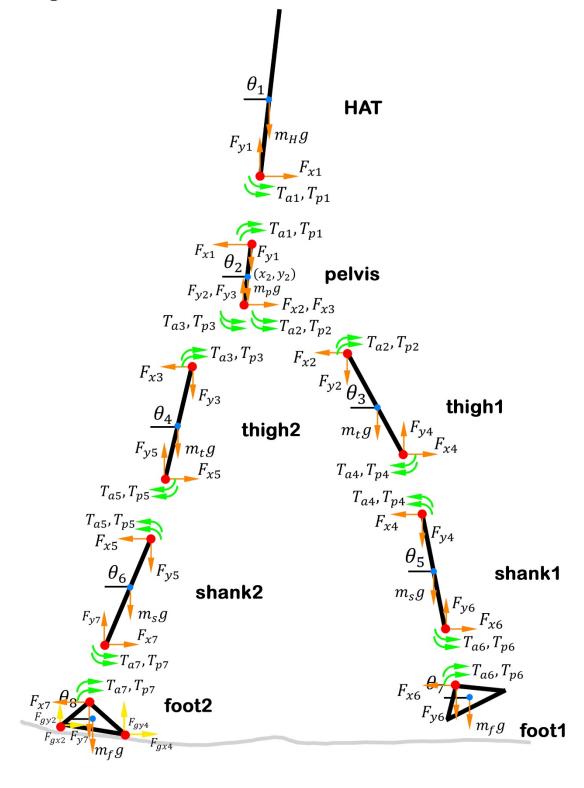


Stage 6

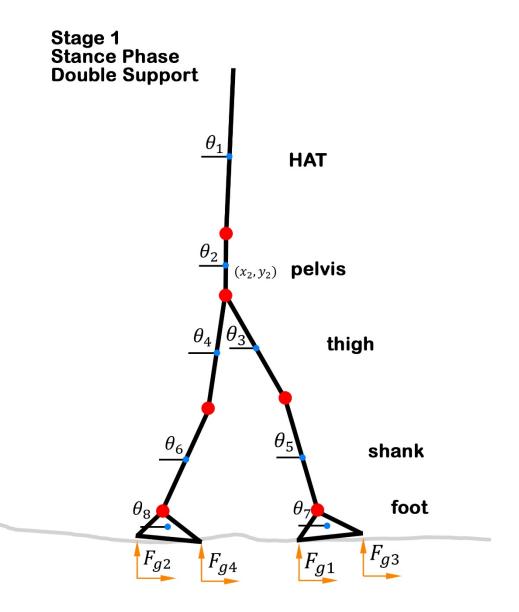


$$\mathbf{D}\ddot{q} + \mathbf{C}\dot{q} + \mathbf{G} = \mathbf{\tau}$$

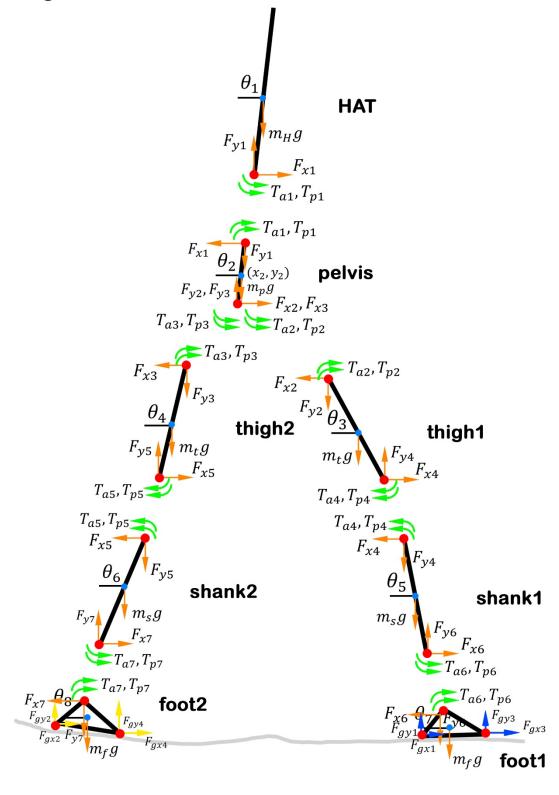
```
\left\{ \left[ \left( m_H + m_p + 2m_t + 2m_s + 2m_f \right) \ddot{x_2} \right] + \left[ m_H l_{H2} \sin \theta_1 \, \ddot{\theta_1} \right] + \left[ \left( m_H - 2m_t - 2m_s - 2m_f \right) l_p \sin \theta_2 \, \ddot{\theta_2} \right] \right\}
                                                                                     +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{3}\,\ddot{\theta}_{3}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{4}\,\ddot{\theta}_{4}\right]
                                                                                     +\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{5}\ddot{\theta}_{5}\right]+\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{6}\ddot{\theta}_{6}\right]+\left[-m_{f}l_{f1}\sin\theta_{7}\ddot{\theta}_{7}\right]
                                                                                   + \left[-m_{f}l_{f1}\sin\theta_{8}\dot{\theta_{8}}\right] + \left[\left[m_{H}l_{H2}\cos\theta_{1}\dot{\theta_{1}}^{2}\right] + \left[\left(m_{H}-2m_{t}-2m_{s}-2m_{f}\right)l_{p}\cos\theta_{2}\dot{\theta_{2}}^{2}\right]
                                                                                   + \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \cos \theta_3 \dot{\theta_3}^2 \right] + \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \cos \theta_4 \dot{\theta_4}^2 \right]
                                                                                   + \left[ (-m_s - 2m_f)l_s \cos\theta_5 \dot{\theta}_5^2 \right] + \left[ (-m_s - 2m_f)l_s \cos\theta_6 \dot{\theta}_6^2 \right] + \left[ -m_f l_{f1} \cos\theta_7 \dot{\theta}_7^2 \right]
                                                                                   +\left[-m_{f}l_{f1}\cos\theta_{8}\dot{\theta_{8}}^{2}\right]\}=F_{gx2}+F_{gx4}
\{[(m_H + m_p + 2m_t + 2m_s + 2m_f)\ddot{y}_2] + [m_H l_{H2} \cos \theta_1 \ddot{\theta}_1] + [(m_H - 2m_t - 2m_s - 2m_f) l_p \cos \theta_2 \ddot{\theta}_2]
                                                                              +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{3}\ddot{\theta_{3}}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{4}\ddot{\theta_{4}}\right]
                                                                             +\left[\left(-m_{s}-2m_{f}\right)l_{s}\cos\theta_{5}\,\ddot{\theta_{5}}\right]+\left[\left(-m_{s}-2m_{f}\right)l_{s}\cos\theta_{6}\,\ddot{\theta_{6}}\right]+\left[-m_{f}l_{f1}\cos\theta_{7}\,\ddot{\theta_{7}}\right]
                                                                            + \left[ -m_{f}l_{f1}\cos\theta_{8}\,\dot{\theta_{8}} \right] + \left\{ \left[ -m_{H}l_{H2}\sin\theta_{1}\,\dot{\theta_{1}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{s} + 2m_{f}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{t} + 2m_{s}\right]l_{p}\sin\theta_{2}\,\dot{\theta_{2}}^{2} + 2m_{f} + 2m_{f}\right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{t} + 2m_{t}\right)l_{p}\sin\theta_{2}\,\dot{\theta_{2}} \right] + \left[ \left( -m_{H} + 2m_{t} + 2m_{t}\right)l
                                                                            + \left[ (m_t + 2m_s + 2m_f) l_t \sin \theta_3 \dot{\theta}_3^2 \right] + \left[ (m_t + 2m_s + 2m_f) l_t \sin \theta_4 \dot{\theta}_4^2 \right]
                                                                            +\left[\left(m_{S}+2m_{f}\right)l_{S}\sin\theta_{5}\dot{\theta}_{5}^{2}\right]+\left[\left(m_{S}+2m_{f}\right)l_{S}\sin\theta_{6}\dot{\theta}_{6}^{2}\right]+\left[m_{f}l_{f1}\sin\theta_{7}\dot{\theta}_{7}^{2}\right]
                                                                            + \left[ m_f l_{f1} \sin \theta_8 \, \dot{\theta_8}^2 \right] + \left\{ m_H g + m_p g + 2 m_t g + 2 m_s g + 2 m_f g \right\} = \frac{F_{av2}}{F_{av4}} + \frac{F_{av4}}{F_{av4}}
                    \left\{ \left[ m_H l_{H2} \sin \theta_1 \, \ddot{x_2} \right] + \left[ m_H l_{H2} \cos \theta_1 \, \ddot{y_2} \right] + \left[ (l_H + m_H l_{H2}^2) \ddot{\theta_1} \right] + \left[ m_H l_{H2} l_p \cos (\theta_1 - \theta_2) \, \ddot{\theta_2} \right] \right\}
                                                                                                  +\left\{ \left[m_{H}l_{H2}l_{p}\sin(\theta_{1}-\theta_{2})\dot{\theta_{2}}^{2}\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] = 0
\{[(m_H - 2m_t - 2m_s - 2m_f)l_n \sin\theta_2 \ddot{x_2}] + [(m_H - 2m_t - 2m_s - 2m_f)l_n \cos\theta_2 \ddot{y_2}]
                                                                            +\left[m_{H}l_{H2}l_{n}\cos(\theta_{1}-\theta_{2})\ddot{\theta_{1}}\right]+\left[\left(l_{n}+m_{H}l_{n}^{2}+2m_{t}l_{n}^{2}+2m_{s}l_{n}^{2}+2m_{t}l_{n}^{2}\right)\ddot{\theta_{2}}\right]
                                                                            +\left[\left(m_t+2m_s+2m_f\right)l_pl_t\cos(\theta_2-\theta_3)\dot{\theta}_3\right]
                                                                            +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{4})\ddot{\theta_{4}}\right]+\left[\left(m_{s}+2m_{f}\right)l_{n}l_{s}\cos(\theta_{2}-\theta_{5})\ddot{\theta_{5}}\right]
                                                                            +\left[\left(m_{S}+2m_{f}\right)l_{p}l_{S}\cos(\theta_{2}-\theta_{6})\dot{\theta_{6}}\right]+\left[m_{f}l_{p}l_{f1}\cos(\theta_{2}-\theta_{7})\dot{\theta_{7}}\right]
                                                                             +\left[m_f l_n l_{f1} \cos(\theta_2 - \theta_8) \ddot{\theta}_8\right]
                                                                            + \left\{ \left[ -m_H l_{H2} l_p \sin(\theta_1 - \theta_2) \dot{\theta_1}^2 \right] + \left[ \left( m_t + 2m_s + 2m_f \right) l_p l_t \sin(\theta_2 - \theta_3) \dot{\theta_3}^2 \right] \right\}
                                                                            +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{4})\dot{\theta_{4}}^{2}\right]+\left[\left(m_{s}+2m_{f}\right)l_{p}l_{s}\sin(\theta_{2}-\theta_{5})\dot{\theta_{5}}^{2}\right]
                                                                            + \left[ (m_s + 2m_f) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta_6}^2 \right] + \left[ m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta_7}^2 \right]
                                                                            + \left[ m_f l_p l_{f1} \sin(\theta_2 - \theta_8) \dot{\theta_8}^2 \right] - T_{m1} + T_{m2} + T_{m2} + T_{m3} + T_{m2} + T_{m3} + T_{m3} + T_{m4} + T_{m3} + T_{m4} +
                                                                             =-(F_{av2}+F_{av4})l_n\sin\theta_2-(F_{av2}+F_{av4})l_n\cos\theta_2
```

```
\{[(-m_t - 2m_s - 2m_f)l_t \sin\theta_3 \ddot{x_2}] + [(-m_t - 2m_s - 2m_f)l_t \cos\theta_3 \ddot{y_2}]
                                                               + \left[ \left( m_t + 2m_s + 2m_f \right) l_n l_t \cos(\theta_2 - \theta_3) \ddot{\theta}_2 \right] + \left[ \left( l_t + m_t l_t^2 + 4m_s l_t^2 + 4m_f l_t^2 \right) \ddot{\theta}_3 \right]
                                                              + [(2m_s + 4m_f)l_tl_s\cos(\theta_3 - \theta_5)\ddot{\theta}_5] + [2m_fl_tl_{f1}\cos(\theta_3 - \theta_7)\ddot{\theta}_7]
                                                              +\{[(-m_t-2m_s-2m_f)l_nl_t\sin(\theta_2-\theta_3)\dot{\theta_2}^2]\}
                                                              +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{3}-\theta_{5})\dot{\theta_{5}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{3}-\theta_{7})\dot{\theta_{7}}^{2}\right]-T_{s}
                                                              -1 +\{(-m_t-2m_s-2m_f)l_t\cos\theta_3g\}=0
    \{ \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \sin \theta_4 \, \ddot{x_2} \right] + \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \cos \theta_4 \, \ddot{y_2} \right] 
                                                              +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{4})\ddot{\theta_{2}}\right]+\left[\left(l_{t}+m_{t}l_{t}^{2}+4m_{s}l_{t}^{2}+4m_{f}l_{t}^{2}\right)\ddot{\theta_{4}}\right]
                                                              +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\cos(\theta_{4}-\theta_{6})\ddot{\theta_{6}}\right]+\left[2m_{f}l_{t}l_{f1}\cos(\theta_{4}-\theta_{8})\ddot{\theta_{8}}\right]
                                                              +\{[(-m_t-2m_s-2m_f)l_pl_t\sin(\theta_2-\theta_4)\theta_2^2]\}
                                                              +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{4}-\theta_{6})\dot{\theta_{6}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{4}-\theta_{8})\dot{\theta_{8}}^{2}\right]-T_{s}
                                                              - + \{(-m_t - 2m_s - 2m_f)l_t \cos \theta_4 g\}
                                                              = -2(F_{av2} + F_{av4})l_t \sin\theta_4 - 2(F_{av2} + F_{av4})l_t \cos\theta_4
      \left\{ \left[ \left( -m_s - 2m_f \right) l_s \sin \theta_5 \, \ddot{x_2} \right] + \left[ \left( -m_s - 2m_f \right) l_s \cos \theta_5 \, \ddot{y_2} \right] + \left[ \left( m_s + 2m_f \right) l_p l_s \cos (\theta_2 - \theta_5) \ddot{\theta_2} \right] \right\}
                                                                + [(2m_s + 4m_f)l_t l_s \cos(\theta_3 - \theta_5)\ddot{\theta}_3] + [(l_s + m_s l_s^2 + 4m_f l_s^2)\ddot{\theta}_5]
                                                                + [2m_f l_s l_{f1} \cos(\theta_5 - \theta_7) \ddot{\theta_7}]
                                                               + \left\{ \left[ \left( -m_s - 2m_f \right) l_p l_s \sin(\theta_2 - \theta_5) \dot{\theta_2}^2 \right] + \left[ \left( -2m_s - 4m_f \right) l_t l_s \sin(\theta_3 - \theta_5) \dot{\theta_3}^2 \right] \right\}
                                                               +\left[2m_{f}l_{s}l_{f1}\sin(\theta_{5}-\theta_{7})\dot{\theta_{7}}^{2}\right]+T_{ss}+T_{ss}+T_{ss}+T_{ss}
                                                               +\{(-m_s-2m_f)l_s\cos\theta_5\ a\}=0
\left\{ \left[ \left( -m_{s}-2m_{f}\right) l_{s}\sin\theta_{6}\,\ddot{x_{2}} \right] + \left[ \left( -m_{s}-2m_{f}\right) l_{s}\cos\theta_{6}\,\ddot{y_{2}} \right] + \left[ \left( m_{s}+2m_{f}\right) l_{p}l_{s}\cos(\theta_{2}-\theta_{6})\ddot{\theta_{2}} \right] \right\} + \left[ \left( m_{s}+2m_{f}\right) l_{p}l_{s}\cos(\theta_{2}-\theta_{6})\ddot{\theta_{2}} \right] + \left[ \left( m_{s}+2m_{f}\right) l_{p}l_{s}\cos(\theta_{2}-\theta_{6}) + \left( m_{f}+2m_{f}\right) l_{p}l_{s} \right] + \left[ \left( m_{s}+2m_{f}\right) l_{p}l_{s}
                                                          + [(2m_s + 4m_f)l_tl_s\cos(\theta_4 - \theta_6)\ddot{\theta}_4] + [(l_s + m_sl_s^2 + 4m_fl_s^2)\ddot{\theta}_6]
                                                           + [2m_f l_s l_{f1} \cos(\theta_6 - \theta_8) \ddot{\theta_8}]
                                                          +\left\{\left[\left(-m_{s}-2m_{f}\right)l_{p}l_{s}\sin(\theta_{2}-\theta_{6})\dot{\theta_{2}}^{2}\right]+\left[\left(-2m_{s}-4m_{f}\right)l_{t}l_{s}\sin(\theta_{4}-\theta_{6})\dot{\theta_{4}}^{2}\right]\right\}
                                                          + \left[2m_f l_s l_{f1} \sin(\theta_6 - \theta_8) \dot{\theta_8}^2\right] + \frac{T_{ss}}{T_{ss}} + \frac{T_{ss}}{T_{ss}} + \frac{T_{ss}}{T_{ss}}
                                                          +\{(-m_s-2m_f)l_s\cos\theta_6\,g\} = -2(F_{gx2}+F_{gx4})l_s\sin\theta_6 - 2(F_{gy2}+F_{gy4})l_s\cos\theta_6
 \left\{ \left[ -m_f l_{f1} \sin \theta_7 \ddot{x_2} \right] + \left[ -m_f l_{f1} \cos \theta_7 \ddot{y_2} \right] + \left[ m_f l_p l_{f1} \cos(\theta_2 - \theta_7) \ddot{\theta_2} \right] + \left[ 2m_f l_t l_{f1} \cos(\theta_3 - \theta_7) \ddot{\theta_3} \right] \right\}
                                                           + \left[ 2m_f l_s l_{f1} \cos(\theta_5 - \theta_7) \ddot{\theta}_5 \right] + \left[ \left( I_f + m_f l_{f1}^2 \right) \ddot{\theta}_7 \right] 
                                                           + \left\{ \left[ -m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta_2}^2 \right] + \left[ -2m_f l_t l_{f1} \sin(\theta_3 - \theta_7) \dot{\theta_3}^2 \right] \right\}
                                                           + \left[ -2m_f l_s l_{f1} \sin(\theta_5 - \theta_7) \dot{\theta}_5^2 \right] - \frac{1}{100} - \frac{1}{100} + \left\{ -m_f l_{f1} \cos \theta_7 g \right\} = 0
```

```
\begin{split} \left\{ \left[ -m_{f}l_{f1}\sin\theta_{8}\,\ddot{x_{2}} \right] + \left[ -m_{f}l_{f1}\cos\theta_{8}\,\ddot{y_{2}} \right] + \left[ m_{f}l_{p}l_{f1}\cos(\theta_{2} - \theta_{8})\ddot{\theta_{2}} \right] + \left[ 2m_{f}l_{t}l_{f1}\cos(\theta_{4} - \theta_{8})\ddot{\theta_{4}} \right] \\ + \left[ 2m_{f}l_{s}l_{f1}\cos(\theta_{6} - \theta_{8})\ddot{\theta_{6}} \right] + \left[ (I_{f} + m_{f}l_{f1}^{2})\ddot{\theta_{8}} \right] \right\} \\ + \left\{ \left[ -m_{f}l_{p}l_{f1}\sin(\theta_{2} - \theta_{8})\,\dot{\theta_{2}}^{2} \right] + \left[ -2m_{f}l_{t}l_{f1}\sin(\theta_{4} - \theta_{8})\,\dot{\theta_{4}}^{2} \right] \\ + \left[ -2m_{f}l_{s}l_{f1}\sin(\theta_{6} - \theta_{8})\,\dot{\theta_{6}}^{2} \right] - \cdots \right\} + \left\{ -m_{f}l_{f1}\cos\theta_{8}\,g \right\} \\ = \left( l_{f1}\sin\theta_{8} - l_{f2}\sin(\alpha_{1} - \theta_{8}) \right) F_{gx2} + \left( l_{f1}\cos\theta_{8} + l_{f2}\cos(\alpha_{1} - \theta_{8}) \right) F_{gy2} \\ + \left( l_{f1}\sin\theta_{8} + l_{f3}\sin(\alpha_{2} + \theta_{8}) \right) F_{gx4} + \left( l_{f1}\cos\theta_{8} + l_{f3}\cos(\alpha_{2} + \theta_{8}) \right) F_{gy4} \end{split}
```



Stage 1

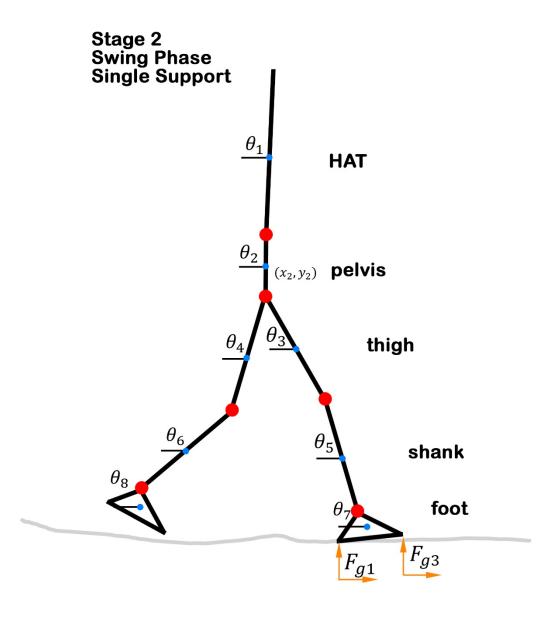


$$\mathbf{D}\ddot{q} + \mathbf{C}\dot{q} + \mathbf{G} = \mathbf{\tau}$$

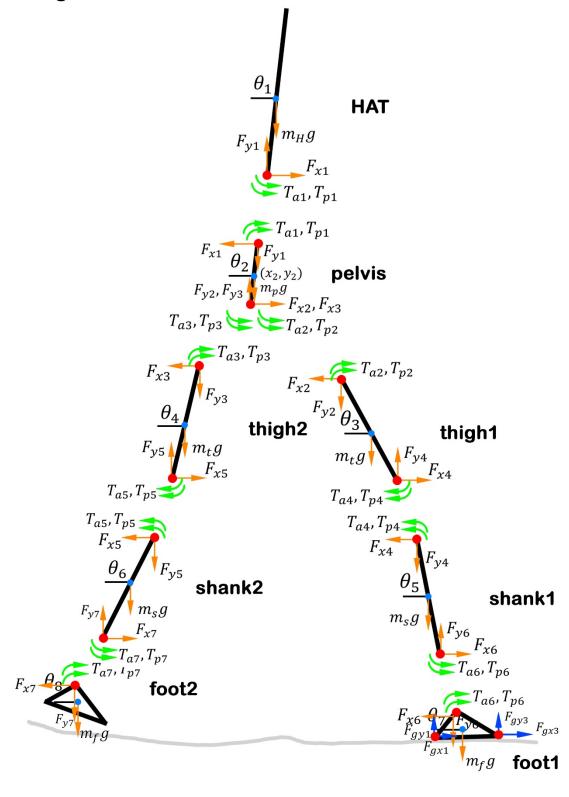
```
\left\{ \left[ \left( m_H + m_p + 2m_t + 2m_s + 2m_f \right) \ddot{x_2} \right] + \left[ m_H l_{H2} \sin \theta_1 \, \ddot{\theta_1} \right] + \left[ \left( m_H - 2m_t - 2m_s - 2m_f \right) l_p \sin \theta_2 \, \ddot{\theta_2} \right] \right\}
                                         +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{3}\ddot{\theta}_{3}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{4}\ddot{\theta}_{4}\right]
                                         +\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{5}\ddot{\theta}_{5}\right]+\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{6}\ddot{\theta}_{6}\right]+\left[-m_{f}l_{f1}\sin\theta_{7}\ddot{\theta}_{7}\right]
                                        + \left[-m_{f}l_{f1}\sin\theta_{8}\ddot{\theta_{8}}\right] + \left\{\left[m_{H}l_{H2}\cos\theta_{1}\dot{\theta_{1}}^{2}\right] + \left[\left(m_{H}-2m_{t}-2m_{s}-2m_{f}\right)l_{p}\cos\theta_{2}\dot{\theta_{2}}^{2}\right]\right\}
                                        + \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \cos \theta_3 \dot{\theta_3}^2 \right] + \left[ \left( -m_t - 2m_s - 2m_f \right) l_t \cos \theta_4 \dot{\theta_4}^2 \right]
                                        +\left[(-m_{s}-2m_{f})l_{s}\cos\theta_{5}\dot{\theta}_{5}^{2}\right]+\left[(-m_{s}-2m_{f})l_{s}\cos\theta_{6}\dot{\theta}_{6}^{2}\right]+\left[-m_{f}l_{f}\cos\theta_{7}\dot{\theta}_{7}^{2}\right]
                                        +\left[-m_{f}l_{f1}\cos\theta_{8}\dot{\theta_{8}}^{2}\right]\} = F_{ax1} + F_{ax2} + F_{ax3} + F_{ax4}
\{[(m_H + m_p + 2m_t + 2m_s + 2m_f)\ddot{y}_2] + [m_H l_{H2}\cos\theta_1\ddot{\theta}_1] + [(m_H - 2m_t - 2m_s - 2m_f)l_p\cos\theta_2\ddot{\theta}_2]
                                   +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{3}\ddot{\theta}_{3}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{4}\ddot{\theta}_{4}\right]
                                   + [(-m_s - 2m_f)l_s\cos\theta_5\,\ddot{\theta}_5] + [(-m_s - 2m_f)l_s\cos\theta_6\,\ddot{\theta}_6] + [-m_fl_{f1}\cos\theta_7\,\ddot{\theta}_7]
                                   +\left[-m_{f}l_{f1}\cos\theta_{8}\ddot{\theta_{8}}\right]+\left\{\left[-m_{H}l_{H2}\sin\theta_{1}\dot{\theta_{1}}^{2}\right]+\left[\left(-m_{H}+2m_{t}+2m_{s}+2m_{f}\right)l_{p}\sin\theta_{2}\dot{\theta_{2}}^{2}\right]
                                   + \left[ (m_t + 2m_s + 2m_f) l_t \sin \theta_3 \dot{\theta}_3^2 \right] + \left[ (m_t + 2m_s + 2m_f) l_t \sin \theta_4 \dot{\theta}_4^2 \right]
                                   +\left[\left(m_{S}+2m_{f}\right)l_{S}\sin\theta_{5}\dot{\theta_{5}}^{2}\right]+\left[\left(m_{S}+2m_{f}\right)l_{S}\sin\theta_{6}\dot{\theta_{6}}^{2}\right]+\left[m_{f}l_{f1}\sin\theta_{7}\dot{\theta_{7}}^{2}\right]
                                   +\left[m_{f}l_{f1}\sin\theta_{8}\dot{\theta_{8}}^{2}\right]+\left\{m_{H}g+m_{p}g+2m_{t}g+2m_{s}g+2m_{f}g\right\}=F_{qy1}+F_{qy2}+F_{qy3}+F_{qy4}
            \{[m_H l_{H2} \sin \theta_1 \ddot{x_2}] + [m_H l_{H2} \cos \theta_1 \ddot{y_2}] + [(l_H + m_H l_{H2}^2) \ddot{\theta_1}] + [m_H l_{H2} l_n \cos(\theta_1 - \theta_2) \ddot{\theta_2}]\}
                                               +\left\{ \left[m_{H}l_{H2}l_{p}\sin(\theta_{1}-\theta_{2})\dot{\theta_{2}}^{2}\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] = 0
 \{[(m_H - 2m_t - 2m_s - 2m_f)l_n \sin\theta_2 \ddot{x_2}] + [(m_H - 2m_t - 2m_s - 2m_f)l_n \cos\theta_2 \ddot{y_2}]
                                    + \left[m_H l_{H2} l_p \cos(\theta_1 - \theta_2) \ddot{\theta_1}\right] + \left[\left(l_p + m_H l_p^2 + 2m_t l_p^2 + 2m_s l_p^2 + 2m_f l_p^2\right) \ddot{\theta_2}\right]
                                    + [(m_t + 2m_s + 2m_f)l_n l_t \cos(\theta_2 - \theta_3) \dot{\theta}_3]
                                    +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{4})\ddot{\theta_{4}}\right]+\left[\left(m_{s}+2m_{f}\right)l_{n}l_{s}\cos(\theta_{2}-\theta_{5})\ddot{\theta_{5}}\right]
                                    +\left[\left(m_{s}+2m_{f}\right)l_{n}l_{s}\cos(\theta_{2}-\theta_{6})\ddot{\theta_{6}}\right]+\left[m_{f}l_{n}l_{f1}\cos(\theta_{2}-\theta_{7})\ddot{\theta_{7}}\right]
                                    + \left[m_f l_p l_{f1} \cos(\theta_2 - \theta_8) \ddot{\theta_8}\right]
                                    +\left\{\left[-m_{H}l_{H2}l_{p}\sin(\theta_{1}-\theta_{2})\dot{\theta_{1}}^{2}\right]+\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{3})\dot{\theta_{3}}^{2}\right]\right\}
                                    + \left[ \left( m_t + 2m_s + 2m_f \right) l_p l_t \sin(\theta_2 - \theta_4) \theta_4^{\ 2} \right] + \left[ \left( m_s + 2m_f \right) l_p l_s \sin(\theta_2 - \theta_5) \theta_5^{\ 2} \right]
                                    + \left[ (m_s + 2m_f) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta_6}^2 \right] + \left[ m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta_7}^2 \right]
                                    +\left[m_{f}l_{p}l_{f1}\sin(\theta_{2}-\theta_{8})\dot{\theta_{8}}^{2}\right]-T_{p1}+T_{p2}+T_{p3}-T_{a3}+T_{a2}+T_{a3}
                                    +\{(m_H-2m_t-2m_s-2m_f)l_n\cos\theta_2\,g\}
                                    = -(F_{gx1} + F_{gx2} + F_{gx3} + F_{gx4})l_p \sin \theta_2 - (F_{gy1} + F_{gy2} + F_{gy3} + F_{gy4})l_p \cos \theta_2
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\{[(-m_t - 2m_s - 2m_f)l_t \sin\theta_3 \ddot{x_2}] + [(-m_t - 2m_s - 2m_f)l_t \cos\theta_3 \ddot{y_2}]
                                                                                   +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{3})\dot{\theta_{2}}\right]+\left[\left(I_{t}+m_{t}l_{t}^{2}+4m_{s}l_{t}^{2}+4m_{f}l_{t}^{2}\right)\dot{\theta_{3}}\right]
                                                                                  + [(2m_s + 4m_f)l_tl_s\cos(\theta_3 - \theta_5)\ddot{\theta}_5] + [2m_fl_tl_{f1}\cos(\theta_3 - \theta_7)\ddot{\theta}_7]
                                                                                  +\left\{\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{n}l_{t}\sin(\theta_{2}-\theta_{3})\dot{\theta_{2}}^{2}\right]\right\}
                                                                                  +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{3}-\theta_{5})\dot{\theta_{5}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{3}-\theta_{7})\dot{\theta_{7}}^{2}\right]-T_{s}
                                                                                  -100 - 100 + \{(-m_t - 2m_s - 2m_f)l_t \cos\theta_3 g\}
                                                                                  = -2(F_{ax1} + F_{ax3})l_t \sin\theta_3 - 2(F_{ay1} + F_{ay3})l_t \cos\theta_3
     \{[(-m_t - 2m_s - 2m_f)l_t \sin\theta_4 \ddot{x_2}] + [(-m_t - 2m_s - 2m_f)l_t \cos\theta_4 \ddot{y_2}]
                                                                                   + \left[ \left( m_t + 2m_s + 2m_f \right) l_p l_t \cos(\theta_2 - \theta_4) \ddot{\theta_2} \right] + \left[ \left( l_t + m_t l_t^2 + 4m_s l_t^2 + 4m_f l_t^2 \right) \ddot{\theta_4} \right]
                                                                                  +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\cos(\theta_{4}-\theta_{6})\ddot{\theta_{6}}\right]+\left[2m_{f}l_{t}l_{f1}\cos(\theta_{4}-\theta_{8})\ddot{\theta_{8}}\right]
                                                                                  +\left\{\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{4})\dot{\theta_{2}}^{2}\right]\right\}
                                                                                  +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{4}-\theta_{6})\dot{\theta_{6}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{4}-\theta_{8})\dot{\theta_{8}}^{2}\right]-\frac{1}{4}
                                                                                  - - + \{(-m_t - 2m_s - 2m_f)l_t \cos\theta_4 g\}
                                                                                  = -2(F_{av2} + F_{av4})l_t \sin\theta_4 - 2(F_{av2} + F_{av4})l_t \cos\theta_4
\{[(-m_s - 2m_f)l_s \sin \theta_5 \ddot{x_2}] + [(-m_s - 2m_f)l_s \cos \theta_5 \ddot{y_2}] + [(m_s + 2m_f)l_n l_s \cos(\theta_2 - \theta_5)\ddot{\theta_2}]
                                                                             + [(2m_s + 4m_f)l_tl_s\cos(\theta_3 - \theta_5)\ddot{\theta_3}] + [(l_s + m_sl_s^2 + 4m_fl_s^2)\ddot{\theta_5}]
                                                                              + [2m_f l_s l_{f1} \cos(\theta_5 - \theta_7) \ddot{\theta_7}]
                                                                             +\left\{ \left[ \left( -m_{s}-2m_{f}\right) l_{p}l_{s}\sin(\theta_{2}-\theta_{5})\,\dot{\theta_{2}}^{2}\right] +\left[ \left( -2m_{s}-4m_{f}\right) l_{t}l_{s}\sin(\theta_{3}-\theta_{5})\,\dot{\theta_{3}}^{2}\right] \right.
                                                                            + \left[ 2m_{f}l_{s}l_{f1}\sin(\theta_{5} - \theta_{7})\dot{\theta_{7}}^{2} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{s} + T_{s} + T_{s} + T_{s} + T_{s} \right] + \left[ T_{s} + T_{
\left\{ \left[ \left( -m_s - 2m_f \right) l_s \sin \theta_6 \, \ddot{x_2} \right] + \left[ \left( -m_s - 2m_f \right) l_s \cos \theta_6 \, \ddot{y_2} \right] + \left[ \left( m_s + 2m_f \right) l_p l_s \cos (\theta_2 - \theta_6) \ddot{\theta_2} \right] \right\}
                                                                             + [(2m_s + 4m_f)l_tl_s\cos(\theta_4 - \theta_6)\ddot{\theta_4}] + [(l_s + m_sl_s^2 + 4m_fl_s^2)\ddot{\theta_6}]
                                                                             + [2m_f l_s l_{f1} \cos(\theta_6 - \theta_8) \ddot{\theta_8}]
                                                                             + \left\{ \left[ \left( -m_s - 2m_f \right) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta_2}^2 \right] + \left[ \left( -2m_s - 4m_f \right) l_t l_s \sin(\theta_4 - \theta_6) \dot{\theta_4}^2 \right] \right\}
                                                                             +\left[2m_{f}l_{s}l_{f1}\sin(\theta_{6}-\theta_{8})\dot{\theta_{8}}^{2}\right]+\left[1+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac{1}{1+1}+\frac
                                                                             +\{(-m_s - 2m_f)l_s\cos\theta_6 g\} = -2(F_{ax2} + F_{ax4})l_s\sin\theta_6 - 2(F_{ay2} + F_{ay4})l_s\cos\theta_6
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 \begin{split} \left\{ \left[ -m_f l_{f1} \sin \theta_7 \, \ddot{x_2} \right] + \left[ -m_f l_{f1} \cos \theta_7 \, \ddot{y_2} \right] + \left[ m_f l_p l_{f1} \cos (\theta_2 - \theta_7) \ddot{\theta_2} \right] + \left[ 2m_f l_t l_{f1} \cos (\theta_3 - \theta_7) \ddot{\theta_3} \right] \right. \\ &\quad + \left[ 2m_f l_s l_{f1} \cos (\theta_5 - \theta_7) \ddot{\theta_5} \right] + \left[ \left( l_f + m_f l_{f1}^2 \right) \ddot{\theta_7} \right] \right\} \\ &\quad + \left\{ \left[ -m_f l_p l_{f1} \sin (\theta_2 - \theta_7) \, \dot{\theta_2}^2 \right] + \left[ -2m_f l_t l_{f1} \sin (\theta_3 - \theta_7) \, \dot{\theta_3}^2 \right] \right. \\ &\quad + \left[ -2m_f l_s l_{f1} \sin (\theta_5 - \theta_7) \, \dot{\theta_5}^2 \right] - \left[ -m_f l_{f1} \cos \theta_7 + l_{f2} \cos (\alpha_1 - \theta_7) \right] \right. \\ &\quad + \left. \left( l_{f1} \sin \theta_7 - l_{f2} \sin (\alpha_1 - \theta_7) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_7 + l_{f3} \cos (\alpha_2 + \theta_7) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \sin \theta_7 + l_{f3} \sin (\alpha_2 + \theta_7) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 \, \ddot{y_2} \right) + \left[ m_f l_p l_{f1} \cos (\theta_2 - \theta_8) \ddot{\theta_2} \right] + \left[ 2m_f l_t l_{f1} \cos (\theta_4 - \theta_8) \ddot{\theta_4} \right] \\ &\quad + \left. \left[ 2m_f l_s l_{f1} \cos (\theta_6 - \theta_8) \ddot{\theta_6} \right] + \left[ \left( l_f + m_f l_{f1}^2 \right) \ddot{\theta_8} \right] \right. \\ &\quad + \left. \left[ -m_f l_p l_{f1} \sin (\theta_2 - \theta_8) \, \dot{\theta_2}^2 \right] + \left[ -2m_f l_t l_{f1} \sin (\theta_4 - \theta_8) \, \dot{\theta_4}^2 \right] \\ &\quad + \left[ -2m_f l_s l_{f1} \sin (\theta_6 - \theta_8) \, \dot{\theta_6}^2 \right] - \left. \left( l_{f1} \sin \theta_8 - l_{f2} \sin (\alpha_1 - \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \sin \theta_8 + l_{f3} \sin (\alpha_2 + \theta_8) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \sin \theta_8 + l_{f3} \sin (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \sin \theta_8 + l_{f3} \sin (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \right. \\ &\quad + \left. \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ \\ &\quad + \left. \left( l_{f1} \cos \theta_8 + l_{f3} \cos (\alpha_2 + \theta_8) \right) \right. \\ \\
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Stage 2



$$\mathbf{D}\ddot{q} + \mathbf{C}\dot{q} + \mathbf{G} = \mathbf{\tau}$$

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\{[(m_H + m_p + 2m_t + 2m_s + 2m_f)\ddot{x_2}] + [m_H l_{H2} \sin \theta_1 \ddot{\theta_1}] + [(m_H - 2m_t - 2m_s - 2m_f) l_p \sin \theta_2 \ddot{\theta_2}]
                                                             +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{3}\ddot{\theta}_{3}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\sin\theta_{4}\ddot{\theta}_{4}\right]
                                                            +\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{5}\ddot{\theta}_{5}\right]+\left[\left(-m_{s}-2m_{f}\right)l_{s}\sin\theta_{6}\ddot{\theta}_{6}\right]+\left[-m_{f}l_{f1}\sin\theta_{7}\ddot{\theta}_{7}\right]
                                                           + \left[-m_{f}l_{f1}\sin\theta_{8}\ddot{\theta_{8}}\right] + \left\{\left[m_{H}l_{H2}\cos\theta_{1}\dot{\theta_{1}}^{2}\right] + \left[\left(m_{H}-2m_{t}-2m_{s}-2m_{f}\right)l_{p}\cos\theta_{2}\dot{\theta_{2}}^{2}\right]\right\}
                                                           +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{3}\dot{\theta_{3}}^{2}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{4}\dot{\theta_{4}}^{2}\right]
                                                           +\left[(-m_s-2m_f)l_s\cos\theta_5\dot{\theta_5}^2\right]+\left[(-m_s-2m_f)l_s\cos\theta_6\dot{\theta_6}^2\right]+\left[-m_fl_{f1}\cos\theta_7\dot{\theta_7}^2\right]
                                                           + \left[ -m_f l_{f1} \cos \theta_8 \, \dot{\theta_8}^2 \right] = F_{ax1} + F_{ax3}
\{[(m_H + m_p + 2m_t + 2m_s + 2m_f)\ddot{y}_2] + [m_H l_{H2} \cos \theta_1 \ddot{\theta}_1] + [(m_H - 2m_t - 2m_s - 2m_f) l_p \cos \theta_2 \ddot{\theta}_2]
                                                       +\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{3}\ddot{\theta_{3}}\right]+\left[\left(-m_{t}-2m_{s}-2m_{f}\right)l_{t}\cos\theta_{4}\ddot{\theta_{4}}\right]
                                                      + [(-m_s - 2m_f)l_s\cos\theta_5\ddot{\theta}_5] + [(-m_s - 2m_f)l_s\cos\theta_6\ddot{\theta}_6] + [-m_fl_{f1}\cos\theta_7\ddot{\theta}_7]
                                                      +\left[-m_{f}l_{f1}\cos\theta_{8}\ddot{\theta_{8}}\right]+\left\{\left[-m_{H}l_{H2}\sin\theta_{1}\dot{\theta_{1}}^{2}\right]+\left[\left(-m_{H}+2m_{t}+2m_{s}+2m_{f}\right)l_{p}\sin\theta_{2}\dot{\theta_{2}}^{2}\right]
                                                      +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{t}\sin\theta_{3}\dot{\theta_{3}}^{2}\right]+\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{t}\sin\theta_{4}\dot{\theta_{4}}^{2}\right]
                                                      + \left[ (m_s + 2m_f) l_s \sin \theta_5 \dot{\theta}_5^2 \right] + \left[ (m_s + 2m_f) l_s \sin \theta_6 \dot{\theta}_6^2 \right] + \left[ m_f l_{f1} \sin \theta_7 \dot{\theta}_7^2 \right]
                                                      + \left[m_f l_{f1} \sin \theta_8 \dot{\theta_8}^2\right] + \left\{m_H g + m_p g + 2m_t g + 2m_s g + 2m_f g\right\} = F_{qv1} + F_{qv3}
              \{[m_H l_{H2} \sin \theta_1 \ddot{x_2}] + [m_H l_{H2} \cos \theta_1 \ddot{y_2}] + [(l_H + m_H l_{H2}^2) \ddot{\theta_1}] + [m_H l_{H2} l_n \cos(\theta_1 - \theta_2) \ddot{\theta_2}]\}
                                                                      +\left\{ \left[m_{H}l_{H2}l_{p}\sin(\theta_{1}-\theta_{2})\dot{\theta_{2}}^{2}\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] + \left[m_{H}l_{H2}\cos\theta_{1}g\right] = 0
\{[(m_H - 2m_t - 2m_s - 2m_f)l_n \sin\theta_2 \ddot{x_2}] + [(m_H - 2m_t - 2m_s - 2m_f)l_n \cos\theta_2 \ddot{y_2}]
                                                      + \left[m_H l_{H2} l_p \cos(\theta_1 - \theta_2) \ddot{\theta_1}\right] + \left[\left(l_p + m_H l_p^2 + 2m_t l_p^2 + 2m_s l_p^2 + 2m_f l_p^2\right) \ddot{\theta_2}\right]
                                                      + [(m_t + 2m_s + 2m_f)l_n l_t \cos(\theta_2 - \theta_3) \dot{\theta}_3]
                                                       +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{4})\ddot{\theta_{4}}\right]+\left[\left(m_{s}+2m_{f}\right)l_{n}l_{s}\cos(\theta_{2}-\theta_{5})\ddot{\theta_{5}}\right]
                                                      +\left[\left(m_{s}+2m_{f}\right)l_{n}l_{s}\cos(\theta_{2}-\theta_{6})\dot{\theta_{6}}\right]+\left[m_{f}l_{n}l_{f1}\cos(\theta_{2}-\theta_{7})\dot{\theta_{7}}\right]
                                                      +\left[m_f l_n l_{f1} \cos(\theta_2 - \theta_8) \ddot{\theta_8}\right]
                                                      +\left\{\left[-m_{H}l_{H2}l_{p}\sin(\theta_{1}-\theta_{2})\dot{\theta_{1}}^{2}\right]+\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{3})\dot{\theta_{3}}^{2}\right]\right\}
                                                      +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{4})\dot{\theta_{4}}^{2}\right]+\left[\left(m_{s}+2m_{f}\right)l_{p}l_{s}\sin(\theta_{2}-\theta_{5})\dot{\theta_{5}}^{2}\right]
                                                      + \left[ \left( m_s + 2m_f \right) l_p l_s \sin(\theta_2 - \theta_6) \dot{\theta_6}^2 \right] + \left[ m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta_7}^2 \right]
                                                       + \left[ m_{f} l_{p} l_{f1} \sin(\theta_{2} - \theta_{8}) \dot{\theta_{8}}^{2} \right] - \frac{1}{T_{eff}} + 
                                                       =-(F_{ax1}+F_{ax3})l_n\sin\theta_2-(F_{ay1}+F_{ay3})l_n\cos\theta_2
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\{[(-m_t - 2m_s - 2m_f)l_t \sin\theta_3 \ddot{x_2}] + [(-m_t - 2m_s - 2m_f)l_t \cos\theta_3 \ddot{y_2}]
                                  + \left[ \left( m_t + 2m_s + 2m_f \right) l_n l_t \cos(\theta_2 - \theta_3) \ddot{\theta}_2 \right] + \left[ \left( l_t + m_t l_t^2 + 4m_s l_t^2 + 4m_f l_t^2 \right) \ddot{\theta}_3 \right]
                                 + [(2m_s + 4m_f)l_tl_s\cos(\theta_3 - \theta_5)\ddot{\theta}_5] + [2m_fl_tl_{f1}\cos(\theta_3 - \theta_7)\ddot{\theta}_7]
                                 +\{[(-m_t-2m_s-2m_f)l_nl_t\sin(\theta_2-\theta_3)\dot{\theta_2}^2]\}
                                  +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{3}-\theta_{5})\dot{\theta_{5}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{3}-\theta_{7})\dot{\theta_{7}}^{2}\right]-T_{s}
                                 -100 - 100 + \{(-m_t - 2m_s - 2m_f)l_t \cos\theta_3 g\}
                                 = -2(F_{ax1} + F_{ax3})l_t \sin\theta_3 - 2(F_{ay1} + F_{ay3})l_t \cos\theta_3
  \{[(-m_t - 2m_s - 2m_f)l_t \sin\theta_4 \ddot{x_2}] + [(-m_t - 2m_s - 2m_f)l_t \cos\theta_4 \ddot{y_2}]
                                  +\left[\left(m_{t}+2m_{s}+2m_{f}\right)l_{n}l_{t}\cos(\theta_{2}-\theta_{4})\ddot{\theta_{2}}\right]+\left[\left(l_{t}+m_{t}l_{t}^{2}+4m_{s}l_{t}^{2}+4m_{f}l_{t}^{2}\right)\ddot{\theta_{4}}\right]
                                 +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\cos(\theta_{4}-\theta_{6})\ddot{\theta_{6}}\right]+\left[2m_{f}l_{t}l_{f1}\cos(\theta_{4}-\theta_{8})\ddot{\theta_{8}}\right]
                                 +\left\{\left|\left(-m_{t}-2m_{s}-2m_{f}\right)l_{p}l_{t}\sin(\theta_{2}-\theta_{4})\dot{\theta_{2}}^{2}\right|\right\}
                                 +\left[\left(2m_{s}+4m_{f}\right)l_{t}l_{s}\sin(\theta_{4}-\theta_{6})\dot{\theta_{6}}^{2}\right]+\left[2m_{f}l_{t}l_{f1}\sin(\theta_{4}-\theta_{8})\dot{\theta_{8}}^{2}\right]-\frac{1}{4}
                                 -100 - 100 + \{(-m_t - 2m_s - 2m_f)l_t \cos \theta_4 g\} = 0
\{ \left[ \left( -m_s - 2m_f \right) l_s \sin \theta_5 \, \ddot{x_2} \right] + \left[ \left( -m_s - 2m_f \right) l_s \cos \theta_5 \, \ddot{y_2} \right] + \left[ \left( m_s + 2m_f \right) l_p l_s \cos (\theta_2 - \theta_5) \ddot{\theta_2} \right]
                                + [(2m_s + 4m_f)l_t l_s \cos(\theta_3 - \theta_5)\ddot{\theta}_3] + [(l_s + m_s l_s^2 + 4m_f l_s^2)\ddot{\theta}_5]
                                + [2m_f l_s l_{f1} \cos(\theta_5 - \theta_7) \ddot{\theta_7}]
                               +\left\{\left[\left(-m_{s}-2m_{f}\right)l_{p}l_{s}\sin(\theta_{2}-\theta_{5})\dot{\theta_{2}}^{2}\right]+\left[\left(-2m_{s}-4m_{f}\right)l_{t}l_{s}\sin(\theta_{3}-\theta_{5})\dot{\theta_{3}}^{2}\right]\right\}
                               +\{(-m_s - 2m_f)l_s\cos\theta_5 g\} = -2(F_{ax1} + F_{ax3})l_s\sin\theta_5 - 2(F_{av1} + F_{av3})l_s\cos\theta_5
   \left\{ \left[ \left( -m_s - 2m_f \right) l_s \sin \theta_6 \, \ddot{x_2} \right] + \left[ \left( -m_s - 2m_f \right) l_s \cos \theta_6 \, \ddot{y_2} \right] + \left[ \left( m_s + 2m_f \right) l_p l_s \cos (\theta_2 - \theta_6) \ddot{\theta_2} \right] \right\}
                                  + [(2m_s + 4m_f)l_t l_s \cos(\theta_4 - \theta_6)\ddot{\theta}_4] + [(l_s + m_s l_s^2 + 4m_f l_s^2)\ddot{\theta}_6]
                                  + [2m_f l_s l_{f1} \cos(\theta_6 - \theta_8) \ddot{\theta_8}]
                                  +\left\{ \left[ \left( -m_{s}-2m_{f}\right) l_{p}l_{s}\sin(\theta_{2}-\theta_{6})\,\dot{\theta_{2}}^{2}\right] +\left[ \left( -2m_{s}-4m_{f}\right) l_{t}l_{s}\sin(\theta_{4}-\theta_{6})\,\dot{\theta_{4}}^{2}\right] \right.
                                  +\left[2m_{f}l_{s}l_{f1}\sin(\theta_{6}-\theta_{8})\dot{\theta_{8}}^{2}\right]+r_{ss}+r_{ss}+r_{ss}+r_{ss}
                                  +\{(-m_s-2m_f)l_s\cos\theta_6\,q\}=0
\left\{ \left[ -m_f l_{f1} \sin \theta_7 \ddot{x_2} \right] + \left[ -m_f l_{f1} \cos \theta_7 \ddot{y_2} \right] + \left[ m_f l_p l_{f1} \cos (\theta_2 - \theta_7) \ddot{\theta_2} \right] + \left[ 2m_f l_t l_{f1} \cos (\theta_3 - \theta_7) \ddot{\theta_3} \right] \right\}
                                + \left[ 2m_f l_s l_{f1} \cos(\theta_5 - \theta_7) \ddot{\theta}_5 \right] + \left[ \left( I_f + m_f l_{f1}^2 \right) \ddot{\theta}_7 \right] 
                                + \left\{ \left[ -m_f l_p l_{f1} \sin(\theta_2 - \theta_7) \dot{\theta_2}^2 \right] + \left[ -2m_f l_t l_{f1} \sin(\theta_3 - \theta_7) \dot{\theta_3}^2 \right] \right\}
                                + \left[-2m_{f}l_{s}l_{f1}\sin(\theta_{5}-\theta_{7})\dot{\theta}_{5}^{2}\right] - \left[-m_{f}l_{f1}\cos\theta_{7}g\right]
                                = (l_{f1}\sin\theta_7 - l_{f2}\sin(\alpha_1 - \theta_7)) \frac{F_{gx1}}{F_{gx1}} + (l_{f1}\cos\theta_7 + l_{f2}\cos(\alpha_1 - \theta_7)) \frac{F_{gy1}}{F_{gy1}}
                                +(l_{f1}\sin\theta_7 + l_{f3}\sin(\alpha_2 + \theta_7))F_{gx3} + (l_{f1}\cos\theta_7 + l_{f3}\cos(\alpha_2 + \theta_7))F_{gy3}
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