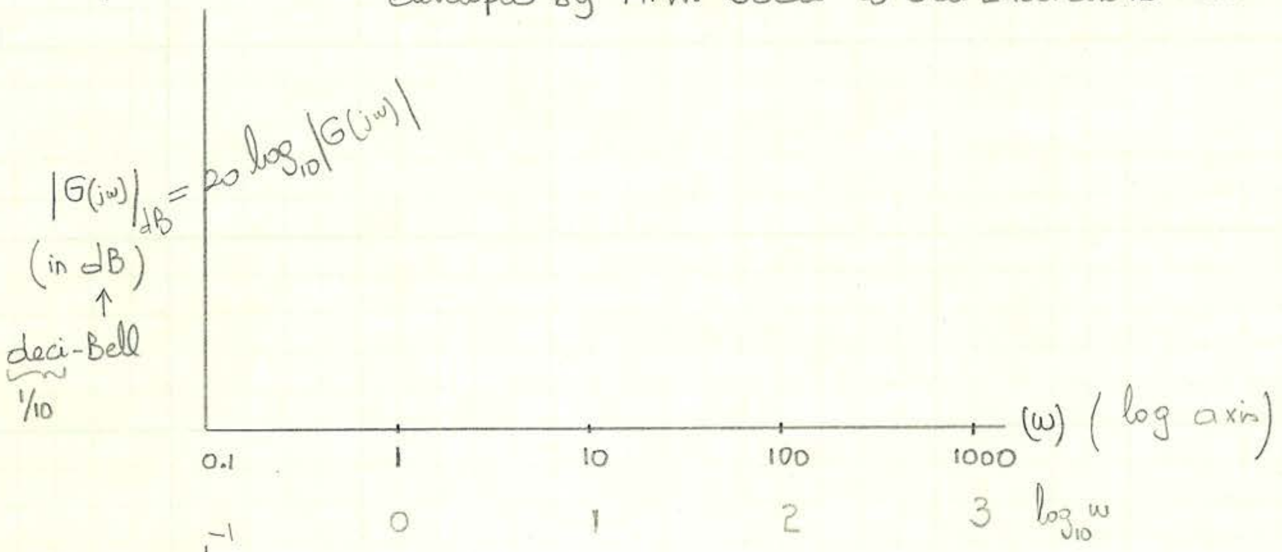
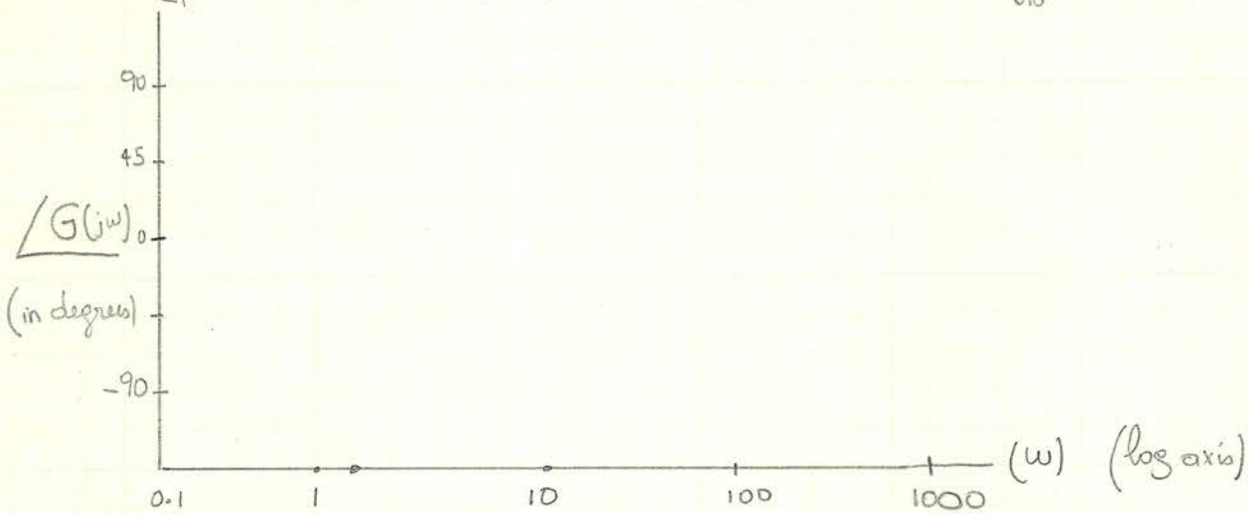
Bode Plots

$G(s) \xrightarrow{s=j\omega} G(j\omega)$

developed by H. W. Bode @ Bell Laboratories 1932-1942





Let $G(s) = \dfrac{K(1+T_1 s)}{s(1+T_2 s)(1+T_3 s)}$

$$|G(j\omega)|_{dB} = 20\log_{10}|G(j\omega)|$$

$$= 20\log_{10}\left|\frac{K(1+T_1 j\omega)}{s(1+T_2 j\omega)(1+T_3 j\omega)}\right| = 20\log_{10}\left[|K(1+T_1 j\omega)|\right] - 20\log_{10}\left[|j\omega(1+T_2 j\omega)(1+T_3 j\omega)|\right]$$

$$= 20\log_{10}|K| + 20\log_{10}|1+T_1 j\omega| - 20\log_{10}|j\omega| - 20\log_{10}|1+T_2 j\omega| - 20\log_{10}|1+T_3 j\omega|$$

Conclusion $|G(j\omega)|_{dB} \leftarrow$ summation & substraction of $20\log_{10}$ (of basic elements)

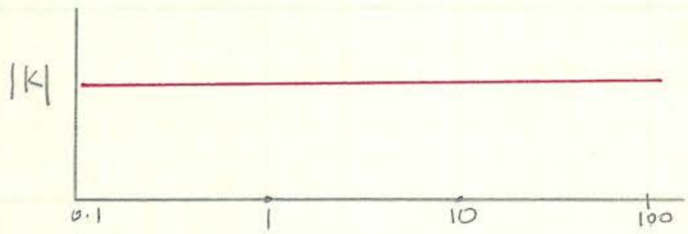
$\angle G(j\omega) = \angle K + \angle 1+T_1 j\omega - \angle j\omega - \angle 1+T_2 j\omega - \angle 1+T_3 j\omega$

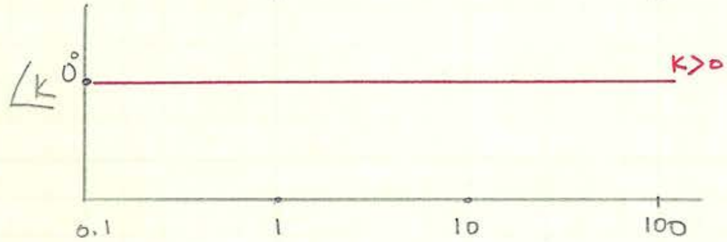
Conclusion : $\angle G(j\omega)$ $\Leftarrow$ summation & substraction of $\angle$ of basic element

- Constant K

$K_{dB} = 20 \log_{10}(K) = cte$

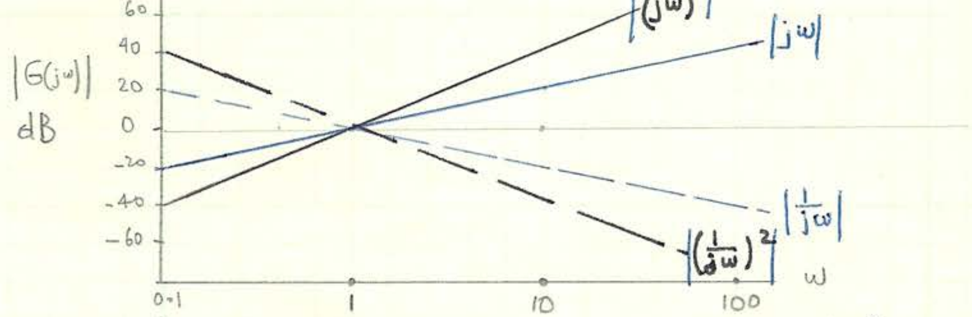
$\angle K = \begin{cases} 0^\circ & K > 0 \\ 180^\circ & K < 0 \end{cases}$

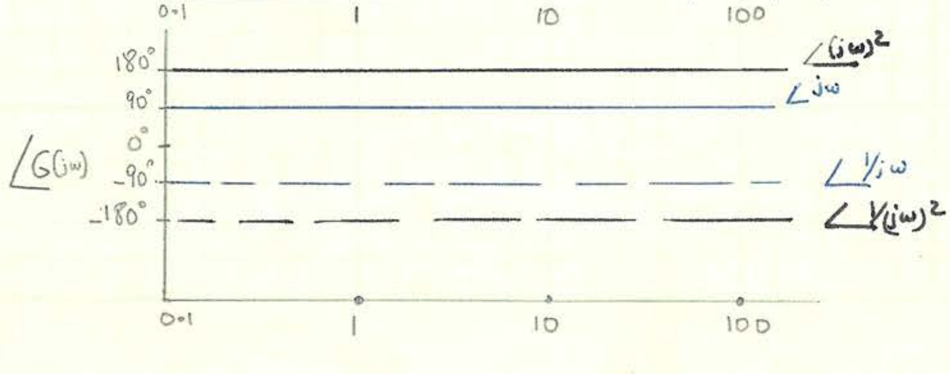




- $(j\omega)^{\pm n}$ [Poles & Zeros @ origin]

magnitude $20 \log_{10} |(j\omega)^{\pm n}| = \underbrace{\pm 20n}_{slope} \underbrace{\log_{10} \omega}$ $\qquad \boxed{\log_{10}(10)} = 1$ $\quad 10 = 10$





- $1+j\omega T$ [Simple zero]

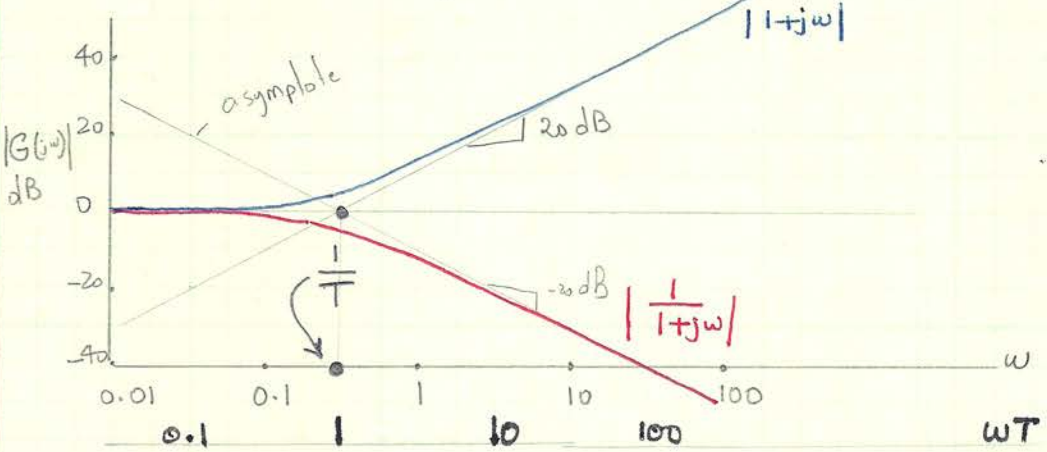
$G(j\omega) = 1 + j\omega T$

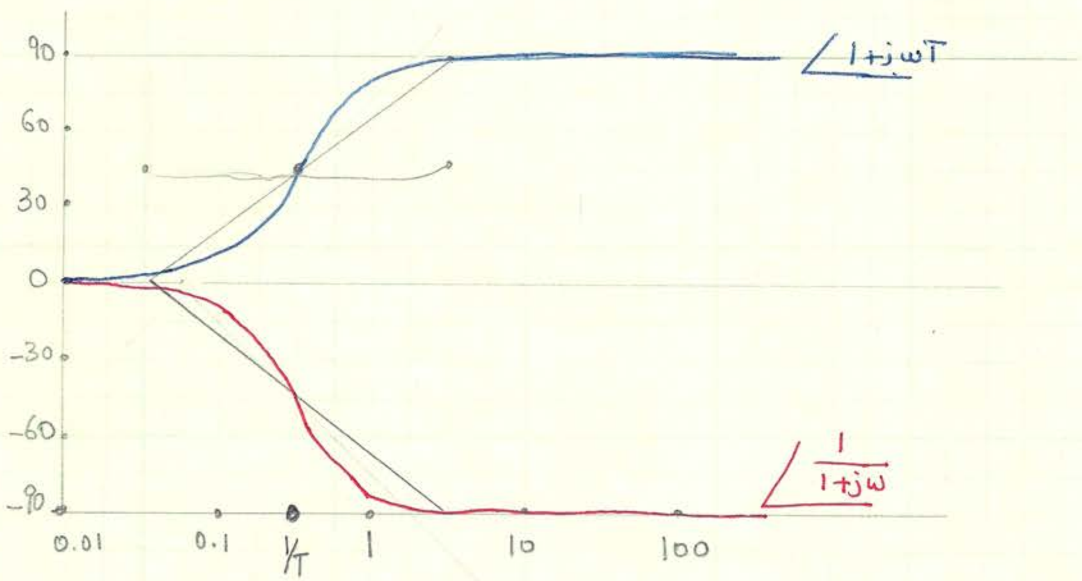
- $|G(j\omega)|_{dB} = 20 \log_{10} |G(j\omega)| = 20 \log_{10} \sqrt{1+\omega^2 T^2}$
  - For $\omega T \ll 1 \Rightarrow 1 + j\omega T \simeq 1 \Rightarrow |G(j\omega)|_{dB} = 20 \log_{10}(1) = 0$
  - For $\omega T \gg 1 \Rightarrow 1 + j\omega T \simeq j\omega T \Rightarrow |G(j\omega)|_{dB} = 20 \log_{10} \omega T$

  Intersection of 2 asymptotes : $0 = 20 \log_{10} \omega T \Rightarrow \omega = \frac{1}{T}$

  corner frequency

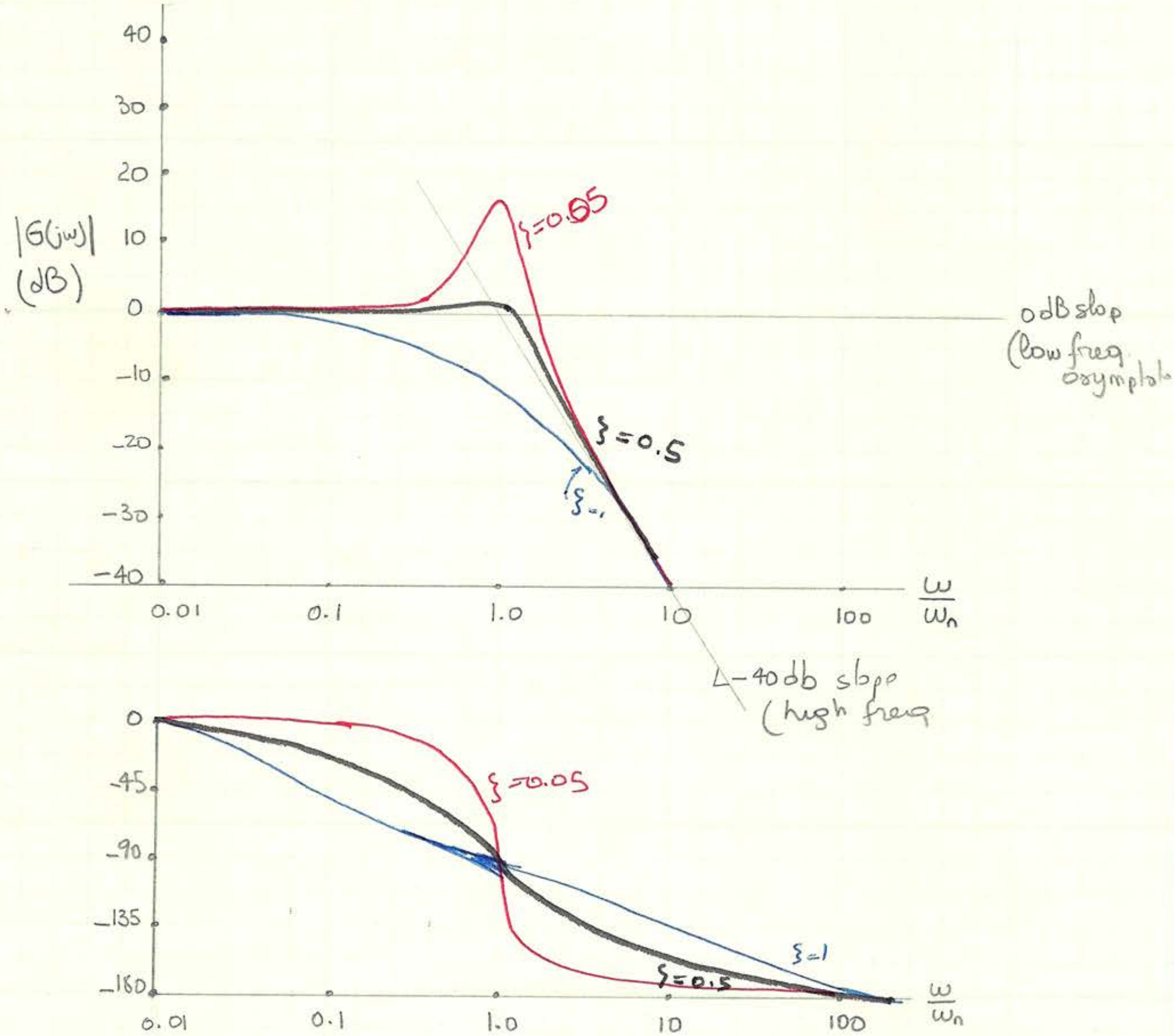
- $\angle G(j\omega) = \tan^{-1} \omega T$
  - For $\omega T \ll 1 \Rightarrow \angle 1 + j\omega T = \angle 1 = 0°$
  - For $\omega T \gg 1 \Rightarrow \angle 1 + j\omega T = \angle j\omega T = 90°$
  - For $\omega T \simeq 1 \Rightarrow \angle 1 + j\omega T = \angle 1 + j = 45°$





- Quadratic Poles (& Zeros)

Consider $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$



NOTE For Quadratic Zeros invert above Bode plot.

Ex: $G(s) = \dfrac{2000(s+0.5)}{s(s+10)(s+50)} = \dfrac{2\,(2s+1)}{s\,(0.1j\omega+1)(0.02j\omega+1)}$

$$= \frac{2\left(\frac{s}{0.5}+1\right)}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{50}+1\right)}\Bigg|_{s=j\omega} = \frac{2\left[\frac{j\omega}{0.5}+1\right]}{j\omega\left[\frac{j\omega}{10}+1\right]\left[\frac{j\omega}{50}+1\right]}$$