Electrical Power Engineering Department

Electromagnetic Fields (EPE112&EPM2142)

Tutorial

Week(3)

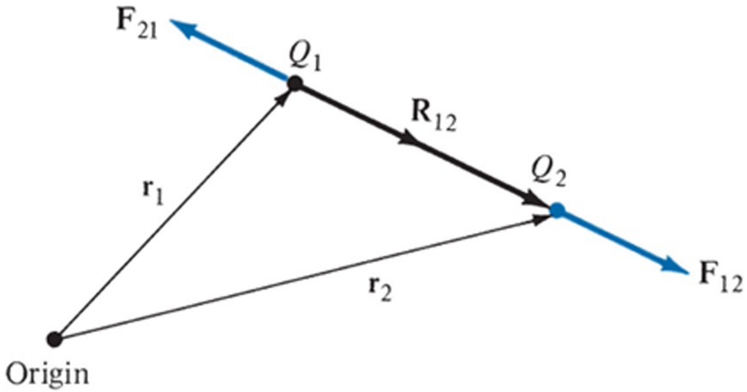
ELECTROSTATIC FIELDS

Tutorial Objectives

- Coulomb’s law and field intensity
- Electric fields due to continuous charge distributions
  - A Line Charge
  - A Surface Charge
  - A Volume Charge
- Gauss’s law

Coulomb's law and field intensity

- Coulomb's law states that the force $\boldsymbol{F}$ between two point charges Q_1 and Q_2 is:
  1. Along the line joining them
  2. Directly proportional to the product $Q_1 Q_2$ of the charges
  3. Inversely proportional to the square of the distance R between them.



$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\mathbf{a}_{R_{12}}$$

Coulomb's law and field intensity

Notes

- Like charges (charges of the same sign) repel each other, while unlike charges attract.
- Q1 and Q2 must be static (at rest).
- The signs of Q1 and Q2 must be taken into account.
- Charges cannot be created or destroyed; the quantity of total charge remains constant.

Coulomb's law and field intensity

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_N$$

$$= \frac{QQ_1(\mathbf{r} - \mathbf{r}_1)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|^3} + \frac{QQ_2(\mathbf{r} - \mathbf{r}_2)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|^3} + \cdots + \frac{QQ_N(\mathbf{r} - \mathbf{r}_N)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_N|^3}$$

$$\boxed{\mathbf{F} = \frac{Q}{4\pi\varepsilon_o} \sum_{k=1}^{N} \frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}}$$

Coulomb's law and field intensity

The ***electric field*** intensity (or ***electric field strength***) **E** is the force that a unit positive charge experiences when placed in an electric field.

$$\boxed{\mathbf{E} = \frac{\mathbf{F}}{Q}} \longrightarrow \boxed{\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r}$$

Coulomb’s law and field intensity

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots + \mathbf{E}_N$$

$$= \frac{Q_1(\mathbf{r} - \mathbf{r}_1)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|^3} + \frac{Q_2(\mathbf{r} - \mathbf{r}_2)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|^3} + \cdots + \frac{Q_N(\mathbf{r} - \mathbf{r}_N)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_N|^3}$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}$$

EXERCISES

1. A $20\,\mathrm{nC}$ point charge is located at $\boldsymbol{P}(2, 4, -3)$ in free space. The field $\overline{\mathbf{E}}$ at $\boldsymbol{A}(-3, 2, 0)$ is:

5. Point charges Q_1 and Q_2 are, respectively, located at $(4, 0, -3)$ and $(2, 0, 1)$. If $Q_2 = 4\ \mathrm{nC}$, which is the value of Q_1 such that:

(i) the $\overline{\mathbf{E}}$ at (5, 0, 6) has no z-component.

(i) The force on a test charge at $(5, 0, 6)$ has no x-component.

7. Point charges $Q_1 = 5\ \mu\text{C}$ and $Q_2 = -4\ \mu\text{C}$ are placed at $(3, 2, 1)$ and $(-4, 0, 6)$, respectively. Determine the force on Q_1.

Electric fields due to continuous charge distributions

$$\mathbf{E} = \int_L \frac{\rho_L \, dl}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(line charge)}$$

$$\mathbf{E} = \int_S \frac{\rho_S \, dS}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(surface charge)}$$

$$\mathbf{E} = \int_v \frac{\rho_v \, dv}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(volume charge)}$$

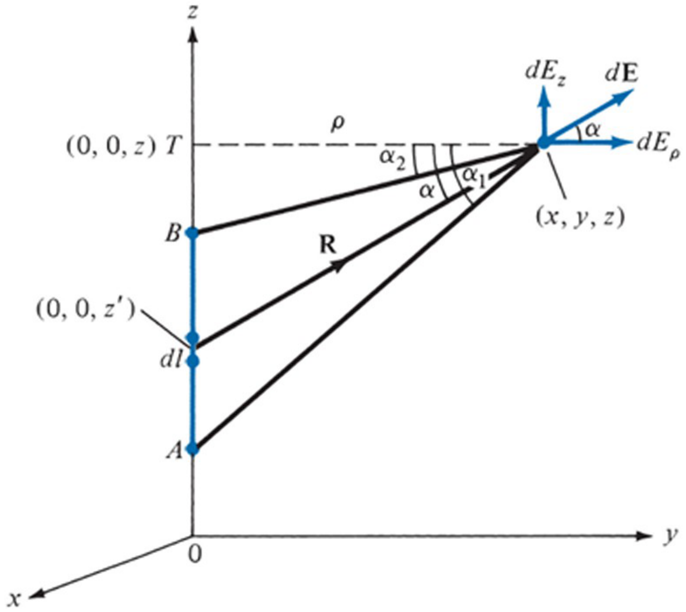
A Line Charge

Thus for a *finite line charge,*

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_o\rho}[-(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z]$$

for an *infinite line charge,*

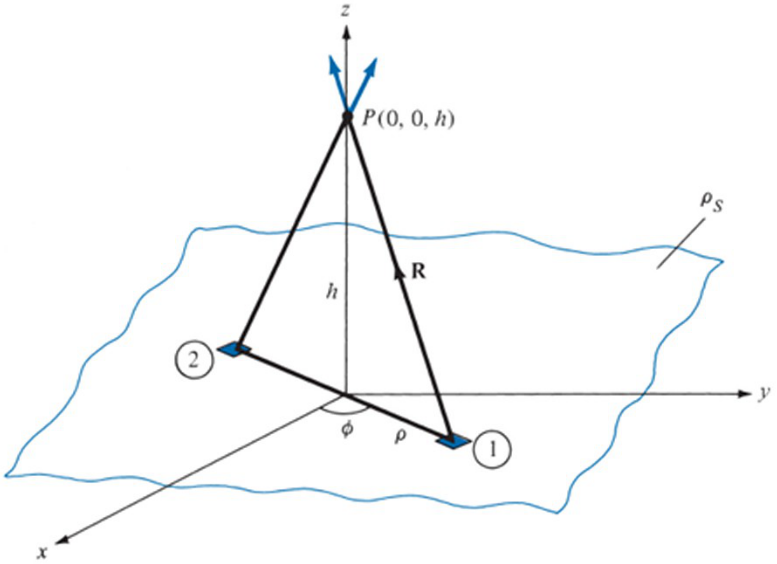
$$\boxed{\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\mathbf{a}_\rho}$$



A Surface Charge

for an *infinite sheet* of charge

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\,\mathbf{a}_n}$$



EXERCISES

4. Plane $z = -10$ m carries charge $-20\,\mathrm{nC/m^2}$. The electric field intensity at the origin is ——- V/m.

6. A sheet of charges $\rho_s = 2\ \mathrm{nC/m^2}$, is present at the plane $x = 3$ in free space, and a line charge $\rho_\ell = 2\ \mathrm{nC/m}$ is located at $x = 1$, $z = 4$. Find:

   (i) The magnitude of electric field intensity at the origin.

   (ii) The direction of $\overline{\mathbf{E}}$ at (4, 5, 6).

   (iii) What is the force per meter length on the line charge?

Gauss’s law

- ***Gauss’s law*** states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface.

$$\Psi = Q_{\text{enc}}$$

EXERCISES

2. Electric —— at a point may be defined as equal to lines of force passing normally through a unit cross-section at that point.

(a) field density

(b) field intensity

(c) flux density

(d) potential

3. Point charges $30\,\mathrm{nC}$, $-20\,\mathrm{nC}$, and $10\,\mathrm{nC}$ are located at $(-1, 0, 2)$, $(0, 0, 0)$, and $(1, 5, -1)$, respectively. The total flux leaving a cube of side 6 m centered at the origin is:

QUESTIONS ?

THANKS

References

- ELEMENTS OF ELECTROMAGNETICS, MATTHEW N. O. SADIKU