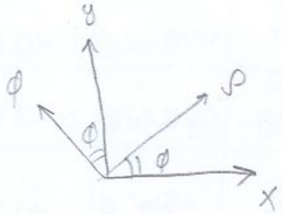
Week (4)



8 Given: $\overrightarrow{CD} = D - C = (-4, -6, 9)$

(A) Express $\overrightarrow{CD}$ in cylin. Cordin.

	-4 $\vec{a}_x$	-6 $\vec{a}_y$	9 $\vec{a}_z$
$\vec{a}_\rho$	$\cos(\phi)$	$\sin(\phi)$	0
$\vec{a}_\phi$	$-\sin(\phi)$	$\cos(\phi)$	0
$\vec{a}_z$	0	0	1

$$\overrightarrow{CD} = (-4\cos(\phi) - 6\sin(\phi))\,\vec{a}_\rho + (+4\sin(\phi) - 6\cos(\phi))\,\vec{a}_y + 9\,\vec{a}_z$$

\# a

(B) $\overrightarrow{DC} = -\overrightarrow{CD}$, $|\overrightarrow{CD}| = \sqrt{4^2 + 6^2 + 9^2} = \sqrt{133}$

$\therefore \vec{a}_{DC} = \frac{1}{\sqrt{133}}(-\overrightarrow{CD})$, at D. $\phi = 180 + \tan^{-1}\left(\frac{4}{1}\right) = 255.964^\circ$

$$\boxed{\therefore \vec{a}_{DC} = -0.5889\,\vec{a}_\rho + 0.3155\,\vec{a}_\phi - 0.7804\,\vec{a}_z}$$

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(C) at D $\phi = 255.964^\circ$

$\overrightarrow{DO} = (1, 4, -2)$. , $|\overrightarrow{DO}| = \sqrt{21}$

$$\overrightarrow{DO} = (\cos(\phi) + 4\sin(\phi))\,\vec{a}_\rho + (-1\sin(\phi) + 4\cos(\phi))\,\vec{a}_\phi + (-2)\,\vec{a}_z$$

$\therefore \longrightarrow \vec{a}_{DO} = \frac{\overrightarrow{DO}}{|\overrightarrow{DO}|}$

$$\boxed{\therefore \vec{a}_{DO} = -0.8997\,\vec{a}_\rho + 0\,\vec{a}_\phi - 0.4364\,\vec{a}_z}$$

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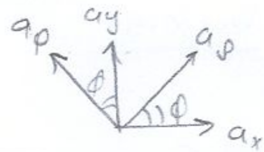
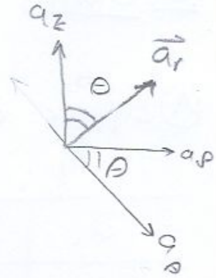
$\boxed{10}$

	$\vec{a}_x$	$\vec{a}_y$	$\vec{a}_z$
$\vec{a}_r$	$\sin(\theta)\cos(\phi)$	$\sin(\theta)\sin(\phi)$	$\cos(\theta)$
$\vec{a}_\theta$	$\cos(\theta)\cos(\phi)$	$\cos(\theta)\sin(\phi)$	$-\sin(\theta)$
$\vec{a}_\phi$	$-\sin(\phi)$	$\cos(\phi)$	0





ⓐ at $\theta = 1^r$, $\phi = 0.8^r$, $r = 2$

$$\vec{a}_x = \sin(\theta)\cos(\phi)\,\vec{a}_r + \cos(\theta)\cos(\phi)\,\vec{a}_\theta - \sin(\phi)\,\vec{a}_\phi = \underbrace{0.5863\,\vec{a}_r + 0.3764\,\vec{a}_\theta - 0.7174\,\vec{a}_\phi}$$ ⋕

ⓑ at $(3, 2, -1)$ $\rightarrow$ $\phi = \tan^{-1}\left(\frac{2}{3}\right) = 0.588^r$

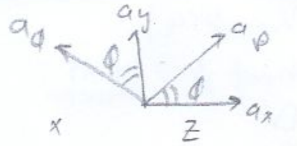
$$\theta = \pi - \tan^{-1}\left(\frac{\sqrt{4+9}}{1}\right) = 1.8413^r$$

$$\therefore \underbrace{\vec{a}_x = 0.8018\,\vec{a}_r - 0.2224\,\vec{a}_\theta - 0.5547\,\vec{a}_\phi}$$ ⋕

ⓒ at $\phi = 0.7^r$, $\theta = \tan^{-1}\left(\frac{\rho}{z}\right) = 1.0304^r$

$$\underbrace{\vec{a}_x = 0.6558\,\vec{a}_r + 0.3935\,\vec{a}_\theta - 0.6442\,\vec{a}_\phi}$$ ⋕

11 $\vec{H} = xy^2z\,\vec{a}_x + x^2yz\,\vec{a}_y + xyz^2\,\vec{a}_z$



(a) In Cylinderical Coord

$$\vec{H} = xyz\begin{bmatrix} y\cos(\phi) + x\sin(\phi) & \vec{a}_\rho \\ -y\sin(\phi) + x\cos(\phi) & \vec{a}_\phi \\ z & \vec{a}_z \end{bmatrix}$$

	y a_x	x a_y	z a_z
a_ρ	$\cos(\phi)$	$\sin(\phi)$	0
a_ϕ	$-\sin(\phi)$	$\cos(\phi)$	0
a_z	0	0	1

$x = \rho\cos(\phi)$
$y = \rho\sin(\phi)$
$z = z$

$$\vec{H} = \frac{\rho^2}{2}\sin(2\phi)\,z\begin{bmatrix} \frac{\rho}{2}\sin(2\phi) + \frac{\rho}{2}\sin(2\phi) & \vec{a}_\rho \\ -\rho\sin^2(\phi) + \rho\cos^2(\phi) & \vec{a}_\phi \\ z & \vec{a}_z \end{bmatrix}$$

$$\therefore \vec{H} = \frac{\rho^2}{2}\sin(2\phi)\,z\begin{bmatrix} \rho\sin(2\phi) & \vec{a}_\rho \\ \rho\cos(2\phi) & \vec{a}_\phi \\ z & \vec{a}_z \end{bmatrix} = \boxed{\frac{\rho^3}{2}z\sin^2(2\phi)\,\vec{a}_\rho + \frac{\rho^3}{4}z\sin(4\phi)\,\vec{a}_\phi + \frac{\rho^2}{2}z^2\sin(2\phi)\,\vec{a}_z}$$

(b) Exercise for students.

12 Exercise

13 Exercise

Week(4)
Sheet 2 - DRAFT

[1]

Given: $Q_1 = 20\,nC$, $P(2,4,-3)$, Req: Calc $\vec{E}$ at $(-3,2,0)$

Sol

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{a_r}$$

$\vec{r} = (-3,2,0) - (2,4,-3) = (-5,-2,3)$

$R = |\vec{r}| = \sqrt{38}$, $\vec{a_r} = -0.8111\vec{a_x} - 0.3244\vec{a_y}$ $0.4867\vec{a_z}$

$$\vec{E} = -3.8368\vec{a_x} - 1.534\vec{a_y} + 2.302\vec{a_z}\ N/C$$

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[3] According to Gauss's law, the total electric flux through any closed surface is equal to the total charge enclosed by that surface.

$\therefore Q_{enc} = 30 - 20 = 10\,C$

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[4]

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\vec{a_n}, \quad \vec{a_n} = \vec{a_z}, \quad \therefore \vec{E} = \frac{-20\times10^{-9}}{2\,\varepsilon_0}\vec{a_z} = -360\pi\vec{a_z}\ V/m$$

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[5] $Q_1 = ??$ @ $(4,0,-3)$ $\quad Q_2 = 4\,nC$ @ $(2,0,1)$

(i) E @ (5,0,6) has no z-component.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(Q_1\frac{\vec{r_1}}{r_1^3} + Q_2\frac{\vec{r_2}}{r_2^3}\right)$$

$\vec{r_1} = (1,0,9)$, $\vec{r_2} = (3,0,5)$

$$E_z = 0 \quad \therefore Q_1 \cdot \frac{9}{(\sqrt{82})^3} + (4\times10^{-9}) \cdot \frac{5}{(\sqrt{34})^3} = 0 \longrightarrow \therefore Q_1 = -8.323\,nC$$

#

ii)

$$Q_1 \cdot \frac{1}{(\sqrt{82})^3} + (4\times10^{-9}) \cdot \frac{3}{(\sqrt{34})^3} = 0 \longrightarrow \therefore Q_1 = -45\,nC$$

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6 $\rho_s = 2\,nC/m^2$ @ $x=3$, $\rho_L = 2\,nC/m$ @ $x=1$, $z=4$

i) find $\underline{E}$ @ origin:

• due to sheet of charges:

$\hookrightarrow \vec{E}_1 = \frac{-\rho_s}{2\varepsilon_0}\vec{a}_x = -112.941\vec{a}_x\,V/m$

• Due to line of Charges

$\hookrightarrow \vec{E}_2 = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$

$= \frac{2\times10^{-9}}{2\pi\varepsilon_0}\cdot\frac{\vec{\rho}}{\rho^2}$

$= -2.1147\vec{a}_x - 8.4588\vec{a}_z\,V/M$

$P_{\text{on the line}} = (1, \lambda, 4)$

$\vec{\rho} = \vec{PO} = (-1, -\lambda, -4)$

$\because \vec{PO} \perp$ line of charges $\rightarrow \therefore -1\times0 + -\lambda\times1 + -4\times0 = 0$

$\therefore \vec{\rho} = -1\,\vec{a}_x - 4\,\vec{a}_z$

$\rho = \sqrt{17}$

$\therefore \vec{E}_t = \vec{E}_1 + \vec{E}_2 = -115.0557\vec{a}_x - 8.4588\vec{a}_z$ $\therefore |\vec{E}_t| = \underline{115.366}$ V/m or N/C

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(ii)

• Due to plane:

$\vec{E}_1 = \frac{\rho_s}{2\varepsilon_0}\vec{a}_x = 112.941\,\vec{a}_x$ V/m

• Due to line:

$\vec{E}_2 = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$

, $P(1, \lambda, 4)$ $\vec{PA} = (3, 5-\lambda, 2)$

$\therefore \vec{PA} \perp$ Line $\rightarrow \therefore 3\times0 + (5-\lambda)\times1 + 2\times0 = 0$

$\therefore \lambda = 5 \longrightarrow \therefore \vec{PA} = (3, 0, 2)$

$\vec{E}_2 = 8.2962\,\vec{a}_x + 5.5308\vec{a}_z$

$\therefore \vec{E}_t = 121.2372\,\vec{a}_x + 5.5308\vec{a}_z$ $|\vec{E}_t| = 121.363$ V/m

$\therefore \vec{a}_{\vec{E}_t} = \frac{\vec{E}_t}{|\vec{E}_t|} = \vec{\square}$ #

iii) $\vec{E}$ on the line:

$$\vec{E} = \frac{-\rho_s}{2\varepsilon_0}\vec{a}_x = \frac{-2\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}\vec{a}_x = -36\pi\,\vec{a}_x$$

$\because \vec{F} = Q\vec{E}$ $\quad \therefore F/L = \rho_L \vec{E} = -0.2262\,\vec{a}_x \ \mu N/m$

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[7] $Q_1 = 5\mu C$ @ $(3,2,1)$, $Q_2 = -4\mu C$ @ $(-4,0,6)$

$$\vec{F}_{Q_1} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\cdot\frac{\vec{R}_{21}}{|\vec{R}_{21}|^3}$$

$\vec{R}_{21} = -(-4,0,6) + (3,2,1) = (+7, +2, -5)$

$|\vec{R}_{21}| = \sqrt{78}$ m

$\therefore \vec{F}_{Q_1} = (-1.8265\,\vec{a}_x - 0.5219\,\vec{a}_y + 1{,}3047\,\vec{a}_z)$ mN