

Package ‘ars’

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Title A package for stat243 final project

Description An adaptive-rejection sampler for a log-concave function $f(x)$ with domain D .

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NeedsCompilation no

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abscissaeSummary	<i>abscissaeSummary</i>
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Description

Given a function and a sequence of points, function computes the value of the given function and its derivative at each point.

Usage

abscissaeSummary(x, h)

Arguments

x	a numeric vector composed of the abscissae
h	the original function in log scale

Value

a numeric matrix, with the three columns being the x's, $h(x)$'s, and derivatives of the $h(x)$'s respectively. If $h(x)$ is Inf, the corresponding x and $h'(x)$ will be dropped.

Examples

```
x <- seq(-2, 2, length.out = 10)
abscissaeSummary(x, function(x) dnorm(x))
```

ars	<i>A function that returns samples from unnormalized density based on reject sampling</i>
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Description

A function that returns samples from unnormalized density based on reject sampling

Usage

```
ars(n, g, k = 30, left = -Inf, right = Inf, stepsize = 10)
```

Arguments

n	a numeric value as sample size.
g	a log-concave function.
k	a numeric value as the max size of initial abscissae.
left	a numeric value as the left bound of input h; default is -Inf.
right	a numeric value as the right bound of input h; default is Inf.
stepsize	an integer that indicates the number of samples generated in the first iteration of sampling; it increases along iterations at a predetermined rate 1.1.

Value

a numeric vector of length n that was sampled from the normalized density function d.

Examples

```
ars(n = 1000, g = dnorm)
```

computeZ

computeZ

Description

A function that computes the intersections of the tangent lines, provided the abscissae matrix

Usage

```
computeZ(abscissae_summary)
```

Arguments

abscissae_summary

the abscissae matrix with the three columns being the x 's, $h(x)$'s, and derivatives of the $h(x)$'s respectively. There should be no Inf's in the third column

Value

a numeric vector consists of the x -coordinates of the intersection points (i.e. z in paper by Gilks et al.)

findInitAbsc

A function that returns a matrix with the three columns being the abscissae x_1, \dots, x_k and their corresponding $h(x)$'s and $h'(x)$'s respectively.

Description

A function that returns a matrix with the three columns being the abscissae x_1, \dots, x_k and their corresponding $h(x)$'s and $h'(x)$'s respectively.

Usage

```
findInitAbsc(g, k, left = -Inf, right = Inf, c = 3)
```

Arguments

g a log-concave function.

k a numeric value that indicates the max size of the abscissae.

$left$ a numeric value that indicates the left bound for domain of function g ; default value is $-\text{Inf}$

$right$ a numeric value that indicates the right bound for domain of function g ; default value is Inf

c a numeric value by which we shift the finite bound to find initial value for the optim function; default value is 3

Value

mat the matrix of interest

Examples

```
findInitAbsc(function(x){
  return(dnorm(x, mean=3, sd=2))
},6)

# Chi-square distribution
g <- function(x){
  return(dchisq(x, 10, ncp=3))
}
findInitAbsc(g,4,3)

# Uniform distribution
g <- function(x){
  return(dunif(x, min=2, max=5))
}
findInitAbsc(g,6,2,5)

# Piecewise constant distribution
h <- function(x){
  a <- sapply(x,function(x){if(x<1) return (x)
    else if(x>=1 & x <=2) return (1)
    else if(x>2) return (-x+3)}})
  return (a)
}
g <- function(x){
  return(exp(h(x)))
}
findInitAbsc(g,6)
```

findMode

A function that returns the mode of a function

Description

A function that returns the mode of a function

Usage

```
findMode(g, left = -Inf, right = Inf, c = 3)
```

Arguments

<code>g</code>	a log-concave function.
<code>left</code>	a numeric value that indicates the left bound for domain of function <code>g</code> ; default value is <code>-Inf</code>
<code>right</code>	a numeric value that indicates the right bound for domain of function <code>g</code> ; default value is <code>Inf</code>
<code>c</code>	a numeric value by which we shift the finite bound to find initial value for the optim function; default value is 3

Value

mode a numeric value that is the mode of the function $h=\log(g)$

Examples

```
# Normal distribution
g <- function(x){
  return(dnorm(x, mean=3, sd=2))
}
findMode(g)

# Chi-square distribution
g <- function(x){
  return(dchisq(x, 10, ncp=3))
}
findMode(g,3)

# Uniform distribution
g <- function(x){
  return(dunif(x, min=2, max=5))
}
findMode(g,2,5)

# Piecewise constant distribution
h <- function(x){
  a <- sapply(x,function(x){if(x<1) return (x)
    else if(x>=1 & x <=2) return (1)
    else if(x>2) return (-x+3)})
  return (a)
}
g <- function(x){
  return(exp(h(x)))
}
findMode(g)
```

lowerHull

A function that, given a vector x, returns the corresponding values of the upper hull function

Description

A function that, given a vector x, returns the corresponding values of the upper hull function

Usage

```
lowerHull(x, abscissae_summary)
```

Arguments

x a numeric vector consists of points on the x-axis

abscissae_summary a matrix with the three columns being x, h(x), and h'(x) at the support points respectively

Value

U: a numeric vector that contains the corresponding upper hull values, with same length as x

Examples

```

h <- function(x){
  return(log(dnorm(x)))
}

xSupport <- seq(-3, 3, length.out = 10)

abscissae_summary = abscissaeSummary(xSupport,h)

x <- seq(-3, 3, length.out = 10)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

x <- seq(-3, 3, length.out = 30)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

```

sampleUpper

sampleUpper

Description

Sample from upper hull based on inverse CDF

Usage

```
sampleUpper(n, CDFInverse)
```

Arguments

n a positive integer as the number of samples
CDFInverse an inverse CDF function of the upper hull

Value

a numeric vector sampled from upper hull, which has size n.

upperCDF

upperCDF

Description

Given a numeric vector and the abscissae matrix, function returns the CDF function of the upper hull.

Usage

```
upperCDF(x, abscissae_summary, xlow = -Inf, xhigh = Inf)
```

Arguments

x	a numeric vector that takes values in [0,1] as the input for the CDF
abscissae_summary	a numeric matrix comprised of abscissae together with $h(x)$ and $h'(x)$
xlow	a numeric value as the left bound of the support of g
xhigh	a numeric value as the right bound of the support of g

Value

A numeric vector consists of the values of CDF for the upper hull at each abscissa.

upperCDFInverse	<i>upperCDFInverse</i>
-----------------	------------------------

Description

Given a numeric vector and the abscissae matrix, function returns the inverse CDF function of the upper hull.

Usage

```
upperCDFInverse(x, abscissae_summary, xlow = -Inf, xhigh = Inf)
```

Arguments

x	a numeric vector that takes values in [0,1] as the input for the CDF
abscissae_summary	a numeric matrix comprised of abscissae together with $h(x)$ and $h'(x)$
xlow	a numeric value as the left bound of the support of g
xhigh	a numeric value as the right bound of the support of g

Value

A numeric vector consists of the values of the inverse CDF for the upper hull at each abscissa.

upperHull	<i>A function that, given a vector x, returns the corresponding function values from the upper hull</i>
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Description

A function that, given a vector x, returns the corresponding function values from the upper hull

Usage

```
upperHull(x, abscissae_summary)
```

Arguments

x a numeric vector consists of points on the x-axis

abscissae_summary a matrix with the three columns being x , $h(x)$, and $h'(x)$ at the support points respectively

Value

U: a numeric vector that contains the corresponding upper hull values, with same length as x

Examples

```
# Test1: log of normal distribution
h <- function(x){
  return(log(dnorm(x)))
}

xSupport <- seq(-3, 3, length.out = 10)

abscissae_summary = abscissaeSummary(xSupport,h)

x <- seq(-3, 3, length.out = 10)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

x <- seq(-3, 3, length.out = 30)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

h <- function(x){
  return(log(dchisq(x,df = 2)))
}

xSupport <- seq(-3, 3, length.out = 10)
abscissae_summary = abscissaeSummary(xSupport,h)
x <- seq(-3, 3, length.out = 10)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

# Test2: log of uniform distribution
h <- function(x){
  return(log(dunif(x,0,1)))
}

xSupport <- seq(-3, 3, length.out = 100)
abscissae_summary = abscissaeSummary(xSupport,h)
x <- seq(-3, 3, length.out = 10)
x <- seq(0.2, 0.9, length.out = 10)
upperHull(x,abscissae_summary)
lowerHull(x,abscissae_summary)

# Test3: log of piecewise constant distribution
h <- function(x){
  a= sapply(x,function(x){if(x<1) return (x)
  else if(x>=1 & x <=2) return (1)
  else if(x>2) return (-x+3)})
}
```



```
    return (a)
  }

  xSupport <- seq(-3, 3, length.out = 15)
  abscissae_summary = abscissaeSummary(xSupport,h)
  x <- seq(-3, 3, length.out = 10)
  x <- seq(0.2, 0.9, length.out = 10)
  upperHull(x,abscissae_summary)
  lowerHull(x,abscissae_summary)
```

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