Week(4) - DRAFT

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a) $Q = \int \rho_L \, \partial l = \int_0^5 12x^2 \, \partial x = \underline{500 \text{ mC}}$ ※

b) $Q = \iint \rho_s \, \partial s = \int_0^4 \int_0^{2\pi} \rho z^2 \, \rho \, \partial \phi \, \partial z = \int_0^4 9z^2 . 2\pi \, \partial z$ , $\rho = 3$

$= \underline{384 \pi \text{ nC}}$ ※

c) $Q = \iiint \rho_v \, \partial v = \iiint \frac{10}{\cancel{r \sin(\theta)}} . r^{\cancel{2}} \cancel{\sin(\theta)} \, \partial r \, \partial \theta \, \partial \phi$ ※

$= 10 \int_0^4 \int_0^{2\pi} \int_0^{\pi} r \, \partial \theta \, \partial \phi \, \partial r = \frac{10 . \pi . 2\pi}{2} . (4)^2 = \underline{1579.1367 \text{ C}}$

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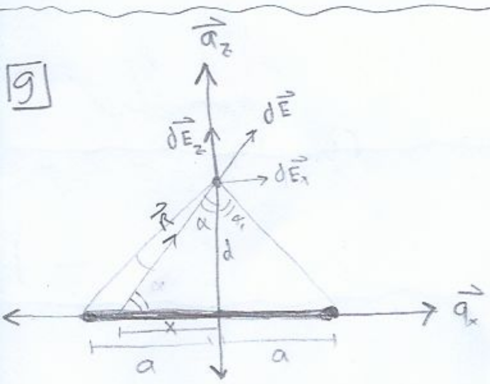


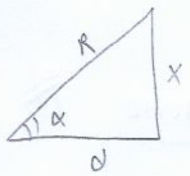

• Due to symmetry, there will be no component in the $(\vec{a}_x)$ direction.

$d\vec{E} = \frac{\rho_L \, \partial l}{4\pi\varepsilon_0 R^3} \vec{R}$ , $\vec{R} = -x \, \vec{a}_x + d \, \vec{a}_z$

$R = \sqrt{x^2 + d^2}$

$\therefore d\vec{E}_z = \frac{\rho_L . d}{4\pi\varepsilon_0 (\sqrt{x^2+d^2})^3} \, \partial l \, \vec{a}_z$ , $\therefore dL = dx$

$\therefore E_z = \frac{\rho_L . d}{4\pi\varepsilon_0} \int_{-a}^{a} \frac{1}{(x^2+d^2)^{3/2}} \, dx$




$\longrightarrow \tan(\alpha) = \frac{x}{d}$ $\therefore x = d\tan(\alpha)$ $\longrightarrow \therefore dx = d \sec^2(\alpha) \, d\alpha$

$\therefore \frac{1}{d^3} \int_{-a}^{a} \frac{d^3}{R^3} \, dx \longrightarrow \therefore \frac{1}{d^{\cancel{3}}} \int_{\alpha_1}^{\alpha_2} \cos^{\cancel{3}}(\alpha) . \cancel{d} . \frac{1}{\cos^2(\alpha)} \, d\alpha = \frac{1}{d^2} \int_{\alpha_1}^{\alpha_2} \cos(\alpha) \, d\alpha = \frac{\sin(\alpha_2) - \sin(\alpha_1)}{d^2}$

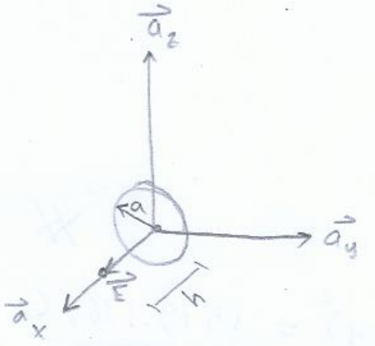
$\sin(\alpha_2) = \frac{a}{\sqrt{a^2+d^2}}$ , $\sin(\alpha_1) = \frac{-a}{\sqrt{a^2+d^2}}$ $\longrightarrow \therefore \vec{E}_z = \frac{\rho_L . d}{4\pi\varepsilon_0} . \frac{1}{d^2} . \frac{2a}{\sqrt{a^2+d^2}} \vec{a}_z$

$\therefore \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 d} . \frac{a}{\sqrt{a^2+d^2}} \vec{a}_z$

[10] Given: a Ring of charge, $(y^2+z^2=4)$ @ $x=0$

$\rho_L = 5\ \mu C/m$

Req: ⓐ find $\vec{E}$ @ $P(3,0,0)$ ⓑ exercise




Sol

$$\vec{E} = \frac{\rho_L}{2\varepsilon_0} \cdot \frac{a h}{(\sqrt{a^2+h^2})^3}\ \vec{a}_x$$

$a = \sqrt{4} = 2m$ , $h = 3m$

$\boxed{\therefore \vec{E} = 36143.27\ \vec{a}_x}$ V/m #

ⓑ Exercise

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[11] repeated

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[12] Exercise

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Sheet(1)-[9] given: $r\ (2 \to 4)$ , $\theta\ (30^\circ \to 50^\circ)$ , $\phi\ (20^\circ \to 60^\circ)$ $dL = dr\,\vec{a}_r + r\,d\theta\,\vec{a}_\theta + r\sin(\theta)\,d\phi\,\vec{a}_\phi$

ⓐ enclosed Volume $= \iiint dv = \int_2^4\int_{20}^{60}\int_{30}^{50} r^2\sin(\theta)\,d\theta\,d\phi\,dr = \underline{2.9092}$ Unit Volume #

ⓑ total surface area $= \int_{20}^{60}\int_{30}^{50} r^2\sin(\theta)\,d\theta\,d\phi\Big|_{r=2} + \int_{20}^{60}\int_{30}^{50} r^2\sin(\theta)\,d\theta\,d\phi\Big|_{r=4}$

$+ \int_{20}^{60}\int_{2}^{4} r\sin(\theta)\,dr\,d\phi\Big|_{\theta=30^\circ} + \int_{20}^{60}\int_{2}^{4} r\sin(\theta)\,dr\,d\phi\Big|_{\theta=50^\circ}$

$+ \int_{30}^{50}\int_{2}^{4} r\,dr\,d\theta\Big|_{\phi=20^\circ} + \int_{30}^{50}\int_{2}^{4} r\,dr\,d\theta\Big|_{\phi=60}$

$= 3.117 + 5.303 + 4.189$

$= \underline{12.609}$ Unit squared #

(c) total length $= 4\int_{2}^{4} \partial r + 2\int_{30}^{50} r\,\partial\theta\Big|_{r=2} + 2\int_{30}^{50} r\,d\theta\Big|_{r=4}$

$$+ \int_{20}^{60} r\sin(\theta)\,\partial\phi\Big|_{\substack{r=2\\ \theta=30^\circ}} + \int_{20}^{60} r\sin(\theta)\,\partial\phi\Big|_{\substack{r=2\\ \theta=50^\circ}} + \int_{20}^{60} r\sin(\theta)\,\partial\phi\Big|_{\substack{r=4\\ \theta=30^\circ}} + \int_{20}^{60} r\sin(\theta)\,d\phi\Big|_{\substack{r=4\\ \theta=50^\circ}}$$

$$= 4(2) + \frac{4}{3}\pi + \frac{2}{9}\pi + 1.0696 + \frac{4}{9}\pi + 2.1392$$

$= 17.492$ Unit Length