Assignment #4

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1 Problem 1

Draw the curves similar to Fig. 2.4 for the range of $0 < d < 2R$ with the parameters that you are interested in e.g. $R = 15cm$, $\lambda = 0.37\mu m$ or $R = 20cm$, $\lambda = 0.78\mu$m or $R = 10cm$, $\lambda = 0.532\mu m$.

Solution

According to eqaution 2.21 and 2.22 in textbook we know that,

$$\begin{aligned} \omega^2 &= \left(\frac{\lambda R}{n\pi}\right)\sqrt{\frac{d}{2R-d}} \\ \omega_0^2 &= \left(\frac{\lambda}{n\pi}\right)\sqrt{\frac{dR}{2} - \frac{d^2}{4}} \end{aligned} \tag{1}$$

Choose the parameters as $R = 20cm$, $\lambda = 0.78\mu$,we can get the following figure.

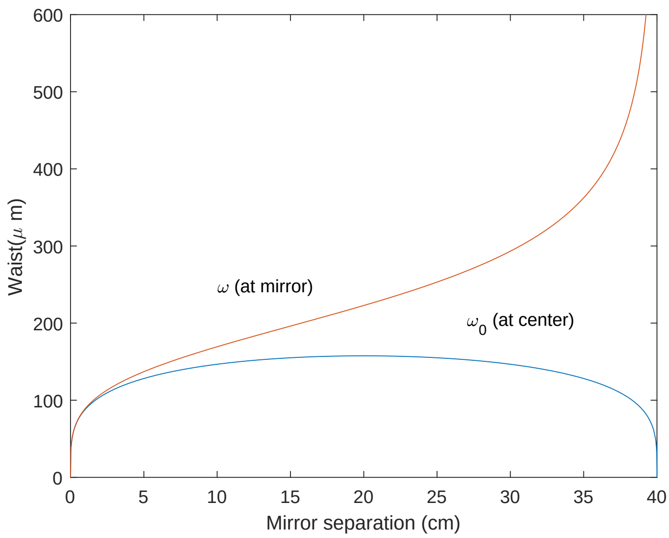

Figure 1: Figure of ω

2 Problem 2

(a) Find the stability condition in terms of $d_3, d_1 + 2d_2$, and R. (b) Find the small waist within d_3 range and the large waist within d_1 in terms of g_1 $(= 1 - (d_1 + 2d_2)/R)$, $g_2 (= 1 - d_3/R)$ and R.

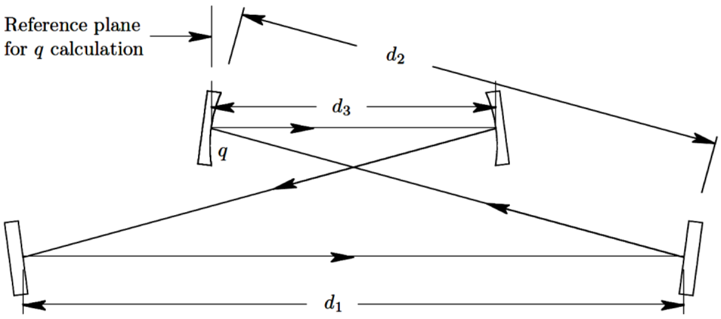

Figure 2: Optical cavity

Solution

2.1 a

According to figure 2 we could get the ABCD matrix as,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 + 2d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_3 \\ 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 2g_1 - 1 & R(g_1 + g_2 - 2g_1 g_2) \\ -\frac{4g_1}{R} & 4g_1 g_2 - 2g_1 - 1 \end{pmatrix} \tag{2}$$

where

$$g_1 = 1 - \tfrac{d_1 + 2d_2}{R}$$
$$g_2 = 1 - \tfrac{d_3}{R} \tag{3}$$

It is the same as what have sloved in two mirror situation which means we could get the stablity condition as,

$$|A + D| \leq 2 \Rightarrow |4g_1 g_2 - 2| \leq 2 \Rightarrow |0 \leq g_1 g_2 \leq 1 \tag{4}$$

Take equation 3 into equation 4 we can get that,

$$R \leq d_3 \leq \frac{R(d_1 + 2d_2)}{d_1 + 2d_2 - R} \tag{5}$$

2.2 b

By solving the satbility condition we can get that,

$$q_1 = \frac{1}{q} = \frac{D - A}{2B} \pm \frac{1}{2B}\sqrt{(A - D)^2 + 4BC}$$
$$= \frac{2g_1(g_2 - 1)}{R(g_1 + g_2 - 2g_1 g_2)} \pm i \frac{2\sqrt{g_1 g_2 (1 - g_1 g_2)}}{R(g_1 + g_2 - 2g_1 g_2)} \tag{6}$$

Due to that $q(z) = iz_R + z$ we can derive that,

$$z_R = -\tfrac{Im(q_1)}{|q_1|^2} = \tfrac{R\sqrt{g_1 g_2 (1 - g_1 g_2)}}{2g_1}$$
$$z_1 = -\tfrac{Re(q_1)}{|q_1|^2} = -\tfrac{R}{2}(g_2 - 1) = \tfrac{d_3}{2} \tag{7}$$

where z_1 is the location of the left hand curved mirror. From z_1 we know that the beam wasit locates at the center of the upper two mirrors. And we can also calculate q_1 as,

$$q_1 = \frac{1}{R} - i\frac{\lambda}{n\pi\omega^2} \tag{8}$$

By equation 8 we can get,

$$R = z + \frac{z_R^2}{z}$$
$$\omega^2 = \frac{\lambda}{n\pi}(z_R + \frac{z^2}{z_R}) \tag{9}$$

the size of beam waist comes to minmum when z^2 comes to minmum($z^2 = 0$). So the largest beam size within d_3 range locates at the center of two mirrors. And,

$$\omega_0 = \sqrt{\frac{\lambda R}{2n\pi g_1}\sqrt{g_1 g_2 (1 - g_1 g_2)}} \tag{10}$$

Assume that the q parameter at the lower mirror is q', the location of q' is shown in figure 3. By using

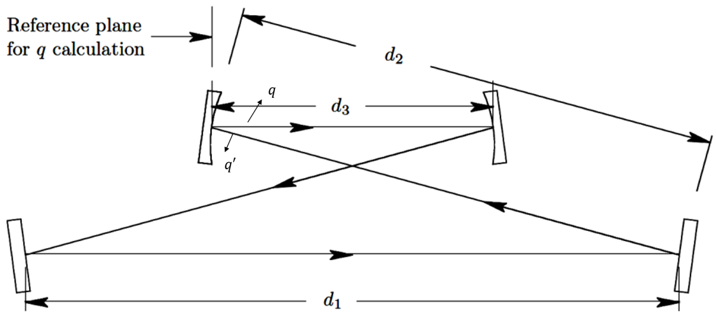

Figure 3: Location of q

ANCD matrix we know when q satisfies the stability condition,

$$q = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 + 2d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_3 \\ 0 & 1 \end{pmatrix} q$$
$$q' = \begin{pmatrix} 1 & d_1 + 2d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_3 \\ 0 & 1 \end{pmatrix} q \tag{11}$$

Solving equation 11 we can get,

$$q' = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}^{-1} q$$
$$= \begin{pmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{pmatrix} q \tag{12}$$

By equation 12 we can get,

$$\begin{aligned} q_1^{'} &= \frac{C + Dq_1}{A + Bq_1} \\ &= \frac{2}{R} + q1 \\ &= \frac{2g_2(1 - g_1)}{R(g_1 + g_2 - 2g_2 g_2)} \pm i \frac{2\sqrt{g_1 g_2 (1 - g_1 g_2)}}{R(g_1 + g_2 - 2g_1 g_2)} \end{aligned} \tag{13}$$

Use equation 7 we can get $z_R^{'}$ and $\omega_0^{'}$ and the location($z_1^{'}$) of the q' .

$$z_R^{'} = \frac{R\sqrt{g_1 g_2 (1 - g_1 g_2)}}{2g_2}$$
$$z_1^{'} = \frac{R}{2}(g_2 - 1) = -\frac{d_1 + 2d_2}{2} \tag{14}$$
$$\omega_0^{'} = \frac{\lambda R\sqrt{g_1 g_2 (1 - g_1 g_2)}}{2g_2}$$

From equation 14 we know that the beam waist locates at the halfway of lower mirror. Use equation 9 we know that,

$$\omega^2 = \frac{\lambda}{n\pi}(z_R + \frac{z^2}{z_R})$$

and z ranges from $[-\frac{d_1}{2}, \frac{d_1}{2}]$ so the largest beam size locates at $z = \pm\frac{d_1}{2}$, which is also the left or right plane lens.

$$\begin{aligned} \omega_{max}^{'2} &= \frac{\lambda}{n\pi}(z_R^{'} + \frac{d_1^2}{4z_R^{'}}) \\ \omega_{max}^{'} &= \sqrt{\frac{\lambda R}{n\pi}\frac{g_1(1-g_1g_2)+4g_2d_1^2}{2\sqrt{g_1g_2(1-g_1g_2)}}} \end{aligned} \tag{15}$$

3 Problem 3

Find and draw the size and the Radius of curvature of the beam everywhere inside the cavities with $R = 6cm, d_3 = 7cm, d_1 + 2d_2 = 18cm, \lambda = 0.74um$. At the curved mirror, how much the radius of curvature is different from R?

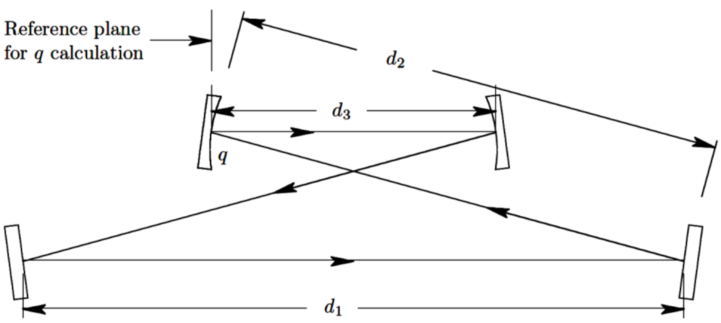

Figure 4:

Solution

To determine the location of beam, choose the reference plane as the zero point of propagation and ste z as the distance of beam propagates from the reference plane. According to what we have obtained in problem 1 and 2, the beam waist of the upper mirrors locates at $z_1 = \frac{d_3}{2}$ and the beam waist of the lower mirrors locate at $z_1^{'} = d_2 + d_3 + \frac{d1}{2}$, using equation 7,9 and 14 we can get the size and radius of curvature while beam propagate btween the mirrors.

$$\begin{aligned} \omega(z) &= \begin{cases} \sqrt{\frac{\lambda}{n\pi}(z_R + \frac{(z-z_1)^2)}{z_R}} & 0 \le z \le d_3 \\ \sqrt{\frac{\lambda}{n\pi}(z_R^{'} + \frac{(z-z_1{'})^2}{z_R^{'}})} & d_3 \le z \le d_1 + 2d_2 + d_3 \end{cases} \\ R(z) &= \begin{cases} z - z_1 + \frac{z_R^2}{z-z_1} & 0 \le z \le d_3 \\ z - z_1^{'} + \frac{z_R^{'2}}{z-z_1^{'}} & d_3 \le z \le d_1 + 2d_2 + d_3 \end{cases} \end{aligned} \tag{16}$$

Finally we can get the figure of beam size and beam curvature as figure 5.And the radius of curvature at the left curved mirror is $r_1 = -3.6429cm$, at the right is $r_2 = 3.6429cm$

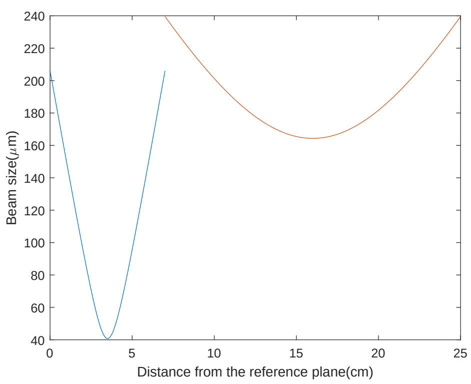

Figure 5: Figure of beam size

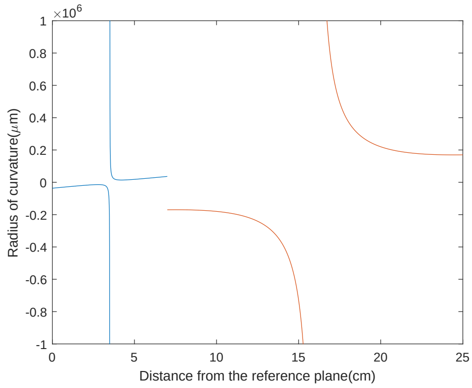

Figure 6: Figure of beam Radius