Assignment #6

Name:Cao Mingming ID:2018311770
cmm18@mails.tsinghua.edu.cn

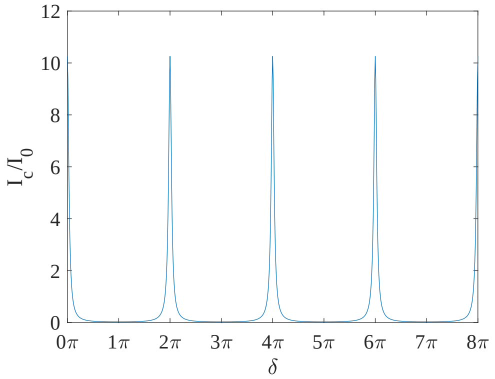
Tsinghua University — November 16, 2018

1 Problem 1

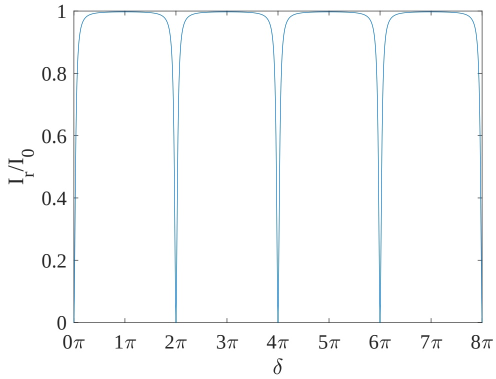
Draw the Eqs. (3.8) and (3.10) in the main text book in terms of δ with $r_1 = r_m = 0.95$ and 0.995

Solution

According the eq 3.8 adn eq 3.10 we can draw the picture as following,
1. $r_1 = r_m = 0.95$

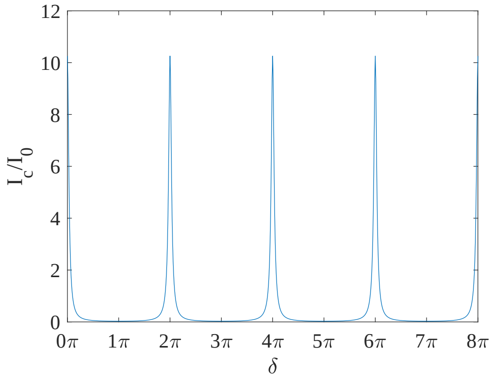


(a) I_c

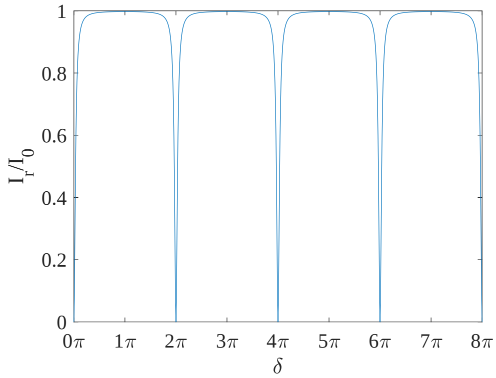


(b) I_r

2. $r_1 = r_m = 0.995$



(c) I_c



(d) I_r

2 Problem 2

A cavity is excited by a 500 nm mode-matched laser beam and has resonances whose separation is 125 MHz and whose width is 2.5 MHz. Determine the following properties of the cavity: a The cavity length b The cavity finesse c The cavity Q d The photon lifetime

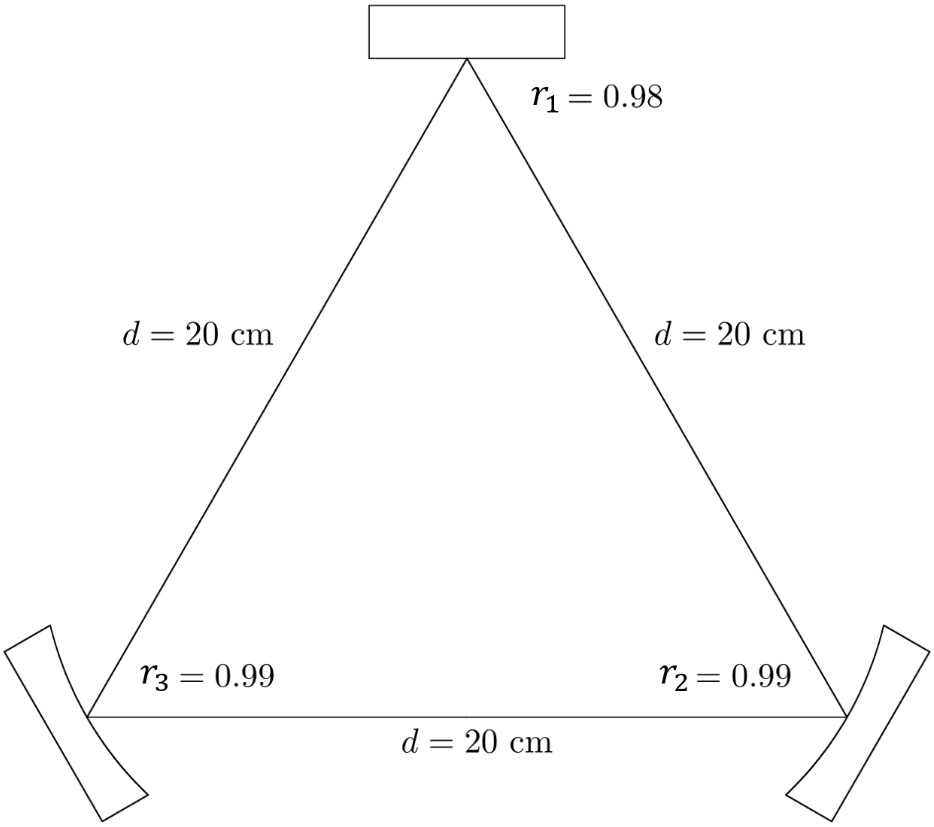
Solution

From the description we know that, $FSR = 125MHz$, $\Delta\nu_{\frac{1}{2}} = 2.5MHz$, therefore we ca get that.

$$\begin{aligned}
& d = \tfrac{c}{2FSR} = 1.2m \\
& F = \tfrac{FSR}{Delta\nu_{\frac{1}{2}}} = 50 \\
& Q = \tfrac{nu}{Delta\nu_{\frac{1}{2}}} = 2.4 \times 10^8 \\
& t_c = Q/\omega = \tfrac{\lambda Q}{2\pi c} = 6.3662 \times 10^{-8}
\end{aligned} \tag{1}$$

3 Problem 3

For the three-mirror ring cavity depicted below, calculate the free spectral range, finesse, Q, and photon lifetime. As shown in the diagram, the three arms are $20cm$ long and have the indicated field reflectivities



Solution

As shown in the figure, we use the same definition of r_m, wecan get that,

$$r_m = r_2 r_3 = 0.9801 \tag{2}$$

Hence we could get the distance of oen round trip is $l = 3 * d = 60cm$ therefore we can get,

$$\begin{aligned} FSR &= \frac{c}{l} = 500MHz \\ F &= \frac{\pi\sqrt{r_1 r_m}}{1 - r_1 r_m} = 77.94 \\ Q &= \frac{\nu}{\Delta\nu_{\frac{1}{2}}} = \frac{lF}{\lambda} \\ t_c &= \frac{l*F}{2\pi c} = 2.481 \times 10^{-8}s \end{aligned} \tag{3}$$

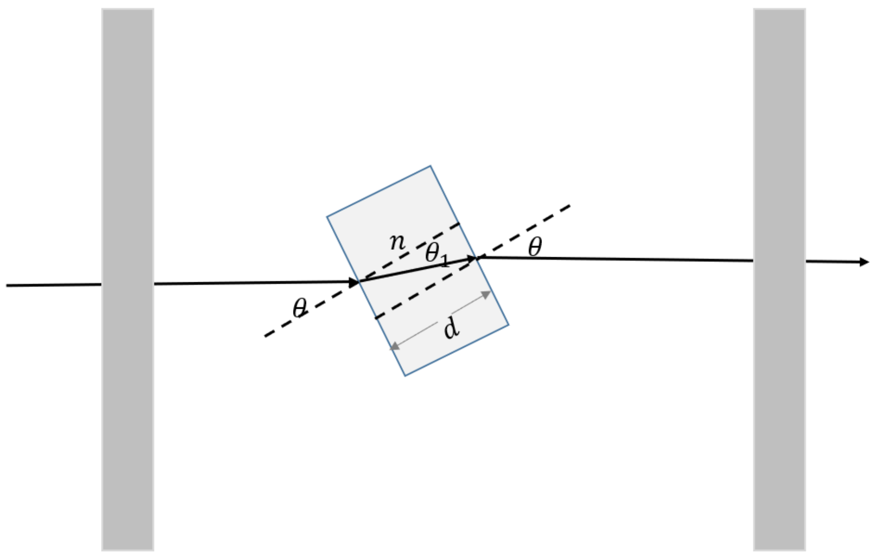
where λ is the wavelength of input beam.

4 Probelm 4

A thin etalon is sometimes used in a laser cavity to help achieve single-mode operation. Such a device consists of a thin plate of quartz which is operated near normal incidence and tilted about an axis normal to the laser beam for tuning purposes. If the index of refraction is 1.5 and the thickness is $0.2cm$, find the linewidth of the etalon and an expression for the frequency as a function of the tilt angle (this dependence is not necessarily linear). The tilt angle is the angle between the normal to the plate and the input k-vector.

Solution

As shown in the figure, suppose that the incident angle is θ and the corresponding refraction angle is θ_1.



Using Snell law we could get that,

$$sin(\theta) = nsin(\theta_1) \tag{4}$$

therefore we can get the distance for a round trip in the etlon as,

$$l = d\sqrt{1 - \frac{sin(\theta^2)}{n^2}} \tag{5}$$

And we can calculate FSR as,

$$FSR = \frac{c}{2nl} = \frac{c}{2\sqrt{n^2 - sin(\theta^2)}d} \tag{6}$$

Suppose that the reflection index of the elton is r therefore we could get the linewidth of elton as,

$$\Delta\nu_{\frac{1}{2}} = \frac{c}{2\pi nd}\frac{1-r}{\sqrt{r}} \tag{7}$$

At the surface of etlon there exists interference therefore only the frequency of light satisfies the conditon of etlon could pass through the etlon, which means that,

$$\begin{aligned} \nu &= k \times FRS \\ &= k\frac{7.5 \times 10^{10}}{\sqrt{n^2 - sin(\theta^2)}}Hz \end{aligned} \tag{8}$$

where k is an integer.