

LINEARIZATION - A SUPPLEMENT

Consider

$$\dot{x} = f(x(t), \tilde{u}(t), t)$$

$$\text{where } x \in \mathbb{R}^n \\ \tilde{u} \in \mathbb{R}^p$$

① Equilibrium

$$\dot{x} = 0 \text{ or } f(x(t), \tilde{u}(t), t) = 0$$

Suppose solution is equilibrium $\begin{cases} x(t) = \bar{x} = \text{constant} \\ \tilde{u}(t) = \bar{u} = \text{constant} \end{cases}$

② Taylor Series Expansion:

$$\dot{x} = f(\bar{x}, \bar{u}, t) + \underbrace{\left. \frac{\partial f}{\partial x} \right|_{\substack{x=\bar{x} \\ u=\bar{u}}}}_A (x - \bar{x}) + \underbrace{\left. \frac{\partial f}{\partial u} \right|_{\substack{x=\bar{x} \\ u=\bar{u}}}}_B (\tilde{u} - \bar{u}) + \text{HOT}$$

$\nearrow u$

③ Jacobian Linearization

$$\dot{y} = A y + B u$$