$$D\ddot{q} + C\dot{q} + G = \tau$$

$$q = [x_2, y_2, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8]^T$$

$$D_{11} = m_H + m_p + 2m_t + 2m_s + 2m_f$$
$$D_{12} = 0$$
$$D_{13} = m_H l_{H2} \sin\theta_1$$
$$D_{14} = (m_H - 2m_t - 2m_s - 2m_f) l_p \sin\theta_2$$
$$D_{15} = (-m_t - 2m_s - 2m_f) l_t \sin\theta_3$$
$$D_{16} = (-m_t - 2m_s - 2m_f) l_t \sin\theta_4$$
$$D_{22} = m_H + m_p + 2m_t + 2m_s + 2m_f$$
$$D_{23} = m_H l_{H2} \cos\theta_1$$
$$D_{24} = (m_H - 2m_t - 2m_s - 2m_f) l_p \cos\theta_2$$
$$D_{25} = (-m_t - 2m_s - 2m_f) l_t \cos\theta_3$$
$$D_{26} = (-m_t - 2m_s - 2m_f) l_t \cos\theta_4$$
$$D_{27} = (-m_s - 2m_f) l_s \cos\theta_5$$
$$D_{28} = (-m_s - 2m_f) l_s \cos\theta_6$$
$$D_{29} = -m_f l_{f1} \cos\theta_7$$
$$D_{37} = 0$$
$$D_{38} = 0$$
$$D_{39} = 0$$
$$D_{310} = 0$$

$$D_{41} = (m_H - 2m_t - 2m_s - 2m_f) l_p \sin\theta_2$$
$$D_{42} = (m_H - 2m_t - 2m_s - 2m_f) l_p \cos\theta_2$$
$$D_{47} = (m_s + 2m_f) l_p l_s \cos(\theta_2 - \theta_5)$$
$$D_{48} = (m_s + 2m_f) l_p l_s \cos(\theta_2 - \theta_6)$$
$$D_{49} = m_f l_p l_{f1} \cos(\theta_2 - \theta_7)$$
$$D_{410} = m_f l_p l_{f1} \cos(\theta_2 - \theta_8)$$

$$D_{51} = (-m_t - 2m_s - 2m_f) l_t \sin\theta_3$$
$$D_{52} = (-m_t - 2m_s - 2m_f) l_t \cos\theta_3$$
$$D_{57} = (2m_s + 4m_f) l_t l_s \cos(\theta_3 - \theta_5)$$
$$D_{58} = 0$$
$$D_{59} = 2m_f l_t l_{f1} \cos(\theta_3 - \theta_7)$$
$$D_{510} = 0$$

$$D_{61} = (-m_t - 2m_s - 2m_f) l_t \sin\theta_4$$
$$D_{62} = (-m_t - 2m_s - 2m_f) l_t \cos\theta_4$$
$$D_{67} = 0$$
$$D_{68} = (2m_s + 4m_f) l_t l_s \cos(\theta_4 - \theta_6)$$
$$D_{69} = 0$$
$$D_{610} = 2m_f l_t l_{f1} \cos(\theta_4 - \theta_8)$$

$$D_{17} = (-m_s - 2m_f) l_s \sin\theta_5$$
$$D_{18} = (-m_s - 2m_f) l_s \sin\theta_6$$
$$D_{19} = -m_f l_{f1} \sin\theta_7$$
$$D_{110} = -m_f l_{f1} \sin\theta_8$$

$$D_{21} = 0$$
$$D_{210} = -m_f l_{f1} \cos\theta_8$$

$$D_{31} = m_H l_{H2} \sin\theta_1$$
$$D_{32} = m_H l_{H2} \cos\theta_1$$
$$D_{33} = (I_H + m_H l_{H2}^2)$$
$$D_{34} = m_H l_{H2} l_p \cos(\theta_1 - \theta_2)$$
$$D_{35} = 0$$
$$D_{36} = 0$$

$$D_{43} = m_H l_{H2} l_p \cos(\theta_1 - \theta_2)$$
$$D_{44} = I_p + m_H l_p^2 + 2m_t l_p^2 + 2m_s l_p^2 + 2m_f l_p^2$$
$$D_{45} = (m_t + 2m_s + 2m_f) l_p l_t \cos(\theta_2 - \theta_3)$$
$$D_{46} = (m_t + 2m_s + 2m_f) l_p l_t \cos(\theta_2 - \theta_4)$$

$$D_{53} = 0$$
$$D_{54} = (m_t + 2m_s + 2m_f) l_p l_t \cos(\theta_2 - \theta_3)$$
$$D_{55} = I_t + m_t l_t^2 + 4m_s l_t^2 + 4m_f l_t^2$$
$$D_{56} = 0$$

$$D_{63} = 0$$
$$D_{64} = (m_t + 2m_s + 2m_f) l_p l_t \cos(\theta_2 - \theta_4)$$
$$D_{65} = 0$$
$$D_{66} = I_t + m_t l_t^2 + 4m_s l_t^2 + 4m_f l_t^2$$

$$\boldsymbol{D_{71}} = \left(-m_s - 2m_f\right) l_s \sin\theta_5$$
$$\boldsymbol{D_{72}} = \left(-m_s - 2m_f\right) l_s \cos\theta_5$$
$$\boldsymbol{D_{73}} = \mathbf{0}$$
$$\boldsymbol{D_{74}} = \left(m_s + 2m_f\right) l_p l_s \cos(\theta_2 - \theta_5)$$
$$\boldsymbol{D_{75}} = \left(2m_s + 4m_f\right) l_t l_s \cos(\theta_3 - \theta_5)$$
$$\boldsymbol{D_{76}} = \mathbf{0}$$
$$\boldsymbol{D_{77}} = I_s + m_s l_s^2 + 4m_f l_s^2$$
$$\boldsymbol{D_{78}} = \mathbf{0}$$
$$\boldsymbol{D_{79}} = 2m_f l_s l_{f1} \cos(\theta_5 - \theta_7)$$
$$\boldsymbol{D_{710}} = \mathbf{0}$$

$$\boldsymbol{D_{81}} = \left(-m_s - 2m_f\right) l_s \sin\theta_6$$
$$\boldsymbol{D_{82}} = \left(-m_s - 2m_f\right) l_s \cos\theta_6$$
$$\boldsymbol{D_{83}} = \mathbf{0}$$
$$\boldsymbol{D_{84}} = \left(m_s + 2m_f\right) l_p l_s \cos(\theta_2 - \theta_6)$$
$$\boldsymbol{D_{85}} = \mathbf{0}$$
$$\boldsymbol{D_{86}} = \left(2m_s + 4m_f\right) l_t l_s \cos(\theta_4 - \theta_6)$$
$$\boldsymbol{D_{87}} = \mathbf{0}$$
$$\boldsymbol{D_{88}} = I_s + m_s l_s^2 + 4m_f l_s^2$$
$$\boldsymbol{D_{89}} = \mathbf{0}$$
$$\boldsymbol{D_{810}} = 2m_f l_s l_{f1} \cos(\theta_6 - \theta_8)$$

$$\boldsymbol{D_{91}} = -m_f l_{f1} \sin\theta_7$$
$$\boldsymbol{D_{92}} = -m_f l_{f1} \cos\theta_7$$
$$\boldsymbol{D_{93}} = \mathbf{0}$$
$$\boldsymbol{D_{94}} = m_f l_p l_{f1} \cos(\theta_2 - \theta_7)$$
$$\boldsymbol{D_{95}} = 2m_f l_t l_{f1} \cos(\theta_3 - \theta_7)$$
$$\boldsymbol{D_{96}} = \mathbf{0}$$
$$\boldsymbol{D_{97}} = 2m_f l_s l_{f1} \cos(\theta_5 - \theta_7)$$
$$\boldsymbol{D_{98}} = \mathbf{0}$$
$$\boldsymbol{D_{99}} = I_f + m_f l_{f1}^2$$
$$\boldsymbol{D_{910}} = \mathbf{0}$$

$$\boldsymbol{D_{101}} = -m_f l_{f1} \sin\theta_8$$
$$\boldsymbol{D_{102}} = -m_f l_{f1} \cos\theta_8$$
$$\boldsymbol{D_{103}} = 0$$
$$\boldsymbol{D_{104}} = m_f l_p l_{f1} \cos(\theta_2 - \theta_8)$$
$$\boldsymbol{D_{105}} = \mathbf{0}$$
$$\boldsymbol{D_{106}} = 2m_f l_t l_{f1} \cos(\theta_4 - \theta_8)$$
$$\boldsymbol{D_{107}} = \mathbf{0}$$
$$\boldsymbol{D_{108}} = 2m_f l_s l_{f1} \cos(\theta_6 - \theta_8)$$
$$\boldsymbol{D_{109}} = \mathbf{0}$$
$$\boldsymbol{D_{1010}} = I_f + m_f l_{f1}^2$$

$$C_{11} = 0$$
$$C_{12} = 0$$
$$C_{13} = m_H l_{H2} \cos\theta_1 \dot{\theta}_1$$
$$C_{14} = \left(m_H - 2m_t - 2m_s - 2m_f\right) l_p \cos\theta_2 \dot{\theta}_2$$
$$C_{15} = \left(-m_t - 2m_s - 2m_f\right) l_t \cos\theta_3 \dot{\theta}_3$$
$$C_{16} = \left(-m_t - 2m_s - 2m_f\right) l_t \cos\theta_4 \dot{\theta}_4$$
$$C_{17} = \left(-m_s - 2m_f\right) l_s \cos\theta_5 \dot{\theta}_5$$
$$C_{18} = \left(-m_s - 2m_f\right) l_s \cos\theta_6 \dot{\theta}_6$$
$$C_{19} = -m_f l_{f1} \cos\theta_7 \dot{\theta}_7$$
$$C_{110} = -m_f l_{f1} \cos\theta_8 \dot{\theta}_8$$

$$C_{21} = 0$$
$$C_{22} = 0$$
$$C_{23} = -m_H l_{H2} \sin\theta_1 \dot{\theta}_1$$
$$C_{24} = \left(-m_H + 2m_t + 2m_s + 2m_f\right) l_p \sin\theta_2 \dot{\theta}_2$$
$$C_{25} = \left(m_t + 2m_s + 2m_f\right) l_t \sin\theta_3 \dot{\theta}_3$$
$$C_{26} = \left(m_t + 2m_s + 2m_f\right) l_t \sin\theta_4 \dot{\theta}_4$$
$$C_{27} = \left(m_s + 2m_f\right) l_s \sin\theta_5 \dot{\theta}_5$$
$$C_{28} = \left(m_s + 2m_f\right) l_s \sin\theta_6 \dot{\theta}_6$$
$$C_{29} = m_f l_{f1} \sin\theta_7 \dot{\theta}_7$$
$$C_{210} = m_f l_{f1} \sin\theta_8 \dot{\theta}_8$$

$$C_{31} = 0$$
$$C_{32} = 0$$
$$C_{33} = 0$$
$$C_{34} = m_H l_{H2} l_p \sin(\theta_1 - \theta_2) \dot{\theta}_2$$
$$C_{35} = 0$$
$$C_{36} = 0$$
$$C_{37} = 0$$
$$C_{38} = 0$$
$$C_{39} = 0$$
$$C_{310} = 0$$

$$C_{41} = 0$$
$$C_{42} = 0$$
$$C_{43} = -m_H l_{H2} l_p \sin(\theta_1 - \theta_2) \dot{\theta}_1$$
$$C_{44} = 0$$
$$C_{45} = \left(m_t + 2m_s + 2m_f\right) l_p l_t \sin(\theta_2 - \theta_3) \dot{\theta}_3$$
$$C_{46} = \left(m_t + 2m_s + 2m_f\right) l_p l_t \sin(\theta_2 - \theta_4) \dot{\theta}_4$$
$$C_{47} = \left(m_s + 2m_f\right) l_p l_s \sin(\theta_2 - \theta_5) \dot{\theta}_5$$
$$C_{48} = \left(m_s + 2m_f\right) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta}_6$$
$$C_{49} = m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta}_7$$
$$C_{410} = m_f l_p l_{f1} \sin(\theta_2 - \theta_8) \dot{\theta}_8$$

$$C_{51} = 0$$
$$C_{52} = 0$$
$$C_{53} = 0$$
$$C_{54} = \left(-m_t - 2m_s - 2m_f\right) l_p l_t \sin(\theta_2 - \theta_3) \dot{\theta}_2$$
$$C_{55} = 0$$
$$C_{56} = 0$$
$$C_{57} = \left(2m_s + 4m_f\right) l_t l_s \sin(\theta_3 - \theta_5) \dot{\theta}_5$$
$$C_{58} = 0$$
$$C_{59} = 2m_f l_t l_{f1} \sin(\theta_3 - \theta_7) \dot{\theta}_7$$
$$C_{510} = 0$$

$$C_{61} = 0$$
$$C_{62} = 0$$
$$C_{63} = 0$$
$$C_{64} = \left(-m_t - 2m_s - 2m_f\right) l_p l_t \sin(\theta_2 - \theta_4) \dot{\theta}_2$$
$$C_{65} = 0$$
$$C_{66} = 0$$
$$C_{67} = 0$$
$$C_{68} = \left(2m_s + 4m_f\right) l_t l_s \sin(\theta_4 - \theta_6) \dot{\theta}_6$$
$$C_{69} = 0$$
$$C_{610} = 2m_f l_t l_{f1} \sin(\theta_4 - \theta_8) \dot{\theta}_8$$

$$C_{71} = 0$$
$$C_{72} = 0$$
$$C_{73} = 0$$
$$C_{74} = \left(-m_s - 2m_f\right) l_p l_s \sin(\theta_2 - \theta_5) \dot{\theta}_2$$
$$C_{75} = \left(-2m_s - 4m_f\right) l_t l_s \sin(\theta_3 - \theta_5) \dot{\theta}_3$$
$$C_{76} = 0$$
$$C_{77} = 0$$
$$C_{78} = 0$$
$$C_{79} = 2m_f l_s l_{f1} \sin(\theta_5 - \theta_7) \dot{\theta}_7$$
$$C_{710} = 0$$

$$C_{81} = 0$$
$$C_{82} = 0$$
$$C_{83} = 0$$
$$C_{84} = \left(-m_s - 2m_f\right) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta}_2$$
$$C_{85} = 0$$
$$C_{86} = \left(-2m_s - 4m_f\right) l_t l_s \sin(\theta_4 - \theta_6) \dot{\theta}_4$$
$$C_{87} = 0$$
$$C_{88} = 0$$
$$C_{89} = 0$$
$$C_{810} = 2m_f l_s l_{f1} \sin(\theta_6 - \theta_8) \dot{\theta}_8$$

$$C_{91} = 0$$
$$C_{92} = 0$$
$$C_{93} = 0$$

$$C_{94} = -m_f l_p l_{f1} \sin(\theta_2 - \theta_7)\, \dot{\theta}_2$$
$$C_{95} = -2m_f l_t l_{f1} \sin(\theta_3 - \theta_7)\, \dot{\theta}_3$$
$$C_{96} = 0$$
$$C_{97} = -2m_f l_s l_{f1} \sin(\theta_5 - \theta_7)\, \dot{\theta}_5$$
$$C_{98} = 0$$
$$C_{99} = 0$$
$$C_{910} = 0$$

$$C_{101} = 0$$

$$C_{102} = 0$$
$$C_{103} = 0$$
$$C_{104} = -m_f l_p l_{f1} \sin(\theta_2 - \theta_8)\, \dot{\theta}_2$$
$$C_{105} = 0$$
$$C_{106} = -2m_f l_t l_{f1} \sin(\theta_4 - \theta_8)\, \dot{\theta}_4$$
$$C_{107} = 0$$
$$C_{108} = -2m_f l_s l_{f1} \sin(\theta_6 - \theta_8)\, \dot{\theta}_6$$
$$C_{109} = 0$$
$$C_{1010} = 0$$

$$G_{11} = 0$$
$$G_{21} = m_H g + m_p g + 2m_t g + 2m_s g + 2m_f g$$
$$G_{31} = m_H l_{H2} \cos\theta_1\, g$$
$$G_{41} = \left(m_H - 2m_t - 2m_s - 2m_f\right) l_p \cos\theta_2\, g$$
$$G_{51} = \left(-m_t - 2m_s - 2m_f\right) l_t \cos\theta_3\, g$$
$$G_{61} = \left(-m_t - 2m_s - 2m_f\right) l_t \cos\theta_4\, g$$
$$G_{71} = \left(-m_s - 2m_f\right) l_s \cos\theta_5\, g$$
$$G_{81} = \left(-m_s - 2m_f\right) l_s \cos\theta_6\, g$$
$$G_{91} = -m_f l_{f1} \cos\theta_7\, g$$
$$G_{101} = -m_f l_{f1} \cos\theta_8\, g$$