PHY517 / AST443: Observational Techniques

Homework 3

1. The Poisson distribution describes the probability to observe x events during a certain measurement interval, given a mean rate μ:

$$P_{\mathrm{P}}(x|\mu) = \frac{\mu^x}{x!}e^{-\mu}$$

Note that x has to be a positive integer. Examples of where the Poisson distribution applies are counting experiments. In optical astronomy, we often *count* the number of electrons registered in the CCD due to incoming photons from a celestial object. The Poisson distribution is asymmetric for low rates $\mu \lesssim 10$, and becomes the Gaussian distribution for high rates $\mu \gg 1$.

   (a) Show that the mean of the Poisson distribution is μ. [1]

   (b) Show that the variance of the Poisson distribution is μ.

   (c) Plot (on a single panel) the Poisson distribution for rates of $\mu = 1, 2, 4, 10$.

   (d) For $\mu = 30$, plot the Poisson distribution, as well as a Gaussian distribution of mean $\mu = 30$. What do you need to set the standard deviation of the Gaussian to?

   (e) You measured $N = 10,000$ electrons from a star. What is the uncertainty on this measurement?

2. For the following, consider the CCD sensor in our STL-1001E camera. When necessary, look up the relevant properties on its spec sheet.

   (a) How many pixels would you expect to fall outside the 1σ interval for a random Gaussian process? How many for the 2σ, 3σ, 4σ, 5σ intervals? You can look up the corresponding integrals of the normal distribution at `https://en.wikipedia.org/wiki/68%E2%80%9395%E2%80%9399.7_rule`.

   (b) The read noise of the camera is fixed - it is not a counting process, and thus does not depend on exposure time or number of counts. It quantifies the standard deviation of a roughly Gaussian distribution of pixel values around the bias level in the absence of any signal. For the following, assume the camera is operated at 0°C.

      i. How large does the background sky level need to be (in counts per pixel), so that statistical noise from the background sky dominates over the read noise?

[1] Hint: the following series identity is useful for Exercise (1):

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{\lambda}$$

ii. If there were no background sky, at what exposure time does noise from the dark current start to dominate over the read noise?