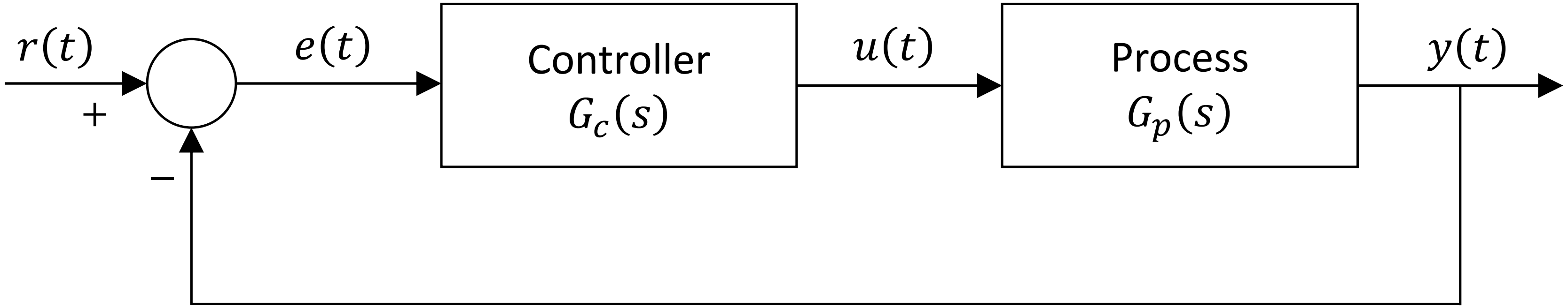
Time Domain Design of Control System

PID Controllers

The **CONTROL** of a mechanical system is the development and application to the system of appropriate forces and moments for operating point control, tracking and stabilization.

This involves designing feedforward and feedback control laws.



Series (cascade) compensation

PID controllers are the most common controllers used in industrial applications

Proportional

Derivative with respect to the error $e(t)$

Integral

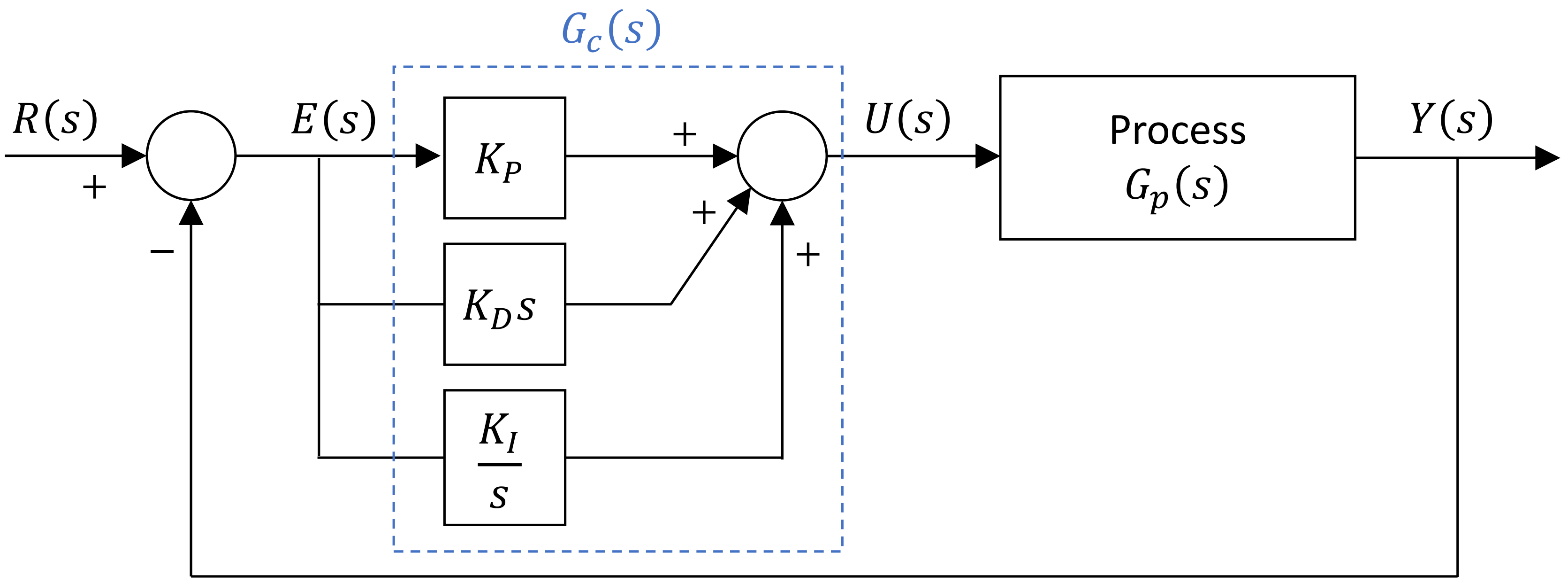
Performance Specifications

Design of a control system in the time domain is performed to meet certain requirements:

- Steady-state accuracy
- Maximum overshoot
- Rise time
- Settling time

step input

PID Controllers



Closed loop transfer function: $\dfrac{Y(s)}{R(s)} = \dfrac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}$

PID controller: $G_c(s) = K_P + K_D s + \dfrac{K_I}{s}$

Control signal: $u(t) = K_P e(t) + K_D \dfrac{\mathrm{d}e(t)}{\mathrm{d}t} + K_I \int_0^t e(\tau)\,\mathrm{d}\tau$

Design Problem Issues

1. Implementation of the PID controller:
   - Analog: operational amplifiers, RC circuits, potentiometers (adjusting the gains)
   - Digital: DAQ (data acquisition systems), computers

2. Choice of the controller gains K_P, K_D, K_I

P-controller

$$G_C(s) = K_P \qquad \text{or} \qquad u(t) = K_P e(t)$$

- Enables feedback
- Gives possibility of modifying the roots of the characteristic equation of the closed loop system for a better performance

PD-controller

$$G_c(s) = K_P + K_D s \qquad \text{or} \qquad u(t) = K_P e(t) + \underbrace{K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t}}$$

Anticipates the increase in error (which usually results in overshoot) and correct it

$\Longrightarrow$ PD (could) reduce the maximum overshoot

PD-controller

$$G_c G_P = (K_P + K_D s) G_P$$

→ PD keeps the same order of the open loop transfer function (OLTF)

⟹ PD does not alter the type of the system

⇨ Derivative or PD control has no effect on the system's steady state error e_{ss} (which depends on the type of the control system) if the error e_{ss} is constant

D or PD control will have an effect on the the error e_{ss} if it varies with time

PD-controller

$\rightarrow$ Adds a simple zero to the OLTF at $s = -\frac{K_P}{K_D}$

$\Longrightarrow$ PD (may) improve damping (i.e., increase the damping ratio ζ)

$\Rightarrow$
- Smaller overshoot
- Smaller settling time

PI-controller

$$G_c(s) = K_P + \frac{K_I}{s} \qquad \text{or} \qquad u(t) = K_P e(t) + K_I \int_0^t e(\tau)\, \mathrm{d}\tau$$

$$G_c G_P = (K_P s + K_I)\frac{G_P}{s}$$

→ PI control adds to the OLTF:

- a zero at $s = -\frac{K_I}{K_P}$
- a pole at $s = 0$ ⟹ PI may cause less stability

PI-controller

$\rightarrow$ Increase the order, hence, the type of the system by 1

$\Longrightarrow$ PI improve the steady state error of the original system (without the I control) by one order

PI is essentially a low-pass filter:

- Slower the rise time
- Longer the settling time

(opposite effect of the PD control)

PID-controller

Combine the features of PD and PI controllers to obtain the best system performance

Strategy for PID control design (to be continued …)