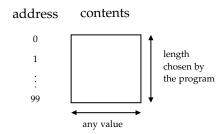
```
(define x (vector 'blue true "you"))
```

is a 3-item vector. From here we know that we can put whatever we want in vector. We can also do

```
1 (define y (make-vector 100))
2 y = '#(0 0 0 ... 0 0 0)
```

which creates a vector of length 100.

Unlike a list, it's like a many slice of memory. So it looks like,



We can also create a vector like

```
(define z (build-vector 100 sqr))
=> 0 1 4 9 ... 99^2
(define y (make-vector 100 5))
=> 5 5 5 5 ... 5 ; a hundred 5's
```

What is different about vectors versus lists is the way we work with them. We already know we work with a list by taking the first item and taking the rest. We process the list recursively. Vectors are quite different. They are not accessed by first and rest. Vector says which item do you want?

```
(define y (make-vector 100 5))
(vector-ref y 7) ; gives 5
(vector-set! y 7 42)
(vector-ref y 42) ; gives 42
```

Thus we access/mutate items by index. Hence the main advantage of vectors over lists is vector-ref and vector-set! run in O(1) time. This is a consequence of the way vector is stored versus the way list is stored. With a list, we can't get to a second item until we have been through the first item. If we want the 100th item in the list, we have to rest 99 times and first. On the other hand, with vectors, which is a slice of memory, if we want 100th item of the vector, all we need to say is where it's start and add 100 \times the slot size, which gives us the exact address where the item is located. Thus we can fetch any item in the vector in constant time.

It turns out these things are cranky to work with. What's wrong with vectors? It has several disadvantages:

- 1. The size is fixed. A list can very easily grow: all we need to do is to put cons at the front. However, with a vector, we have made a choice to designate a particular slice of memory of being part of that vector. The memory before/after it could very well be used for other things. Thus we cannot just take a vector and make it bigger.
- 2. It's difficult to add or remove elements. Everything is stored consecutively. If we want to take something out of the middle, that leaves a gap, we then need to shuffle everything down to close the gap. Similar situation if we want to add something into the middle.

3. Once we start to work with vectors instead of lists, we will find our Racket code doesn't work so well functionally anymore, it kinda force us to the imperative Racket.