A Rather Serious Collection of Study Materials for CVML

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In this brief document, I will begin by suggesting a rather comprehensible list of books and online lectures on several sub-branches of fundamental mathematics. These are meant to contribute to your core understanding of concepts essential to machine learning and computer vision. I will then list a couple foundational books which are tailored towards computer vision and machine learning (CVML). Before delving into the lists, let me mention one monumental effort in bringing together almost all the mathematical background needed in CVML:

Algebra, Topology, Differential Calculus, and Optimization Theory For Computer Science and Machine Learning. Jean Gallier and Jocelyn Quaintance. (October 30, 2022)
https://www.cis.upenn.edu/~jean/math-deep.pdf

It still leaves out certain topics such as measure theory, stochastic differential equations, Riemannian geometry and constrained optimization. Yet, this 2190-page book is still comprehensive in its content.

I would also like to acknowledge the MML (Mathematics for Machine Learning) Book, which formed the basis of our camp for a few years: https://mml-book.github.io/.

Let's dive in.

Analysis:
Real analysis will be one of the most important building blocks giving us perspective on how mathematics is practised. Like many others, I suggest Abbot's book:
https://homel.vsb.cz/~ulc0011/Abbott%20-%20Understanding%20Analysis.pdf

There are solutions to the problems in case you are stuck (yes, it is important to solve the exercises when you read a book):
https://www.uli.rocks/understanding-analysis-solutions/main.pdf

Though, there are some alternatives:
- Jiří's book is neat and has a number of resources: https://www.jirka.org/ra/
  (make sure you cover Volume II)
- I find Spivak's Calculus to be a gem (some differential geometry and topology content is also included): https://www.amazon.com/Calculus-Michael-Spivak/dp/0521867444
  (I'm sure you can dig online for a free pdf.)

Functional Analysis:
I have not deeply covered this area myself. But followed these Youtube Lectures for a while:
https://www.youtube.com/watch?v=yDdxFBcvSGw&list=PLBh2i93oe2qsGKDOsuVVw-OCAfprrnGfr

Kreyszig's Book on Introductory Functional Analysis seems to do a good job in delivering the dense content in a comprehensible fashion: https://tinyurl.com/2p8dmfbm.

Differential / Riemannian Geometry:
Before I taught CS 468 @Stanford (cs468.stanford.edu), Justin Solomon was teaching the same course. As a result of his exhaustive activism, he was able to release his course videos: https://tinyurl.com/yn669etd.
These lectures would make a good starting point for a weakly-informed computer scientist to catch up on the practical notions of differential geometry. If you like to go a bit deeper, then eigencris makes a nice series of youtube videos on Tensors and Tensor Calculus:

- https://www.youtube.com/watch?v=TvxmkZmBa-k&list=PLJHszsWbB6hrkmmq57lX8BV-o-YIOFsiG&index=2
- https://www.youtube.com/watch?v=kGXr1SF3WmA&list=PLJHszsWbB6hpk5h8lSfBkVrpjsqvUGTCx
  (He comes from a physics perspective but if not interested, make sure to strip away the physics related content while still maintaining the perspective.)

Next, any computer scientist / mathematician, who is curious about how Riemannian geometry can aid optimization, could easily read Nicolas Boumal's book on An introduction to optimization on smooth manifolds:
https://www.nicolasboumal.net/book/ (latest update Feb. 11, 2023!)

Note that mathematicians almost exclusively study do Carmo's Riemannian Geometry book. I find this a little involved and cryptic. But feel free to check it out if this is up your alley.

Topology:
Tadashi Tokieda lectures are celebrated: https://tinyurl.com/2p8f4z3b

These can be accompanied by the snoopy-notes and Lia Vas's notes:
https://www.math.colostate.edu/~renzo/teaching/Topology10/Notes.pdf
http://liavas.net/courses/math430/

Group Theory (GT):
This is a little hard for me to write about. GT is often studied either as part of abstract algebra or as part of representation theory. This is because it is foundational to mathematics. Therefore, I will suggest a mix of references, which one should cherry-pick according to their preferences:

- Before we go on to the books, ForYourMath youtube channel gives a good introduction:
https://www.youtube.com/watch?v=zBApU-aq-b0&list=PLQHzlagV3jqJIGcGOC3CFrWz-PY7xe8C7
- As the first book, one could consider "An Infinitely Large Napkin" by Evan Chen (a PhD student at MIT). This is comprehensive and follows a very pedagogical approach. Only at times can be overwhelming in content: https://venhance.github.io/napkin/Napkin.pdf (open source)
- Nathan Carter's Visual Group Theory gives an intuitive explanation of the domain: http://web.bentley.edu/empl/c/ncarter/vgt/gallery.html
  If you search online, you will probably find the book itself as pdf. It has a youtube channel too: https://www.youtube.com/playlist?list=PLwV-9DG53NDxU337smpTwm6sef4x-SCLv as well as the solutions under: https://github.com/dvanderfaeillie/VisualGroupTheory
- Humphrey's group theory can be a good reference too, but maybe a little dense: http://xn--webducation-dbb.com/wp-content/uploads/2021/01/Oxford-science-publications-John-F.-Humphreys-A-course-in-group-theory-Oxford-University-Press-1996.pdf

- Finally, geometric group theory is a little useful in machine learning: https://www.math.ucdavis.edu/~kapovich/EPR/ggt.pdf

Measure Theory:
This is one of the challenging sub-fields. Unfortunately, it is as useful as it is challenging. So I will with reservation suggest:

- Measure & Integration Theory: https://books.google.com.tr/books?id=5mskCf1wKPkC&lpg=PP1&dq=bauer+measure&pg=PP1&redir_esc=y#v=onepage&q&f=false
- Measures, Integrals & Martingales: https://books.google.com.tr/books?id=_OjSIphIrH0C&lpg=PP1&pg=PP1&redir_esc=y&hl=en#v=onepage&q&f=false
- Measure theory youtube lectures: https://www.youtube.com/playlist?list=PLBh2i93oe2qvMVqAzsX1Kuv6-4fjazZ8j (Bright side of mathematics lectures are usually great.)

Further Reading into Computer Vision & Machine Learning (CVML):
I do not suggest reading mathematically grounded machine learning or computer vision books, without exposing yourself to the study of mathematics, at least to a certain level. Such bare minimum typically consists of Advanced Linear Algebra, Advanced Probability Theory (or Measure Theory), Advanced Calculus and hopefully either Group Theory or Differential Geometry. Some basic understanding of topology will be helpful along the way. Forgive me stressing this, but when I say ‘Advanced’, what I mean is complete control of the toolset, and not mere ‘familiarity’.
Though, once you have a good understanding of the required mathematics, there are a number of machine learning and computer vision books at varying levels of difficulty, that you might read with joy. As machine learning is rather broad, different books span different parts of the CVML realm. All these are amazing books and it is hard to go wrong. Pick according to your interests:

- **HDP-Book (High Dimensional Probability)**: This is probably one of the best books on the fundamentals of statistical learning. Might be challenging at times, but if you grasp the material, you should know that you are in a good state: https://www.math.uci.edu/~rvershyn/papers/HDP-book/HDP-book.html

- **GDL-Book (Geometric Deep Learning)**: For those who are interested in the geometric machine learning realm, this book is essential: https://arxiv.org/abs/2104.13478

- **EDG-Book (Convex Optimization & Euclidean Distance Geometry)**: Jan Dattoro himself is a character. He is still at Stanford and making music. This book, according to me, is Dattoro’s masterpiece. Even the footnotes of this book are full of surprising content. Turn to this, if you like to read about some exotic geometric notions and how they relate to optimization: https://ccrma.stanford.edu/~dattorro/0976401304_v2015.04.11.pdf

- **SDE-Book (Stochastic Differential Equations)**: Oksendal has written one of the bibles of stochastic differential equations, which became even more popular with the proliferation of diffusion models. Make sure to know measure theory before attempting: http://www.stat.ucla.edu/~ywu/research/documents/StochasticDifferentialEquations.pdf

- **OT-Book (Topics in Optimal Transportation)**: In case you like to understand optimal transport further, I will recommend a simplified (note, not simple, only simplified) version of the original seminal codex of Villani:
  https://www.math.ucla.edu/~wgangbo/Cedric-Villani.pdf .
  Make sure you have measure theory in your pocket. If this book is a little hard to comprehend, the recent "computational" version of Peyre & Cuturi could still do the job and be more intuitive: https://arxiv.org/abs/1803.00567
  * Cedric Villani is a character too, you could look him up and watch some of his videos. He is a Fields Medal Laureate and former minister of science in France.

- **Red-Book (A Vector Space Approach to Models & Optimization)**: The hidden gem that everyone knows yet nobody talks about. You will find the most important notions of optimization and great perspectives thru vector spaces:
  https://www.amazon.com/Approach-Optimization-Systems-Engineering-Analysis/dp/0471219207 (again, please dig online for some free version)

- **Manifold-Book**: I mentioned this in the differential geometry section, but if you like to learn about optimization on the manifolds and how this can be used for constrained optimization, then consider this recent book: https://www.nicolasboumal.net/book/

- **STL-Book (Statistical Learning Theory)**: Vapnik's seminal 1995 work on statistical learning theory. He introduces a lot of (now abandoned) fundamental concepts of learning: http://lib.ysu.am/disciplines_bk/22cca8eefb24af29d10bbc661e3a5ebf.pdf

- **Optimization-Bible**: Boyd's book of optimization is known by this name:
  https://web.stanford.edu/~boyd/cvxbook/

- **DGM-Book (Deep Generative Modeling)**: Jakub Tomzcak has written a comprehensive review book on Deep Generative Modeling:
  https://link.springer.com/book/10.1007/978-3-030-93158-2
  The codes used in the book can be found online:
  https://github.com/jmtomczak/intro_dgm

- **Prince-Books**: On computer vision, I find Simon Prince's books to be quite useful:
  https://udlbook.github.io/udlbook/ (new book about deep learning)
  http://www.computervisionmodels.com/ (old book)

I know this is a long list. Please get in touch with me if you feel troubled making a choice.

Sit and read books. :)
Believe me, you have all the time in the world to do so.
It will be so rewarding you have no idea.

With love,
/tolga