10.13

Note $\omega_c = \omega_g$

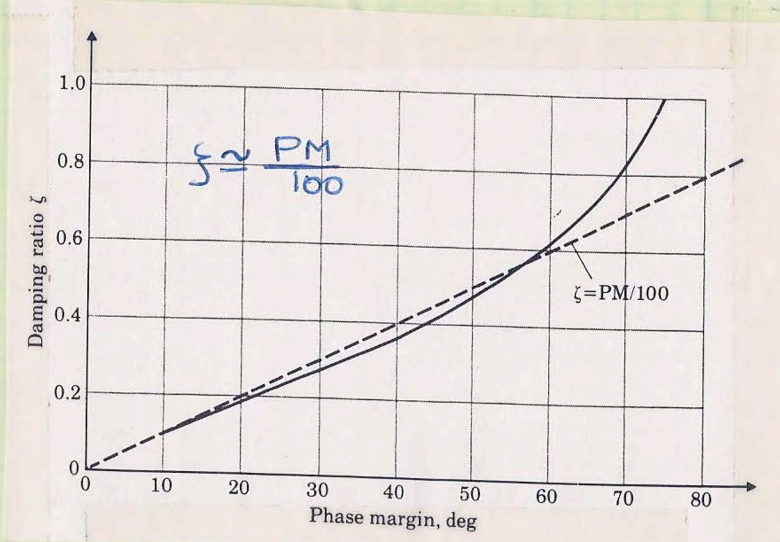
RULES OF THUMB

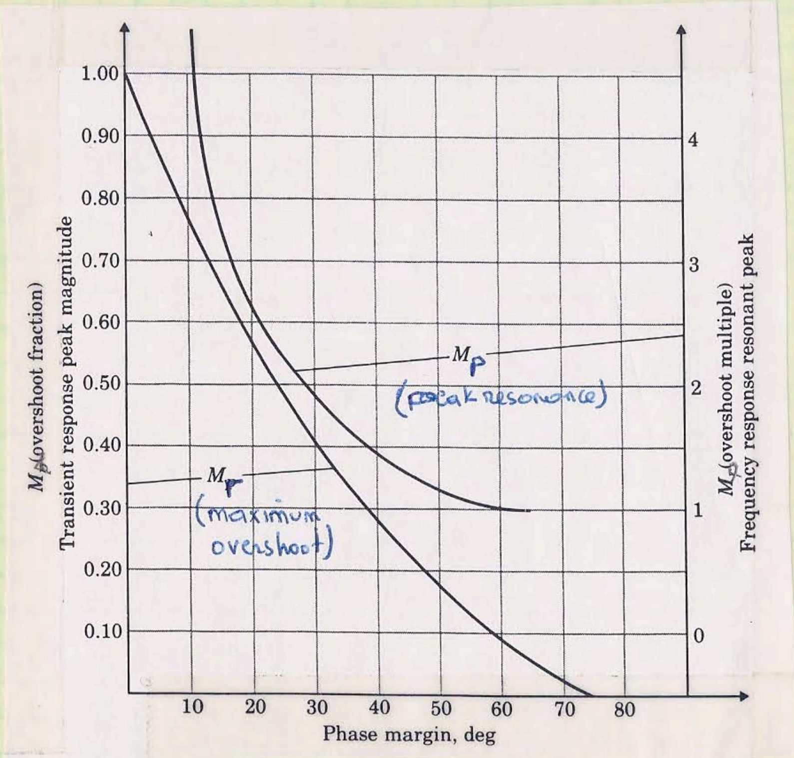
$\boxed{\text{PM and Closed loop system specs}}$

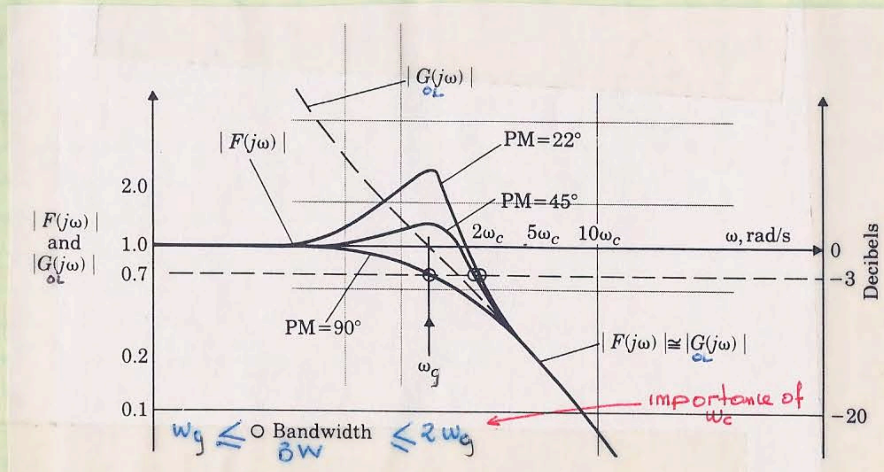
$$u \to \bigcirc \to \boxed{G_{OL}(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}} \to y$$

PM: phase margin from Bode plot of $G_{OL}(s)$

$$G_{CL}(s) = F(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$







Bode's Gain-Phase Relationship

A minimum-phase transfer function does not have poles or zeros in the right half s-plane or on the $j\omega$ axis excluding the origin

One of Bode's important contributions is his theorem that states:

"For any minimum phase system, the phase of $G(j\omega)$ is uniquely related to the magnitude of $G(j\omega)$"

Approximation $\angle G(j\omega) \simeq n \times 90^\circ$

where n is the slope of $|G(j\omega)|$ in units of $\frac{\text{decade of amplitude}}{\text{per decade of frequency}}$

so a slope of $-20\,dB \Rightarrow n=-1$

$-40\,dB \Rightarrow n=-2$

$40\,dB \Rightarrow n=2$

$20\,dB \Rightarrow n=1$

Consequences:

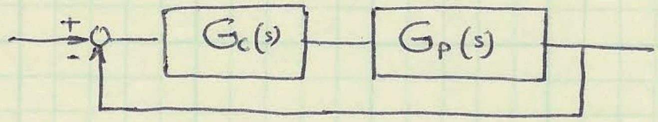
A compensator which results in a magnitude plot of $-20\,dB$ slope @ crossover frequency is better than a compensator which results in a magnitude plot of $-40\,dB$

why?

A: slope of $-20\,dB \Rightarrow PM = +90$ $(\angle G(j\omega) \simeq -90)$

$-40\,dB \Rightarrow PM = 0$!

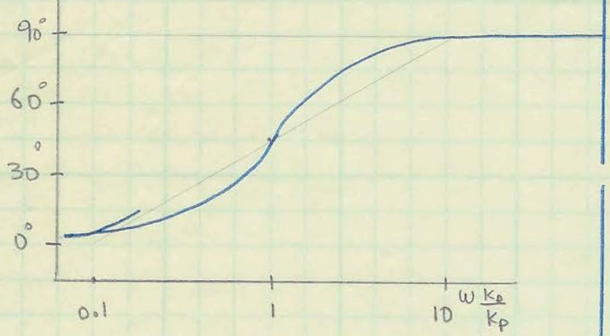
Compensation



positive phase is introduced to the system over some frequency range

PD: $G_c(s) = K_P + K_D s$

$= K_P\left[1 + \frac{K_D}{K_P} s\right]$



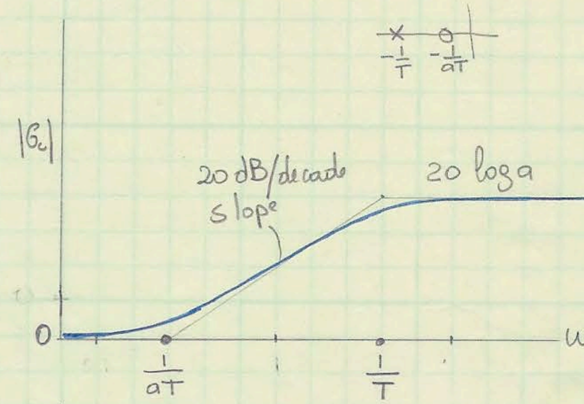
(+): stabilizing effect: ↑ phase

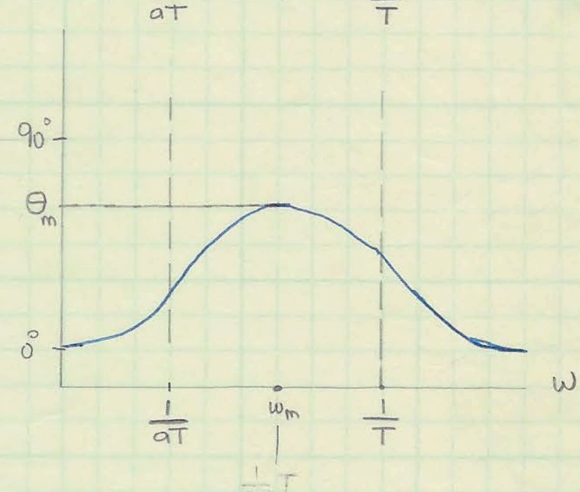
choose (place) $\omega = \frac{K_P}{K_D}$ so that increased phase occurs @ crossover $|G_c(s)G(s)| = 1$

(−): magnitude continues to grow @ high frequencies

Amplification of high frequency noise

Lead Compensation $G_c(s) = \frac{1+aTs}{1+Ts}$ $a>1$





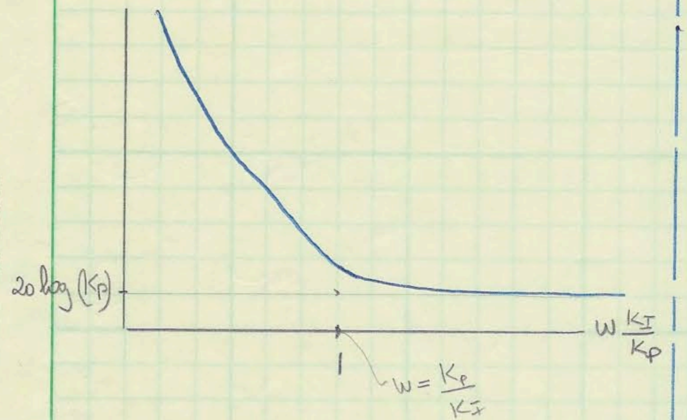
$\omega_m = \frac{1}{\sqrt{a}}T$; $\sin\Theta_m = \frac{a-1}{a+1}$ or $a = \frac{1+\sin\Theta_m}{1-\sin\Theta_m}$

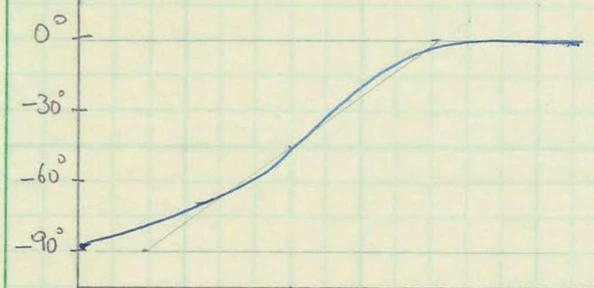
(see textbook (chap 11) for details of choice of a & T)

Objective: increase PM

negative ~~frequency~~ phase is introduced over some frequency range

PI $G_c(s) = K_p + \frac{K_I}{s} = \frac{K_I}{s}\left(1 + \frac{K_P}{K_I}s\right)$





(+) infinite gain at zero frequency

$\Rightarrow e_{ss} \to$ very small ($\simeq 0$)

(−) phase decrease below $\omega = \frac{K_P}{K_I}$

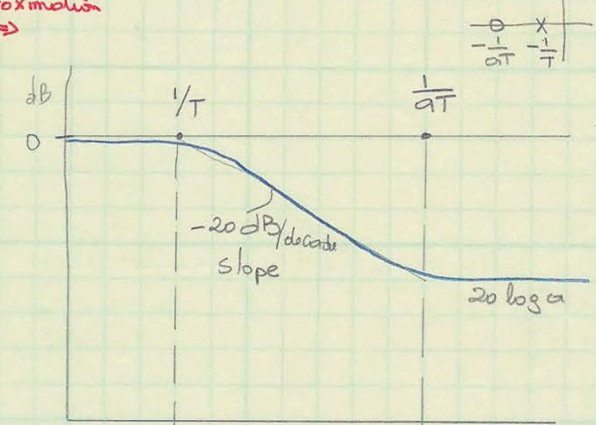
$\Rightarrow \omega = \frac{K_P}{K_I}$ placed substantially below crossover so that phase margin will not be affected very much.

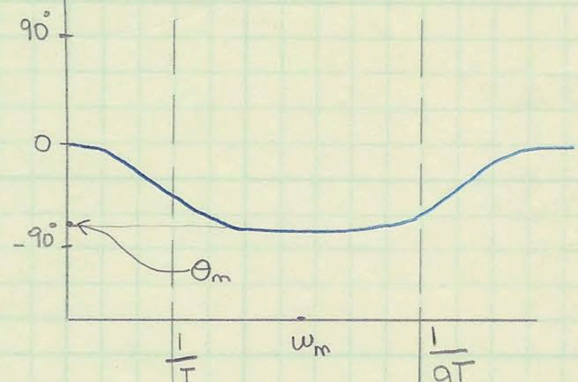
Ex: $G_{OL}(s) = \frac{1}{\cancel{s}(s+1)} \rightarrow e_{ss}' = \lim_{s\to 0} \cancel{s}\left[\frac{1}{1+G_c(s)G_{OL}}\right]\cancel{\frac{1}{s}}$ (due to step) OK

$\Rightarrow e_{ss} = \lim_{s\to 0}\left[\frac{1}{1+G_c\frac{1}{\cancel{s}(s+1)}}\right]\cancel{s} = \lim_{s\to 0}\left[\frac{1}{1+G_c\frac{1}{s+1}}\right] = \frac{1}{G_c(0)+1}$

Lag Compensation $G_c(s) = \frac{1+aTs}{1+Ts}$ $a<1$

approximation





$\omega_m = \frac{1}{\sqrt{a}}T$; $\sin\phi_m = \frac{a-1}{a+1}$

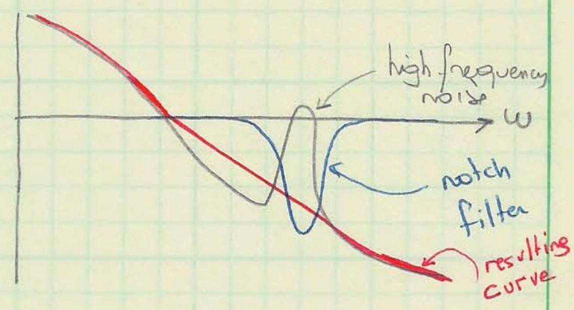
Objective: move gain crossover frequency to a lower frequency while keeping PM relatively unchanged.

$\rightarrow$ reduces high frequency gain

- PID Compensator
- ~~Pho~~ Lead-Lag compensator
- Notch Filter

typically: $\frac{\text{2 complex zeros}}{\text{2 real zeros}}$

useful in such situation



Good Design results in the following Magnitude plot