Representations and Transformations

Lydia E. Kavraki
http://www.cs.rice.edu/~kavraki
http://www.kavrakilab.org

Motion Planning: Representations and Transformations

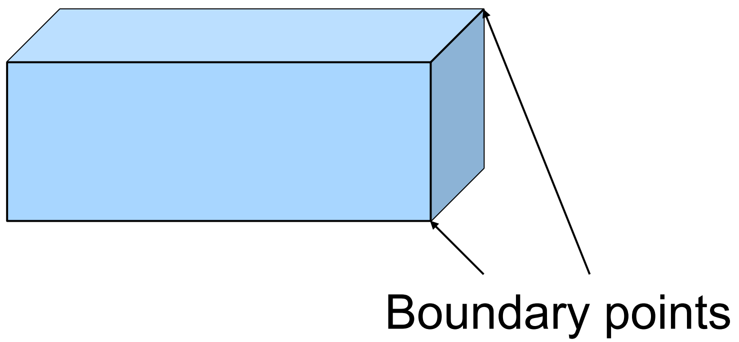
- **Objective:** Plan a path for a robot model, that takes it from some initial configuration to some desired goal configuration.
- **Constraints:** Avoid collisions in the physical environment at all times, respect physics of the problem.
- **Key questions:**
  1. How to represent the robot and the physical world?
  2. How to compute motion of the robot ?
  3. How to do all of this efficiently, and in a mathematically sound way ?

Representation of Physical Objects

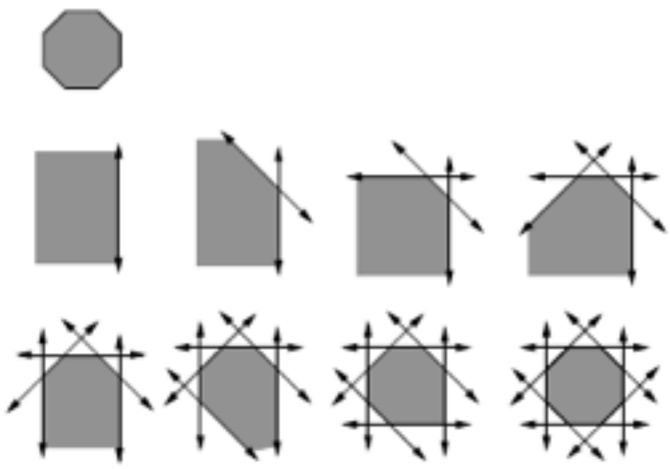
- World $\mathcal{W}$ is the physical space
- $\mathcal{W} = \mathbb{R}^2$ or $\mathbb{R}^3$
- Many alternatives for representing physical objects:
    1. Boundary-point based
    2. Primitive based
    3. Polygon soups
    4. Point Clouds and others representations

Representation of Physical Objects

- **Boundary-point representation:** Represent the object by specifying details about points on the boundary

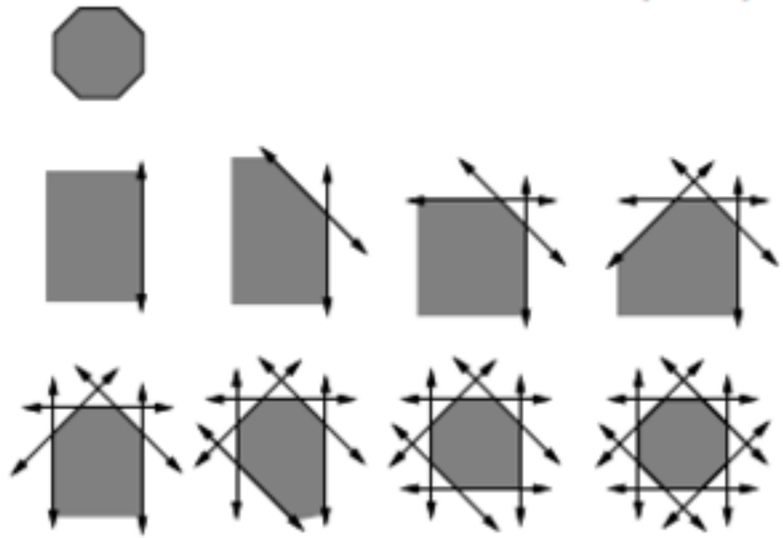


- **Primitive-based representation:** Use primitives for defining objects

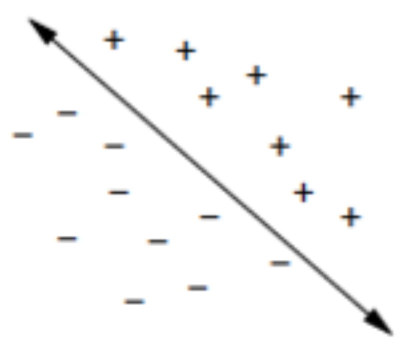


Linear Primitives

$f(x, y) = ax + by + c$ Inside: $f(x, y) \leq 0$



Intersections make convex polygons or polyhedra.



Notions of inside and outside are clear.

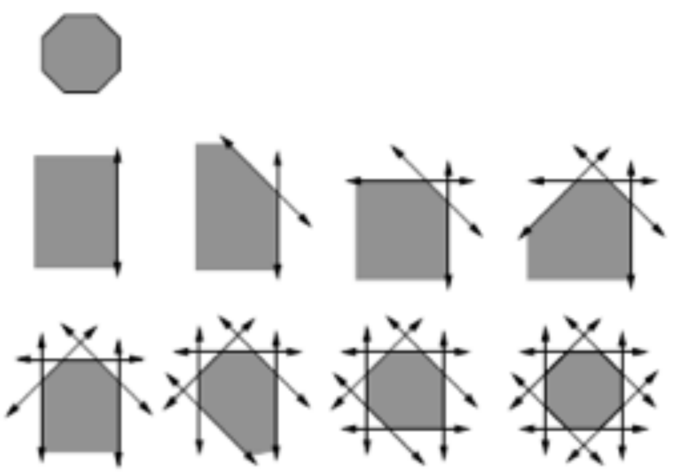
Convex vs. Nonconvex

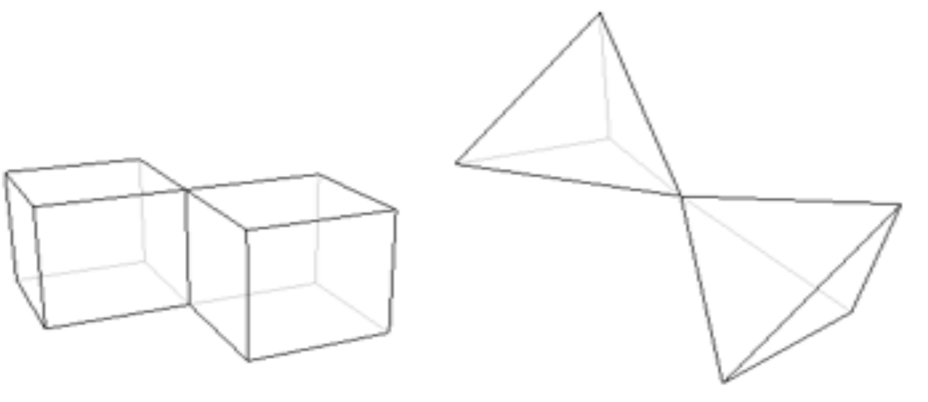
Convexity: A set S is convex if it satisfies the following:

$$x, y \in \mathcal{S} \Rightarrow \lambda x + (1 - \lambda) y \in \mathcal{S}$$

- Convex polygons (polyhedra) easy to describe using intersection of halfplanes (halfspaces)

- Nonconvex polygons are described as union of convex polygons (e.g, triangles)
- Nonconvex polyhedra are described as union of convex polyhedra (e.g., tetrahedra)

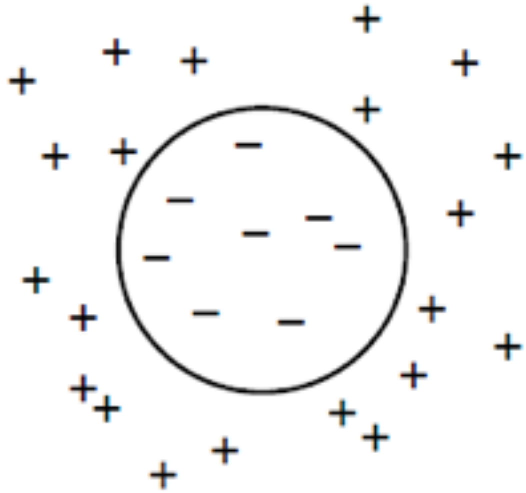




Note: The problem of decomposing a nonconvex set into convex sets is not easy in general

Semi Algebraic Sets

- Polygons and polyhedra use linear primitives (e.g., halfspaces)
- **Algebraic set:** A generalization of polygons and polyhedra that uses **nonlinear primitives**



$$H = \{(x, y) \in \mathcal{W} \mid f(x, y) \leq 0\}, \text{ where,}$$

$$f(x, y) = (x - 2)^2 + (y - 2)^2 - 4$$

Semi Algebraic set: Sets constructed using finite intersection and unions of algebraic sets

?

Center of outer circle=(x1,y1); Radius = r1
Center of left eye: (x2,y2); Radius = r2
Center of right eye: (x3,y3); Radius = r3
Mouth is an ellipse: Center = (x4,y4); Major axis “a”, Minor axis “b”

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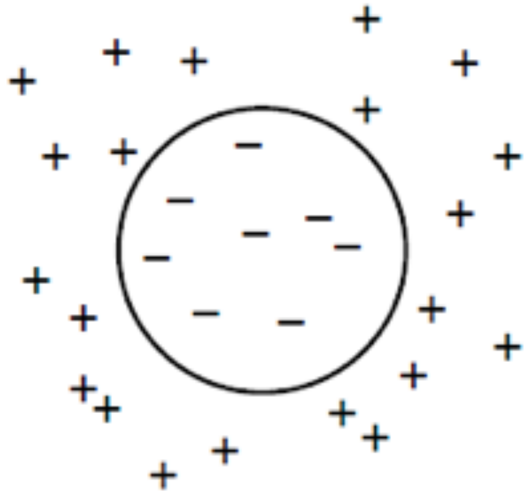


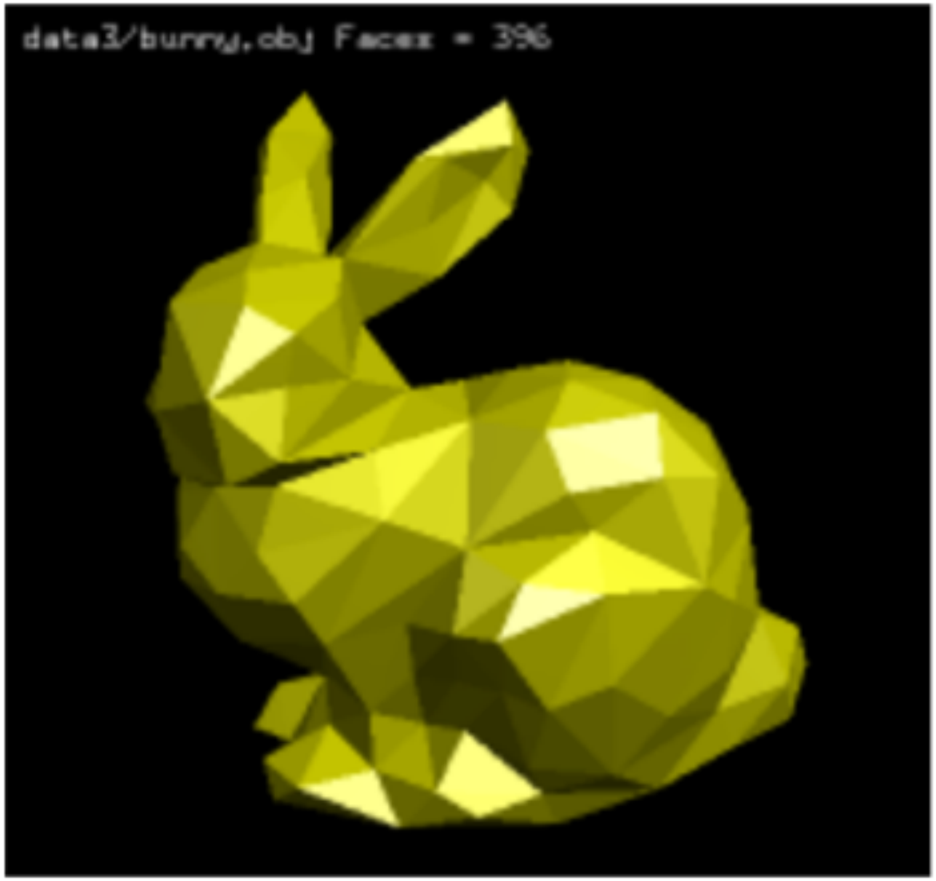
$$H = \{(x, y) \in \mathcal{W} | f(x, y) \leq 0\}, \text{ where,}$$

$$f(x, y) = (x-2)^2 + (y-2)^2 - 4$$

Semi Algebraic set: Sets constructed using finite intersection and unions of algebraic sets

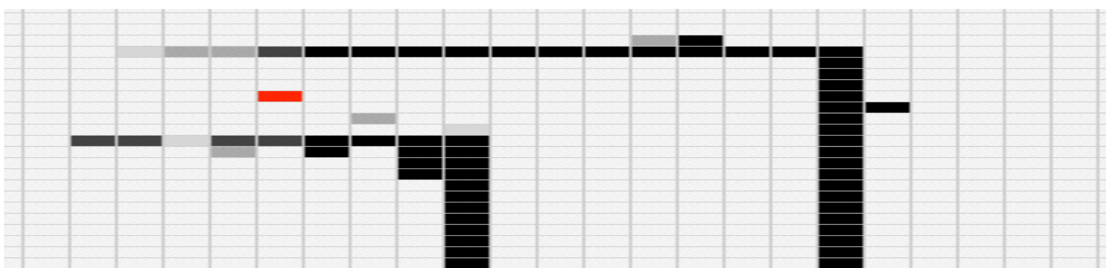
$$\begin{aligned}
f_1 &= x^2 + y^2 - r_1^2, \\
f_2 &= -\left((x - x_2)^2 + (y - y_2)^2 - r_2^2\right), \\
f_3 &= -\left((x - x_3)^2 + (y - y_3)^2 - r_3^2\right), \\
f_4 &= -\left(x^2/a^2 + (y - y_4)^2/b^2 - 1\right). \\
\mathcal{O} &= H_1 \cap H_2 \cap H_3 \cap H_4.
\end{aligned}$$

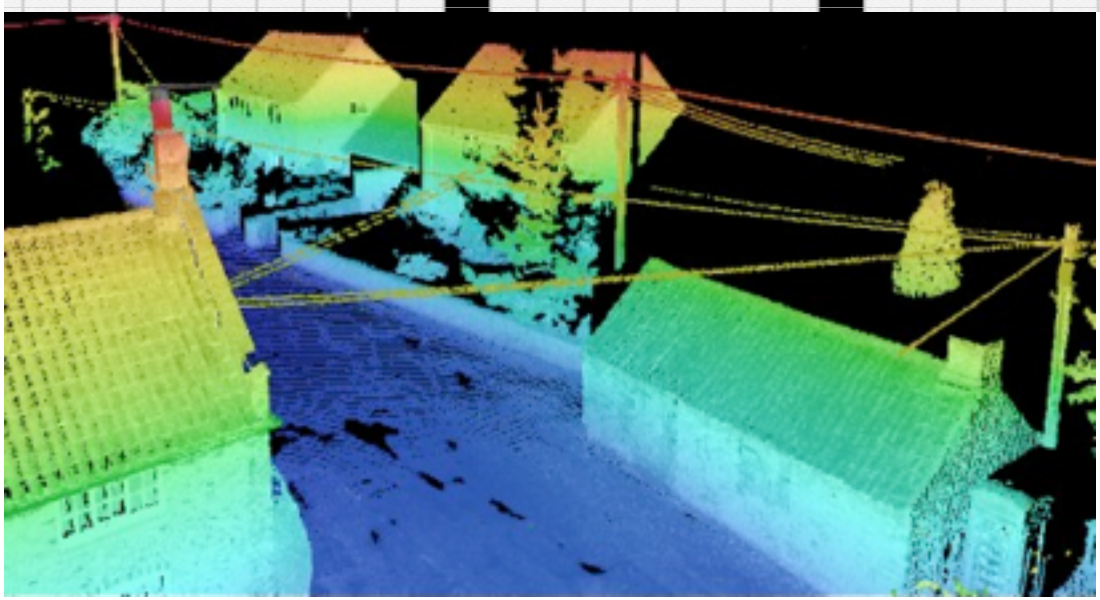
Polygon Soups



Objects identified using sensors

- Real world objects are usually detected using perception module of a robot
- Obstacles are usually detected using variety of sensors
- Obstacles are described using point cloud, occupancy grid, bitmaps



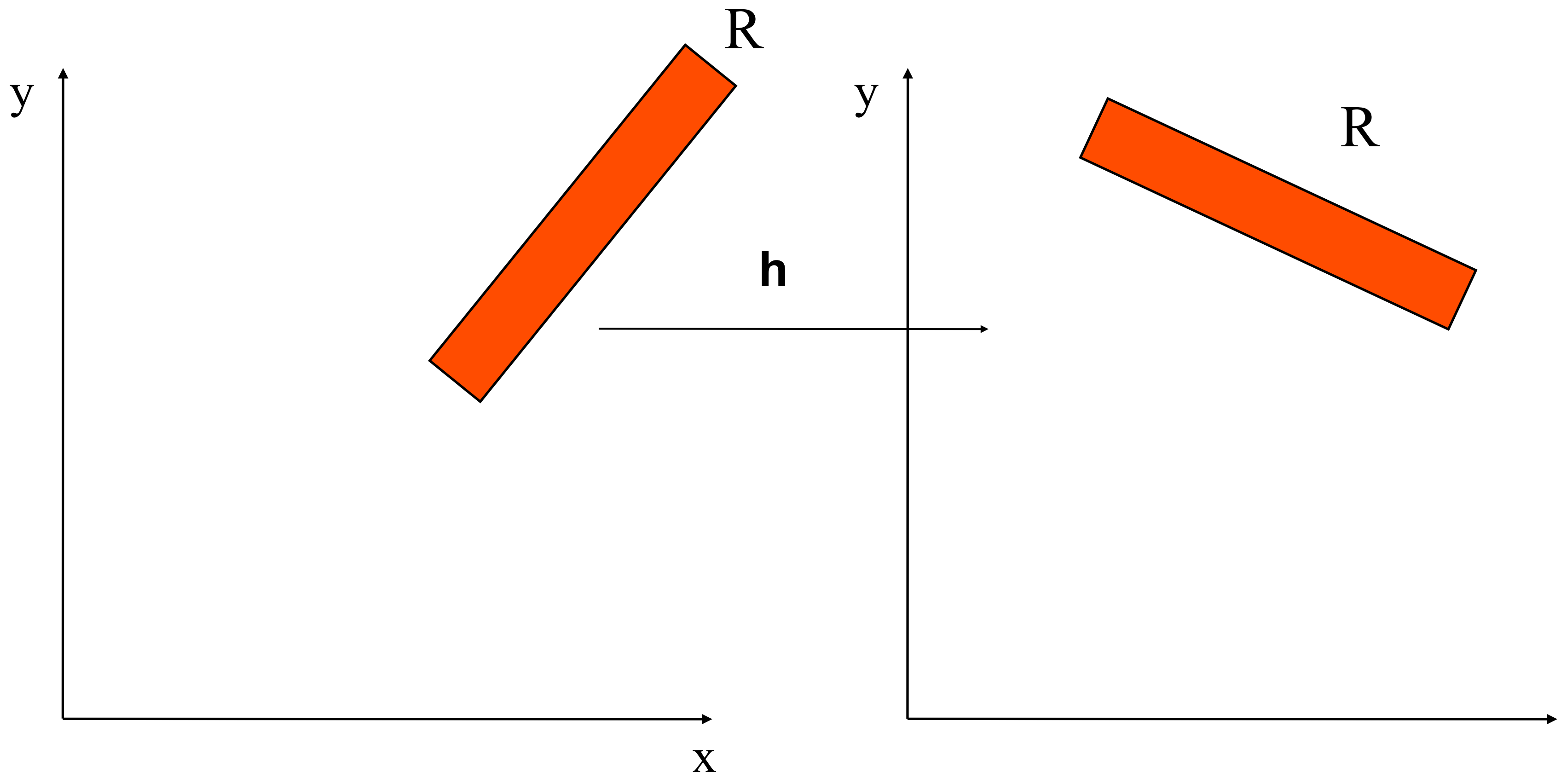


Rigid Body Transformations

- **Rigid body model:** Model of physical object with no deformation
- **Transformation of robot model:** $h(\mathcal{A}) = \{h(a) \in \mathcal{W} | a \in \mathcal{A}\}$

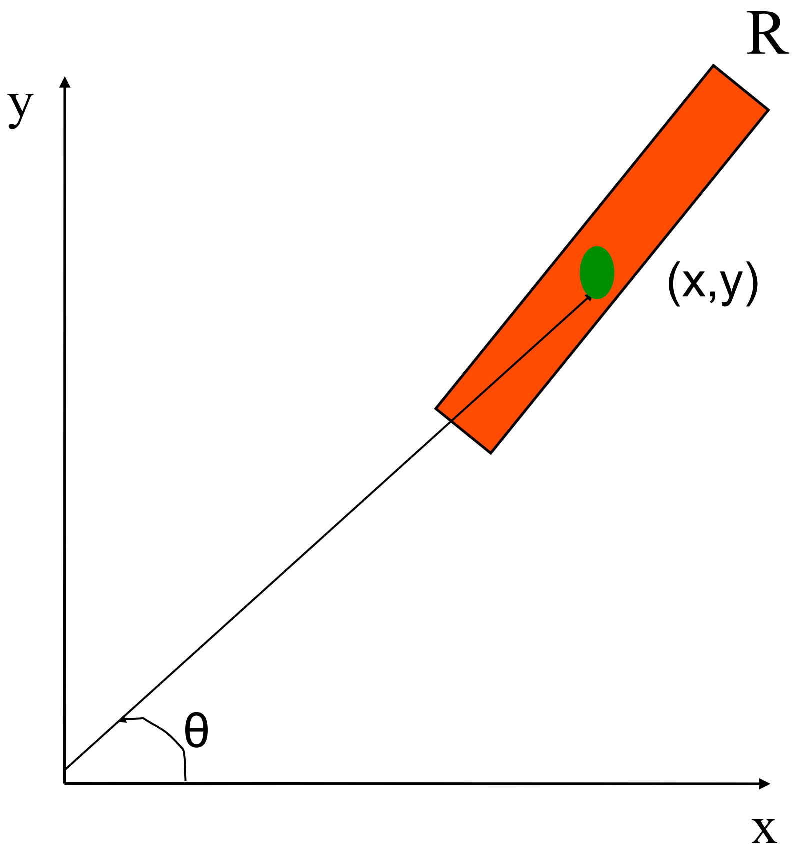
What is a rigid body transformation?

- A transformation ***h*** is a rigid body transformation, if it satisfies the following two conditions:
  1. Distance between points on the object is preserved
  2. Relative orientation does not change, i.e., no mirror images



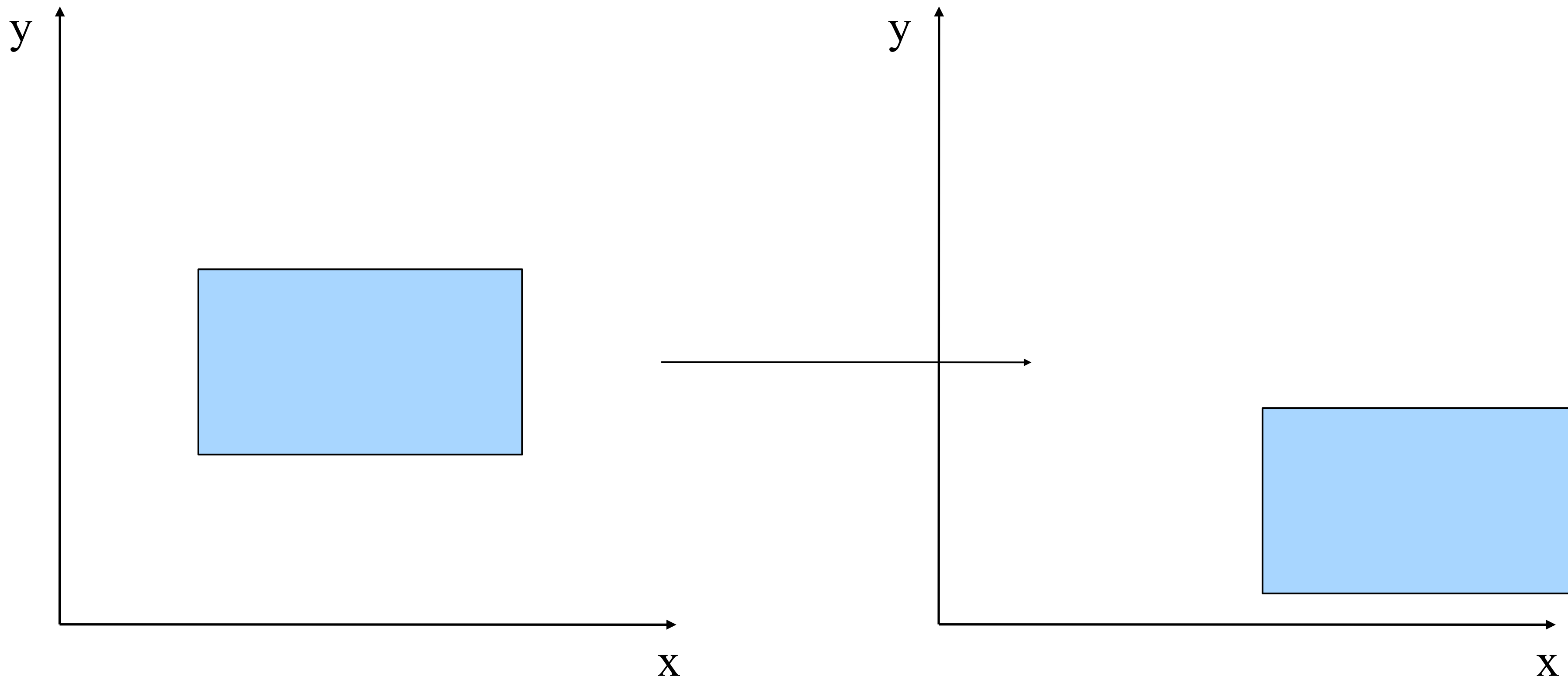
Rigid Body Transformations in 2D

- A rigid body in 2D is described completely by position and orientation of its reference frame **w.r.t a global coordinate frame**
- Two kinds of transformations possible: Translation and Rotation



Rigid Body Translations in 2D

- **Rigid body translation** described by: $$h(x, y) = (x + x_t, y + y_t)$$
- **Boundary representation:** Apply ***h*** to each boundary point



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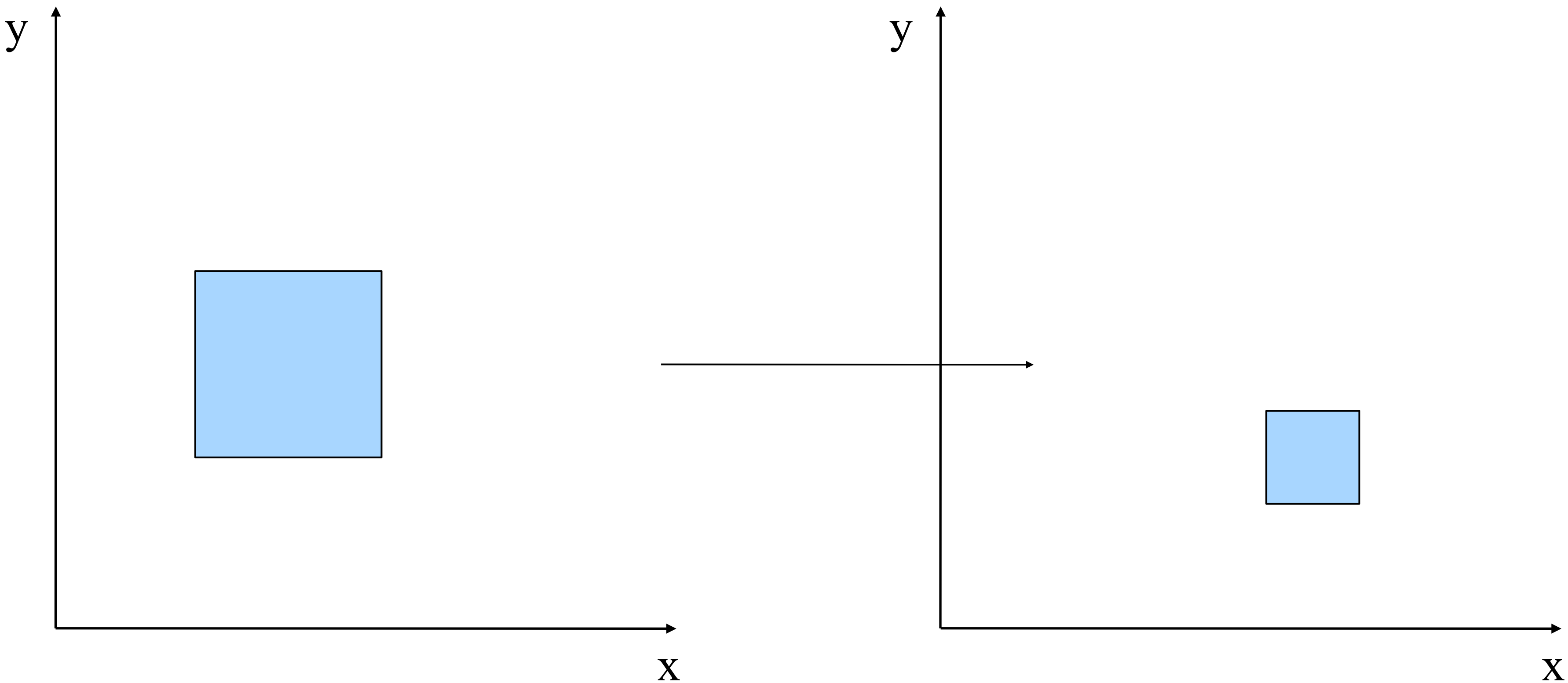
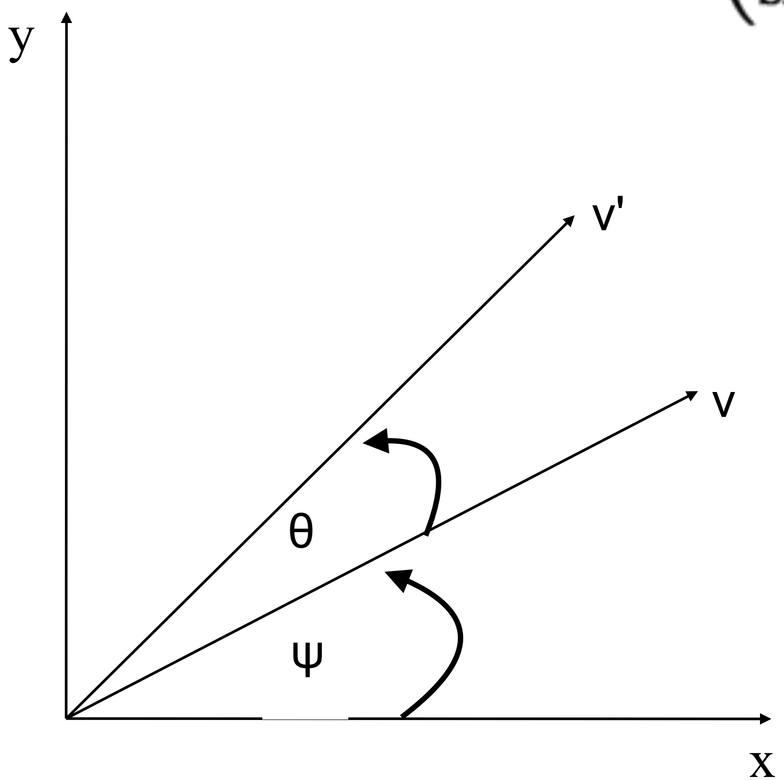


Question: Does the above picture indicate a rigid body translation?

Rigid body Rotations in 2D

- Rotation about origin is given by the transformation:

$$h(\theta) = R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$



$$v = (r\cos(\psi), r\sin(\psi))$$

$$\begin{aligned} v' &= (r\cos(\psi+\theta), r\sin(\psi+\theta)) \\ &= \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\ &= R(\theta)v \end{aligned}$$

- Effect of successive rotations is given by multiplication:

$$v'' = R(\theta_2)v' = R(\theta_2)R(\theta_1)v \quad = R(\theta_2 + \theta_1)v$$

General Rigid Body Motions in 2D

- Given a vector v representing a point on rigid body,

$$(t, R(\theta)) : v \rightarrow R(\theta)v + t$$ **Rotate followed by translate !!**

- Same transformation in homogeneous coordinates:

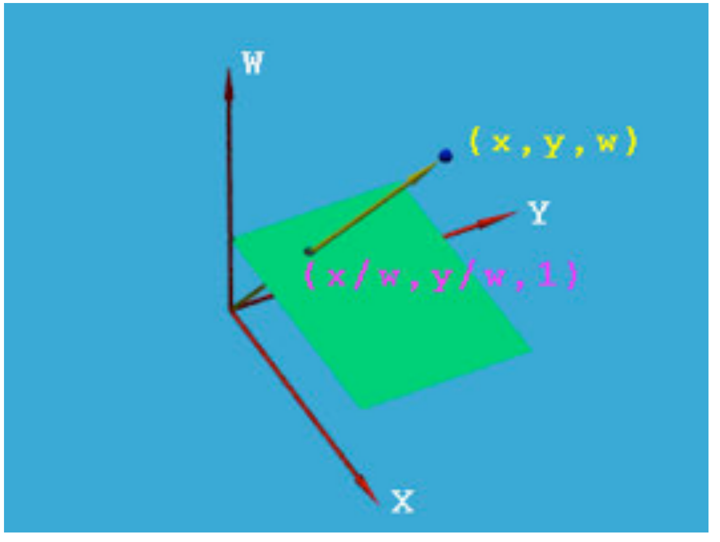
$$\begin{pmatrix} R(\theta) & t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v \\ 1 \end{pmatrix}$$

It is the same thing:

$$\left(\begin{array}{c|c} R(\theta) & t \\ \hline 0 & 1 \end{array}\right) \cdot \begin{pmatrix} v \\ 1 \end{pmatrix} = \begin{pmatrix} R(\theta) \cdot v + t \\ 1 \end{pmatrix}$$

Homogenous Coordinates

- **“∞” cannot be represented in Euclidean coordinate system**
- Lot of geometric concepts greatly simplified if ∞ can be represented
- Advantage that coordinates of points at infinity can also be represented using finite coordinates
- A triplet (x,y,w) represents homogeneous coordinates of a point (x/w,y/w) with $w \neq 0$
- At least one of x,y,w is always nonzero
- (x,y,w) is same as (10x,10y, 10w): multiplying by a non-zero factor does not change the coordinate
- (x,y,0) represents ∞ in the direction of (x,y)



General Rigid Body Motions in 2D

- Given a vector v representing a point on rigid body,

$$(t, R(\theta)) : v \to R(\theta)v + t$$ **Rotate followed by translate !!**

- Same transformation in homogeneous coordinates:

$$\begin{pmatrix} R(\theta) & t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v \\ 1 \end{pmatrix}$$

- Successive transformations can be easily computed in homogeneous coordinates:

$$\text{Rotate by } \theta_1 \to \text{Translate by } t_1 \to \text{Rotate by } \theta_2 \to \text{Translate by } t_2$$

$$= \begin{pmatrix} R(\theta_2) & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R(\theta_1) & t_1 \\ 0 & 1 \end{pmatrix}$$

Question: How to compute rotations about arbitrary point?

Homogeneous coordinates are convenient

Rotate by θ_1
Translate by t_1
Rotate by θ_2
Translate by t_2

Previously: $$R(\theta_2)\Big(R(\theta_1)\,v + t_1\Big) + t_2 =$$
$$R(\theta_2)R(\theta_1)\,v + R(\theta_2)\,t_1 + t_2$$

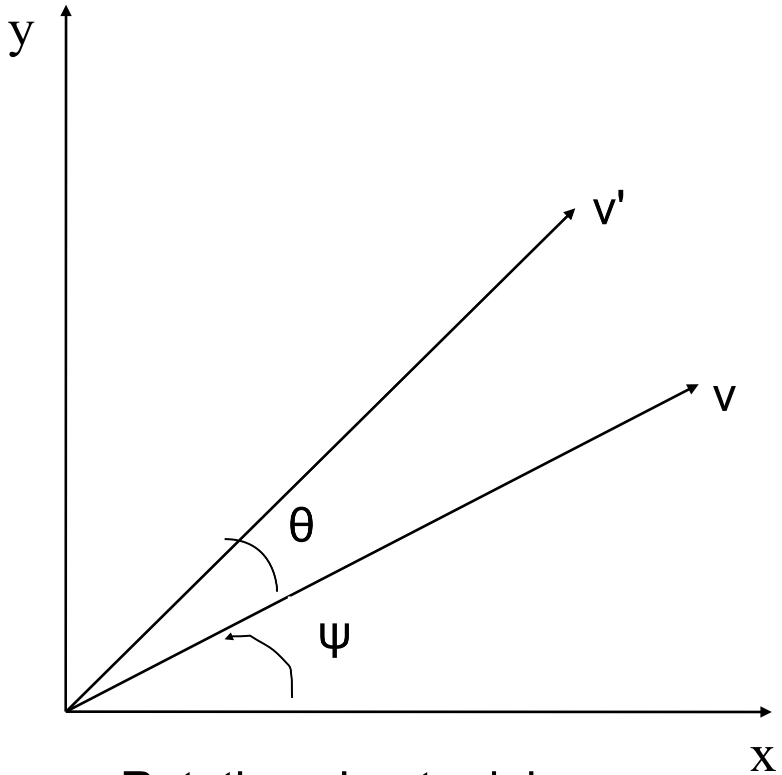
Now:

$$\left(\begin{array}{c:c} R(\theta_2) & t_2 \\ \hdashline 0 & 1 \end{array}\right)\left(\begin{array}{c:c} R(\theta_1) & t_1 \\ \hdashline 0 & 1 \end{array}\right) =$$

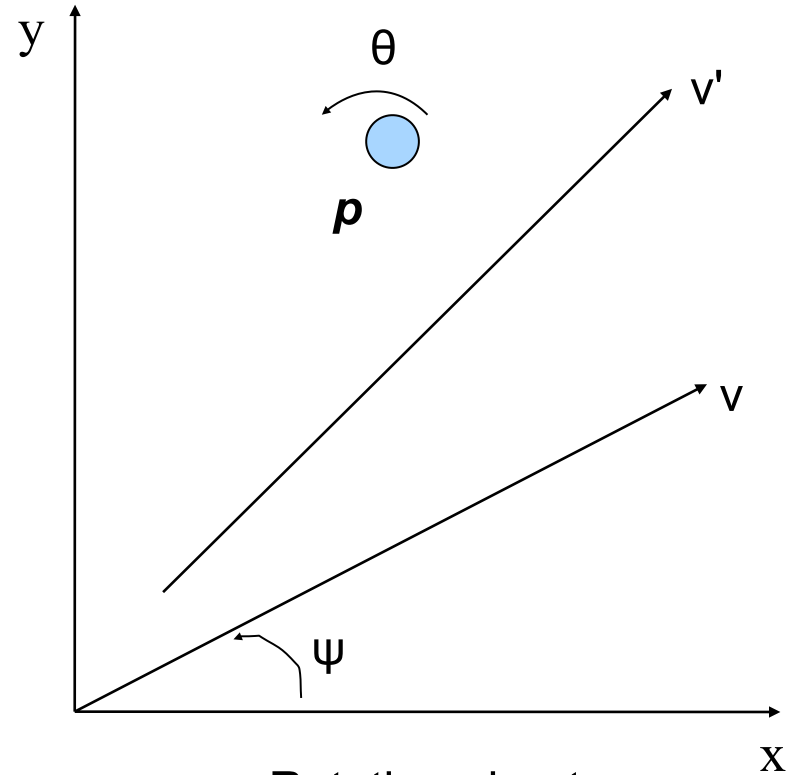
$$\left(\begin{array}{c:c} R(\theta_2)R(\theta_1) & R(\theta_2)\,t_1 + t_2 \\ \hdashline 0 & 1 \end{array}\right)$$

2D Rotations about an arbitrary point

- **Problem:** Find rotation matrix corresponding to rotation about an arbitrary point ***p***
- ***Solution hint:*** Use homogeneous coordinates and framework developed so far.



Rotation about origin



Rotation about ***p***

2D Rotations about an arbitrary point

- **Problem:** Find rotation matrix corresponding to rotation about an arbitrary point ***p***
- ***Solution hint:*** Use homogeneous coordinates and framework developed so far.
- **Solution:** Can be computed by using the following sequence of transformations:

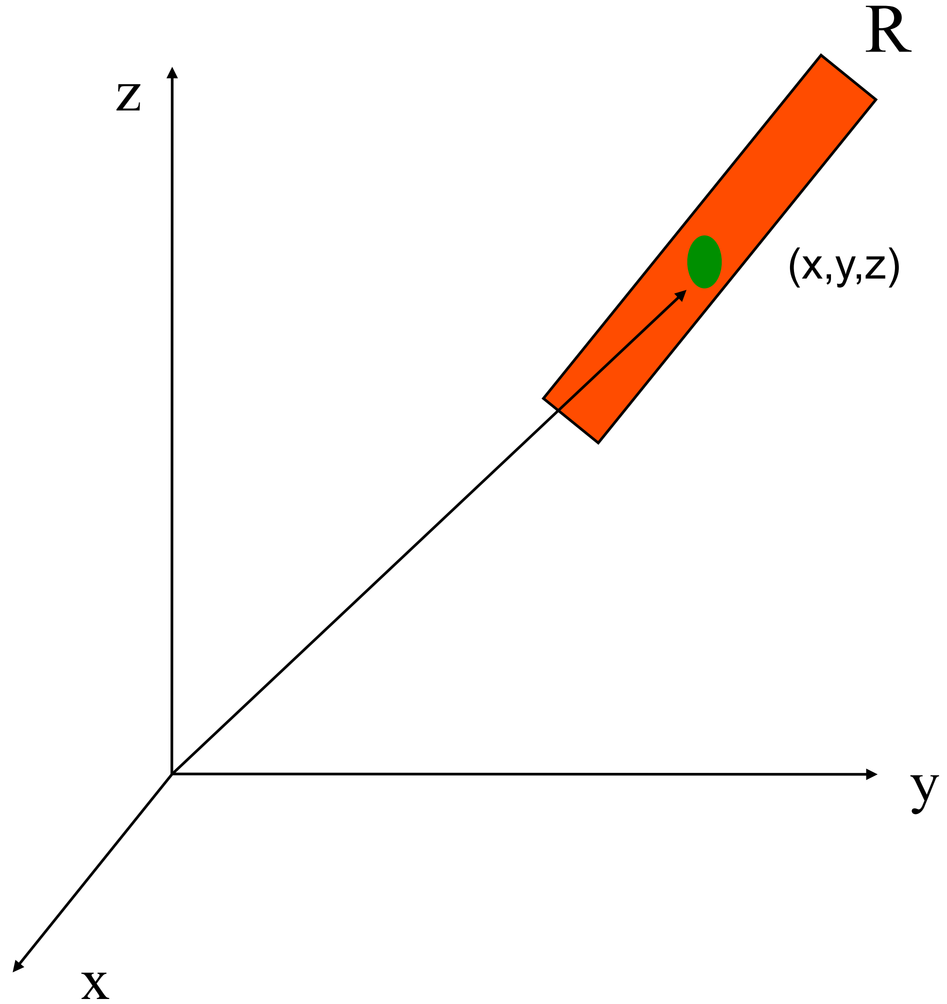
$$\text{Translate by } -p \rightarrow \text{Rotate by } \theta \rightarrow \text{Translate by } p$$

$$= \begin{pmatrix} I & p \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R(\theta) & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} I & -p \\ 0 & 1 \end{pmatrix}$$

Homogenous coordinates simplify things for computations in path planning

Rigid Body Transformations in 3D

- A rigid body in 3D is described completely by position and orientation of its reference frame **w.r.t a global coordinate frame**
- Two kinds of transformations possible: Translation and Rotation

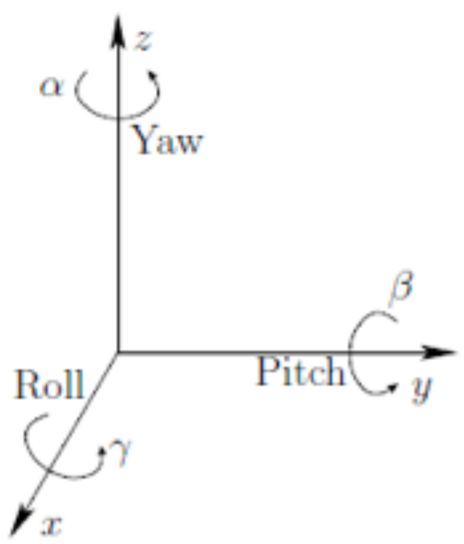


Rigid Body Translations in 3D

- **Rigid body translation** described by: $h(x, y, z) = (x + x_t, y + y_t, z + z_t)$
- **Boundary representation:** Apply ***h*** to each boundary point

Rigid body Rotations in 3D

- Rotation is considered about the origin
- There are 3 axis of rotations: x, y, z
- Each rotation is **counter**clockwise
- A commonly used convention is yaw-pitch-roll referring to rotations about z, y, x axis respectively.



$$R_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

yaw

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$$

pitch

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}$$

roll

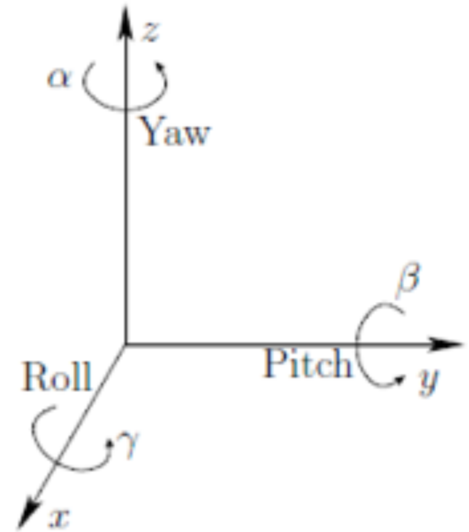
Successive Rotations in 3D

- Yaw, Pitch, Roll can be used to place 3D body in any orientation
- Successive rotations applied the same way as in 2D
- Order of rotations is important

Rotate by γ about x axis: $\boldsymbol{R_x(\gamma)}$

Rotate by β about y axis: $\boldsymbol{R_y(\beta)}$

Rotate by α about z axis: $\boldsymbol{R_z(\alpha)}$



$$R(\alpha,\beta,\gamma) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) =$$

$$\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}$$

(Note)

"Euler angles"

Euler's rotation theorem states that any rotation can be represented by no more than three rotations about coordinate axes, - no two successive rotations are about the same axis

Hence we can define

XYX, XZX, YXY, YZY, ZXZ, ZYZ

XYZ, XZY, YZX, YZX, ZYX, ZYX

typically referred as the yaw,pitch,roll

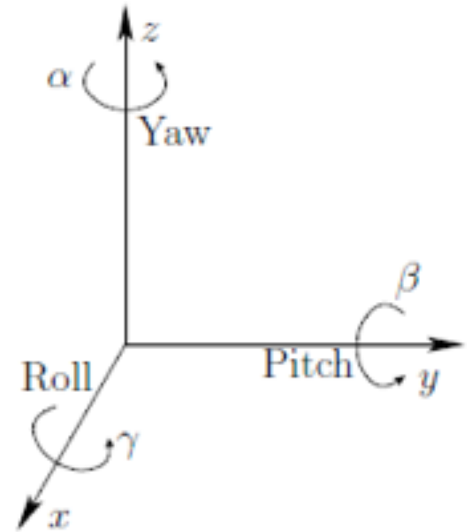
Successive Rotations in 3D

- Roll, Pitch, Yaw can be used to place 3D body in any orientation
- Successive rotations applied the same way as in 2D
- Order of rotations is important

Rotate by γ about x axis: $\boldsymbol{R_x(\gamma)}$

Rotate by β about y axis: $\boldsymbol{R_y(\beta)}$

Rotate by α about z axis: $\boldsymbol{R_z(\alpha)}$

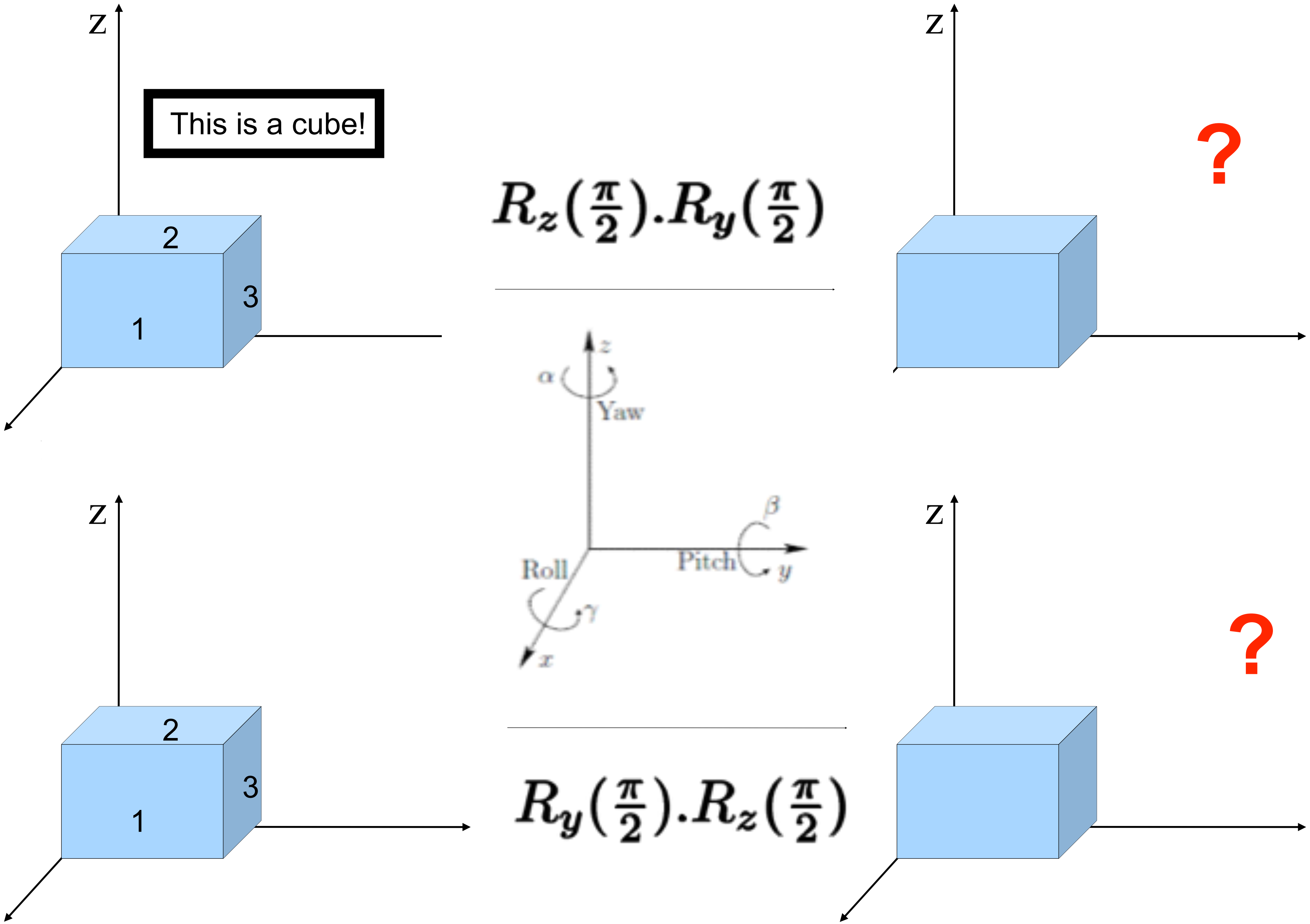


$$R(\alpha,\beta,\gamma) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) =$$

$$\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}$$

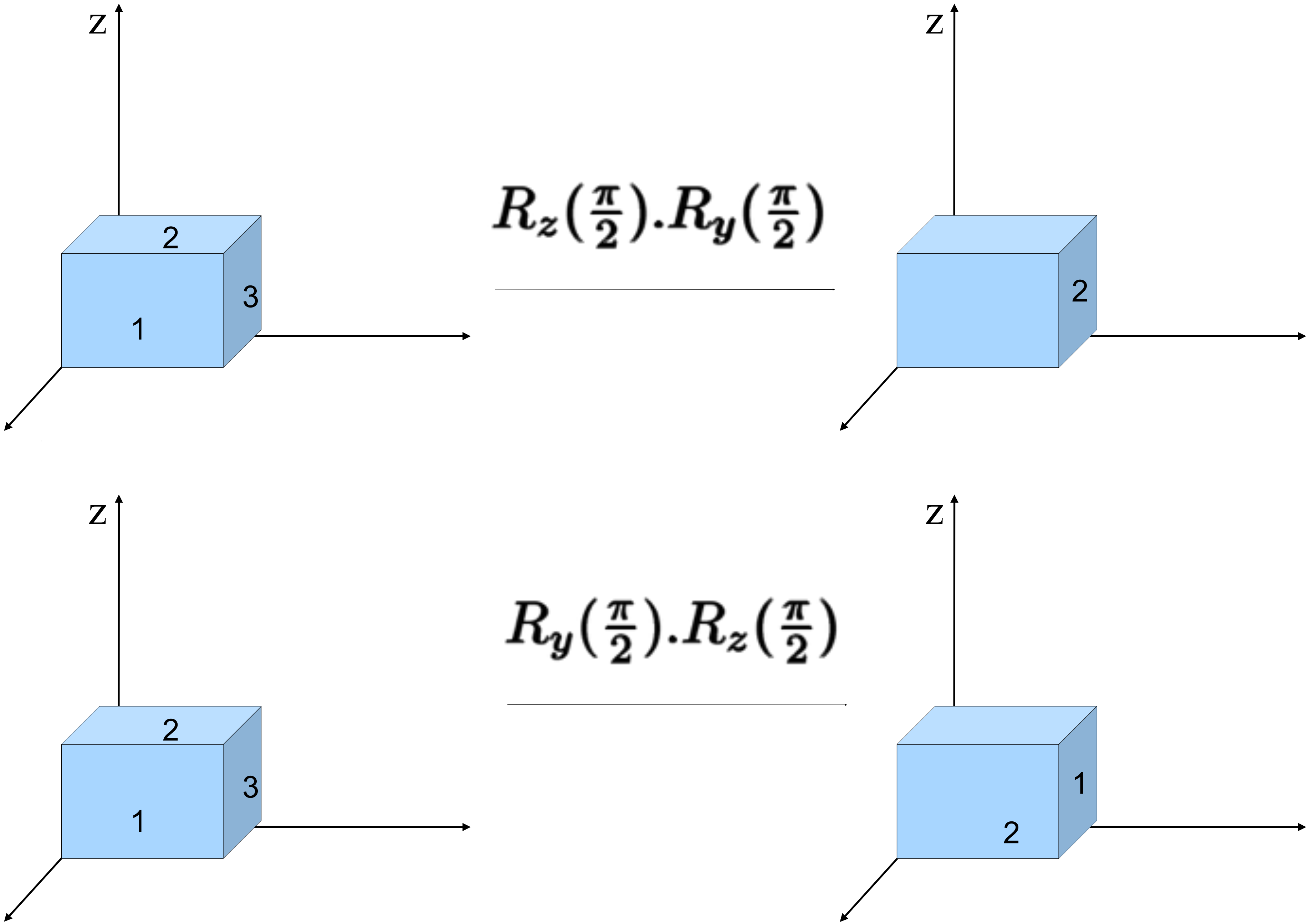
Question: Do rotations in 3D commute ?

Do 3D rotations commute?



Try to estimate which side will be where side 3 is now

Do 3D rotations commute?



3D rotations do not commute while 2D rotations do commute !

General Rigid Body Motions in 3D

- Given a vector v representing a point on rigid body,

$$(R(\alpha, \beta, \gamma), t) : v \rightarrow R(\alpha, \beta, \gamma)v + t$$

- Same transformation in homogeneous coordinates:

$$\begin{pmatrix} R(\alpha, \beta, \gamma) & t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v \\ 1 \end{pmatrix}$$

- Successive transformations can be easily computed in homogeneous coordinates (yaw,pitch,roll):

$$\text{Rotate by } (\gamma_1 \rightarrow \beta_1 \rightarrow \alpha_1) \rightarrow \text{Translate by } t_1$$

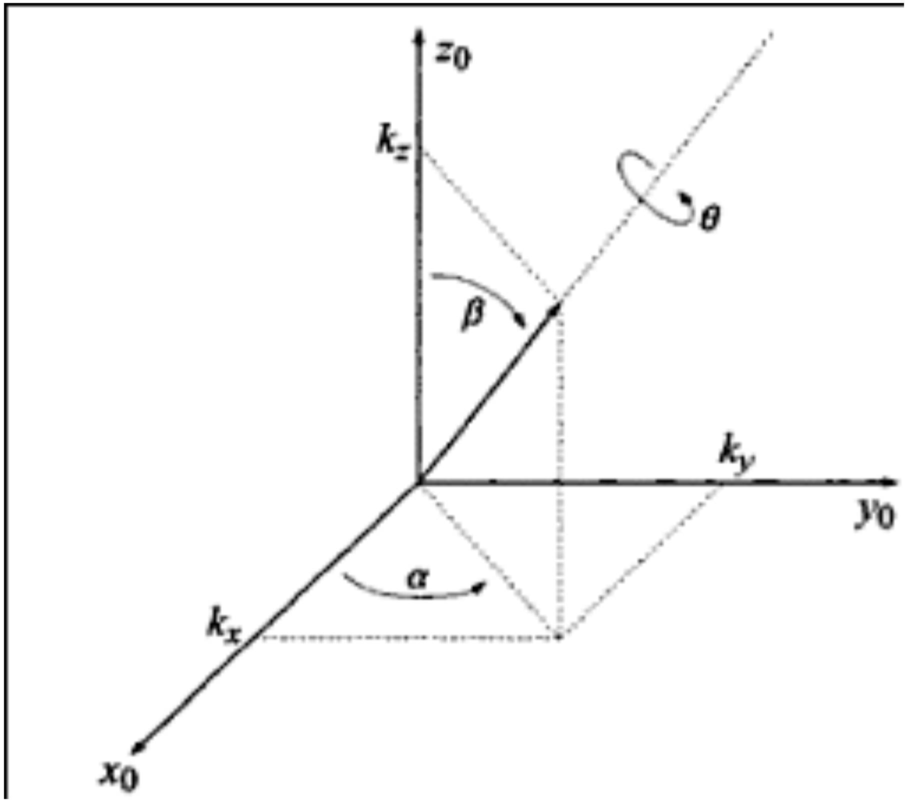
$$\text{Rotate by } (\gamma_2 \rightarrow \beta_2 \rightarrow \alpha_2) \rightarrow \text{Translate by } t_2$$

$$= \begin{pmatrix} R(\alpha_2 \beta_2, \gamma_2) & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R(\alpha_1, \beta_1, \gamma_1) & t_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} v \\ 1 \end{pmatrix}$$

Question: How to compute rotations about arbitrary axis?

3D Rotations about an arbitrary axis

- **Problem:** Find rotation matrix corresponding to rotation about an arbitrary axis ***k*** ***(also called Axis-Angle Parametrization)***
- ***Solution: Transform to the (x,y,z) coordinate axis, apply the rotation, and then reverse the transformations.***



$$\text{Unit vector along rotation axis} = (k_x, k_y, k_z)$$
$$R_k(\theta) = R_z(\alpha)R_y(\beta)R_z(\theta)R_y(-\beta)R_z(-\alpha)$$

Properties of Rotation Matrices

- Columns of ***R*** are mutually orthonormal: $$r_i^T r_j = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } i \neq j. \end{cases}$$

$$\Rightarrow R^T R = R R^T = I \Rightarrow \det(R) = \pm 1$$

- We are using Right-handed coordinate system: $\det(R) = +1$

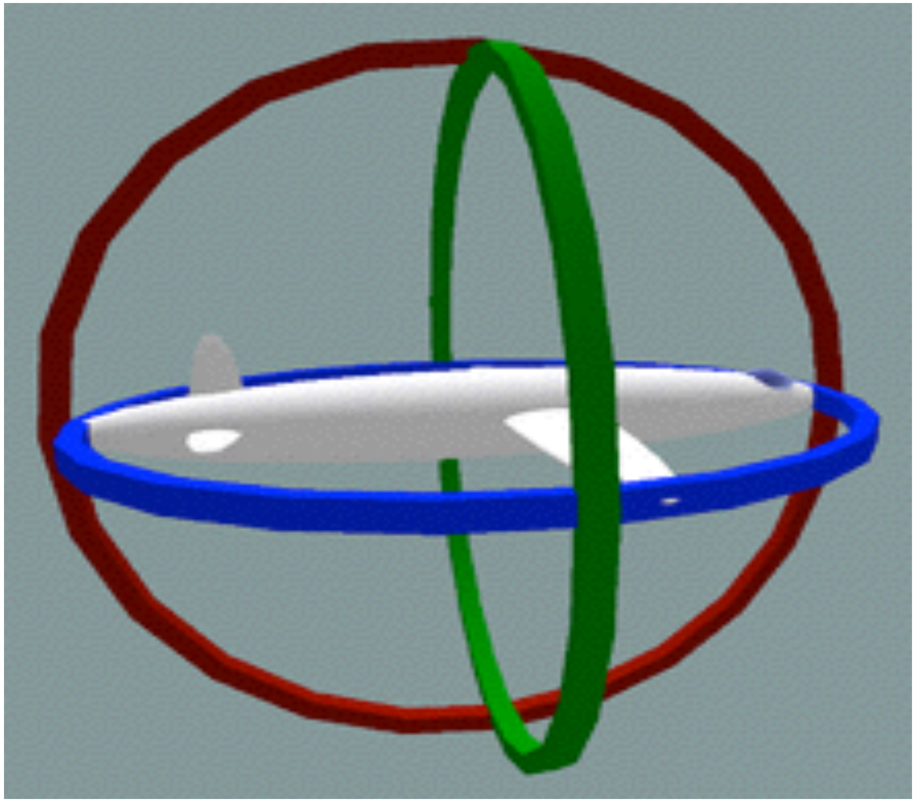
- **Special Orthogonal Matrices:**

$$SO(n) = \{R \in \mathbb{R}^n \times \mathbb{R}^n : RR^T = I, \det(R) = 1\}$$

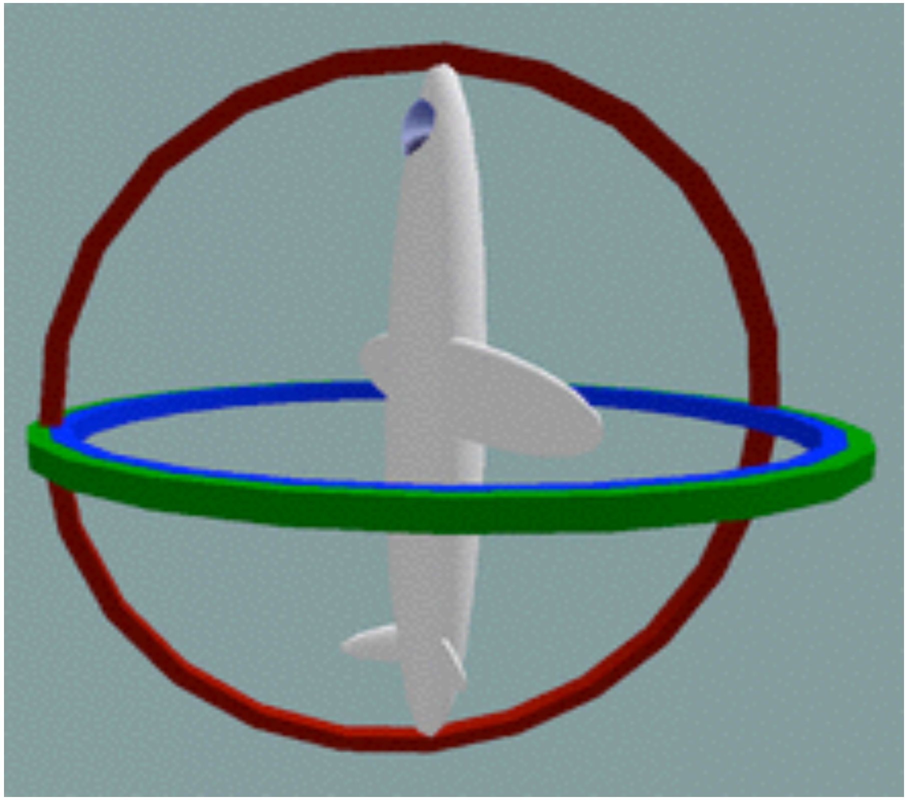
Gimbal Lock Problem

Gimbal Lock Problem

- **Gimbal Lock:** Problem of loss of a degree of freedom in 3D space
- Can occur when using rotation matrices for rotations
- Terminology based on problem arising in aircraft navigation



Normal situation



Gimbal Lock: Two gimbals are in same plane

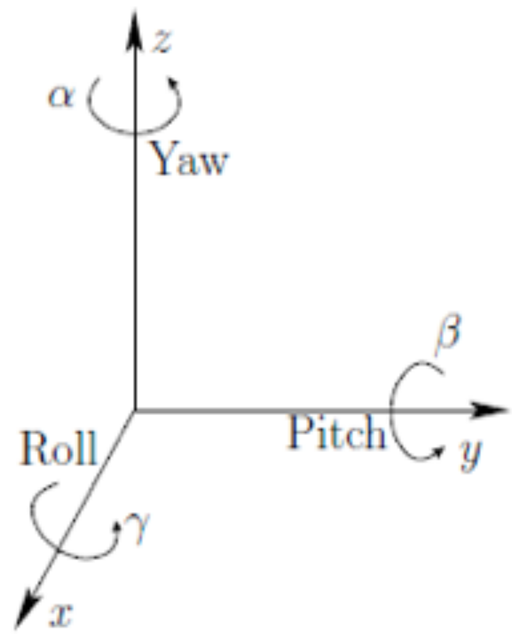
Gimbal Lock

Gimbal locked airplane: When the pitch (green) and yaw (magenta) gimbals become aligned, changes to roll (blue) and yaw (magenta) apply the same rotation to the airplane (from wikipedia).

Check: http://www.youtube.com/watch?v=zc8b2Jo7mno

https://www.youtube.com/watch?v=OmCzZ-D8Wdk
https://www.youtube.com/watch?v=N5PDboNJwks

Quaternions



Roll,Pitch, Yaw
or other combinations have the same problem

- **Quaternions:** An alternate approach using 4 parameters
- Quaternions are a generalization of complex numbers to a 4D space

Quaternion Multiplication

Definition

- $q = a + b\,i + c\,j + dk$
- Sum of a scalar and a vector

Multiplication (Basis Vectors)

- $i^2 = j^2 = k^2 = -1$
- $i\,j = k \qquad j\,k = i \qquad k\,i = j$
- $j\,i = -k \qquad k\,j = -i \qquad i\,k = -j$

Multiplication (Arbitrary Quaternions)

- $(a + \mathbf{v})(c + \mathbf{w}) = (a\,c - \mathbf{v} \cdot \mathbf{w}) + (c\,\mathbf{v} + a\,\mathbf{w} + \mathbf{v} \times \mathbf{w})$

Properties of Quaternion Multiplication

- Associative
- Not Commutative
- Distributes Through Addition

[From Ron Goldman]

Rotations with Quaternions

Conjugate

- $q = a + bi + cj + dk$
- $q^{*} = a - bi - cj - dk$

Unit Quaternion

- $q(N, \theta) = \cos(\theta) + \sin(\theta) N$
- $N =$ Unit Vector

Rotation of Vectors from Quaternion Multiplication (Sandwiching)

- $S_{q(N,\theta/2)}(v) = q(N, \theta/2)\, v\, q^{*}(N, \theta/2)$

  -- θ = Angle of Rotation

  -- N = Axis of Rotation

[From Ron Goldman]

Applications of Quaternions in Robotics

Compact Representation for Rotations of Vectors in 3-Dimensions

- 3×3 Matrices -- 9 Entries
- Unit Quaternions -- 4 Coefficients

Avoids Distortions

- After several matrix multiplications, rotation matrices may no longer be orthogonal due to floating point inaccuracies.
- Non-Orthogonal matrices are difficult to renormalize -- leads to distortions.
- Quaternions are easily renormalized -- avoids distortions.

Interpolation Between Rotations

- Linear Interpolation between two rotation matrices R_1 and R_2 fails to generate another rotation matrix.

  $Lerp(R_1, R_2, t) = (1-t)R_1 + tR_2$ -- not necessarily orthogonal matrices.

- Spherical Linear Interpolation between two unit quaternions always generates a unit quaternion.

  $Slerp(q_1, q_2, t) = \dfrac{\sin((1-t)\phi)}{\sin(\phi)} q_1 + \dfrac{\sin(t\phi)}{\sin(\phi)} q_2$ -- always a unit quaternion.

[From Ron Goldman]

Summary of All the Formulas

Multiplication (Basis Vectors)

- $i^2 = j^2 = k^2 = -1$
- $i\,j = k \qquad j\,k = i \qquad k\,i = j \qquad j\,i = -k \qquad k\,j = -i \qquad i\,k = -j$

Multiplication (Arbitrary Quaternions)

- $(a + \mathbf{v})(c + \mathbf{w}) = (a\,c - \mathbf{v} \cdot \mathbf{w}) + (c\,\mathbf{v} + a\,\mathbf{w} + \mathbf{v} \times \mathbf{w})$

Rotation of Vectors by Angle θ Around Axis N (Sandwiching Formula)

- $S_{q(N,\theta/2)}(v) = q(N,\theta/2)\, v\, q^*(N,\theta/2)$

  -- $q(N,\theta/2) = \cos(\theta/2) + \sin(\theta/2)N \qquad q^*(N,\theta/2) = \cos(\theta/2) - \sin(\theta/2)N$

Interpolation Between Rotations (SLERP)

- $Slerp(q_1, q_2, t) = \dfrac{\sin((1-t)\phi)}{\sin(\phi)} q_1 + \dfrac{\sin(t\phi)}{\sin(\phi)} q_2$

  -- $\phi =$ Angle Between q_1 and q_2

[From Ron Goldman]

Advantages of Quaternion Approach

- Quaternion approach does not suffer from gimbal lock problem
- Concatenating rotations is computationally faster and numerically more stable
- Extracting the angle and axis of rotation is simpler
- Interpolation is more straightforward

Storage requirements

Method	Storage
Rotation matrix	9
Quaternion	4
Angle/axis	3*

Performance comparison of rotation chaining operations

Method	# multiplies	# add/subtracts	total operations
Rotation matrices	27	18	45
Quaternions	16	12	28

Performance comparison of vector rotating operations

Method	# multiplies	# add/subtracts	# sin/cos	total operations
Rotation matrix	9	6	0	15
Quaternions	21	18	0	39
Angle/axis	23	16	2	41

(Source: Wikipedia)

A disadvantage: Not intuitive, cannot visualize. Need to convert to rotation matrix to visualize

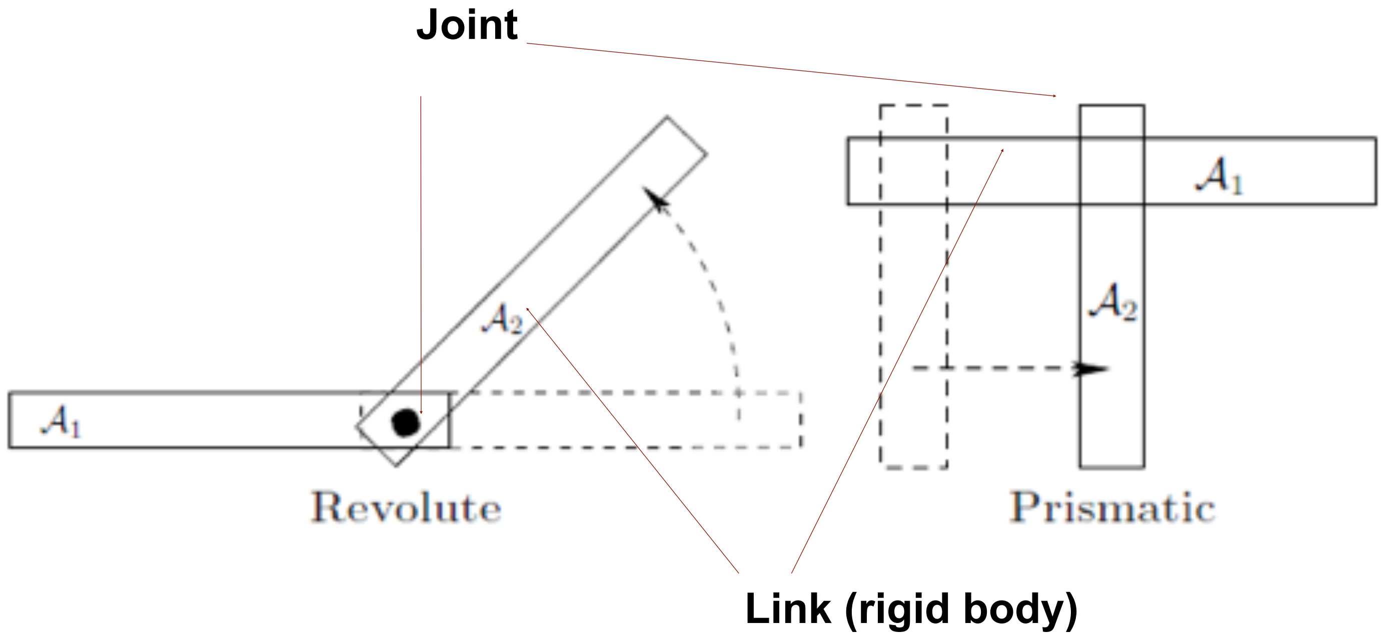
Advantages of Quaternion Approach

- Avoids distortions
  - After several matrix multiplications, rotation matrices may no longer be orthogonal due to floating point inaccuracies.
  - Non-orthogonal matrices are difficult to renormalize -leads to distortions.

- Interpolation (that is motion....)
  - Linear interpolation between two rotation matrices R_1 and R_2 fails to generate another rotation matrix.
  - Spherical linear interpolation between two unit quaternions always represents a unit quaternion - will come back to this point.

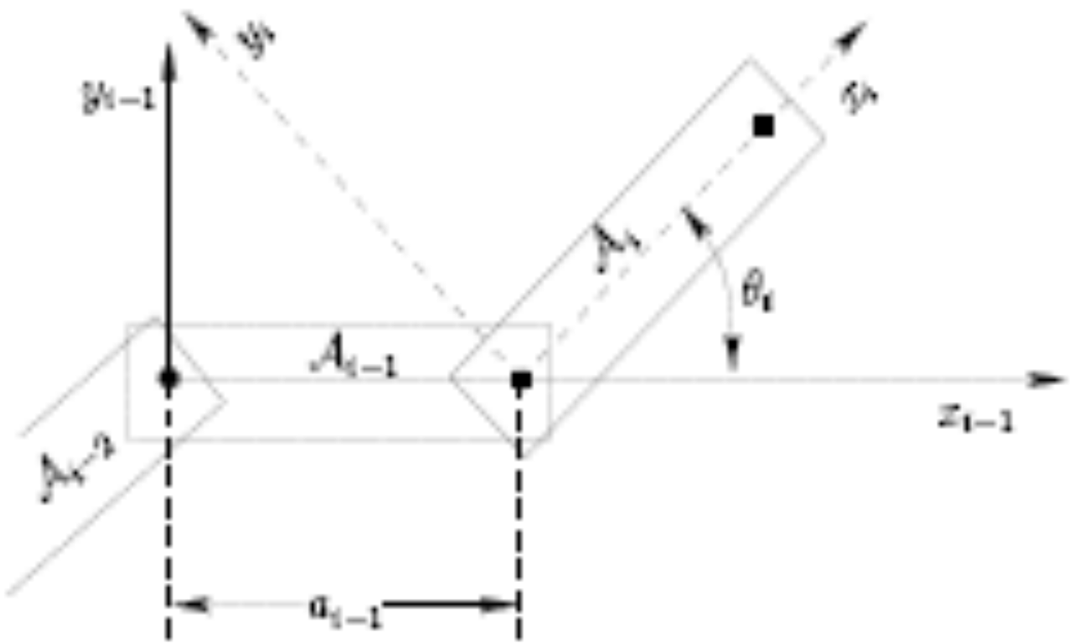
Application: Kinematic Chains of Bodies

- **Kinematics:** Study of possible movements and configurations of a system ``geometry of the system"
- **Link:** Each rigid body in a chain of rigid bodies
- **Joint:** Connects two rigid bodies and enforces constraints
- **Forward kinematics:** Position of the end effector in terms of joint angles
- **Inverse kinematics:** Joint angles in terms of the position of the end effector

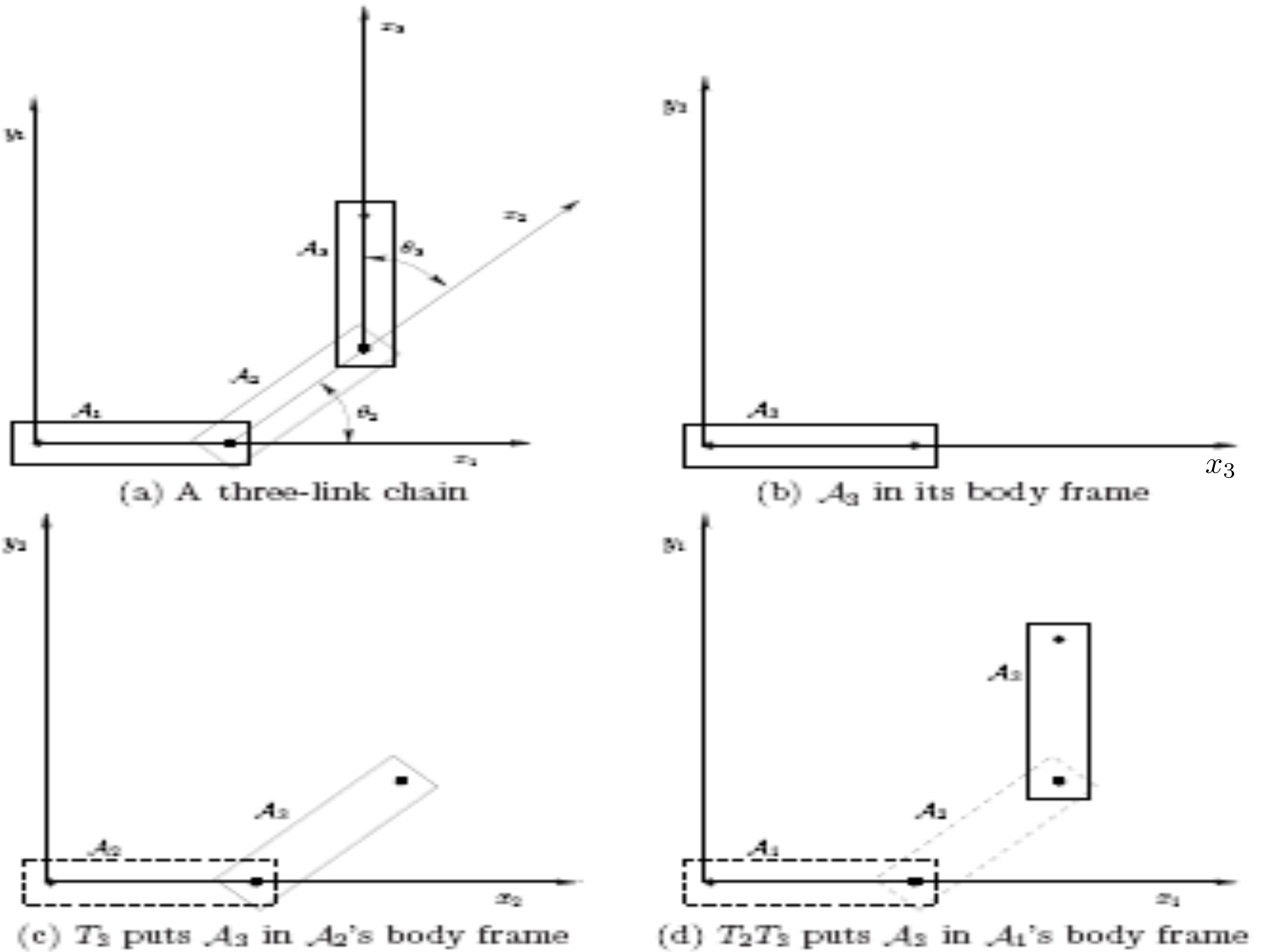


Transforming Kinematic Chains of Bodies

- The coordinates of points of interest are usually expressed in **"local coordinate frames"**
- Specification in terms of **"local coordinate frames"** greatly simplifies lot of computations



Homogenous Transformations for 2D Chains



(a) A three-link chain

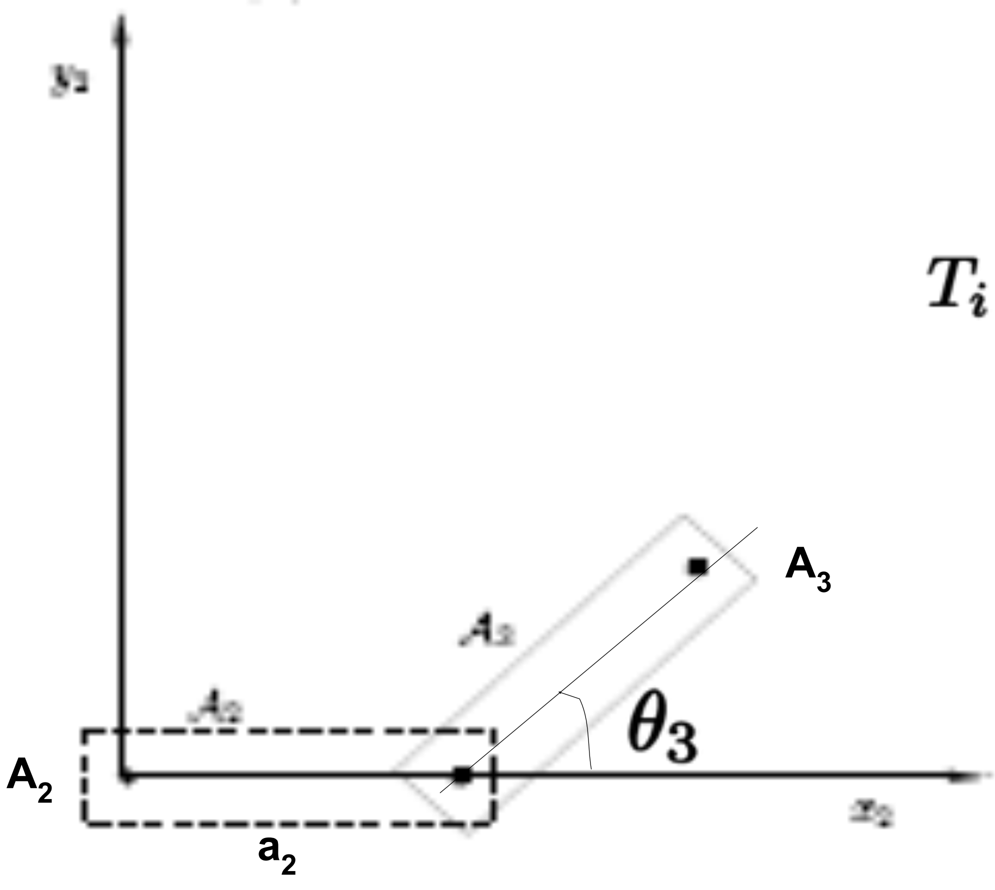
(b) $\mathcal{A}_3$ in its body frame

(c) T_3 puts $\mathcal{A}_3$ in $\mathcal{A}_2$'s body frame

(d) T_2T_3 puts $\mathcal{A}_3$ in $\mathcal{A}_1$'s body frame

Homogenous Transformations for 2D Chains

- $\mathbf{T_i}$ is the transformation matrix for link $\mathbf{A_i}$
- Application of $\mathbf{T_i}$ moves $\mathbf{A_i}$ from its body frame to body frame of $\mathbf{A_{i-1}}$



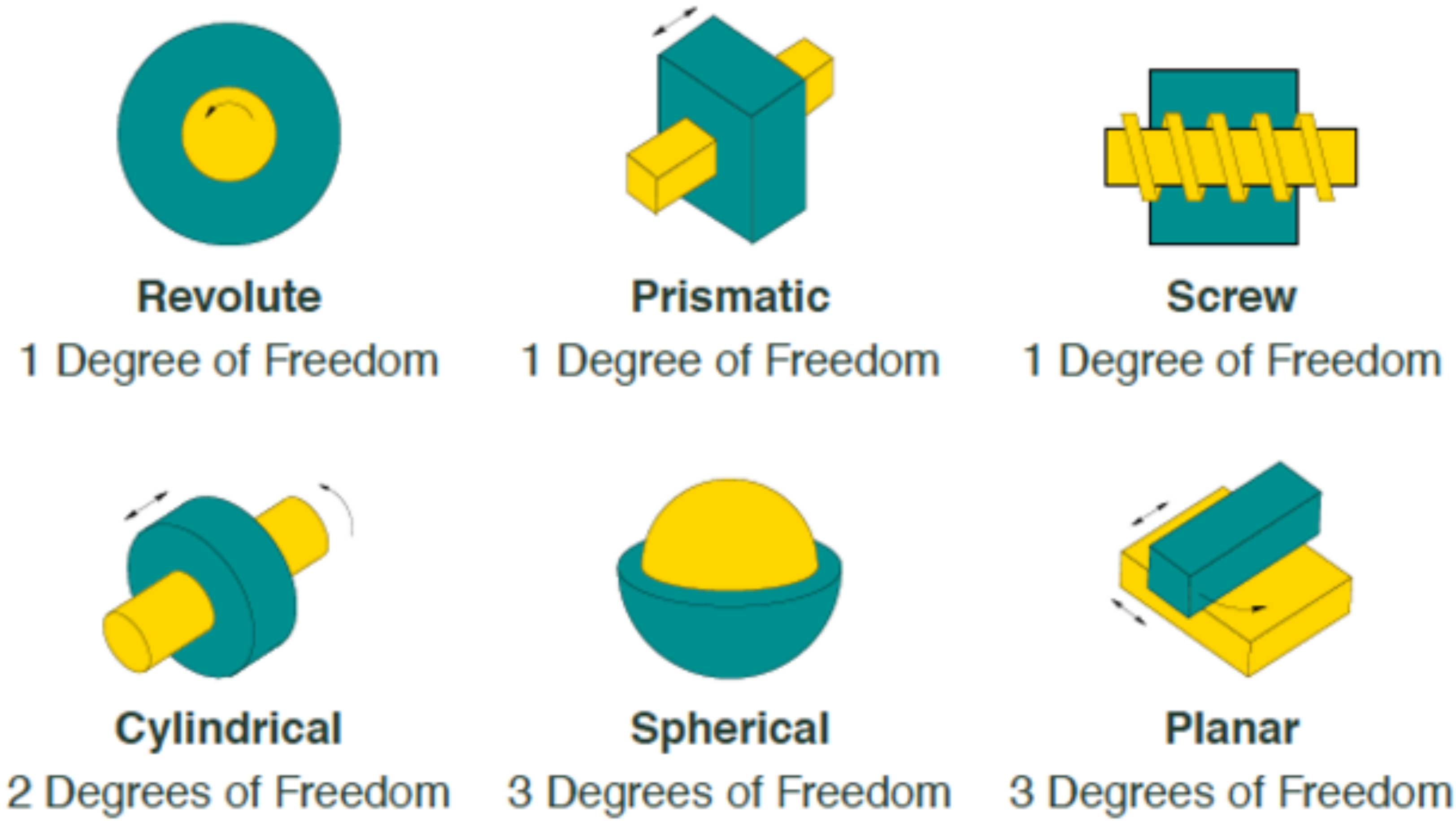
Distance between joints in link $\mathbf{A_{i-1}}$

$$T_i = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) & a_{i-1} \\ \sin(\theta_i) & \cos(\theta_i) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Location of point } (x, y) \in \mathcal{A}_m = T_1 T_2 \ldots T_m \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Multiple Bodies in 3D

In three dimensions, bodies may be non-rigidly attached in many ways:



Nevertheless, systems of parametrizations are developed:

Denavit-Hartenburg, Khalil-Kleinfinger, ...

References - 2D Transformations

- The material covered in this class is based on Section 3.1, 3.2.1 3.2.2, from the “Planning Algorithms” book.
- In case of any discrepancy in formulae and equations used, please let the instructor know of the same. Please consider the textbook version of those to be correct.

References - 3D Transformations

- The material covered in this class is based on Section 3.2.3, 3.3.1 and Section 4.2.2 from the “Planning Algorithms” book. You can also skim 3.2.2
- Some of the related material is also available in Appendix E of the class textbook “Principles of Robot Motion”.
- In case of any discrepancy in formulae and equations used, please let the instructor know of the same. Please consider the textbook version of those to be correct.

End of Tranformations