Assignment #2

Name:Cao Mingming
Student ID:2018311770
cmm18@mails.tsinghua.edu.cn

Tsinghua University — October 17, 2018

1 Question 1

Derive ABCD matrix for a curved mirror, a thin lens and a thick lens is illustrated as figure 1.

Solution

1.1 Thin lens

The sketch map of a thins is illustrated as in figure Suppose that the spot size keeps unchange, by the

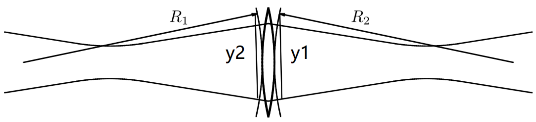

Figure 1: Thin lens

simple lens formula we could get.

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \tag{1}$$

Take the image distance i as R_1, the object distance o is R_2 and f is the focal length.

$$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f} \tag{2}$$

Due to that the ray height is unchangd, and solpe can be obtained by len frmula

$$y_1 = y_2, \quad y_2^{'} = -\frac{y_2}{i}, \quad y_1^{'} = \frac{y_1}{o} \tag{3}$$

Substitute equation 2 into equation 3, we can get

$$\begin{aligned} y_2 &= y_1 + 0 * y_1^{'} \\ y_2^{'} &= -\tfrac{y_1}{f} + y_1^{'} \end{aligned} \tag{4}$$

So the ABCD matrix of thin lens is,

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 0 \end{pmatrix} \tag{5}$$

1.2 Curved mirror

Supposed that the radius of thin mirror is R, which menas that the focal length is $f = \frac{R}{2}$. Substitue f into equation 5 we can get ABCD matrix of curved mirror is,

$$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 0 \end{pmatrix} \tag{6}$$

And the z direction (propagation direction) gets inversed after mirror.

1.3 Thick Lens

As is shown in figure 3 the radii of a thick lens are R_1 and R_2, thickness is d and the refractive index are n_1 and n_2. It can be decomposed into three parts, a refraction part with radius of R_1, a propagation part

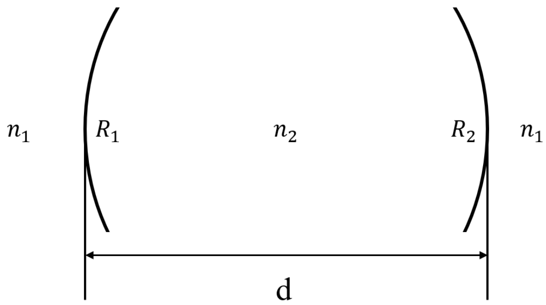

Figure 2: Thick lens

with length d and a refraction part with radius of $-R_2$. Therefore the ABCD matrix can be written as,

$$A = A_3 * A_2 * A_1 \tag{7}$$

where $A_1, \quad A_2, \quad A_3$ are the ABCD matrixes of the three part metntioned above. Substitute

$$A_1 = \begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_1 R_2} & \frac{n_2}{n_1} \end{pmatrix}$$

into equation 7 we can derive that

$$A = \begin{pmatrix} 1 + \frac{d(n_1 - n_2)}{R_1 n_2} & d\frac{n_1}{n_2} \\ \frac{(n_1 - n_2) n_2 (R_2 - R_1) - d(n_1 - n_2)^2}{n_1 n_2 R_1 R_2} & 1 + \frac{d(n_2 - n_1)}{R_2 n_2} \end{pmatrix} \tag{8}$$

2 Question 2

By using ABCD matrix, find the q-parameters after passing through the lens with focal length f depending on d. Find the new beam waist and the distance to it. When $Z_R \ll f$, find the $\omega_0^{'}$ and distance to it in terms of f, ω_0 and λ.

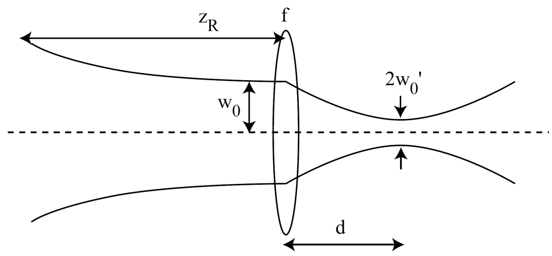

Figure 3: Representation fo question 1

Slolution

The ABCD matrix for the thin lens whit focal lenght f is,

$$\begin{bmatrix} 1 & 0 \\ -1/f & 0 \end{bmatrix} \tag{9}$$

therefore we could get q_1

$$q_2 = \frac{q_0}{1 - q_1/f} \tag{10}$$

q_2 follows that,

$$q_1(z) = z + i\frac{n\pi\omega_0^2}{\lambda} \tag{11}$$

The beam waist locates at the left surface of lens, which means that $z = 0, q_1 = in\pi\omega_0^2/\lambda$. Substitue it into equation 2.

$$q_2 = i\frac{fn\pi\omega_0^2}{\lambda f - in\pi\omega_0^2} \tag{12}$$

We can get the radius and the distance to beam by q parameter.

$$\begin{aligned} \text{Radius of waits:} \quad \omega_0^{'} = \sqrt{\frac{\lambda}{n\pi} Im\{q(z)\}} = \frac{\lambda f \omega_0}{\sqrt{(\lambda f)^2 + (n\pi\omega_0^2)^2}} \\ \text{Distance to the waist:} \quad d = -re\{q(z)\} = \frac{f(n\pi\omega_0^2)^2}{(\lambda f)^2 + (n\pi\omega_0^2)^2} \end{aligned} \tag{13}$$

Substitute $Z_R = n\pi\omega_0^2/\lambda$ into equation 5, $\omega_0^{'}$ and d can been written as,

$$\begin{aligned} \omega_0^{'} &= \frac{\omega_0}{\sqrt{1 + (Z_R/f)^2}} \\ d &= \frac{f}{1 + (f/Z_R)^2} \end{aligned} \tag{14}$$

In the case that $Z_R \gg f$, we ignore the f/Z_R term to get that,

$$\begin{aligned} \omega_0^{'} &= 0 \\ d &= f \end{aligned} \tag{15}$$

which indicates that the beam focuses at focus of the lens.