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PHYS 372
MODERN EXPERIMENTAL PHYSICS

Millikan Oil Drop Experiment

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Abstract

Estimating the fundamental unit of electrical charge (the charge of an electron) by measuring the gravitational and electric forces exerted on free falling droplets of oil between the plates of a capacitor.

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1 Introduction

In 1909 Robert Millikan was the first to perform the oil drop experiment, the purpose being to measure the charge of the electron. The experiment relies on the ability to measure the tiny electric charge carried by small droplets of oil. Droplets are suspended in air between the electrodes of a parallel plate capacitor. By observing a droplet in free fall with a microscope, it is possible to find its size and mass, which permit calculation of the gravitational and buoyant forces it experiences. By then introducing a strong charge to the electrodes, a charged particle may begin to rise in the electric field. The electric force, along with the gravitational and buoyant forces, make possible the calculation of free charge on the droplet. By doing repeated measurements on a large sample of droplets and performing a statistical analysis, we seek to show that the free charge on the droplets is an integer multiple of some fundamental value. That is, the charge of the electron.

2 Equipment Setup

The apparatus used was an ‘out of the box’ experiment from Pasco®. An atomizer was used to create a mist of tiny droplets, a few of which will fall through a pin sized hole in the center of the top capacitor plate at a time. The voltage source for the capacitor was a Kepco high voltage DC power supply, this was monitored with a digital multimeter (DMM). A polarity switch is included to control the direction of the electric field. Droplets between the electrodes can be ionized by exposing the chamber to a weak alpha emitter.

A graded telescope was fixed to the side of the chamber to allow observation of droplet position. Also within the chamber is a small thermistor which produces resistance proportional to the temperature of the chamber. This resistance was measured with a second DMM. The oil used was Squibb Mineral Oil [$\rho = (0.8780 \pm 0.003)g/cm^3$]. Timings of the droplets in motion were made using a stopwatch.

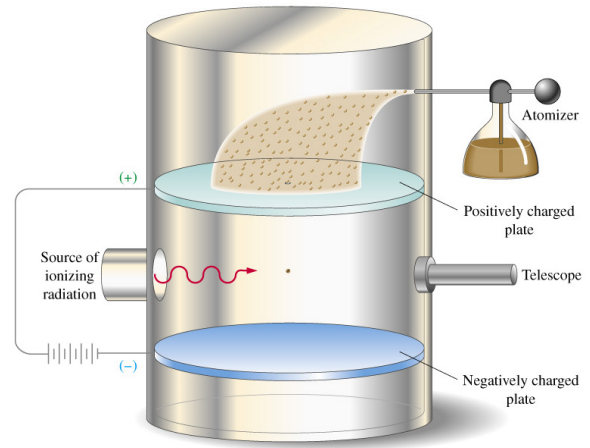


Figure 1: Oil Drop Equipment

3 Theory

The forces acting on a charge as it falls through air at terminal velocity can be seen in Figure 2a. v_f is the terminal velocity and k is the drag coefficient between the air and droplet. Since the forces are equal and opposite,

$$mg = kv_f \quad (1)$$

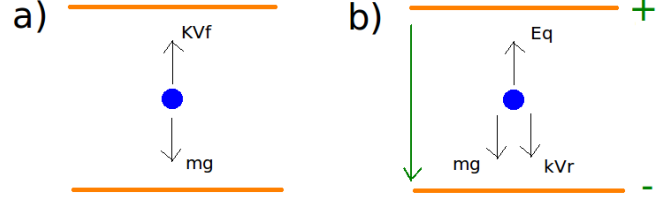


Figure 2: Forces on a) a falling charge b) a rising charge.

Figure 2b depicts the forces acting on a droplet with negative charge (q) as it rises under the influence of the field in the capacitor. E is the field strength and v_r is the terminal rising velocity. Again the net force is zero, thus:

$$Eq = mg + kv_r \quad (2)$$

It is noteworthy that there is a slight buoyant force acting on the droplet in both cases, however, it is negligible since the density of air is 3 orders of magnitude less than that of the oil. Solving equation (1) for k and subbing it into (2) to find q gives:

$$q = \frac{mg(v_f + v_r)}{Ev_f} \quad (3)$$

Direct measurements of v_f and v_r are made in the experiment, but we cannot directly measure m and E . To evaluate these variables, we must express them in terms of measurable quantities. The mass in equation (3) may be expressed in terms of the volume of a sphere:

$$m = \frac{4}{3}\pi a^3 \rho \quad (4)$$

Where a is the radius of the droplet and ρ is the density of the oil. Using Stokes Law, we may express a in terms of the droplets terminal fall velocity in air:

$$a = \sqrt{\frac{p\eta v_f}{2g\rho}} \quad (5)$$

Where η is the coefficient of viscosity for the air. Since the fall velocity of the droplets observed is less than 0.1cm/s, Stokes Law is inaccurate. To account for this inaccuracy, the viscosity must be multiplied by a correction factor which yields a effective viscosity (η_{eff}) of:

$$\eta_{eff} = \eta \left(\frac{1}{1 + \frac{b}{Pa}} \right) \quad (6)$$

Where $b = 8.20 \times 10^{-3} \text{ Pa}\cdot\text{m}$ is a constant, P is the barometric pressure in Pascals and a is the radius of the droplet. Subbing (6) into (5) and solving for the radius gives:

$$a = \sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P} \quad (7)$$

Subbing (7) into (4) to find the mass gives:

$$m = \frac{4}{3}\pi \left[\sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P} \right]^3 \rho \quad (8)$$

The electric field between the capacitor plates is given by:

$$E = \frac{V}{d} \quad (9)$$

Where V is the voltage measured between the plates and d is the plate separation. Using equations (8) and (9) we now have all the ingredients needed to solve for charge (3) in terms of measurable quantities. Substitution yields:

$$q = \frac{4}{3}\pi\rho \left[\sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P} \right]^3 \frac{d(v_f + v_r)}{Vv_f} \quad (10)$$

4 Method

The pressure in the room was periodically recorded using the wall-mounted barometer, and the chamber temperature was monitored using the thermistor resistance. In order to deduce the chamber temperature from the thermistor resistance, the Pasco® manual included a table of resistance values for various temperatures. This data was fitted using a third-order polynomial (see Figure 3) to find that:

$$T = 151.906 - 121.312R + 41.107R^2 - 4.847R^3 \quad (11)$$

As an appendix, Pasco also included a graph of the viscosity of air as a function of temperature. The relationship is linear so by choosing two points on the graph it can be shown that:

$$\eta = [(0.00476471 \text{ } ^\circ\text{C}^{-1})T + 1.7285] \times 10^{-5} \text{ Nsm}^{-2} \quad (12)$$

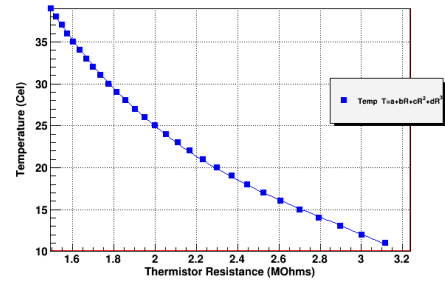


Figure 3: Temperature vs. Thermistor Resistance along with fitting function.

With the apparatus set up and leveled, the capacitor voltage was set to 400 ± 1 Volts for all measurements. Droplets were introduced to the chamber with the atomizer, then the ionizing alpha source (thorium-232) was exposed to the droplets for a few seconds. By changing or neutralizing the capacitor polarity, the observer could control the presence of the electric field. After finding a droplet with a desirable looking rise and fall rate, timings were made using a stopwatch as it moved up and down across the 0.5mm scale, then entered into a spreadsheet. This allowed calculation of rise and fall velocity. For each pair of velocities measured (rise and fall), the spreadsheet utilized equation (10) to calculate the free charge on the droplet. This method was advantageous since it allowed us to see the charge immediately after acquiring each data trial.

As many measurements as possible (typically 10 to 20 pairs) were made of each droplet until moving onto a new one. There was a noticeable tendency for droplets to drift right across the telescope which was typically the limiting factor in the amount of measurements we could make. After compiling data for 80 droplets, the spreadsheet was saved in a .csv format for import by a ROOT®analysis script.

5 Analysis and Results

The basic algorithm followed by the ROOT®analysis script is as follows:

- Retrieves all individual charge measurements for each droplet, then calculates the average charge for that droplet.
- Fills a histogram with the average charge of each droplet and weights each entry by the number of timing measurements made for that droplet. This way, droplets for which many measurements were made are valued above others.
- Fits the histogram with a four-peak Gaussian function of the following form using maximum-likelihood estimation.

$$f(x) = ae^{-\frac{(x-q)^2}{2\sigma_a^2}} + be^{-\frac{(x-2q)^2}{2\sigma_b^2}} + ce^{-\frac{(x-3q)^2}{2\sigma_c^2}} + de^{-\frac{(x-4q)^2}{2\sigma_d^2}} \quad (13)$$

Where: a, b, c, d = amplitudes of peaks 1, 2, 3, 4
 $\sigma_a, \sigma_b, \sigma_c, \sigma_d$ = standard deviations of peaks 1, 2, 3, 4
 q = fundamental unit of electric charge

Notice the fitting function just a superposition of four separate Gaussian functions where the mean of each is an integer multiple of q (the charge on an electron). This is what we would expect since some charges will carry multiple free electrons. If the peaks do occur at integer multiples of q , we can be confident that it is the fundamental unit of charge.

- Calculates the χ^2 per number degrees freedom for the fit and also the error associated with all of the fit parameters list above.

After running the script, Figure 4 was generated along with the following parameters from equation (13):

$$\begin{aligned} a &= 4.5 \pm 1.6 & \sigma_a &= (1.08 \pm 0.38) \times 10^{-20} \\ b &= 9.9 \pm 2.9 & \sigma_b &= (0.67 \pm 0.16) \times 10^{-20} \\ c &= 8.4 \pm 3.8 & \sigma_c &= (0.87 \pm 0.40) \times 10^{-20} \\ d &= 3.0 \pm 1.8 & \sigma_d &= (1.55 \pm 0.56) \times 10^{-20} \end{aligned}$$

$$\underline{q = (1.585 \pm 0.005) \times 10^{-19} \text{ Coulombs}}$$

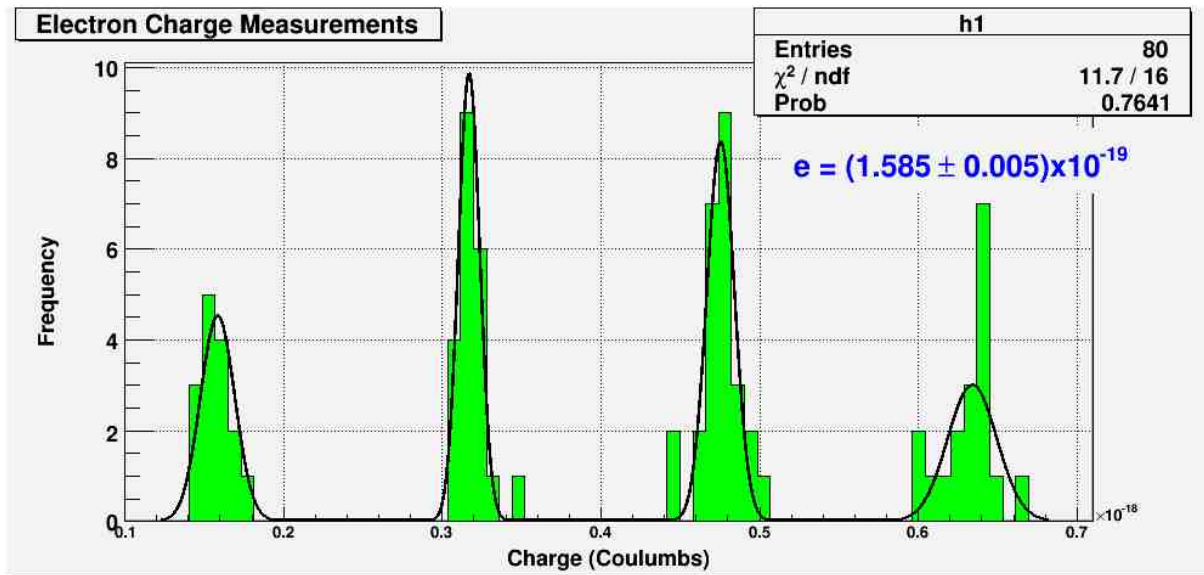


Figure 4: Histogram of measured charges and fitting function.

6 Discussion

The accepted value for the charge of an electron is 6.02×10^{-19} Coulombs. The value we found is 3.4 standard deviations away from this value. The probability of this occurring statistically is less than 0.5%. Two possible explanations for this discrepancy come to mind:

1. Our sample size may have been too small to accurately represent the population, however, this seems like a very unlikely scenario since the quality of our Gaussian fit is high. The χ^2/ndf of the fit is 11.7/16, roughly 0.73, this is considered a 'good' fit by the standards presented in our lectures. While more data would likely improve our fit slightly, it is unlikely to affect our result significantly.

2. There could have been a systematic error present in our data acquisition which shifted our value of q downwards. Since our discrepancy doesn't appear to be statistical in nature, this is a more likely scenario. The trouble remains to determine the source of our systematic error.

One possible source of systematic error, which is very appealing at first glance, is the possibility that the apparatus was not properly leveled at the beginning of the experiment. This would introduce an asymmetric error since tilting the apparatus could only make the distance traveled by electrons appear shorter (see Figure 5). If θ were the angle by which the apparatus was off level and D were the actual distance traveled, then an electron which appeared to travel 0.5mm would've actually traveled:

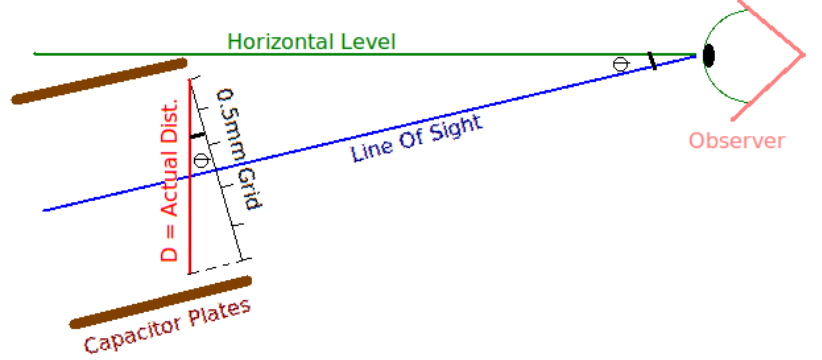


Figure 5: If apparatus is off-level, distance traveled appears shorter.

$$D = \frac{0.5mm}{\cos\theta} \quad (14)$$

While correcting for this angle would bring our final value closer up towards the accepted value, the apparatus would need to have been a full 5° off-level to even bring us within error of the accepted value. It is highly unlikely that we were anywhere near 5° off-level. Even if we assume, in a worst case scenario, that the apparatus was 2° off-level the result would still be 2.6 standard deviations away from the accepted value. Clearly, the leveling of the apparatus has little effect on the results.

Other candidates for the source of our systematic error are pressure fluctuation and air drafts in the room, observer bias, and fringe effects due to the non-uniformity of the electric field between the capacitor plates. Unfortunately, without launching what would be a very time consuming search for the source of our error, we will have to stick with our current results. Given more time and an ideal environment, I would obtain more data with the apparatus isolated in a draft-free area and more closely monitor the environment variables. It could then be possible to determine what variable has impacted our results.

Bibliography

Textbooks and Papers

1. Tipler & Llewellyn *Modern Physics: Fifth Edition*. Freeman Publishing.
2. *Millikan Oil Drop Apparatus User Manual*. Pasco Industries.
3. Griffiths *Introduction to Electrodynamics*. Addison Wesley.