Analysis of Novint Falcon Haptic Device as Manipulator

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Motivation

Novint Falcon is a parallel haptic device like Stewart platform, however, it only has three transitional DOFs. As Figure 1 shows, end effector of the Novint Falcon is attached by three kinematic chians actuated by three motor rotors respectively and each of the chain consists of four links. This haptic device is initially designed to be external device for shooting video games, but in this project, we analyze the Novint Falcon as the manipulator and controls it to solve the pinball maze problem. Pinball maze is a small

Figure 1 Novint Falcon [1]

Methods

1. Forward Kinematics

The kinematic configuration was first introduced by Tsai and Stamper in the 1997 technical research report [2]. Three kinematic chains are connected to the end effector platform which is always parallel to the basement. And each chain contains four links with revolute joints connected. Based on the kinematic configuration, we derive the DH parameter table for each leg, which is shown in Table 1.

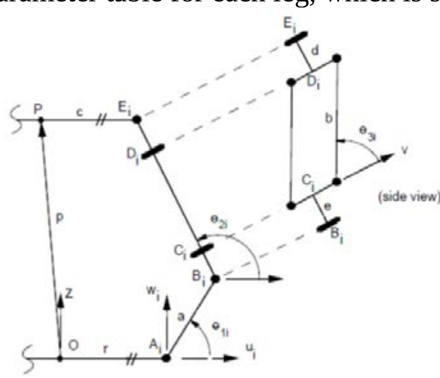

Figure 2 Kinematic Configuration of Each Chain [2]

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	90°	0	0	θ_1
2	0	a	0	$\theta_2 - \theta_1$
3	90°	e	0	$-90° + \theta_3$
4	0	b	0	$90° - \theta_3$
5	−90°	d	0	$180° - \theta_2$

Table 1 DH Parameter of Each Chain

2. Inverse Kinematics [2]

The inverse kinematics of Novint Falcon can be stated as to calculate the angular position of each joint using the position of end effector. Denote $p = (x, y, z)^T$ as the position of center of end effector platform with respect to the center of basement, and subscript i as the i-th kinematic chain. In addition, a coordinate frame (u_i, v_i, w_i) is defined for each kinematic chain, attached at the joint A_i at the end of each kinematic chain. The position of A_i can be expressed as

$$p_i = \begin{vmatrix} \cos(\phi_i) & \sin(\phi_i) & 0 \\ -\sin(\phi_i) & \cos(\phi_i) & 0 \\ 0 & 0 & 1 \end{vmatrix} p + \begin{vmatrix} -r \\ 0 \\ 0 \end{vmatrix}$$

Where ϕ_i represents rotation angle of each A_i with respect to the center of basement and $p_i = (p_{ui}, p_{vi}, p_{wi})^{\mathrm{T}}$.

$$p_{ui} = a\cos(\theta_{1i}) - c + (d + e + b sin(\theta_{3i}))\cos(\theta_{2i})$$
$$p_{\mathrm{vi}} = b cos(\theta_{3i})$$
$$p_{wi} = a\sin(\theta_{1i}) + (d + e + b sin(\theta_{3i}))\sin(\theta_{2i})$$

From above, θ_{3i} can be obtained from $\theta_{3i} = \pm\arccos(\frac{p_{vi}}{b})$. In addition, θ_{1i} can be generated by using the following substitution:

$$\sin(\theta_{1i}) = \frac{2t_{1i}}{1 + t_{1i}^2}, \cos(\theta_{1i}) = \frac{1 - t_{1i}^2}{1 + t_{1i}^2}$$

Where t_{1i} is the root of the equation $l_{2i}t_{1i}^2 + l_{1i}t_{1i} + l_{0i} = 0$, and

$$l_{0i} = p_{wi}^2 + p_{ui}^2 + 2cp_{ui} - 2ap_{ui} + a^2 + c^2 - d^2 - e^2 - b^2 \, sin(\theta_{3i})^2 - 2be \, sin(\theta_{3i}) - 2bd \, sin(\theta_{3i}) - 2de - 2ac$$
$$l_{1i} = -4ap_{wi}$$
$$l_{2i} = p_{wi}^2 + p_{ui}^2 + 2cp_{ui} + 2ap_{ui} + a^2 + c^2 - d^2 - e^2 - b^2 \, sin(\theta_{3i})^2 - 2be \, sin(\theta_{3i}) - 2bd \, sin(\theta_{3i}) - 2de + 2ac$$

Finally, θ_{2i} can be obtained from $\theta_{2i} = \arccos(\frac{p_{ui} - a\cos(\theta_{1i}) + c}{d + e + b sin(\theta_{3i})})$.

What should be noted is that there may be two or more solutions of each kinematic chain, so it is necessary to check which is the best solution, in other word, to find which is the nearest position with respect to the previous one.

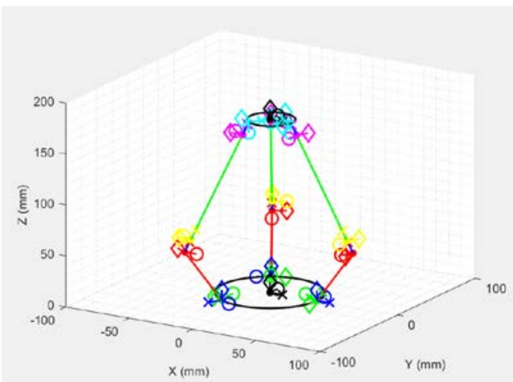

Figure 3 MATLAB Simulation of Kinematic Configuration (p=(0;0;170))

3. Dynamics [3]

The following equation describes the dynamic model:

$$\begin{vmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{vmatrix} = -ag\left(\frac{1}{2}m_a + m_b\right)\begin{vmatrix} \cos(\theta_{11}) \\ \cos(\theta_{12}) \\ \cos(\theta_{13}) \end{vmatrix} + (J^T)^{-1}m\left(a_p + \begin{vmatrix} 0 \\ g \\ 0 \end{vmatrix}\right) + c_d \begin{vmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{12} \\ \dot{\theta}_{13} \end{vmatrix} + I_A \begin{vmatrix} \ddot{\theta}_{11} \\ \ddot{\theta}_{12} \\ \ddot{\theta}_{13} \end{vmatrix}$$

Following unknown parameters is identified as [3]

$$-ag\left(\frac{1}{2}m_a + m_b\right) = -0.0298(kg \cdot m^2 \cdot s^{-2})$$
$$m = 0.8653(kg)$$
$$c_d = -0.0021(\mathrm{kg} \cdot \mathrm{s}^{-1})$$
$$I_A = 0.0004(kg \cdot \mathrm{m})$$

In addition, J represents the Jacobian matrix of the system, and J can be obtained from

$$\mathrm{J_F} = \begin{vmatrix} j_{F11} & j_{F12} & j_{F13} \\ j_{F21} & j_{F22} & j_{F23} \\ j_{F31} & j_{F32} & j_{F33} \end{vmatrix}, J_I = \begin{vmatrix} j_{I1} & 0 & 0 \\ 0 & j_{I2} & 0 \\ 0 & 0 & j_{I3} \end{vmatrix}$$

Where

$$j_{Fi1} = \cos(\theta_{2i})\sin(\theta_{3i})\cos(\phi_i) - \cos(\theta_{3i})\sin(\phi_i)$$
$$j_{Fi2} = \cos(\theta_{3i})\cos(\phi_i) + \cos(\theta_{2i})\sin(\theta_{3i})\sin(\phi_i)$$
$$j_{Fi3} = sin(\theta_{2i})sin(\theta_{3i})$$
$$j_{Ii} = \mathrm{a}\sin(\theta_{2i} - \theta_{1i})\sin(\theta_{3i})$$

And a_p is the acceleration of the end effector, $a_p = J^{-1}\begin{vmatrix} \ddot{\theta}_{11} \\ \ddot{\theta}_{12} \\ \ddot{\theta}_{13} \end{vmatrix} + \frac{d}{dt}(J^{-1})\begin{vmatrix} \dot{\theta}_{11} \\ \dot{\theta}_{12} \\ \dot{\theta}_{13} \end{vmatrix}$.

4. Path Generation

This problem is meant to solve the pinball maze problem in geometric configuration. Pinball maze, also referred to ball-in-a-maze puzzle, is a small gadget that people should rotate or maneuver the maze and makes the ball in the maze reach the goal point.

Figure 4 Pinball Maze

However, Novint Falcon only has three translational DOFs, which means it cannot rotate the maze. So, we add two more devices and make them as a system. Then we attach a platform with pinball maze to the system, when the Falcons move their end effector, they will rotate the maze and maneuver the pinball hit the goal.

This path generation problem can be stated as that given the solution path of pinball in the maze, find the motion path of end effectors of three Novint Falcons.

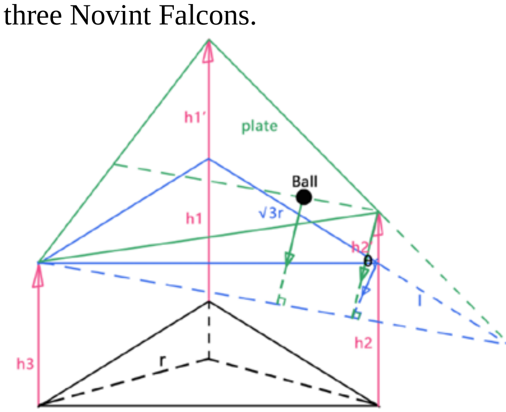

Figure 5 Geometric Configuration of Ball on Plate

To simplify the control strategy, three Novint Falcons only moves in vertical orientation. Figure 5 shows the geometric configuration of the system. h_1, h_2 and h_3 represents the end effectors with respect to the basement. θ represents the ball vector with respect to the origin from overlook view.

$\boldsymbol{\theta}$	$\boldsymbol{l}$	$\boldsymbol{h_1}$	$\boldsymbol{h_2}$	$\boldsymbol{h_3}$
$\mathbf{0}$	N/A	$-h$	$\mathbf{0}$	$\mathbf{0}$
$(\mathbf{0}, \frac{\boldsymbol{\pi}}{\mathbf{3}})$	$\frac{\sqrt{3}r\cos\left(\theta+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}$	$-h$	$-h+\frac{hl}{h+\sqrt{3}r}$	0
$[\frac{\boldsymbol{\pi}}{\mathbf{3}}, \frac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}})$	$\frac{\sqrt{3}r\cos\left(\frac{5\pi}{6}-\theta\right)}{\cos\left(\theta-\frac{\pi}{6}\right)}$	$-h+\frac{hl}{h+\sqrt{3}r}$	$-h$	0
$\frac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}}$	N/A	0	$-h$	0
$(\frac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}}, \boldsymbol{\pi})$	$\frac{\sqrt{3}r\cos\left(\theta-\frac{\pi}{2}\right)}{\cos\left(\frac{7\pi}{6}-\theta\right)}$	0	$-h$	$-h+\frac{hl}{h+\sqrt{3}r}$
$[\boldsymbol{\pi}, \frac{\mathbf{4}\boldsymbol{\pi}}{\mathbf{3}})$	$\frac{\sqrt{3}r\cos\left(\frac{3\pi}{2}-\theta\right)}{\cos\left(\theta-\frac{5\pi}{6}\right)}$	0	$-h+\frac{hl}{h+\sqrt{3}r}$	$-h$
$\frac{\mathbf{4}\boldsymbol{\pi}}{\mathbf{3}}$	N/A	0	0	$-h$
$(\frac{\mathbf{4}\boldsymbol{\pi}}{\mathbf{3}}, \frac{\mathbf{5}\boldsymbol{\pi}}{\mathbf{3}})$	$\frac{\sqrt{3}r\cos\left(\theta-\frac{7\pi}{6}\right)}{\cos\left(\frac{11\pi}{6}-\theta\right)}$	$-h$	0	$-h+\frac{hl}{h+\sqrt{3}r}$
$[\frac{\mathbf{5}\boldsymbol{\pi}}{\mathbf{3}}, \mathbf{2}\boldsymbol{\pi})$	$\frac{\sqrt{3}r\cos\left(\frac{13\pi}{6}-\theta\right)}{\cos\left(\theta-\frac{3\pi}{2}\right)}$	$-h$	$-h+\frac{hl}{h+\sqrt{3}r}$	0

Table 2 Geometric Configuration

Results and Conclusion

We simulate the methods above in MATLAB and test the kinematics and dynamics configurations by making the Novint Falcon manipulator follow the routine of three-dimensional helix line. The equations of helix line:

```
% helix
z = 151 : 180;
theta = linspace(-3*pi,3*pi,30);
x = 10*cos(theta);
y = 10*sin(theta);
s = [x;y;z];
```

First the kinematic configuration is applied, the result turns out to work well, which verifies the feasibility of Tsai and Stamper's research [1]. The motion video of Novint Falcon is shown in *'helix.mp4'*.

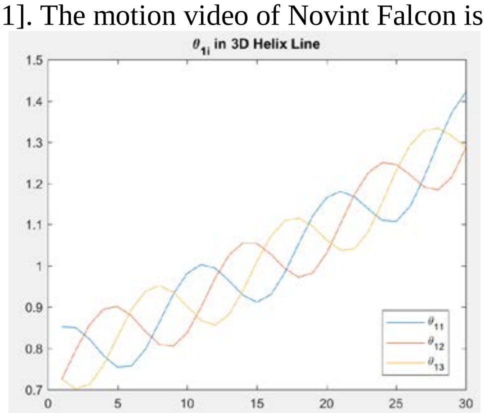

Figure 6 θ_{1i} in 3D Helix Line

Then we simulate the dynamic configuration of Novint Falcon to generate torques of three motors when end effector moves in helix line. While calculating the first and second derivatives, we use fourth order accuracy central and forward (backward) finite difference. In addition, we neglect the effect of gravity on the links. The result is shown in Figure 7.

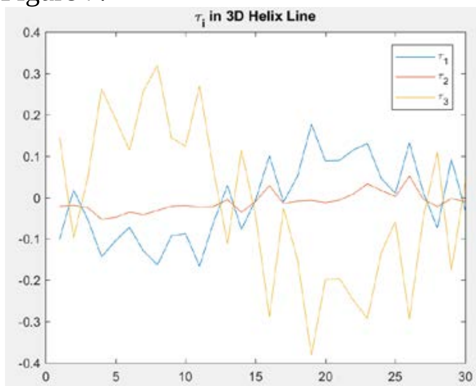

Figure 7 τ_i in 3D Helix Line

After that, we research on the path generation problem to solve the pinball maze problem. First, the system is asked to draw a single circle. And the result shows the system can behave in continuous and expected motion.

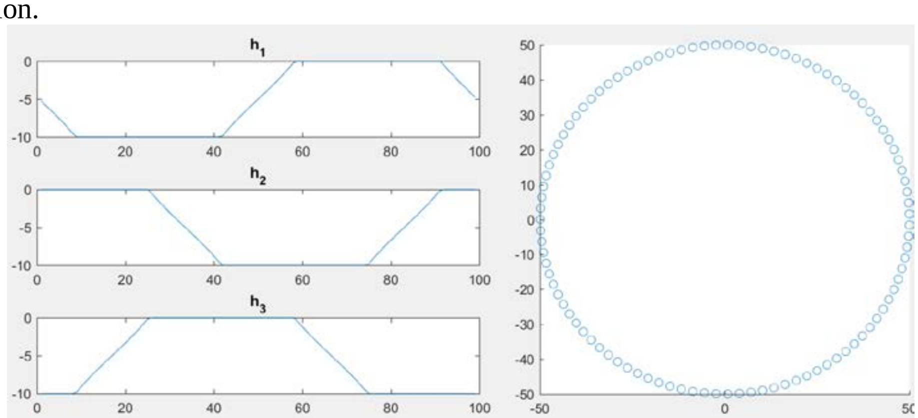

Figure 8 Drawing a Circle

Then we input the maze solution path and generate the variation of *h* to iteration number. The demonstration video is shown in *'maze.mp4'*.

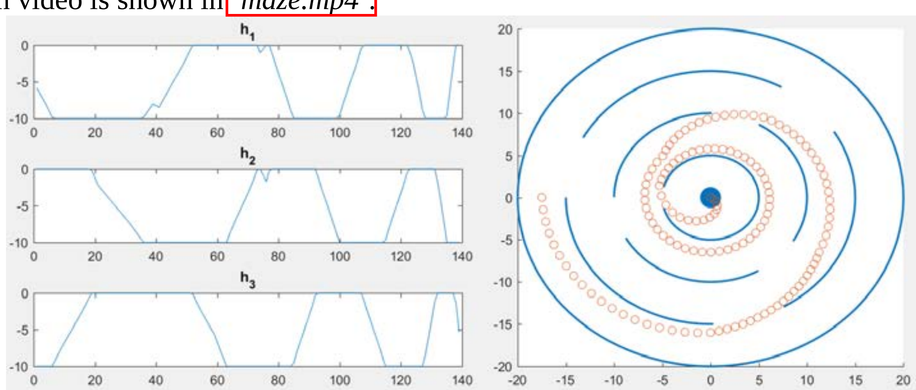

Figure 9 Solving the maze

Reference

[1] Block, Daniel J., Mark B. Michelotti, and Ramavarapu S. Sreenivas. "Application of the Novint Falcon haptic device as an actuator in real-time control." *Paladyn, Journal of Behavioral Robotics* 4.3 (2013): 182-193.
[2] Tsai, Lung-Wen, and Richard E. Stamper. *A parallel manipulator with only translational degrees of freedom*. 1997.
[3] Karbasizadeh, Nima, et al. "Dynamic identification of the Novint Falcon Haptic device." *Robotics and Mechatronics (ICROM), 2016 4th International Conference on*. IEEE, 2016.