

Online Supplement for “An Alternative Globalization Strategy for Unconstrained Optimization”

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The sixth order poynomial function is given by

$$P(\beta) = a_6\beta^6 + a_5\beta^5 + a_4\beta^4 + a_3\beta^3 + a_2\beta^2 + a_1\beta + a_0.$$

Let us define

$$u_1 = (s_k^t)^\top g_k, \quad u_2 = \|s_k^t\|^2, \quad u_3 = \|y_k^t\|^2, \quad u_4 = \|g_k^t\|^2, \quad u_5 = f_k^t - f_k - (g_k^t)^\top s_k^t, \quad u_6 = (y_k^t)^\top s_k^t, \quad u_7 = (y_k^t)^\top g_k.$$

Then, the coefficients of $P(\beta)$ are as follows:

$$a_6 = -16u_2, \quad a_5 = 2u_2(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6) + 2u_2^2(8u_5/u_2 - 4/u_2u_6),$$

$$\begin{aligned} a_4 = & -2u_2(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6)u_1 - 1/4u_2^2(8u_5/u_2 - 4/u_2u_6)(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6) \\ & + 4u_2^2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3) - u_2(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6)u_6 \\ & + 16u_2^2u_4 + 16u_2^2u_7 + 4u_2(-u_2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3) - 2u_6^2 + 2u_2u_3) \\ & + 2u_2^2(8u_5/u_2 - 4/u_2u_6)u_1 + 4u_2^2u_3 + u_2^2(8u_5/u_2 - 4/u_2u_6)u_6, \end{aligned}$$

$$\begin{aligned} a_3 = & -4u_2(-u_2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3) - 2u_6^2 + 2u_2u_3)u_1 \\ & - 1/2u_2^2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3)(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6) \\ & - 2u_2(-u_2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3) - 2u_6^2 + 2u_2u_3)u_6 + 32u_2^2u_6u_4 \\ & + 16u_2^2u_6u_7 - 1/2u_2^2(8u_5/u_2 - 4/u_2u_6)(-u_2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3) \\ & - 2u_6^2 + 2u_2u_3) - 2u_2(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6)u_6u_1 + 2u_2^2(8u_5/u_2 - 4/u_2u_6)u_6u_1 \\ & - 1/2u_2(-2u_1 - 2u_5)(-u_2(8u_5/u_2 - 4/u_2u_6) - 16u_6)u_6 + 4u_2^2(-2u_1 - 2u_5)u_3 + 4u_2^2(2u_6u_5/u_2 \\ & - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3)u_1 + 2u_2^2(2u_6u_5/u_2 - (u_6^2 - u_2u_3)/u_2 - 2u_7 + 1/u_2u_6^2 - u_3)u_6 \\ & + 1/2u_2^2(8u_5/u_2 - 4/u_2u_6)(-2u_1 - 2u_5)u_6 + 8u_2^2(-2u_1 - 2u_5)u_7, \end{aligned}$$

$$\begin{aligned}
a_2 = & 8u_2^2 u_6 (-2u_1 - 2u_5) u_7 - 4u_2 (-u_2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) \\
& - 2u_6^2 + 2u_2 u_3) u_6 u_1 - 1 / 2 u_2 (u_6^2 - u_2 u_3) (-u_2 (8u_5 / u_2 - 4 / u_2 u_6) - 16u_6) u_1 \\
& + 8u_2^2 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_7 + u_2^2 (-2u_1 - 2u_5)^2 u_3 + 16u_2^2 u_6^2 u_4 \\
& + 1 / 2 u_2^2 (8u_5 / u_2 - 4 / u_2 u_6) (u_6^2 - u_2 u_3) u_1 + 4u_2^2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 \\
& - 2u_7 + 1 / u_2 u_6^2 - u_3) u_6 u_1 + 1 / 2 u_2^2 (8u_5 / u_2 - 4 / u_2 u_6) (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_6 \\
& + u_2^2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) (-2u_1 - 2u_5) u_6 \\
& - u_2 (-2u_1 - 2u_5) (-u_2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) - 2u_6^2 + 2u_2 u_3) u_6 \\
& + 1 / 2 u_2^3 (8u_5 / u_2 - 4 / u_2 u_6) ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) \\
& - u_2^2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) (-u_2 (2u_6 u_5 / u_2 \\
& - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) - 2u_6^2 + 2u_2 u_3) - 1 / 2 u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 \\
& - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) (-u_2 (8u_5 / u_2 - 4 / u_2 u_6) - 16u_6) + 8u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 \\
& - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) u_1 + 4u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) u_6 \\
& + 4u_2^2 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_3 - 1 / 2 u_2 (-u_6 (u_1 + u_5) \\
& + u_2 (u_7 + u_6 u_5 / u_2)) (-u_2 (8u_5 / u_2 - 4 / u_2 u_6) - 16u_6) u_6 \\
& + 8u_2^2 (u_6^2 - u_2 u_3) u_4 + 4u_2^2 (u_6^2 - u_2 u_3) u_7,
\end{aligned}$$

$$\begin{aligned}
a_1 = & 8u_2^2 (u_6^2 - u_2 u_3) u_4 u_6 + 2u_2^2 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_3 (-2u_1 - 2u_5) \\
& + 8u_2^2 u_6 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_7 + 2u_2^2 (u_6^2 - u_2 u_3) (-2u_1 - 2u_5) u_7 \\
& + 8u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) u_6 u_1 - u_2 (u_6^2 - u_2 u_3) (-u_2 (2u_6 u_5 / u_2 \\
& - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) - 2u_6^2 + 2u_2 u_3) u_1 + u_2^3 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 \\
& - 2u_7 + 1 / u_2 u_6^2 - u_3) ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) - u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 \\
& - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) (-u_2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) - 2u_6^2 + 2u_2 u_3) \\
& + u_2^2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) (u_6^2 - u_2 u_3) u_1 + u_2^2 (2u_6 u_5 / u_2 \\
& - (u_6^2 - u_2 u_3) / u_2 - 2u_7 + 1 / u_2 u_6^2 - u_3) (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_6 \\
& + 2u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) (-2u_1 - 2u_5) u_6 \\
& - u_2 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) (-u_2 (2u_6 u_5 / u_2 - (u_6^2 - u_2 u_3) / u_2 \\
& - 2u_7 + 1 / u_2 u_6^2 - u_3) - 2u_6^2 + 2u_2 u_3) u_6,
\end{aligned}$$

$$\begin{aligned}
a_0 = & u_2^2 (u_6^2 - u_2 u_3)^2 u_4 + u_2^2 (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2))^2 u_3 + 2u_2^2 (u_6^2 - u_2 u_3) (-u_6 (u_1 + u_5) \\
& + u_2 (u_7 + u_6 u_5 / u_2)) u_7 + 2u_2^2 (u_6^2 - u_2 u_3) ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) \\
& + u_3 (u_1 + u_5)) u_1 + u_2^3 ((u_6^2 - u_2 u_3) u_5 / u_2 - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5))^2 + 2u_2^2 ((u_6^2 - u_2 u_3) u_5 / u_2 \\
& - u_6 (u_7 + u_6 u_5 / u_2) + u_3 (u_1 + u_5)) (-u_6 (u_1 + u_5) + u_2 (u_7 + u_6 u_5 / u_2)) u_6.
\end{aligned}$$