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The Jones matrix of $\frac{\lambda}{2}$ wave plate is.

$$A_{\frac{\lambda}{2}} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Of which we chose horizontal as fast axis.

If we rotate $A_{\frac{\lambda}{2}}$ by angle $-\theta$ and $A_{\frac{\lambda}{4}}$ by angle $-\alpha$ we can get:

$$A_1 = S(+\theta) A_{\frac{\lambda}{2}} S^{-1}(+\theta)$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{bmatrix}$$

$$A_2 = S(\alpha) A_{\frac{\lambda}{4}} S^{-1}(\alpha)$$

$$= \begin{bmatrix} \cos^2\alpha + i\sin^2\alpha & (i-1)\frac{\sin 2\alpha}{2} \\ (i-1)\frac{\sin 2\alpha}{2} & -\sin^2\alpha + i\cos^2\alpha \end{bmatrix}$$

~~If we plan the $A_{\frac{\lambda}{2}}$ before~~

If the light travel through $A_{\frac{\lambda}{2}}$ first the $A_{\frac{\lambda}{4}}$.

Suppose the input beam is.

$$E_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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Therefore we can get the transmission light as.

$$E_t = A_2 A_1 E.$$

$$= A_2 \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + i\sin^2\alpha & (i-1)\frac{\sin 2\alpha}{2} \\ (i-1)\frac{\sin 2\alpha}{2} & -\sin^2\alpha + i\cos^2\alpha \end{bmatrix} \begin{bmatrix} \cos 2\theta \\ -\sin 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos(2\theta-\alpha) + i\sin\alpha\sin(\alpha - 2\theta) \\ -\sin\alpha\cos(2\theta-\alpha) + i\cos\alpha\sin(\alpha-2\theta) \end{bmatrix}$$

By carefully choose α, θ we can get the ~~outp~~ transimssion beam as.

1) $\sin(\alpha - 2\theta) = 0 \Rightarrow \alpha - 2\theta = n\pi \quad (n \in \mathbb{Z})$

$$E_t = \begin{bmatrix} \cos\alpha \\ -\sin\alpha \end{bmatrix}$$

Therefore E_t can represent any linear pollizhzation.

2) $\cos(2\theta - \alpha) = 0 \Rightarrow 2\theta - \alpha = (n + \frac{1}{2})\pi$

$$E_t = \begin{bmatrix} i\sin\alpha \\ i\cos\alpha \end{bmatrix}$$

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Therefore E_t can represent any ~~tt~~ circular polirization light.

2. problem 4.1 in textbook.

4.1

The Johns of a beam splitter has two parts, the reflection parts and transimission part.

Assume that there is no loss in the beam splitter.

$$A_T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_R = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

And the rotation angle is θ; therefore

$$S(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad S(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Therefore, after rotation

$$A_T' = S(\theta) A_T S^{-1}(\theta) = \begin{bmatrix} \cos^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$A_R' = S(\theta) A_R S^{-1}(\theta) = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

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Problem 4.2

From the description we know that the Jones matrix of this element is

$$A = S(\tfrac{\pi}{4}) A_H S(-\tfrac{\pi}{4}) S\tfrac{\pi}{4}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}$$

Of which we choose vertical as fast axis.

If the input beam is a left circular beam for example.

$$E_i = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$E_t = AE_i = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & i \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{2\sqrt{2}}\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore the incident light will be rejected.

If it is a left circular beam

$$E_i = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$E_t = AE_i = \frac{1}{2\sqrt{2}}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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Which indicats that it will become a linear polrization light. If we rotate the device by $-\theta$. Then we can get

$$A_1 = S(\theta) A S^{-1}(\theta)$$

$$= \frac{1}{2}\begin{pmatrix} \cos^2\theta + \sin\theta\cos\theta + i(\sin^2\theta + \sin\theta\cos\theta) & -\sin\theta\cos\theta - \sin^2\theta + i(\sin\theta\cos\theta + \cos^2\theta) \\ -\sin\theta\cos\theta - i\sin^2\theta + \cos^2\theta + i\sin\theta\cos\theta & \sin^2\theta + i\sin\theta\cos\theta - \sin\theta\cos\theta + i\cos^2\theta \end{pmatrix}$$

Therefore we A_1 acts on different circular wave. We can get that.

$$A_1 \frac{\pm 1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} \cos^2\theta + \sin\theta\cos\theta + i(\sin^2\theta + \sin\theta\cos\theta) - i \cdots \\ -\sin\theta\cos\theta - i\sin^2\theta + \cos^2\theta + i\sin\theta\cos\theta + \cdots \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A_1 \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} (\sin\theta + \cos\theta)(\cos\theta + i\sin\theta) \\ (\cos\theta - \sin\theta)(\cos\theta + i\sin\theta) \end{bmatrix}$$

$$= (\cos\theta + i\sin\theta)\begin{bmatrix} \sin\theta + \cos\theta \\ \cos\theta - \sin\theta \end{bmatrix}$$

which is still a linear polarization beam.

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Problem 4.

The zero-order waveplate can be made by align the fast axis of the first plate to the slow axis of the second waveplate. Suppose that the refraction index of these two waveplate are n_e, n_o, n_e', n_o'. For a $k\lambda$ ($k=\frac{1}{2}$ or $\frac{1}{4}$ ...) waveplate, there should be.

$$\begin{cases} d_1(n_e-n_o)=(m+k_1)\lambda \\ d_2(n_e'-n_o')=(n+k_2)\lambda \end{cases}$$

If we want to make a zero-orde ~~pla~~ waveplate. Therefore we can get that.

$$d_1(n_e-n_o)-d_2(n_e'-n_o')=(n-m+k_1-k_2)\lambda=k\lambda$$

That is the rule how we choose d_1 and d_2.

If $n_e=n_e'$, $n_o=n_o'$, we can simplify it as.

$$(d_1-d_2)(n_e-n_o)=k\lambda \quad (k=\frac{1}{2},\frac{1}{4})$$

Suppose that n_e and n_o ~~is~~ are a functions of temperature. and wavelenght. Let $k\lambda=opp(\lambda)$ (optical path difference)

$$\frac{\partial opp(\lambda)}{\partial T}=|d_1-d_2|\left(\frac{\partial n_e}{\partial T}-\frac{\partial n_o}{\partial T}\right)$$

Compare with that of muli-orde waveplat

$$\frac{\partial opp_1(\lambda)}{\partial T}=d_1\left(\frac{\partial n_e}{\partial T}-\frac{\partial n_o}{\partial T}\right)$$

$$\frac{\partial opp_2(\lambda)}{\partial T}=d_2\left(\frac{\partial n_e}{\partial T}-\frac{\partial n_o}{\partial T}\right)$$

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Therefore. $\frac{\partial OPD(\lambda)}{\partial T} = \frac{\partial OPD_1(\lambda)}{\partial T} - \frac{\partial OPD_2(\lambda)}{\partial T}$.

~~Hence it has small~~

Due to that $\begin{cases} d_1 - d_2 < d_1 \\ d_1 - d_2 < d_2 \end{cases}$

Therefore zero-orde wave plate has smaller dependence on temperature.
That's the same for wavelength.

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