VLSI DSP Design 2019 Fall 期中考

NCHU Department of Electrical Engineering Total score 110

盡量給部分分數，除非是亂寫，以提高分數的鑑別度

Part A. 簡答題 (49)

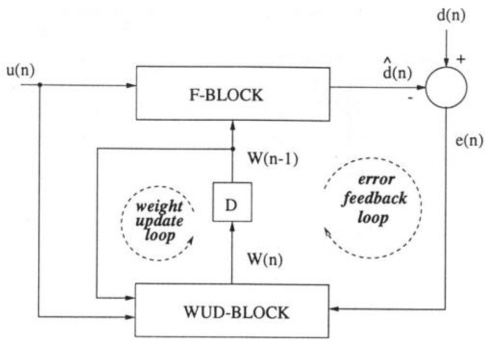
A1. (3) What is the meaning of up sampling

For up sampling factor M, inserting M-1 zeros between two successive data samples

A2. (3) Pseudo inverse of a matrix A

$A^{+} = (A^t A)^{-1} A^t$

A3. (6) Please draw the block diagram of an LMS filter and show how the coefficients are updated



$\mathbf{w}(n) = \mathbf{w}(n) + \mu \cdot e(n) \cdot \mathbf{u}(n)$

A4. (4) What is Wiener-Hopf equation

$R_{xx} \cdot \underline{a} = r_{xy}$ where

$\underline{a} = [a_0, a_1, a_2, \cdots, a_p]^t$

$$R_{xx} = \mathrm{E}\{\underline{x} \cdot \underline{x}^t\} = \begin{bmatrix} x_n \\ x_{n-1} \\ \vdots \\ x_{n-p} \end{bmatrix} \cdot [x_n, x_{n-1}, \cdots, x_{n-p}]$$

$r_{xy} = \mathrm{E}\{\underline{x} \cdot y_n\}$

A5. (3) 2×2 Givens rotation matrix to convert $[5\ 0]^t$ to $[4\ 3]^t$

$$\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} \times \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

A6. (3) What is decimation in frequency (DIF) in Fast Fourier Transform

Input data is of ordered sequence while output data is of shuffled sequence (不必畫圖)

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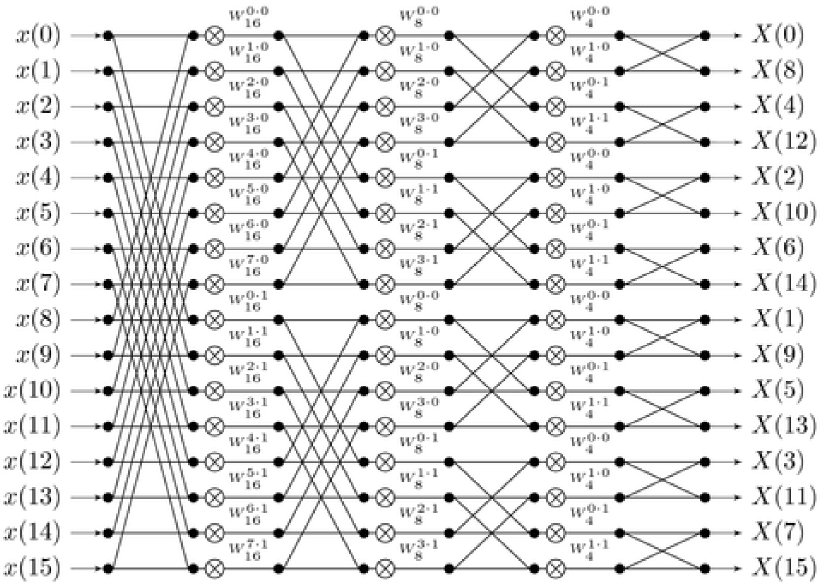


A7. (6) If a matrix **A** is decomposed the product of a lower triangular matrix **L** and an upper triangular matrix **U**, i.e., **A** = **LU**, please show how to use forward and backward substitutions to solve a linear system **Ax**=**b**, where **x** and **b** are column vectors.

$\mathbf{LUx} = \mathbf{b}$, let $\mathbf{Ux} = \mathbf{a}$

$\Longrightarrow \mathbf{La} = \mathbf{b}$, apply forward substitution to solve **a**

$\Longrightarrow \mathbf{Ux} = \mathbf{a}$, apply backward substitution to solve **x**

A8. (3) What is an intra-precedence constraint in a DFG

Indicated by an edge with no delay element associated with it

A9. (3) what is a linear phase filter

A filter whose phase shift is proportional to frequency. The coefficients of a linear phase filter are symmetric.

A10. (6) For the lattice filter shown below, please derive the equations for $e_f(n|m)$ and $e_b(n|m)$, where n is the time index and m is the stage index

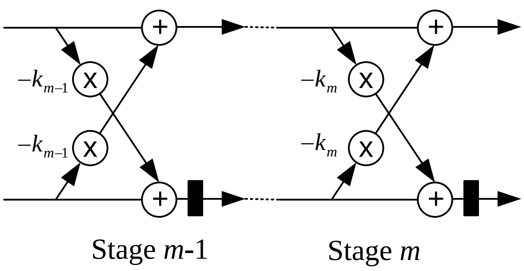


Fig. 1

$$e_f(n|m) = e_f(n|m-1) - k_m(n)e_b(n-1|m-1)$$
$$e_b(n|m) = e_b(n-1|m-1) - k_m(n)e_f(n|m-1)$$

A11. (6) please explain the differences between a DG and a DFG

- DFG node: represents computation in one iteration and executed repetitively from one iteration to another
- DG node: represents a single computation instance
- DFG contains computations in one iteration
- DG contains computations for all iterations in an algorithm
- DFG edge: may contain a delay
- DG edge: contains no delay

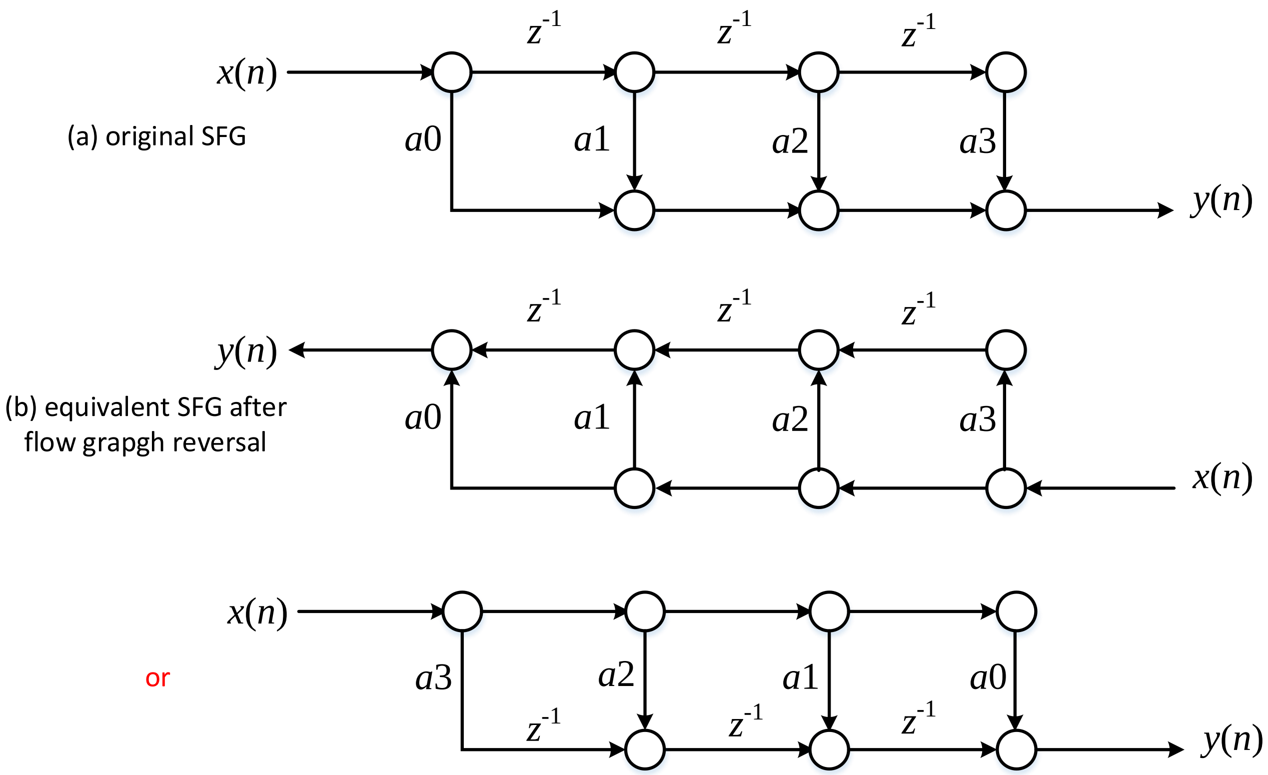
Part B. 計算與設計題 (58)

B1. (10) For a 4-tap FIR filter

$y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) + a_3 x(n-3)$,

a) Please draw its signal flow graph

b) Use flow graph reversal scheme to derive its equivalent SFG



B2. (12) consider the DFG shown in Fig. 2, assume addition and multiplication require 1 u.t. and 2 u.t., respectively

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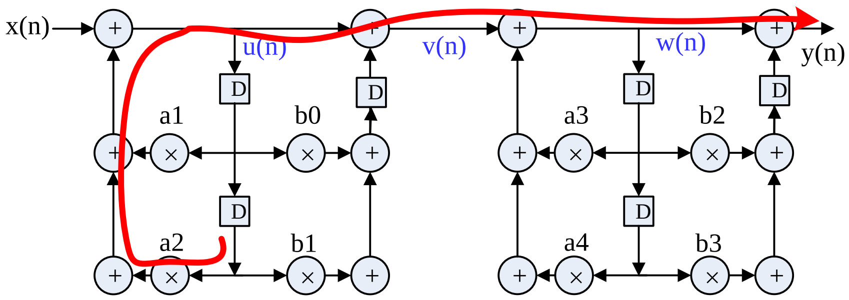


Fig. 2

a) Please write down the difference equation of y(n)?

u(n) = x(n)+a1*u(n-1)+a2*u(n-2)

v(n) = u(n)+b0*u(n-2)+b1*u(n-3)

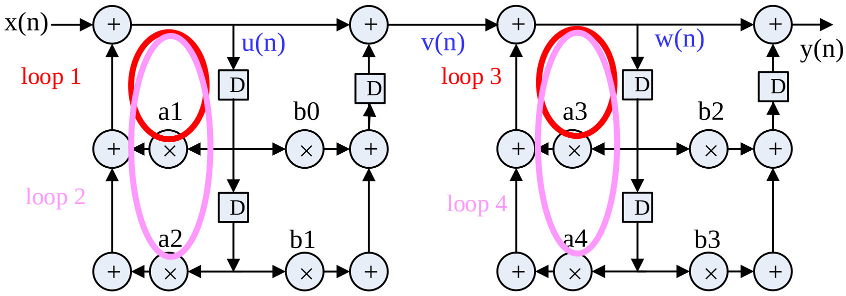
w(n) = v(n)+a3*v(n-1)+a4*v(n-2)

y(n) = w(n)+b2*w(n-2)+b3*w(n-3)

b) What is the critical path of the design?

Indicated by the red line (Note: since the lowest left adder is redundant, a critical path starting from multiplier a1 is also correct)

c) List all the loop bounds of the design



Loop 1: (Tm + 2Ta) / 1 = 4

Loop 2: (Tm + 3Ta) / 2 = 5/2

Loop 3: (Tm + 2Ta) / 1 = 4

Loop 4: (Tm + 3Ta) / 2 = 5/2

B3. (12) For the DFG shown in Fig. 3, assume addition and multiplication require 1 u.t. and 3 u.t., respectively

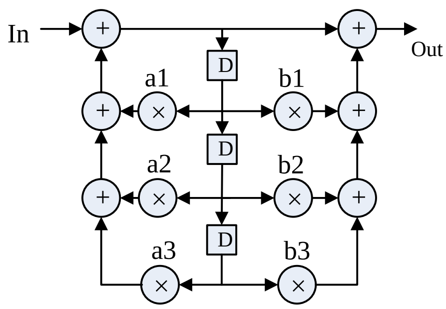


Fig. 3

a) please derive its transfer function

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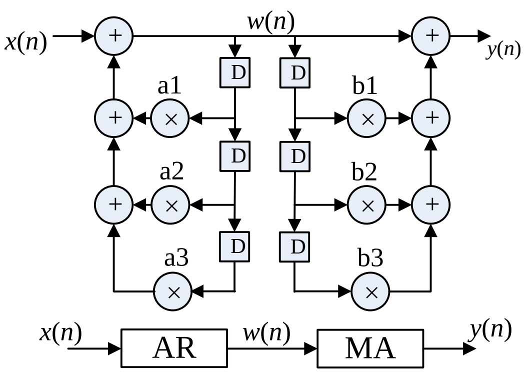


$$w(n) = a_1 w(n-1) + a_2 w(n-2) + a_3 w(n-3) + x(n)$$

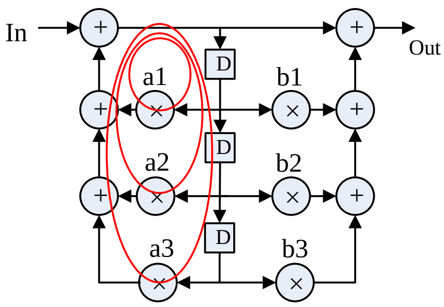
$$\frac{W(z)}{X(z)} = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}}$$

$$y(n) = w(n) + b_1 w(n-1) + b_2 w(n-2) + b_3 w(n-3)$$

$$\frac{Y(z)}{W(z)} = 1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

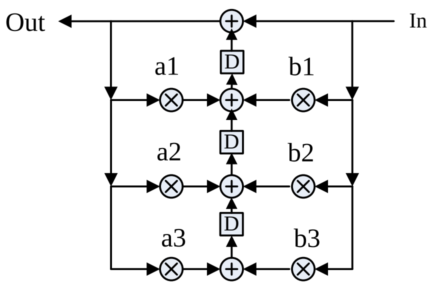
$$H(z) = \frac{Y(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = \frac{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3}}$$

b) please derive its iteration bound



iteration bound = max{5/1, 6/2, 6/3} = 5

c) use the flow graph traversal scheme to derive its transpose form



B4. (12) Consider a 2-level pipelined all pass 4th order IIR digital filter shown in Fig 4. Assume multiplication and addition require 3 u.t. and 1 u.t., respectively.

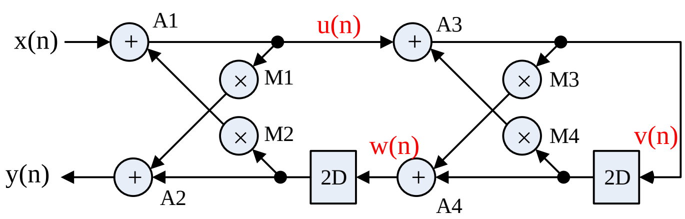


Fig. 4v

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a) please derive its transfer function

v(n) = u(n) + M4*v(n-2) => v(z) = u(z) + M4*v(z)*z^{-2} => v(z) = u(z)/(1-M4* z^{-2})

w(n) = M3*v(n) + v(n-2) => w(z) = M3*v(z) + v(z)*z^{-2} = (M3 + z^{-2})v(z)

$$w(z) = \frac{M3 + z^{-2}}{1 - M4 \cdot z^{-2}} \cdot u(z)$$

let $P(z) = \frac{M3+z^{-2}}{1-M4\cdot z^{-2}}$ and w(z) = P(z) * u(z)

u(n) = x(n) + M2*w(n-2) => u(z) = x(z) + M2*z^{-2} *w(z) = x(z) + M2*z^{-2} * P(z) * u(z)

⇨ $u(z) = \frac{x(z)}{1-M2\cdot z^{-2}\cdot P(z)}$

y(n) = M1*u(n) + w(n-2) => y(z) = M1*u(z) + w(z)*z^{-2}

⇨ y(z) = M1*u(z) + P(z) * u(z)* z^{-2}

⇨ y(z) = (M1+ P(z) * z^{-2})*u(z)

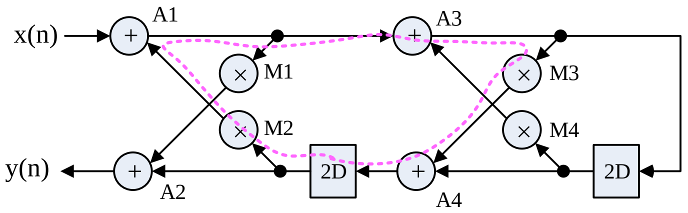
⇨ $y(z) = \frac{M1+P(z)\cdot z^{-2}}{1-M2\cdot P(z)\cdot z^{-2}} x(z)$

⇨ $H(z) = \frac{y(z)}{x(z)} = \frac{M1+P(z)\cdot z^{-2}}{1-M2\cdot P(z)\cdot z^{-2}} = \frac{M1+\frac{M3+z^{-2}}{1-M4\cdot z^{-2}}\cdot z^{-2}}{1-M2\cdot\frac{M3+z^{-2}}{1-M4\cdot z^{-2}}\cdot z^{-2}}$

⇨ $H(z) = \frac{M1\cdot(1-M4\cdot z^{-2})+(M3+z^{-2})\cdot z^{-2}}{1-M4\cdot z^{-2}-M2\cdot(M3+z^{-2})\cdot z^{-2}} = \frac{M1+(M1\cdot M4+M3)\cdot z^{-2}+z^{-4}}{1-(M2\cdot M3+M4)\cdot z^{-2}-M2\cdot z^{-4}}$

因為這題的推導比較難，有大概的樣子就可以給 50%以上的部分分數

b) calculate the iteration period bound of the filter



iteration bound = (2T_m+3T_a)/2=9/2 u.t.

c) what is the critical path

critical path is identical to critical loop indicated in b) and the total delay is 9 u.t.

B5. (12) please derive the dependence graph of the following filters (draw up to 5 iterations)

a) y(n) = a0*x(n) + a1*x(n-1)+a2*x(n-2)

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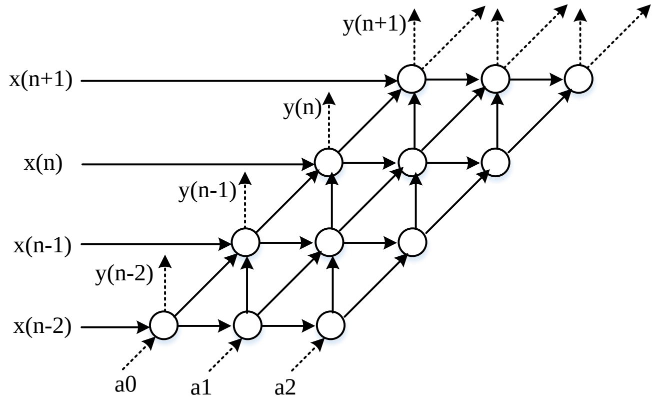

b) y(n) = b0*x(n) + a1*y(n-2) + a2*y(n-3)