Assignment #1

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1 Problem 1

After solving the stability condition, the Eq. (2.3) of the textbook show two solutions. However,the both of solutions cannot give a proper q-parameter, since there should be only on the size and the Radius curvature of the beam shown in the Eq. (2.4) and Eq. (2.5). Then what would be the proper way of understanding the two solutions in the Eq. (2.3)?

Sloution

From the previous chapter q can be written in the form of,

$$q_1 = \frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}. \tag{1}$$

The solution of stable condition indicates that,

$$\begin{aligned} q_1 &= \tfrac{D-A}{2B} \pm \tfrac{1}{2B}\sqrt{\Delta^2} \\ \Delta^2 &= (A-D)^2 + 4BC \end{aligned} \tag{2}$$

If $\Delta^2 0$, q_1 is a real number which cannot be q parameter. So we could get that $\Delta^2 0$. And due to the determinant of ABCD matrix is 1, we can rewrite $\Delta^2 = (A+D)^2 - 4$. Now we can rewrite the solution as,

$$q_1 = q_1 = \frac{D-A}{2B} \pm i\frac{1}{2B}\sqrt{4-(A+D)^2}. \tag{3}$$

Compare equation 3 with equation 1 we can get that,

$$\begin{aligned} R &= \tfrac{2B}{D-A} \\ \omega &= \sqrt{\tfrac{2\lambda|B|}{4-(A+D)^2}} \end{aligned} \tag{4}$$

The pluse one of the equation 2 can ignore for that its propagation direction is oppsite to real propagation direction.

2 Problem 2

Is the complex q parameter in the cavity dependent on the direction of the laser beam? As an example, in the above figure, the q parameter in the case of (a) would be same or different from the q_r parameter in the case of (b)? Find both q and q r parameters from the self-consistent solution and conclude that they are same or different.

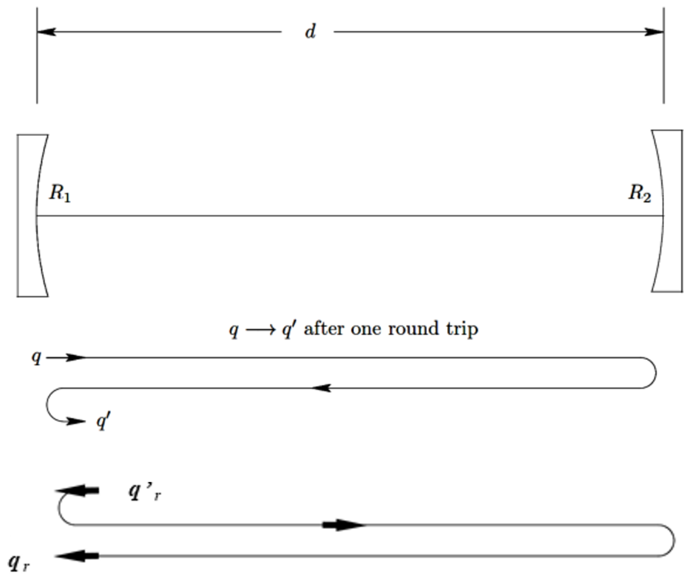

Figure 1: Question 2

Solution

For the case (a), ABCD matrix can be written as,

$$M_1 = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_1} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \tag{5}$$

Let,

$$g1 = 1 - \frac{d}{R_1} \quad g_2 = 1 - \frac{d}{R_2} \tag{6}$$

we can get,

$$M_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2g_2 - 1 & 2g_2 d \\ \frac{2}{d}(2g_1g_2 - g_1 - g_2) & 4g_1g_2 - 2g_2 - 1 \end{pmatrix} \tag{7}$$

Take equation 7 into equation 2,

$$q_1 = \frac{g_1 - 1}{d} \pm i\frac{1}{g_2 d}\sqrt{g_1g_2(1 - g_1g_2)}. \tag{8}$$

If we change the direction of laser beam, the ABCD matrix along the propagation direction is,

$$M_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R_1} & 1 \end{pmatrix} \tag{9}$$

simplifying above equation we could get,

$$M_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 4g_1g_2 - 2g_2 - 12g_2 - 1 & 2g_2 d \\ \frac{2}{d}(2g_1g_2 - g_1 - g_2) & 2g_2 - 1 \end{pmatrix} \tag{10}$$

Compare with M_1 we find that A exchanges position with D in M_2, and using equaiton 3 we could get,

$$q_{r1} = -\frac{g_1 - 1}{d} \pm i\frac{1}{g_2 d}\sqrt{g_1g_2(1 - g_1g_2)}. \tag{11}$$

So we can find that q_1 and q_{r1} has the same beam size but their radius of curvature are the oppsite number which is caused by the opposite propagtion direction.

3 Probelm 3

In the following optical resonator, find the radius of curvature $R(z)$ and width $w(z)$ in terms of z and draw graphs such as $R(z)$ vs z and $w(z)$ vs z. You may choose the values of R_1, R_2 , and d in the stable condition (e.g. $R_1 = 10cm, R_2 = 12cm, d = 8cm, \lambda = 1\mu m$).

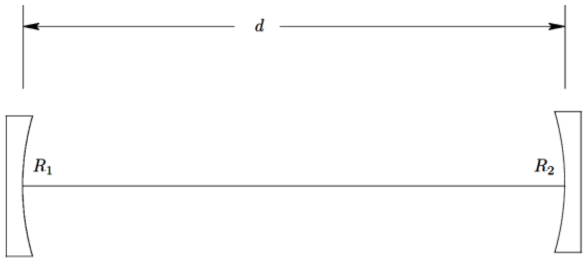

Figure 2: Question 3

Solution

Assume that the laser beam starts from the left hand of the cavity, the distance there is z_1, according to chapter 1,

$$q(z) = q_0 + z = iz_R + z \tag{12}$$

From equation 8 we can get that,

$$q_1 = \frac{1}{q(z_1)} = \frac{g_1 - 1}{d} - i\frac{1}{g_2 d}\sqrt{g_1 g_2(1 - g_1 g_2)}. \tag{13}$$

So we can get,

$$\begin{aligned} z_1 &= -\tfrac{Re(q_1)}{|q_1|^2} = d\tfrac{g_2(g_1-1)}{g_1+g_2-2g_1g_2} \\ z_R &= d\tfrac{\sqrt{g_1g_2(1-g_1g_2)}}{g_1+g_2-2g_1g_2} \end{aligned} \tag{14}$$

And,

$$\begin{aligned} q(z) &= iz_R + z \\ &= id\frac{\sqrt{g_1 g_2(1 - g_1 g_2)}}{g_1 + g_2 - 2g_1 g_2} + z \end{aligned} \tag{15}$$

compare eqaution 15 with

$$q_1 = \frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2}$$

we can get,

$$\begin{aligned} R(z) &= z + \tfrac{z_R^2}{z} \\ \omega(z) &= \sqrt{\tfrac{\lambda}{n\pi}(z_R + \tfrac{z^2}{z_R})} \end{aligned} \tag{16}$$

By eqaution 15 we can draw the picture as following.

4 Problem 4

In the above optical resonator, discuss the location of the beam waist in the case of $R_1 \leq R_2$. The location is clear to mirror R_1 or R_2 ?

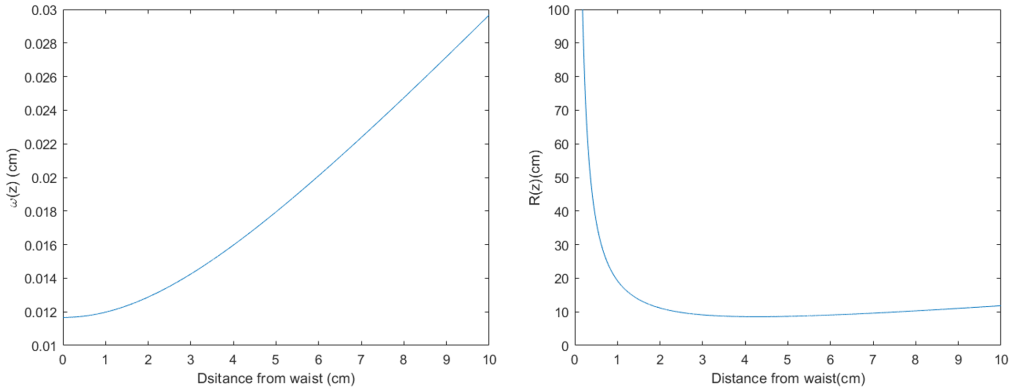

Figure 3: Figure of $R(z)$ and $\omega(z)$

Solution

We have set location of beam waist as the zero point of z In the above optical cavity. And we have calculated the distance of mirror R_1 from the beam waist as equation 14. The length of the cavity is d,so we could get the distance of mirror R_2 from the waist as $z_1 + d$,

$$\begin{aligned} z_2 &= z_1 + d \\ &= d\frac{g_1(1-g_2)}{g_1+g_2-2g_1g_2} \end{aligned} \tag{17}$$

To compare which one is closer to the beam waist, let

$$\begin{aligned} k &= \frac{|z_1|}{|z_2|} \\ &= \frac{|1-\frac{1}{g_1}|}{|1-\frac{1}{g_2}|} \\ &= \frac{|R_2-d|}{|R_1-d|} \end{aligned} \tag{18}$$

So we could get the conclusion that,

- case 1: $d < R_1 < R_2$ k>1, z_2 is closer.
- case 2: $R_1 < R_2 < d$ k<1, z_1 is closer.