



Fields and Galois Theory

PMATH 348



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Preface

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Introduction

One way to motivate Galois theory is to consider polynomial equations over real numbers. The simplest polynomial equation that we can solve is linear equation: $ax + b = 0$ with $a, b \in \mathbb{R}$ and $a \neq 0$. The solution to this equation is $x = -\frac{b}{a}$.

We can also solve the quadratic equations: $ax^2 + bx + c = 0$ with $a, b, c \in \mathbb{R}$ and $a \neq 0$. Its solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

When $b^2 - 4ac < 0$, we need the notion of complex numbers to such an equation.

We notice that the solutions of linear and quadratic equations only use addition, subtraction, multiplication, division and the n th root. This motivates the definition of radical expression.

radical expression

An expression involving only $+, -, *, \div, \sqrt[n]{}$ is called **radical**.

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