Godel's Incompleteness Theorems

Submitted by Sibi Yadav

July 2021

1 Summary

Incompleteness, as defined by Gödel The limits of provability in formal axiomatic systems are addressed by two theorems of mathematical logic. Kurt Gödel published these results in 1931, and they are significant in both mathematical logic and mathematical philosophy. The theorems are often, but not always, taken as demonstrating that Hilbert's programme to find a full and consistant set of functions is correct.The theorems are commonly, but not always, regarded as demonstrating that Hilbert's programme to create a full and consistent set of axioms for all mathematics is impossible.

According to the first incompleteness theorem, no consistent set of axioms whose theorems can be listed by an effective technique (i.e., an algorithm) can prove all truths about natural number arithmetic. There will always be claims about natural numbers that are true but unprovable within the system for any such consistent formal system. The second part is still missing.The system cannot demonstrate its own consistency, according to the second incompleteness theorem, which is an extension of the first.

Under basic assumptions, Gödel's second incompleteness theorem demonstrates that this canonical consistency statement Cons(F) will not be provable in F. In Gödel's work "On Formally Undecidable Propositions in Principia Mathematica and Related Systems I," the theorem was first published as "Theorem XI." The term "formalised system" in the following sentence also incorporates the assumption that F is effectively axiomatized. "Assume F is a consistent formalised system that contains elementary arithmetic," says the second incompleteness theorem.

This theorem is more powerful than the first incompleteness theorem since the first incompleteness theorem's statement does not directly convey the system's consistency. Formalizing the proof of the first incompleteness theorem within the system F yields the demonstration of the second incompleteness theorem.

The incompleteness results have an impact on mathematical philosophy, particularly forms of formalism that specify their principles using a single system of formal logic. Incompleteness theorems imply that there can be no universal mathematical theory, no unification of what is provable and true. What mathe-

maticians can prove is determined by their beginning assumptions, rather than some basic ground truth from which all answers are derived.

Mathematicians have discovered the kinds of unanswerable issues that Gödel's theorems predicted. For example, Gödel helped prove that the continuum hypothesis, which concerns the sizes of infinity, and the halting issue, which asks whether a computer programme supplied with a random input will continue forever or eventually cease, are both undecidable.

Unanswered issues have even arisen in physics, implying that Gödelian incompleteness affects not only math, but reality in some unfathomable way.