Rings and Fields

PMATH 334

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Preface

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Contents

1 Introduction

Fermat's Last Theorem

The equation $x^m + y^m = z^m$ has no non-trivial solutions in integers for $m \geq 3$.

For example, $(1, 0, 1)$, $(-1, 0, 1)$ for m even, are trivial solutions.

In 1897, Gabriel Lamé assumed that m is a prime

$$z^p = x^p + y^p = (x + y)(x + \zeta_p y)(x + \zeta_p^2 y) \cdots (x + \zeta_p^{p-1} y)$$

where $\zeta_p = \cos(\frac{2\pi}{p}) + i \sin(\frac{2\pi}{p})$.

Then the next step is to show that $(x + \zeta_p^? y)$ are coprime.

$$q_1^{\alpha_1} q_2^{\alpha_2} \cdots = z^p = (x + y)(x + \zeta_p y)(\quad) \cdots (\quad)$$