

# PHY517 / AST443: Observational Techniques

## Homework 3

1. The Poisson distribution describes the probability to observe  $x$  events during a certain measurement interval, given a mean rate  $\mu$ :

$$P_P(x|\mu) = \frac{\mu^x}{x!} e^{-\mu}$$

Note that  $x$  has to be a positive integer. Examples of where the Poisson distribution applies are counting experiments. In optical astronomy, we often *count* the number of electrons registered in the CCD due to incoming photons from a celestial object. The Poisson distribution is asymmetric for low rates  $\mu \lesssim 10$ , and becomes the Gaussian distribution for high rates  $\mu \gg 1$ .

- (a) Show that the mean of the Poisson distribution is  $\mu$ .<sup>1</sup>
  - (b) Show that the variance of the Poisson distribution is  $\mu$ .
  - (c) Plot (on a single panel) the Poisson distribution for rates of  $\mu = 1, 2, 4, 10$ .
  - (d) For  $\mu = 30$ , plot the Poisson distribution, as well as a Gaussian distribution of mean  $\mu = 30$ . What do you need to set the standard deviation of the Gaussian to?
  - (e) You measured  $N = 10,000$  electrons from a star. What is the uncertainty on this measurement?
2. For the following, consider the CCD sensor in our STL-1001E camera. When necessary, look up the relevant properties on its spec sheet.
    - (a) How many pixels would you expect to fall outside the  $1\sigma$  interval for a random Gaussian process? How many for the  $2\sigma$ ,  $3\sigma$ ,  $4\sigma$ ,  $5\sigma$  intervals? You can look up the corresponding integrals of the normal distribution at [https://en.wikipedia.org/wiki/68%E2%80%939395%E2%80%939399.7\\_rule](https://en.wikipedia.org/wiki/68%E2%80%939395%E2%80%939399.7_rule).
    - (b) The read noise of the camera is fixed - it is not a counting process, and thus does not depend on exposure time or number of counts. It quantifies the standard deviation of a roughly Gaussian distribution of pixel values around the bias level in the absence of any signal. For the following, assume the camera is operated at  $0^\circ\text{C}$ .
      - i. How large does the background sky level need to be (in counts per pixel), so that statistical noise from the background sky dominates over the read noise?

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<sup>1</sup>Hint: the following series identity is useful for Exercise (1):

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^\lambda$$

- ii. If there were no background sky, at what exposure time does noise from the dark current start to dominate over the read noise?
- 3. For groups doing the exoplanet transient lab:
  - (a) (As a group:) For each transit: submit the mid-transit time, the transit duration, and the magnitude dip, as well as the StarAlt plot with these times marked. State when you should arrive at the telescope.
  - (b) (Individually:) Submit a plot of transit depth vs. host star brightness. Which systems are the best targets?