# Introduction to Game Theory

ECON 212

Joel Blit

LaTeXed by Sibelius Peng

# Preface

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Sibelius Peng

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# Introduction

What is game theory? It is study of strategic interactions between agents (players). One player is maximizing its utility subject to other constraints. The outcome for an agent depends not only on his choice of action but also on the action of the other player(s).

What do we mean by game? Unlike common definition of game: a competitive activity played according to rules like chess, monopoly, in this course, we mean strategic interactions between agents in any arena: politics, economic competition.

We are going to analyze these situations by building models (which are necessarily an incomplete abstraction). If we understand the "game", more likely to "win" at it. If we don't like it, we can sometimes change it, or not play it at all.

## 1.1 Strategic Game

For a complete mathematical definition, please refer to chapter 2 of CO 456.

**strategic game**

A **strategic game** consists of

- a set of **players**,
- a set of **actions** for each player,
- **preferences** over the set of action profiles for each player.

Action profile is an action for each different player, and there are many action profiles, which lead to different outcomes for the game.

Note that in a strategic game, "time" is absent. In other words, all players choose an action simultaneously. Once an action is chosen it cannot be changed This is different from the extensive games. In extensive games, there is a sequence of actions happening in different time. Then the players can actually see what other players played before.

### 1.1.1 Prison's Dilemma

A famous example of strategic game is **prison's dilemma** (PD). Two suspects are held in separate cells. The police has enough evidence to convict each of them of a minor offense. If one of them finks they can convict the other of a major offense (and they set free the one who finked). If they both fink, they are both convicted of a major offense, but the punishment is slightly less harsh because they cooperated.

Now let's formalize this model. There are two players (suspects) $p_1, p_2$. There are two actions for each player: {Quiet, Fink}, or $\{Q, F\}$ for $i = 1, 2$. Thus there are four action profiles, and players have preferences over these four action profiles as follows:

$$u_1(F, Q) > u_1(Q, Q) > u_1(F, F) > u_1(Q, F)$$
$$u_2(Q, F) > u_2(Q, Q) > u_2(F, F) > u_2(F, Q)$$

Here $u_i(ap_1, ap_2)$ denotes the utility or payoff for player $i$ in the action profile $(ap_1, ap_2)$. We can also express this using a payoff table:

| | | Suspect 2 | |
|---|---|---|---|
| | | Fink | Quiet |
| Suspect 1 | Fink | $b, b$ | $d, a$ |
| | Quiet | $a, d$ | $c, c$ |

Here we typically have $a > b > c > d$. Take the top left cell as an example: first $b$ is payoff to suspect 1 if they are both quiet, and second $b$ refers to payoff to suspect 2 if they are both quiet. Similarly, second $a$ in the top right cell is payoff to suspect 2 if suspect 1 is quiet and suspect 2 finks. First $c$ in the bottom right cell is payoff to suspect 1 if they both fink.

As we can see here, prisoner's dilemma is quite simple, but it captures something really important. It models a situation where there are gains from cooperation, but each player is better off being uncooperative regardless of what the other does. Many real-life situations can be represented by this type of strategic interaction.

Some selected examples include arms race, two countries are deciding between no nuke or build nuke. It is true that there is no way for these two countries to coordinate, unless we go outside of this game: two countries can get together and say: "we realize that we are in prison's dilemma. The outcome would not be good for either one of us." Then two countries can sign some legally binding agreement, which can increase the cost of building nuclear weapons or diplomatic cost.

#### Tariff Wars

Another examples include advertising in duopoly (ads/no ads), tariff wars. In tariff wars, there are two countries engaging in free trade. Each country can choose whether to impose a tariff. From https://en.wikipedia.org/wiki/Tariff,

> A tariff is a tax imposed by a government of a country or of a supranational union on imports or exports of goods.

Tariff results in the country paying lower prices for imports and therefore a gain of 8 at the other country's expense. But Tariffs also result in a deadweight loss of 5 for the country that imposes them. From https://www.investopedia.com/terms/d/deadweightloss.asp,

> A deadweight loss is a cost to society created by market inefficiency, which occurs when supply and demand are out of equilibrium. Mainly used in economics, deadweight loss can be applied to any deficiency caused by an inefficient allocation of resources.

Then we can depict the tariff wars with the following payoff table:

| | | Canada | |
|---|---|---|---|
| | | No Tariff | Tariff |
| U.S. | No Tariff | $0, 0$ | $-8, 3$ |
| | Tariff | $3, -8$ | $-5, -5$ |

Consider the bottom left cell. Imagine the U.S. imposes tariff on Canada, not vice versa. The U.S. gets a benefit of 8, but also suffers from economic inefficiency of 5, so the U.S. is 3 better off than they were in free trade. Canada doesn't have inefficiency cost, but 8 dollars are taken from the U.S., thus the payoff is 8 dollars lower than they were in free trade.

Now consider the bottom right cell. Both countries are taking 8 from the other. Then net-net, the result is 0 for both. And they both have $-5$ inefficiency cost.

## Carbon Emissions

There are two countries both of which are currently emitting lots of carbon. Each country can choose whether to continue to emit at current levels or whether to curb their emissions. Assume the cost of curbing emissions is 6, and benefit of a country curbing emissions is 5 (to each country in the world). This can be described in the following payoff table:

| | | Canada | |
|---|---|---|---|
| | | Curb Emissions | Emit |
| U.S. | Curb Emissions | $4, 4$ | $-1, 5$ |
| | Emit | $5, -1$ | $0, 0$ |

## Location of Production and IP Theft

Over the last 10 years, lots of high-tech U.S. companies have moved to China. They found that local competitors learned the technologies quickly from these companies, then they are going to have lower costs. The explanation lies in the game theory.

Assume there is an industry with two U.S. firms (duopoly) that are splitting the world market equally (earning $10M each). Each firm can choose whether to continue producing in the U.S. or shift production to China. Because producing in China lowers the costs, firm can drop prices and drive the other firm out of business. But when a firm produces in China their product will be imitated and they will face new Chinese entrants in their industry (which decreases profit by $5M).

| | | Firm 2 | |
|---|---|---|---|
| | | U.S. | China |
| Firm 1 | U.S. | 10M, 10M | 0M, 15M |
| | China | 15M, 0M | 5M, 5M |

## Joint Project

There are two students Alice and Bob working on a joint project. Each student can choose whether to put effort into their part of the project. If neither student works, they get 50%. If one works, they get 70%. If both work, they get 90%. But note that effort is costly. Each student is indifferent between working and getting a grade $X$, and shirking and getting $X - 30$.

| | | Bob | |
|---|---|---|---|
| | | Work | Shirk |
| Alice | Work | 60, 60 | 40, 70 |
| | Shirk | 70, 40 | 50, 50 |

To break this scenario, professor could observe how much effort each student has put in. Another way is that we can turn it into a repeated game. Then the outcome of this game will affect the games in the future.

### 1.1.2 Battle of the Sexes (BoS)

Early before the invention of the internet, Barb and Sam would like to get together in the evening. The "what's on" section of the paper says that there are two events going on in Waterloo that evening: Ballet and Soccer. Barb really likes watching Ballet, and Sam really likes watching Soccer. But they both really like each other and would most of all like to meet up. The problem is that they forgot to coordinate on the venue (and cell phones were not yet invented).

| | | Sam | |
|---|---|---|---|
| | | Ballet | Soccer |
| Barb | Ballet | 2, 1 | 0, 0 |
| | Soccer | 0, 0 | 1, 2 |

Observer that BoS is a coordination game. Players want to cooperate, so they both pick what's best for both of them. Once they cooperate, they are not incentive to do something else. This is quite different from the prison's dilemma where each player will deviate from the cooperation.

BoS can also model a number of different real-life situations. For example, the Prime Minister and the Finance Minister trying to decide which position to take on an issue. Another example would be that two firms trying to agree on an interface between their respective products.

### 1.1.3 Matching Pennies

Each of two players has a coin. They simultaneously choose whether to show the head or the tail of the coin If they show the same side. then player 2 pays player 1 a dollar. If they show a different side, then player 1 pays player 2 a dollar.

| | | PII | |
|---|---|---|---|
| | | Head | Tail |
| PI | Head | $1, -1$ | $-1, 1$ |
| | Tail | $-1, 1$ | $1, -1$ |

Notice that this is an example of game where one wins and the other loses. In other words, this is a "strictly competitive" game (the interests of the two players are diametrically opposed). A typical example would be penalties kick in football, in which a player is allowed to take a single shot on the goal while it is defended only by the opposing team's goalkeeper (from wiki).

### 1.1.4 Stag Hunt

A group of hunters is trying to catch a stag. Each hunter can focus on catching the stag, or he can instead catch a hare. If all hunters pursue the stag, then they catch it and share it equally. If any hunter goes for a hare (which he catches and keeps to himself), then the stag escapes All hunters prefer a stag, to a hare, to nothing.

Now let's formalize this model. Players are all the hunters. For each player, the set of actions is {Stag, Hare}. The preferences are: for each hunter the action profile in which they all choose Stag is ranked highest, followed by any profile in which he chooses Hare, followed by any profile in which he chooses Stag and at least one other hunter chooses Hare. If we have two hunters, we can draw a payoff table:

| | | HII | |
|---|---|---|---|
| | | Stag | Hare |
| HI | Stag | 2, 2 | 0, 1 |
| | Hare | 1, 0 | 1, 1 |

Stag Hunt is similar to the prison's dilemma except that players would rather cooperate as long as everyone else is cooperating.

Perhaps this is a better model of an arm's race. Country would rather not have arms when the other country doesn't. In other words, we don't care the military superiority over other countries, but we do care other countries not having the stronger militaries. Perhaps it is also a better model of working on a joint project for the similar reason. Student would rather put in effort as long as it is not futile.