Introduction to Game Theory

ECON 212

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Preface

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Introduction

What is game theory? It is study of strategic interactions between agents (players). One player is maximizing its utility subject to other constraints. The outcome for an agent depends not only on his choice of action but also on the action of the other player(s).

What do we mean by game? Unlike common definition of game: a competitive activity played according to rules like chess, monopoly, in this course, we mean strategic interactions between agents in any arena: politics, economic competition.

We are going to analyze these situations by building models (which are necessarily an incomplete abstraction). If we understand the "game", more likely to "win" at it. If we don't like it, we can sometimes change it, or not play it at all.

1.1 Strategic Game

For a complete mathematical definition, please refer to chapter 2 of CO 456.

strategic game

A **strategic game** consists of

- a set of **players**,
- a set of **actions** for each player,
- **preferences** over the set of action profiles for each player.

Action profile is an action for each different player, and there are many action profiles, which lead to different outcomes for the game.

Note that in a strategic game, "time" is absent. In other words, all players choose an action simultaneously. Once an action is chosen it cannot be changed This is different from the extensive games. In extensive games, there is a sequence of actions happening in different time. Then the players can actually see what other players played before.

A famous example of strategic game is **prison's dilemma** (PD). Two suspects are held in separate cells. The police has enough evidence to convict each of them of a minor offense. If one of them finks they can convict the other of a major offense (and they set free the one who finked). If they both fink,

they are both convicted of a major offense, but the punishment is slightly less harsh because they cooperated.

Now let's formalize this model. There are two players (suspects) p_1, p_2. There are two actions for each player: {Quiet, Fink}, or $\{Q, F\}$ for $i = 1, 2$. Thus there are four action profiles, and players have preferences over these four action profiles as follows:

$$u_1(F, Q) > u_1(Q, Q) > u_1(F, F) > u_1(Q, F)$$
$$u_2(Q, F) > u_2(Q, Q) > u_2(F, F) > u_2(F, Q)$$

Here $u_i(ap_1, ap_2)$ denotes the utility or payoff for player i in the action profile (ap_1, ap_2). We can also express this using a payoff table:

		Suspect 2	
		Fink	Quiet
Suspect 1	Fink	b, b	d, a
	Quiet	a, d	c, c

Here we typically have $a > b > c > d$. Take the top left cell as an example: first b is payoff to suspect 1 if they are both quiet, and second b refers to payoff to suspect 2 if they are both quiet. Similarly, second a in the top right cell is payoff to suspect 2 if suspect 1 is quiet and suspect 2 finks. First c in the bottom right cell is payoff to suspect 1 if they both fink.

As we can see here, prisoner's dilemma is quite simple, but it captures something really important. It models a situation where there are gains from cooperation, but each player is better off being uncooperative regardless of what the other does. Many real-life situations can be represented by this type of strategic interaction.

Some selected examples include arms race, two countries are deciding between no nuke or build nuke. It is true that there is no way for these two countries to coordinate, unless we go outside of this game: two countries can get together and say: "we realize that we are in prison's dilemma. The outcome would not be good for either one of us." Then two countries can sign some legally binding agreement, which can increase the cost of building nuclear weapons or diplomatic cost.

Another examples include advertising in duopoly (ads/no ads), tariff wars. In tariff wars, there are two countries engaging in free trade.