KAUNO TECHNOLOGIJOS UNIVERSITETAS

INFORMATIKOS FAKULTETAS

Algoritmų sudarymas ir analizė (P170B400)

Namų darbų ataskaita

(13 variantas)

Atliko:

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Turinys

1. Palyginkite funkcijas (a ir b formulių poros), kai $n \to \infty$ (1 lentelė).

25.

$n^5 + 7n^3 - 1 = f(n)$ $\qquad \frac{f(n)}{g(n)}$ dalinti!

$n^7 + n^3 + 5n = g(n)$

$$\lim_{n\to\infty} \frac{n^5 + 7n^3 - 1}{n^7 + n^3 + 5n} = \lim_{n\to\infty} \frac{\frac{n^5}{n^7} + \frac{7n^3}{n^7} - \frac{1}{n^7}}{1 + \frac{n^3}{n^7} + \frac{5n}{n^7}} =$$

$$\lim_{n\to\infty} \frac{\frac{1}{n^2} + \frac{7}{n^4} - \frac{1}{n^7}}{1 + \frac{1}{n^4} + \frac{5}{n^6}} = \frac{0 + 0 - 0}{1 + 0 + 0} = \frac{0}{1} = 0$$

Ats.: $n^5 + 7n^3 - 1 = O(n^7 + n^3 + 5n)$; $\cancel{g(n)}$ $n^7 + n^3 + 5n = \Omega(n^5 + 7n^3 - 1)$

37.

$\log_2^2(n^2) = f(n)$ $\qquad \frac{f(n)}{g(n)}$ dalinti!

$\sqrt{n^3}\log_3(n) = g(n)$

Išvestinės:

$$\log_2^2(n^2) = (2\log_2(n^2))' = 2\cdot\frac{1}{n^2\ln 2}2n = \frac{4n}{n^2\ln 2} = \frac{4}{n\ln 2}$$

$$(\sqrt{n^3}\log_3 n)' = (n^{\frac{3}{2}})'\cdot\log_3 n + n^{\frac{3}{2}}\cdot(\log_3 n)' = \frac{3}{2}\cdot\sqrt{n}\log_3 n + \frac{n^{\frac{3}{2}}}{n\ln 3}$$

$$\lim_{n\to\infty} \frac{\log_2^2(n^2)}{\sqrt{n^3}\log_3(n)} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{\frac{4}{n\ln 2}}{\left(\frac{3}{2}\sqrt{n}\log_3 n + \frac{n^{\frac{3}{2}}}{n\ln 3}\right)^{\cdot n\ln 2}} =$$

$$= \lim_{n\to\infty} \frac{4}{\left(\frac{3}{2}\sqrt{n}\log_3 n + \frac{n^{\frac{3}{2}}}{n\ln 3}\right)\cdot n\ln 2} = \frac{4}{\infty} = 0$$

Ats.: $\log_2^2(n^2) = O(\sqrt{n^3}\log_3(n))$; $\sqrt{n^3}\log_3(n) = \Omega(\log_2^2(n^2))$

2. Suprastinkite funkcionalą (2 lentelė).

14. $O\left((\ln(n^2))^2 + \sqrt[4]{n^3} \right)$

$f(n) = (\ln(n^2))^2$ dalyba:

$g(n) = \sqrt[4]{n^3}$

$$\lim_{n\to\infty} \frac{(\ln n^2)^2}{\sqrt[4]{n^3}} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{\frac{8\ln n}{n}}{\frac{3}{4\sqrt[4]{n}}} =$$

$$= \lim_{n\to\infty} \frac{8\ln n \cdot 4\sqrt[4]{n}}{3n} = \left[\frac{\infty}{\infty}\right] = \lim_{n\to\infty} \frac{32\sqrt[4]{n} + \frac{8\ln n}{\sqrt[4]{n^3}}}{3} =$$

$$= \lim_{n\to\infty} \frac{32 + 8\ln(n)}{3\cdot\sqrt[4]{n^3}} = \left[\frac{\infty}{\infty}\right] = \frac{\frac{8}{n}}{\frac{9}{4\sqrt[4]{n}}} = \lim_{n\to\infty} \frac{32\sqrt[4]{n}}{9n} =$$

$$= \left[\frac{\infty}{\infty}\right] = \frac{\frac{8}{\sqrt[4]{n^3}}}{9} = \frac{8}{9\sqrt[4]{n^3}} = \frac{8}{\infty} = 0$$

Atsakymas: $O((\ln(n^2)))$

Išvestinės:

1) $((\ln n^2)^2)' = (2\ln n)^2 = (4\ln^2 n)' = 4\cdot 2\ln n \cdot \frac{1}{n} = \frac{8\ln n}{n}$

$\sqrt[4]{n^3} = (n^{\frac{3}{4}})' = \frac{3}{4} n^{\frac{3}{4}-1} = \frac{3}{4} n^{-\frac{1}{4}} = \frac{\frac{3}{4}}{n^{\frac{1}{4}}} = \frac{3}{4\sqrt[4]{n}}$

2) $8\ln n \cdot 4\sqrt[4]{n} = (8\ln n)'\cdot 4\sqrt[4]{n} + 8\ln n \cdot (4\sqrt[4]{n})' =$

$= 32\sqrt[4]{n} + \frac{\cancel{4}\cdot 8\ln n}{\cancel{4}\sqrt[4]{n^3}}$; $(3n)' = 3$

$n^{\frac{1}{4}} = \frac{1}{4} n^{-\frac{3}{4}} = \frac{1}{4\sqrt[4]{n^3}}$

3) $\frac{32\sqrt[4]{n}, \sqrt[4]{n^3}}{n} + \frac{8\ln n}{4\sqrt{n^3}} = \frac{32n + 8n\ln n}{n(4\sqrt{n^3})} = \frac{32 + 8\ln(n)}{\sqrt[4]{n^3}} =$

$= \frac{32 + 8\ln(n)}{\sqrt[4]{n^3}}$

4) $32 + 8\ln(n) = \frac{8}{n}$

$3\sqrt[4]{n^3} = 3 \cdot n^{\frac{3}{4}} = 3 \cdot \frac{3}{4} n^{-\frac{1}{4}} = 9 n^{-\frac{1}{4}} = \frac{9}{4\sqrt[4]{n}}$

5) $g_n = g$

$32\sqrt[4]{n} = 32 \cdot \frac{1}{4} n^{-\frac{3}{4}} = \frac{8}{\sqrt[4]{n^3}}$

3. Įvertinkite eilutės sumą (3 lentelė).

① $\sum_{i=1}^{n} i \log_2 i^3$ – monotoniškai didėjanti.

$$\int_0^n f(x\log_2 x^3)\,dx \le \sum_{i=1}^{n} i\log_2 i^3 \le \int_1^{n+1} (x\log_2 x^3)\,dx$$

1) $\int_0^n (x\log_2 x^3)\,dx = 3\int_0^n x\log_2 x\,dx =$

$$= \begin{bmatrix} u=\log_2 x & dv = x \\ du = \frac{1}{x\ln 2} & v = \frac{x^2}{2} \end{bmatrix} = uv - \int v\,du =$$

$$= \left(\frac{\log_2 x\cdot x^2}{2} - \int \frac{x^2}{2x\ln 2}\right) = \frac{\log_2 x\cdot x^2}{2} -$$

$$- \frac{x^2}{4\ln 2}\Big|_0^n = 3\left(\frac{\log_2 n\cdot n^2}{2} - \frac{n^2}{4\ln 2}\right) =$$

$$= 3\left(\frac{\log_2 n\cdot n^2}{2} - \frac{n^2}{4\ln 2}\right)$$

2) $\int_1^{n+1} (x\log_2 x^3)\,dx = 3\int_1^{n+1} x\log_2 x\,dx =$

$$= \begin{bmatrix} u=\log_2 x & dv = x \\ du = \frac{1}{x\ln 2} & v = \frac{x^2}{2} \end{bmatrix} = uv - \int v\,du =$$

$$= \left(\frac{\log_2 x\cdot x^2}{2} - \int\frac{x^2}{2x\ln 2}\right) = \frac{\log_2 x\cdot x^2}{2} - \frac{x^2}{4\ln 2}\Big|_1^{n+1} =$$

$$= 3\left(\frac{\log_2 (n+1)\cdot (n+1)^2}{2} - \left(\frac{(n+1)^2}{4\ln 2} - \frac{\cancel{n+1}\,1^2}{4\ln 2}\right)\right) =$$

$$= 3\left(\frac{\log_2 (n+1)\cdot (n+1)^2}{2} - \frac{(n+1)^2+1}{4\ln 2}\right)$$

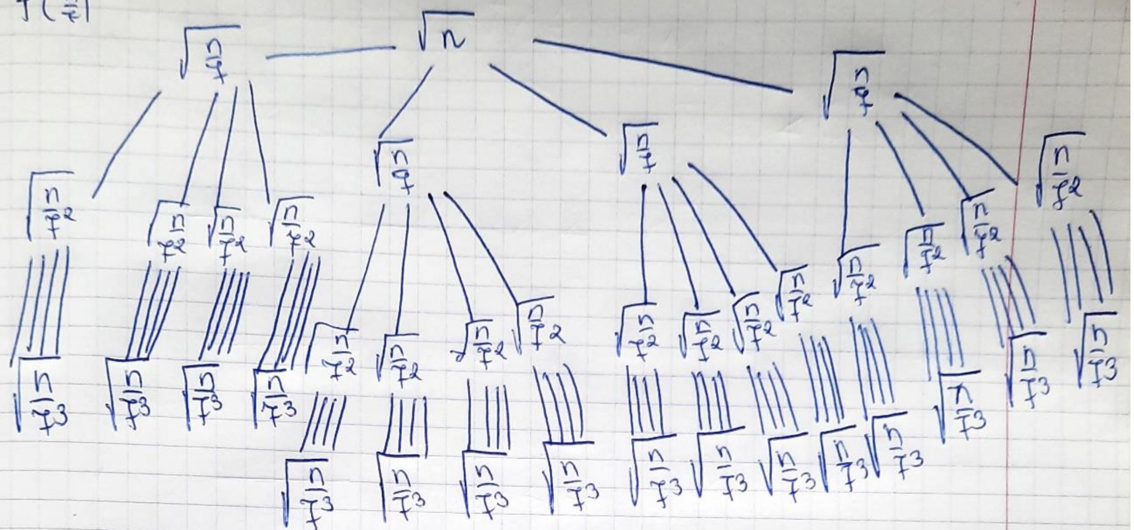
Atsakymas: $3\left(\frac{\log_2 n \cdot n^2}{2} - \frac{n^2}{4\ln 2}\right) \le \sum_{i=1}^{n} i \log i^3 \le$

$$\le 3\left(\frac{\log_2 (n+1) \cdot (n+1)^2}{2} - \frac{(n+1)^2+1}{4\ln 2}\right)$$

4. Raskite lygties sprendinį medžio metodu, kai $T(1) = 1$ ir galioja 4 lentelėje duotas rekurentinis sąryšis, kai $n \geq 2$

(14) $T(n) = 4 \cdot T\left(\frac{n}{7}\right) + \sqrt{n}$

$T\left(\frac{n}{7}\right)$



$\sqrt{n};\ 4\sqrt{\frac{n}{7}};\ 16\sqrt{\frac{n}{7^2}};\ 64\sqrt{\frac{n}{7^3}}$ $\quad 4^i \cancel{\frac{\sqrt{n}}{\sqrt{7^i}}} = 4^i \cancel{\frac{\sqrt{n}}{7^{\frac{i}{2}}}}$

$\sqrt{n}\left(\frac{16}{7}\right)^{\frac{i}{2}} = \sqrt{n}\left(\left(\frac{16}{7}\right)^{\frac{1}{2}}\right)^i$

$h = \log_7 n$

$$T(n) = \sqrt{n}\sum_{i=0}^{\log_7 n}\left(\left(\frac{16}{7}\right)^{\frac{1}{2}}\right)^i \leq \sqrt{n}\sum_{i=0}^{\log_7 n+1}\left(\left(\frac{16}{7}\right)^{\frac{1}{2}}\right)^i = \sqrt{n}\,\frac{\left(\left(\frac{16}{7}\right)^{\frac{1}{2}}\right)^i - 1}{\left(\frac{16}{7}\right)^{\frac{1}{2}} - 1} =$$

$$= \sqrt{n}\,\frac{\sqrt{7}}{4}\left(\left(\frac{16}{7}\right)^{\frac{\log_7 4}{2}+1} - 1\right) = \sqrt{n}\,\frac{\sqrt{7}}{4}\left(\frac{16^{\log_7\sqrt{n}+1} - 7\sqrt{n}}{7\sqrt{n}}\right) =$$

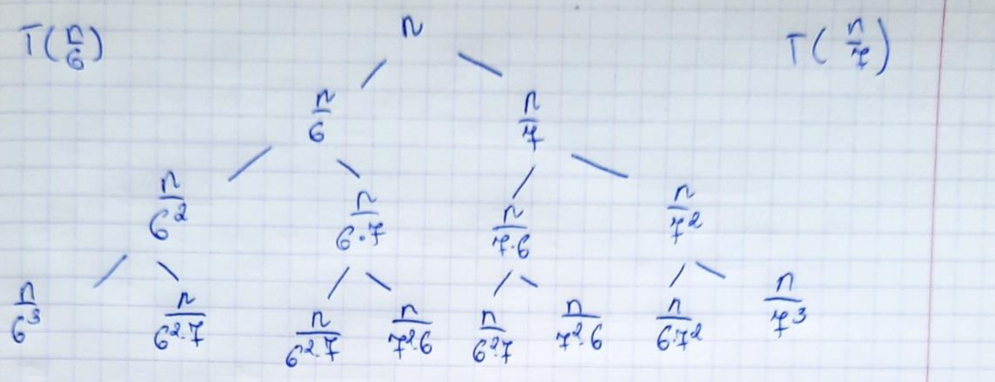
$$= \sqrt{n}\cdot\frac{\sqrt{7}}{4}\left(\frac{16^{\log_7\sqrt{n}}\cdot 16 - 7\sqrt{n}}{7\sqrt{n}}\right) = \frac{16^{\log_7\sqrt{n}}\cdot 16 - 7\sqrt{n}}{4} =$$

$$= \frac{n^{\log_7 4}\cdot 16 - 7\sqrt{n}}{4}$$

Atsakymas: $n^{\log_7 4}$, nes auga greičiau už $7\sqrt{n}$

5. Raskite lygties sprendinį medžio metodu, kai $T(1) = 1$ ir galioja 5 lentelėje duotas rekurentinis sąryšis, kai $n \geq 2$.

(14.) $T(n) = T(\frac{n}{6}) + T(\frac{n}{7}) + n$



$n;\ \frac{n}{6} + \frac{n}{7};\ \frac{n}{6^2} + 2\frac{n}{6\cdot 7} + \frac{n}{7^2};\ \frac{n}{6^3} + 3\frac{n}{6^2\cdot 7} + 3\frac{n}{7^2\cdot 6} + \frac{n}{7^3}$

$n\left(\frac{1}{6^3} + 3\frac{1}{6^2\cdot 7} + 3\frac{1}{7^2\cdot 6} + \frac{n}{7^3}\right)$

$n\left(\frac{1}{6} + \frac{1}{7}\right)^i = n\left(\frac{13}{42}\right)^i$

$T(n) = n\sum_{i=0}^{n}\left(\frac{13}{42}\right)^i < n\sum_{i=0}^{\infty}\left(\frac{13}{42}\right)^i = \frac{n}{1-\frac{13}{42}} = \frac{42}{29}n$

$T(n) = O(n)$

$T(n) = n\sum_{i=0}^{n}\left(\frac{13}{42}\right)^i \geq n\sum_{i=0}^{\log_7 n}\left(\frac{13}{42}\right)^i = n\frac{\left(\frac{13}{42}\right)^{\log_7 n+1} - 1}{\frac{13}{42} - 1} =$

$= \frac{42}{29}n\left(1 - \frac{13}{42}^{\log_7 n+1}\right)$

$1 - \left(\frac{13}{42}\right)^{\log_7 n+1} > 1 - \left(\frac{13}{42}\right)^{\log_7 n + 1} = \frac{29}{42}$, kai $n > 1$

$T(n) \geq \frac{42}{29}n \cdot \frac{29}{42} = n \qquad T(n) = \Omega(n)$

Galioja ir $\Theta(n)$, nes $n \leq T(n) < \frac{42}{29}n$

Visiems > 1

6. 4 uždavinyje (4 lentelė) gautą rekurentinės lygties atsakymą patikrinkite taikydami pagrindinę teoremą.

(14.) $T(n) = 4 \cdot T\left(\frac{n}{7}\right) + \sqrt{n}$

$a = 4$, $b = 7$, $f(n) = \sqrt{n}$ Tikriname 1-ąjį atvejį:

$$\lim_{n \to \infty} \frac{\sqrt{n}}{n^{\log_7 4 - \varepsilon}} = \lim_{n \to \infty} \frac{n^{\frac{1}{2}}}{n^{\log_7 4 - \varepsilon}} = \lim_{n \to \infty} n^{\frac{1}{2} - \log_7 4 - \varepsilon} =$$

$$= 0 \quad \text{Jei} \quad \frac{1}{2} - \log_7 4 - \varepsilon < 0$$

$$\varepsilon > -0{,}21$$

Atsakymas: $T(n) = \Theta\left(n^{\log_7 4}\right)$

7. Patikrinkite, ar 5 uždavinyje (5 lentelė) spręstą rekurentinę lygtį tenkina sprendinys $T(n) = O(n^2)$

(14.) $T(n) = O(n^2)$

$$T(n) = T\left(\frac{n}{6}\right) + T\left(\frac{n}{7}\right) + n$$

$$T\left(\frac{n}{6}\right) < c\left(\frac{n}{6}\right)^2 \quad ; \quad T\left(\frac{n}{7}\right) < c\left(\frac{n}{7}\right)^2$$

$$\frac{cn^2}{36} + \frac{cn^2}{49} + n < cn^2$$

$$\frac{85cn^2}{1764} + n < cn^2 \quad | : n^2$$

$$\frac{85c}{1764} + \frac{1}{n} < c$$

$$\frac{85cn + 1764}{1764n} < c$$

$$85cn + 1764 < 1764cn$$

$$1764 < 1679cn$$

$$c > \frac{1764}{1679}, \text{ visiems } n > 1$$

8. Pagal duotą uždavinio sąlygą (6 lentelė) užrašykite sprendimą rekurentiniu sąryšiu bendrajam atvejui ir pateikite uždavinio sprendimo pavyzdį taikydami dinaminį programavimą.

13

n x n

	1	1	4	5	2 → Pradedame
	1	2	3	1	6
Baigiame	4	1	5	3	2
	3	1	2	1	4
←	2	1	4	5	2

	i = 4	3	2	1	0	j
	13	12	11	7	2	0
	14/17	14/16	14/12	8/9	8	1
	21/24	17/20	13/18	12/13	10	2
	27/25	21/22	21/17	14/15	14	3
	29/28	23/26	25/25	20/21	16	4

if $i = 0$

sum[0, j] = matrix[0, j] + sum[0, j-1]

if $j = 0$

sum[i, 0] = matrix[i, 0] + sum[i-1, 0]

Others

sum[i, j] = matrix[i, j] + Max(sum[i-1, j], sum[i, j-1])

sum[size-1, size-1] = 29

Sudėtingumas – n^2.

Sprendžiant rekursiškai sudėtingumas būtų 2^n