Solow Model: Logistic Population Growth

Growth & Development: Theory
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Introduction and Methodology

The canonical Solow model (Solow 1956) assumes that the population size grows at a constant rate. In this project, I consider a logistic population growth function instead that converges to a finite amount of people in the economy. The goal is to understand how transitional dynamics of the Solow Growth Model are affected in light of a converging population size.
I compare model simulations for two economies that are, except for the population evolution, identical to each other. Labor force size is equal to population, production occurs in a constant returns to scale production function with capital share $\alpha = 1/3$ and capital deprecates with a rate of $\delta = 0.1$. I assume a savings rate of 20 percent.
The baseline model exhibits exponential population growth of the form $L_t^{exp} = L_0 e^{nt}$, with a constant growth rate $n = 0.053$ and $t \in \{1, 200\}$. Worker size of the benchmark model are assumed to grow according to an s-shaped function, analytically

$$L_t^{log} = \frac{L_{final}}{1 + e^{b(m-t))}}, \tag{1}$$

where the growth rate of the population increases over time until a turning point m is reached, and then decreases. $b = 0.1$ governs the steepness of the s-shape and L_{final} the final population size, respectively. The model is calibrated such that the final population sizes of both economies are equal to each other and the turning point m occurs at half-time.

Results

The top right graph of figure 1 shows the trajectories of per capita consumption, output and capital respectively, with population evolving according to equation 1. In comparison to the baseline model, the exponential growth path of absolute variables (c.f. bottom graphs of figure 1) is not maintained in the benchmark scenario because of slowing population growth.

Top two graphs of figure 2 show the evolution of returns to labor and capital input to production, respectively. They quickly approach a stable value due to the constant growth rate of population. Wages and interest rate experience a jump in the second half of the time frame due to the kick-off in

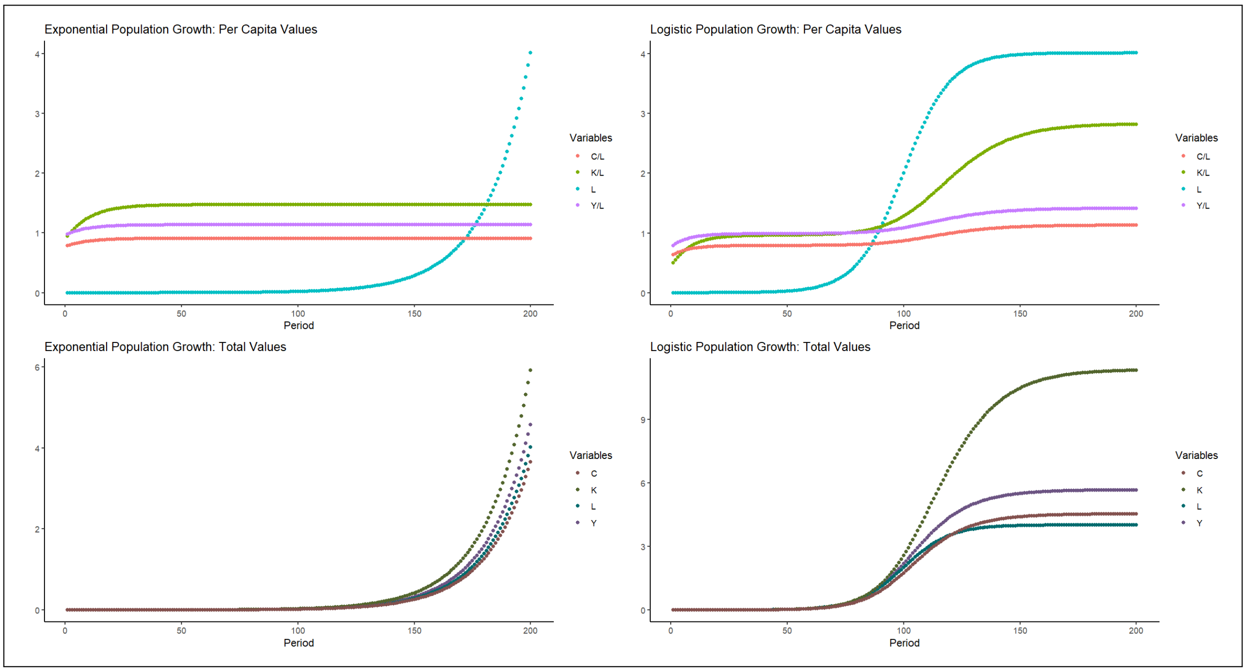

Figure 1: Transitional dynamics of total and per capita output, consumption, and capital, respectively. Left column: Exponential population growth scenario, right column: Logistic population growth scenario. Savings rate is set to 20 percent.

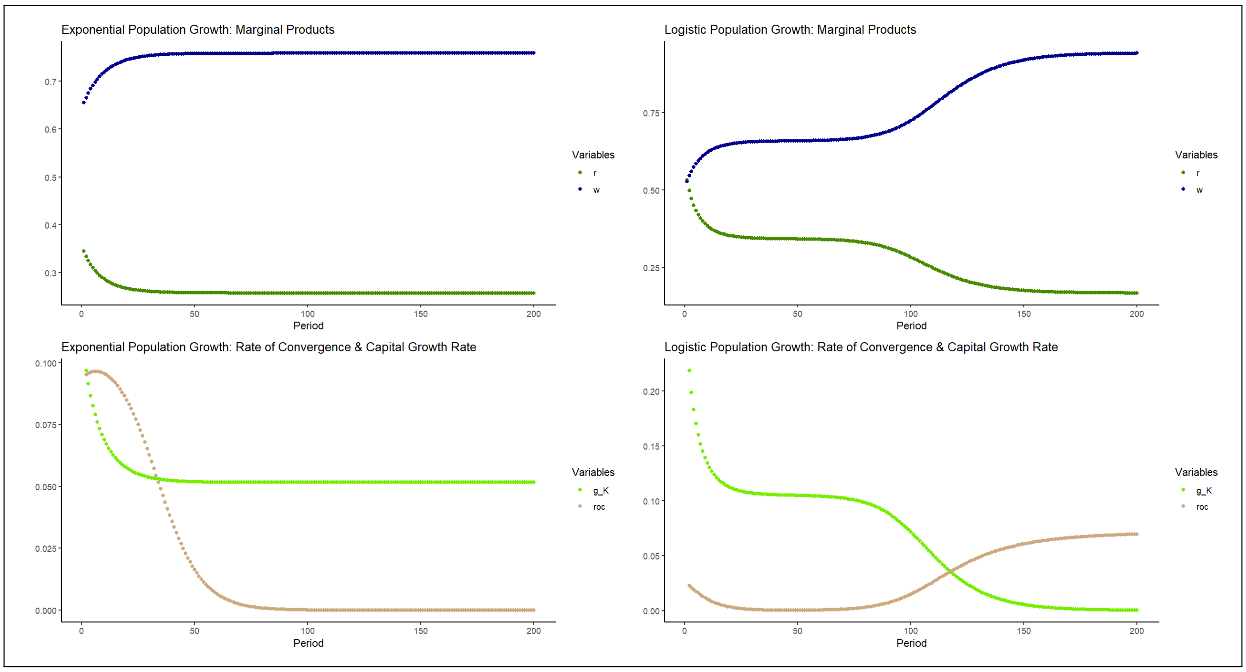

Figure 2: Transitional dynamics of wages, interest rates, capital growth rates and rates of convergence, respectively. Left column: Exponential population growth scenario, right column: Logistic population growth scenario. Savings rate is set to 20 percent.

population growth in the logistic population growth scenario.

Discussion

From the previous observations it gets clear that until the turning point (at period 100 in figure 1 and 2), the quality of the variable's movements are similiar in both scenarios. This shows that the benchmark model suits as an extension to the baseline scenario, where in the first half population grows exponentially. In the second half, growth declines and population approaches a final value. Looking at the rate of convergence (bottom graphs of figure 2), we can see that in the baseline scenario, convergence to the steady sate level of capital has almost finished, while the economy with logistic population growth is still in a converging transitional phase with a positive rate of convergence value in the final period. The steady state value of capital per person is $\frac{\tilde{K}}{L} = (\frac{s}{\delta})^{\frac{1}{1-\alpha}}$, which is a special case of the baseline scenario with population growth rate $n = 0$. In this calibration, capital per person is 2.820377 in the final period, which is close to the numerical steady state value of $\frac{\tilde{K}}{L} \sim 2.828427$. Repeating the simulation with a longer time horizon shows that capital per person does eventually approach the steady state value k_{ss}^{log}.

References

Solow, Robert M (1956). "A contribution to the theory of economic growth". In: *The quarterly journal of economics* 70.1, pp. 65–94.