Tanta University
Faculty of Engineering
Electrical Power and Machines Engineering Dept.

Electromagnetic Fields
Sheet 1 - Vector Analysis

1. The vector from the origin to point **A** is given as $(6, -2, -4)$, and the unit vector directed from the origin toward point **B** is $(2, -2, 1)/3$. If points **A** and **B** are ten units apart, find the coordinates of point **B**.

2. Find the actual angle between the two vectors $\overline{\mathbf{A}} = 2\overline{\mathbf{a}}_\mathbf{x} + \overline{\mathbf{a}}_\mathbf{y} + 3\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{B}} = \overline{\mathbf{a}}_\mathbf{x} - 3\overline{\mathbf{a}}_\mathbf{y} + 2\overline{\mathbf{a}}_\mathbf{z}$.

3. Given the points **M** $(0.1, -0.2, -0.1)$, **N** $(-0.2, 0.1, 0.3)$, and **P** $(0.4, 0, 0.1)$.

   (1) the vector $\overline{\mathbf{R}}_{MP}$ is:
   - (a) $(0.3, 0.2, 0.2)$
   - (b) $(2.0, 0.2, 0.3)$
   - (c) $-0.3\overline{\mathbf{a}}_\mathbf{x} + 0.2\overline{\mathbf{a}}_\mathbf{y} - 0.2\overline{\mathbf{a}}_\mathbf{z}$
   - (d) $0.3\overline{\mathbf{a}}_\mathbf{x} + 0.2\overline{\mathbf{a}}_\mathbf{y}$

4. Given the vectors $\overline{\mathbf{M}} = -10\overline{\mathbf{a}}_\mathbf{x} + 4\overline{\mathbf{a}}_\mathbf{y} - 8\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{N}} = 8\overline{\mathbf{a}}_\mathbf{x} + 7\overline{\mathbf{a}}_\mathbf{y} - 2\overline{\mathbf{a}}_\mathbf{z}$, find:

   (a) a unit vector in the direction of $-\overline{\mathbf{M}} + 2\overline{\mathbf{N}}$

   (b) the magnitude of $5\overline{\mathbf{a}}_x + \overline{\mathbf{N}} - 3\overline{\mathbf{M}}$ and

   (c) the value of $|M||2N|(\overline{\mathbf{M}} + \overline{\mathbf{N}})$.

5. Vector $\overline{\mathbf{A}}$ extends from the origin to point $(1, 2, 3)$, and vector $\overline{\mathbf{B}}$ extends from the origin to $(2, 3, -2)$. Find:

   (a) the unit vector in the direction of $(\overline{\mathbf{A}} - \overline{\mathbf{B}})$;

   (b) the unit vector in the direction of the line extending from the origin to the midpoint of the line joining the ends of $\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$.

6. Given the two vector $\overline{\mathbf{F}} = 10\overline{\mathbf{a}}_\mathbf{x} - 6\overline{\mathbf{a}}_\mathbf{y} + 5\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{G}} = 0.1\overline{\mathbf{a}}_\mathbf{x} + 0.2\overline{\mathbf{a}}_\mathbf{y} + 0.3\overline{\mathbf{a}}_\mathbf{z}$, find:

   (a) the vector component of $\overline{\mathbf{F}}$ that is parallel to $\overline{\mathbf{G}}$;

   (b) the vector component of $\overline{\mathbf{F}}$ that is perpendicular to $\overline{\mathbf{G}}$;

   (c) the vector component of $\overline{\mathbf{G}}$ that is perpendicular to $\overline{\mathbf{F}}$.

7. Three vectors extending from the origin are given as $\overline{\mathbf{R}}_1 = 7\overline{\mathbf{a}}_\mathbf{x} + 3\overline{\mathbf{a}}_\mathbf{y} - 2\overline{\mathbf{a}}_\mathbf{z}$, $\vec{R}_2 = -2\overline{\mathbf{a}}_\mathbf{x} + 7\overline{\mathbf{a}}_\mathbf{y} - 3\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{R}}_3 = 2\overline{\mathbf{a}}_\mathbf{y} + 3\overline{\mathbf{a}}_\mathbf{z}$. Find:

   (a) a unit vector perpendicular to $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

   (b) a unit vector perpendicular to the vectors $\overline{\mathbf{R}}_1 - \overline{\mathbf{R}}_2$ and $\overline{\mathbf{R}}_2 - \overline{\mathbf{R}}_3$;

   (c) the area of the triangle defined by $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

   (d) the area of the triangle defined by the heads of $\overline{\mathbf{R}}_1$, $\overline{\mathbf{R}}_2$, and $\overline{\mathbf{R}}_3$.

8. Express in cylindrical components:

   (a) the vector from $\mathtt{C}(3, 2, -7)$ to $\mathtt{D}(-1, -4, 2)$;

   (b) a unit vector at $\mathtt{D}$ directed toward $\mathtt{C}$;

   (c) a unit vector at $\mathtt{D}$ directed toward the origin.

9. The surfaces $r = 2$ and 4, $\theta = 30^\circ$ and 50°, and $\phi = 20^\circ$ and 60° identify a closed surface. Find:

   (a) the enclosed volume;

   (b) the total area of the enclosing surface;

   (c) the total length of the twelve edges of the surface;

10. Express the unit vector $\overline{\mathbf{a}}_x$ in spherical components at the point:

    (a) $r = 2\text{m}$, $\theta = 1\text{rad}$, $\phi = 0.8\text{rad}$;

    (b) $x = 3\text{m}$, $y = 2\text{m}$, $z = -1\text{m}$;

    (c) $\rho = 2.5\text{m}$, $\phi = 0.7\text{rad}$, $z = 1.5\text{m}$.

11. Express the field vector

$$\overline{\mathtt{H}} = xy^2z\overline{\mathbf{a}}_\mathbf{x} + x^2yz\overline{\mathbf{a}}_\mathbf{y} + xyz^2\overline{\mathbf{a}}_\mathbf{z}$$

    (a) In cylindrical and spherical coordinates.

    (b) In spherical coordinates

12. Given $\overline{\mathtt{A}} = 2\overline{\mathbf{a}}_\mathbf{x} + 4\overline{\mathbf{a}}_\mathbf{y} + 10\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathtt{B}} = -5\overline{\mathbf{a}}_\rho + \overline{\mathbf{a}}_\phi - 3\overline{\mathbf{a}}_\mathbf{z}$, find:

    (a) $\overline{\mathtt{A}} + \overline{\mathtt{B}}$ at $\mathtt{P}(0, +2, -5)$.

    (b) The angle between $\overline{\mathtt{A}}$ and $\overline{\mathtt{B}}$ at $\mathtt{P}$.

    (c) The scalar component of $\overline{\mathtt{A}}$ along $\overline{\mathtt{B}}$ at $\mathtt{P}$.

13. A vector field in mixed coordinate variables is given by:

$$\overline{\mathtt{G}} = \frac{x\cos\phi}{\rho}\overline{\mathbf{a}}_\mathbf{x} + \frac{2yz}{\rho^2}\overline{\mathbf{a}}_\mathbf{y} + \left(1 - \frac{x^2}{\rho^2}\right)\overline{\mathbf{a}}_\mathbf{z}$$

    Express $\overline{\mathtt{G}}$ completely in spherical system.

With All Best Wishes