Midterm Exam of the Quantum Electronics

Nov. 8, 2016

1. Gaussian Beam

In a region of space distant from currents and charges, it can be shown (using Maxwell's equations) that the electric and magnetic fields satisfy the scalar wave equation

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0,$$

where u is any field component.

(a) Explain the paraxial approximation and derive the paraxial wave equation from the above wave equation. You may introduce a function $\psi(x, y, z)$ that satisfies the paraxial wave equation, and set the relation between ψ and u.

(b) By using the following trial solution of the paraxial wave equation,

$$\psi(x, y, z) = \exp\left\{-i\left(P(z) + \frac{k}{2q(z)}r^2\right)\right\}, r^2 = x^2 + y^2,$$

find the equations on $P(z)$ and $q(z)$ by comparing terms of equal power in r.

(c-e) At given arbitrary $q(z)$ with the wavelength of the laser beam λ, and beam waist ω_0, find the following parameters.

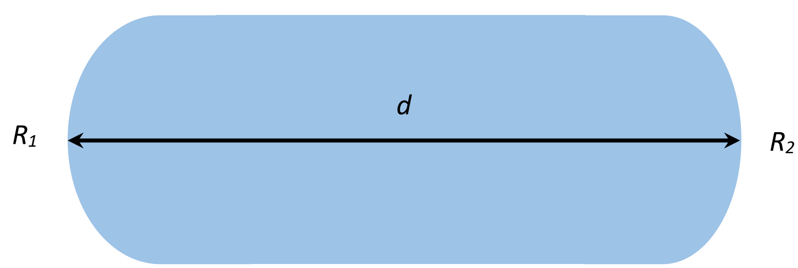
(c) Find the distance to the beam waist ω_0.

(d) Find the radius of the beam $\omega(z)$.

(e) Find the Radius curvature $R(z)$.

2. Optical Resonators

Let's consider the optical material with the index of refraction *n* and mirror coating at both of ends as shown in the following figure.



(a) Find the condition for the stable cavity in terms of R_1, R_2, *d* and *n*.

(b-d) Let's consider the parameters of the cavity as R_1 = 9 cm, R_2 = 12 cm, *d* = 6 cm, *n* = 1.5, and $\lambda = 1\mu$m

(b) Find the Radius of curvature and the beam radius near mirror R_1.

(c) Find the Radius of curvature and the beam radius near mirror R_2.

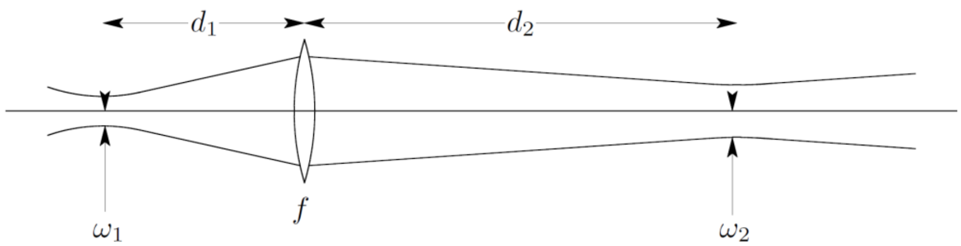
(d) Find ω_0 of the cavity and the distance between the mirror R_1 and the location of ω_0.

(e) What will be the largest distance of *d* for the stable cavity condition?

(f) For such largest distance, find ω_0 of the cavity and the distance between the mirror R_2 and the location of ω_0.

3. Spatial Mode Matching

We'd like to coupling the material by using a single lens for the mode matching.



(a) Defining $f_0 = \frac{\pi\omega_1\omega_2}{\lambda} = \sqrt{z_{R1}z_{R2}}$ and using a lens with $f > f_0$, show the relation

$$d_1 = f \pm \frac{\omega_1}{\omega_2}\sqrt{f^2 - f_0^2}$$

$$d_2 = f \pm \frac{\omega_2}{\omega_1}\sqrt{f^2 - f_0^2}$$

(b) If you want to make $\omega_2 = \omega_0$ of the problem 2(d) with given $\omega_1 = 1\ \mathrm{mm}$, find the profer focal length of the lens f and d_1, d_2 without considering the index of refraction *n* of the matrial in Problem 2.

(c) If you include the effect of the index of refraction *n*, what will be the distances of d_1, d_2 for the same mode matching condition?

4. Energy relations in optical cavities and Laser oscillation.

Let's think the same optical material shown in the figure of problem 2. The reflection coefficients of the mirrors R_1 and R_2, are r_1 and r_2, respectively. The power loss of laser beam in the material is given as $\frac{dP}{dz} = -\alpha P$, where α is the absorption coefficient.

(a) Find the electric field inside of the material near the mirror R_1.

(b) Find the intensity of the field inside of the cavity near the mirror R_1.

(c) Find the full width at half maximum (FWHM), $\Delta\nu_{1/2}$, of the internal field and calculate the Finesse that is defined by $2\pi/\Delta\nu_{1/2}$.

(d) The thickness of the material *d* is 6 cm, the absorption coefficient is 1/1200 cm^-1, and the reflection coefficients of the mirrors $r_1 = r_2 = 0.99$. Find the free-spectral range and the Finesse of the material.

(e) For the case that the impedance is matched, find the amount of enhancement on the intensity of the circulating laser.

(f) When the input laser beam is turned off instantaneously, what will be the decay time of the laser power inside the material?