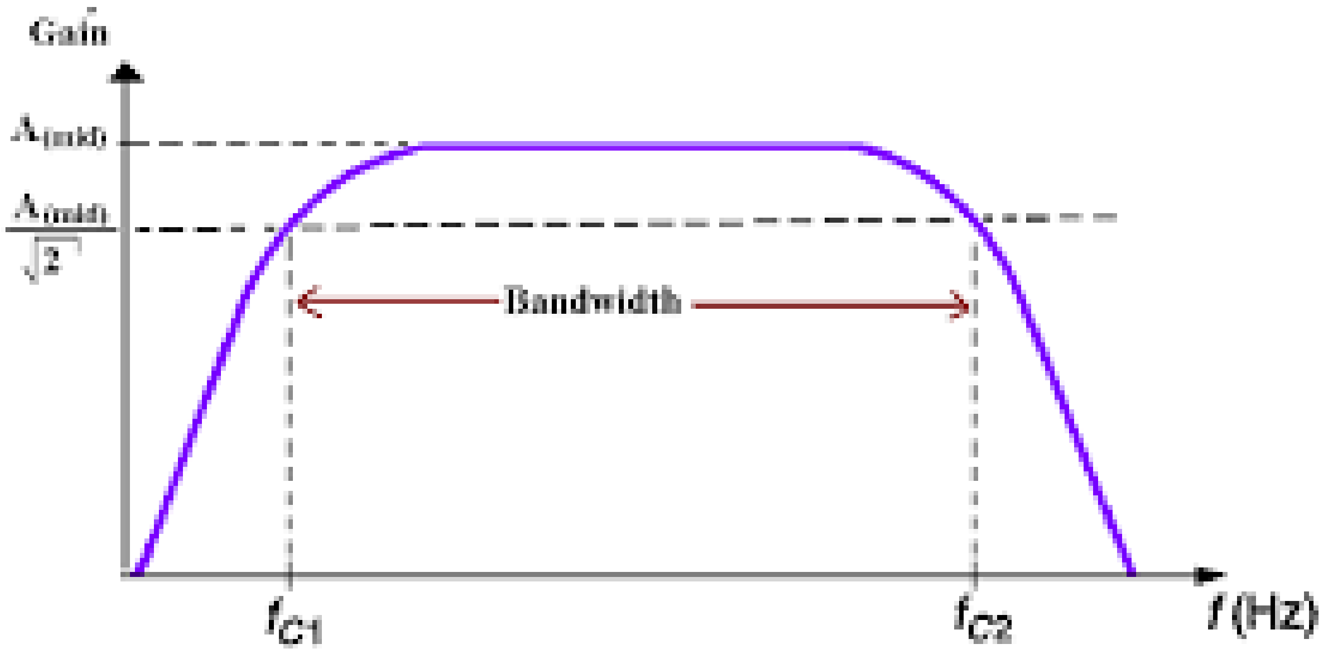
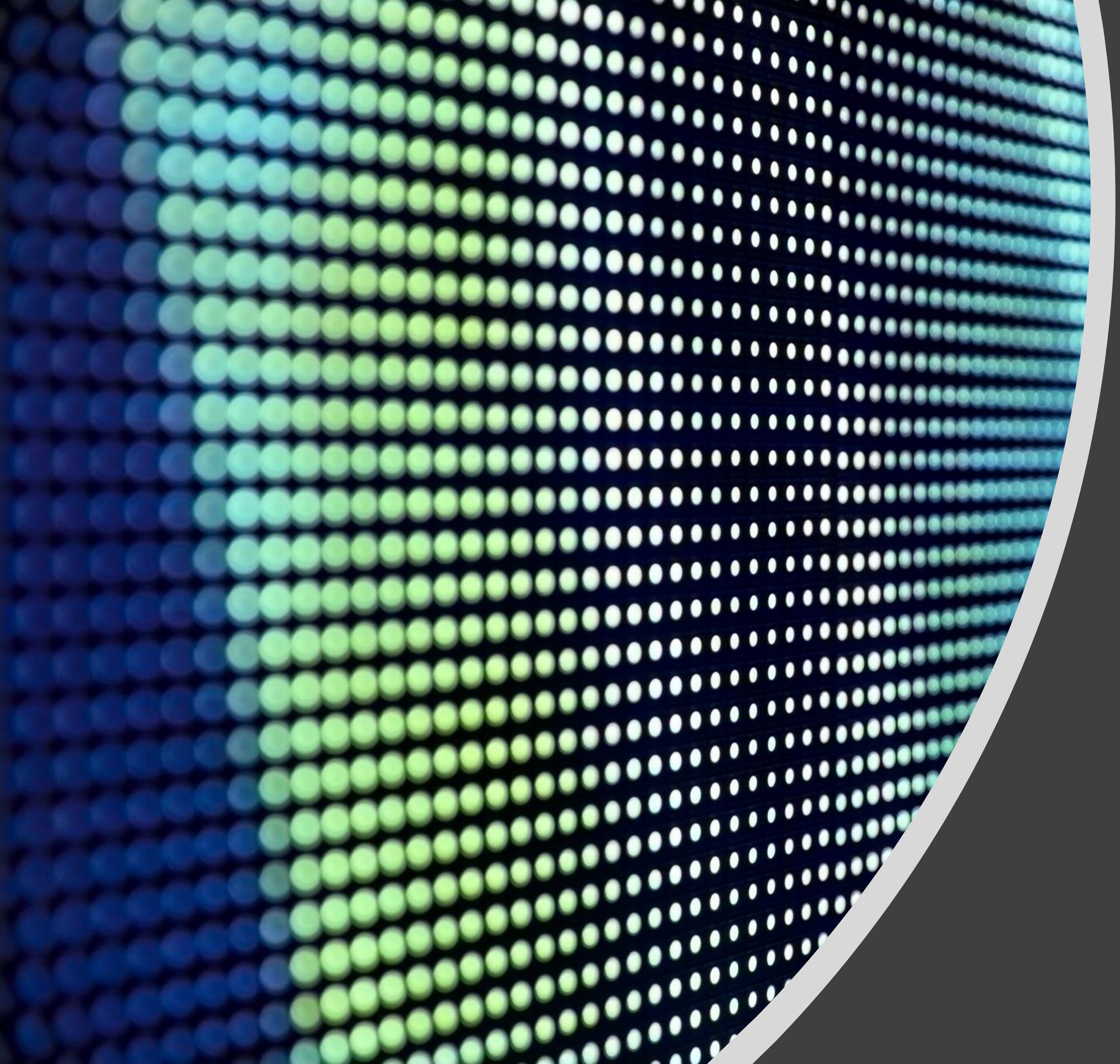
# Electronic Circuits (1) EEC2103




LEC (4) BJT Frequency Response

Dr. Nancy Alshaer



# Low Frequency Amplifier Response

# BJT Amplifier Low Frequency Response

Assuming that the coupling and bypass capacitors are ideal shorts at the midrange signal frequency, you can determine the midrange voltage gain using Equation 10–5, where $R_c = R_C \| R_L$.

$$A_{v(mid)} = \frac{R_c}{r'_e}$$

If a swamping resistor ($R_{E1}$) is used, it appears in series with $r'_e$ and the equation becomes

$$A_{v(mid)} = \frac{R_c}{r'_e + R_{E1}}$$

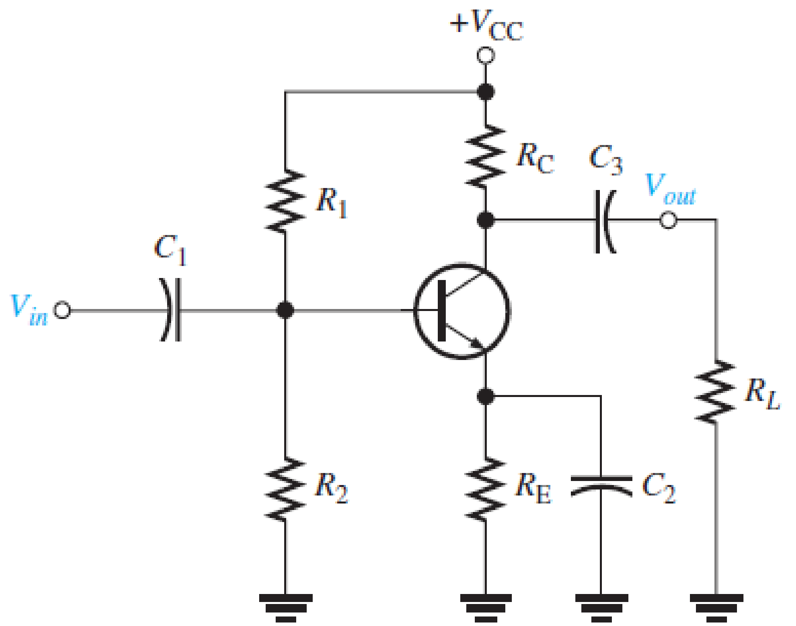



FIGURE 10–8

A capacitively coupled BJT amplifier.

# BJT Amplifier Low Frequency Response

The BJT amplifier in Figure 10–8 has three high-pass *RC* circuits that affect its gain as the frequency is reduced below midrange. These are shown in the low-frequency ac equivalent circuit in Figure 10–9. Unlike the ac equivalent circuit used in previous chapters, which represented midrange response ($X_C \cong 0\ \Omega$), the low-frequency equivalent circuit retains the coupling and bypass capacitors because $X_C$ is not small enough to neglect when the signal frequency is sufficiently low.

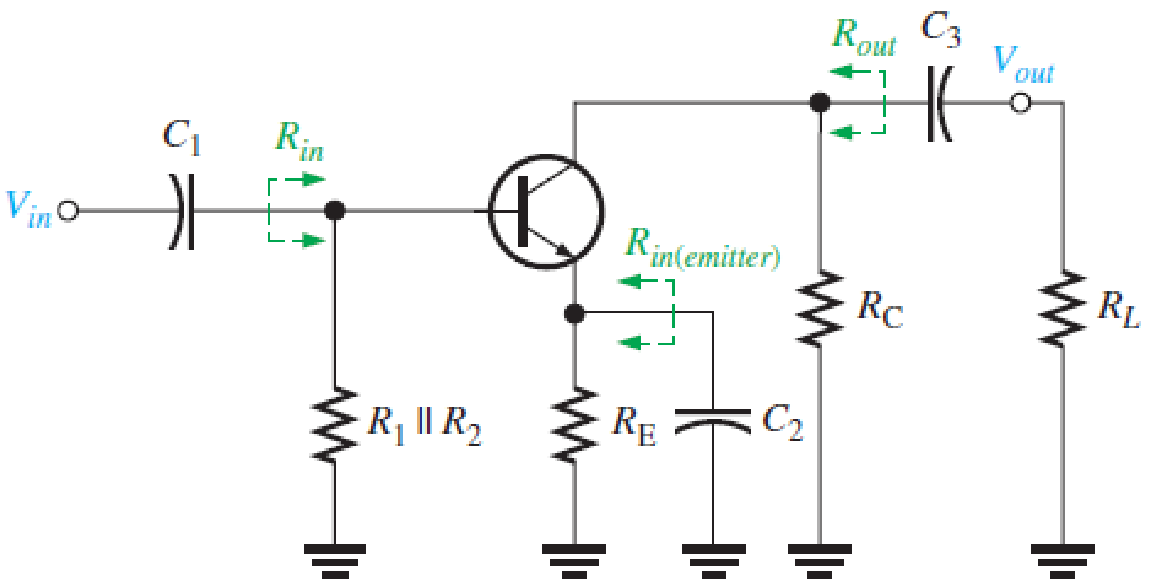



**FIGURE 10–9**

The low-frequency ac equivalent circuit of the amplifier in Figure 10–8 consists of three high-pass *RC* circuits.

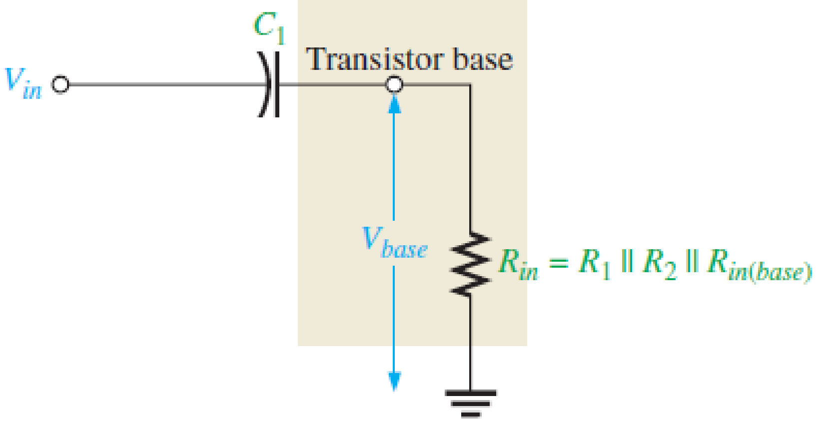
# BJT Amplifier Low Frequency Response

## The Input *RC* Circuit

$$V_{base} = \left(\frac{R_{in}}{\sqrt{R_{in}^2 + X_{C1}^2}}\right)V_{in}$$

**▶ FIGURE 10–10**

Input *RC* circuit formed by the input coupling capacitor and the amplifier's input resistance.




As previously mentioned, a critical point in the amplifier's response occurs when the output voltage is 70.7% of its midrange value. This condition occurs in the input *RC* circuit when $X_{C1} = R_{in}$.

$$V_{base} = \left(\frac{R_{in}}{\sqrt{R_{in}^2 + R_{in}^2}}\right)V_{in} = \left(\frac{R_{in}}{\sqrt{2R_{in}^2}}\right)V_{in} = \left(\frac{R_{in}}{\sqrt{2}R_{in}}\right)V_{in} = \left(\frac{1}{\sqrt{2}}\right)V_{in} = 0.707V_{in}$$

In terms of measurement in decibels,

$$20\log\left(\frac{V_{base}}{V_{in}}\right) = 20\log(0.707) = -3\text{ dB}$$

# BJT Amplifier Low Frequency Response

***Lower Critical Frequency*** The condition where the gain is down 3 dB is logically called the $-3$ *dB point* of the amplifier response; the overall gain is 3 dB less than at midrange frequencies because of the attenuation (gain less than 1) of the input *RC* circuit. The frequency, $f_{cl}$, at which this condition occurs is called the *lower critical frequency* (also known as the *lower cutoff frequency, lower corner frequency,* or *lower break frequency*) and can be calculated as follows:

$$X_{C1} = \frac{1}{2\pi f_{cl(input)} C_1} = R_{in}$$

$$\boxed{f_{cl(input)} = \frac{1}{2\pi R_{in} C_1}}$$

If the resistance of the input source is taken into account, Equation 10–6 becomes

$$\boxed{f_{cl(input)} = \frac{1}{2\pi (R_s + R_{in}) C_1}}$$

Study Example 10-3

$f_{cl(input)}$=282 Hz

# BJT Amplifier Low Frequency Response

***Voltage Gain Roll-Off at Low Frequencies*** As you have seen, the input $RC$ circuit reduces the overall voltage gain of an amplifier by 3 dB when the frequency is reduced to the critical value $f_c$. As the frequency continues to decrease below $f_c$, the overall voltage gain also continues to decrease. The rate of decrease in voltage gain with frequency is called **roll-off**. For each ten times reduction in frequency below $f_c$, *there is a 20 dB reduction in voltage gain.*

Let's consider a frequency that is one-tenth of the critical frequency ($f = 0.1f_c$). Since $X_{C1} = R_{in}$ at $f_c$, then $X_{C1} = 10R_{in}$ at $0.1f_c$ because of the inverse relationship of $X_{C1}$ and $f$. The attenuation of the input $RC$ circuit is, therefore,

$$\text{Attenuation} = \frac{V_{base}}{V_{in}} = \frac{R_{in}}{\sqrt{R_{in}^2 + X_{C1}^2}} = \frac{R_{in}}{\sqrt{R_{in}^2 + (10R_{in})^2}} = \frac{R_{in}}{\sqrt{R_{in}^2 + 100R_{in}^2}}$$

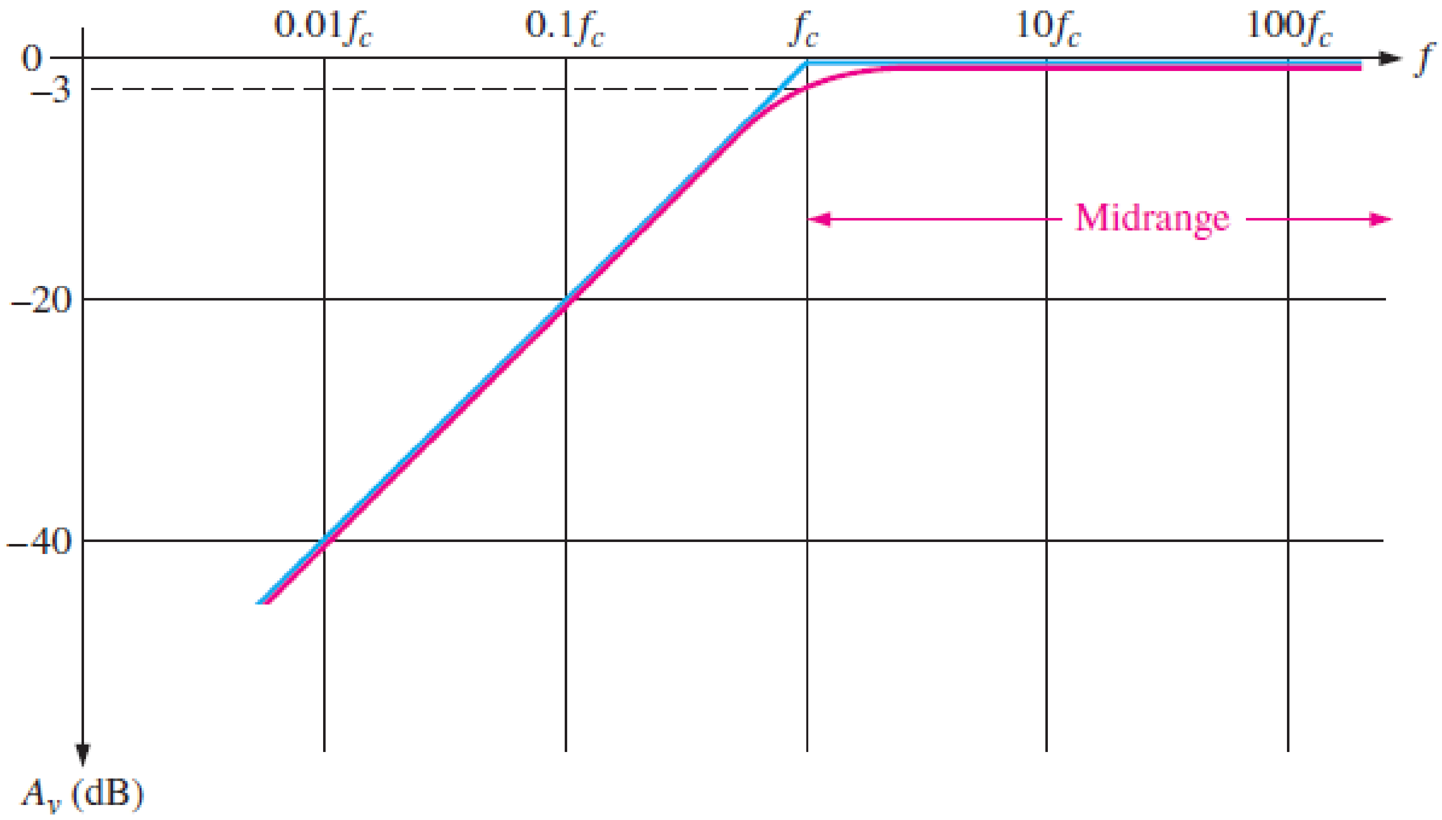
$$= \frac{R_{in}}{\sqrt{R_{in}^2(1 + 100)}} = \frac{R_{in}}{R_{in}\sqrt{101}} = \frac{1}{\sqrt{101}} \cong \frac{1}{10} = 0.1$$

The dB attenuation is

$$20\log\left(\frac{V_{base}}{V_{in}}\right) = 20\log(0.1) = -20\text{ dB}$$

# BJT Amplifier Low Frequency Response

***The Bode Plot*** A ten-times change in frequency is called a **decade**. So, for the input *RC* circuit, the attenuation is reduced by 20 dB for each decade that the frequency decreases below the critical frequency. This causes the overall voltage gain to drop 20 dB per decade.




Study Example 10-4

▲ FIGURE 10–12

Bode plot. (Blue is ideal; red is actual.)

# BJT Amplifier Low Frequency Response

- Sometimes, the voltage gain roll-off of an amplifier is expressed in **dB/octave** rather than **dB/decade**.
- A decade is defined as any 10-to-1 frequency range.
- An octave is defined as any 2-to-1 frequency range
- An **octave** corresponds to a doubling or halving of the frequency.
- For example, an increase in frequency from 100 Hz to 200 Hz is an octave.
- Likewise, a decrease in frequency from 100 kHz to 50 kHz is also an octave.
- **-20 dB/decade ≅ -6 dB/octave**, **-40 dB/decade ≅ -12 dB/octave**, and so on.
- Notes:

- Number of octaves= $log_2\left(\frac{f_2}{f_1}\right)$
- Number of decades= $log_{10}\left(\frac{f_2}{f_1}\right)$

```
dB/octave = (dB/decade) / 3.322
```

$Log_2(10)/Log_2(2)=3.322$

# BJT Amplifier Low Frequency Response

***Phase Shift in the Input RC Circuit*** In addition to reducing the voltage gain, the input *RC* circuit also causes an increasing phase shift through an amplifier as the frequency decreases.

$$\theta = \tan^{-1}\left(\frac{X_{C1}}{R_{in}}\right)$$

For midrange frequencies, $X_{C1} \cong 0\ \Omega$, so

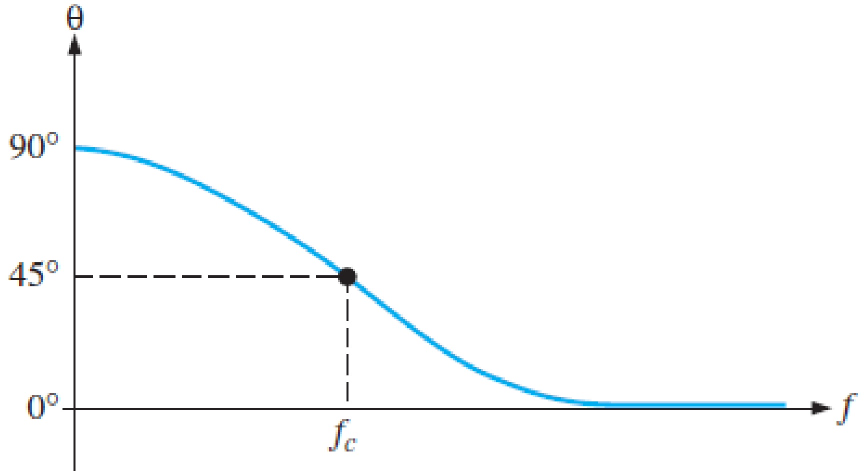
$$\theta = \tan^{-1}\left(\frac{0\ \Omega}{R_{in}}\right) = \tan^{-1}(0) = 0°$$

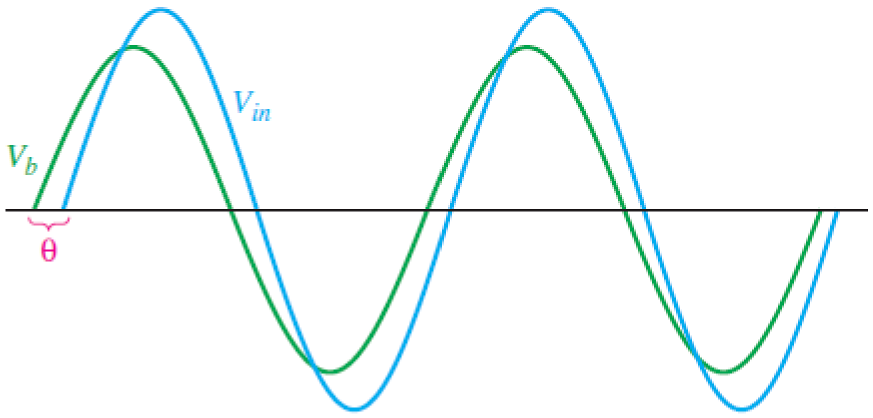
At the critical frequency, $X_{C1} = R_{in}$, so

$$\theta = \tan^{-1}\left(\frac{R_{in}}{R_{in}}\right) = \tan^{-1}(1) = 45°$$

At a decade below the critical frequency, $X_{C1} = 10R_{in}$, so

$$\theta = \tan^{-1}\left(\frac{10R_{in}}{R_{in}}\right) = \tan^{-1}(10) = 84.3°$$



# BJT Amplifier Low Frequency Response

## The Output *RC* Circuit

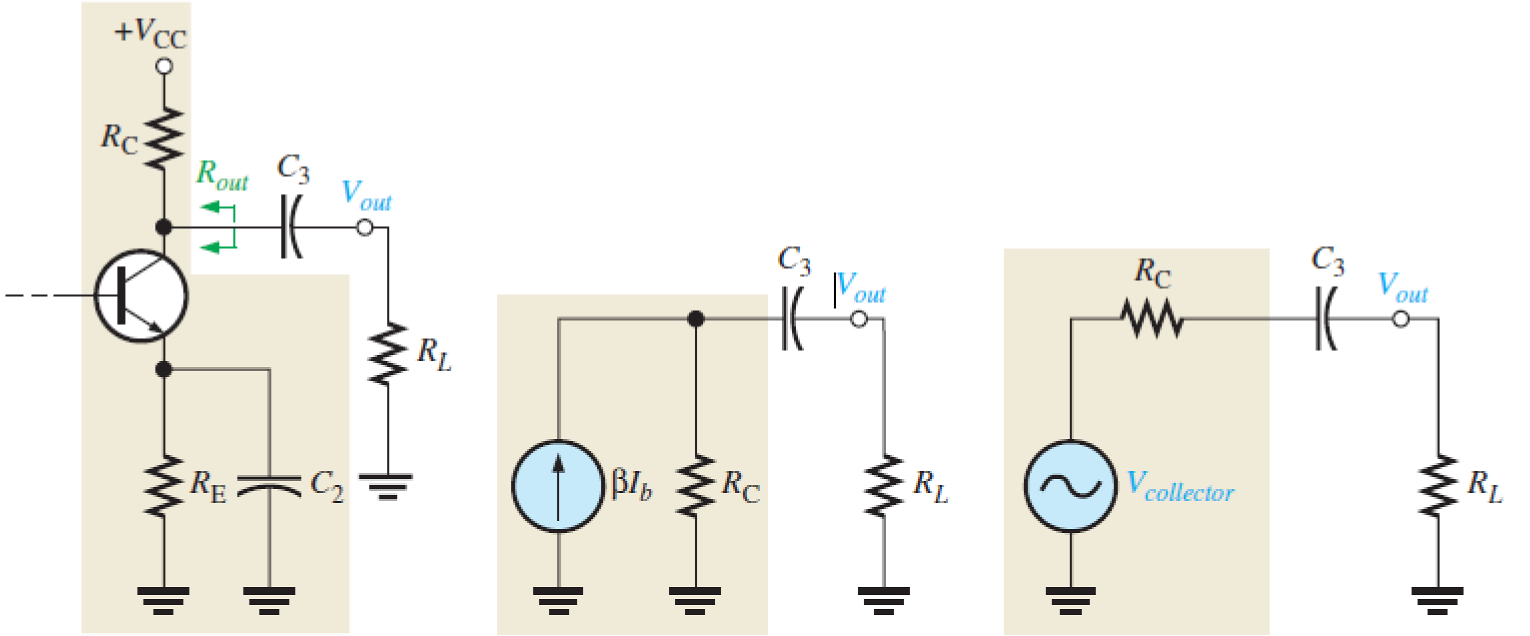
the transistor is treated as an ideal current source (with infinite internal resistance), and the upper end of $R_C$ is effectively at ac ground, as shown in Figure 10–15(b).

The lower critical frequency of this output *RC* circuit is

$$f_{cl(output)} = \frac{1}{2\pi(R_C + R_L)C_3}$$

Study Example 10-5

$f_{cl(ouputt)}$=50.8 Hz




(a) (b) (c)

# BJT Amplifier Low Frequency Response

***Phase Shift in the Output RC Circuit*** The phase angle in the output $RC$ circuit is

$$\theta = \tan^{-1}\left(\frac{X_{C3}}{R_C + R_L}\right)$$

$\theta \cong 0°$ for the midrange frequencies and approaches 90° as the frequency approaches zero ($X_{C3}$ approaches infinity). At the critical frequency $f_c$, the phase shift is 45°.

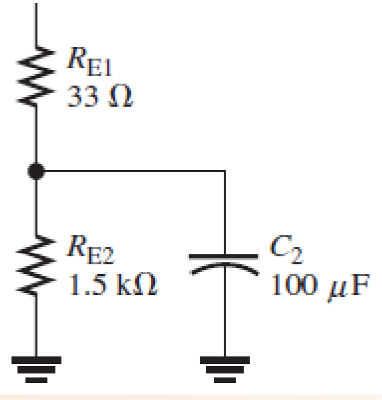
# BJT Amplifier Low Frequency Response

## The Bypass *RC* Circuit

$$f_{cl(bypass)} = \frac{1}{2\pi[(r'_e + R_{th}/\beta_{ac}) \parallel R_E]C_2}$$

If a swamping resistor is used,

$$f_{cl(bypass)} = \frac{1}{2\pi[(r'_e + \frac{R_{th}}{\beta_{ac}} + R_{E1})||R_{E2}]C_2}$$




Study Example 10-6

$f_{cl(bypass)}$=36 Hz

# BJT Amplifier Low Frequency Response

## Total Low-Frequency Response of an Amplifier

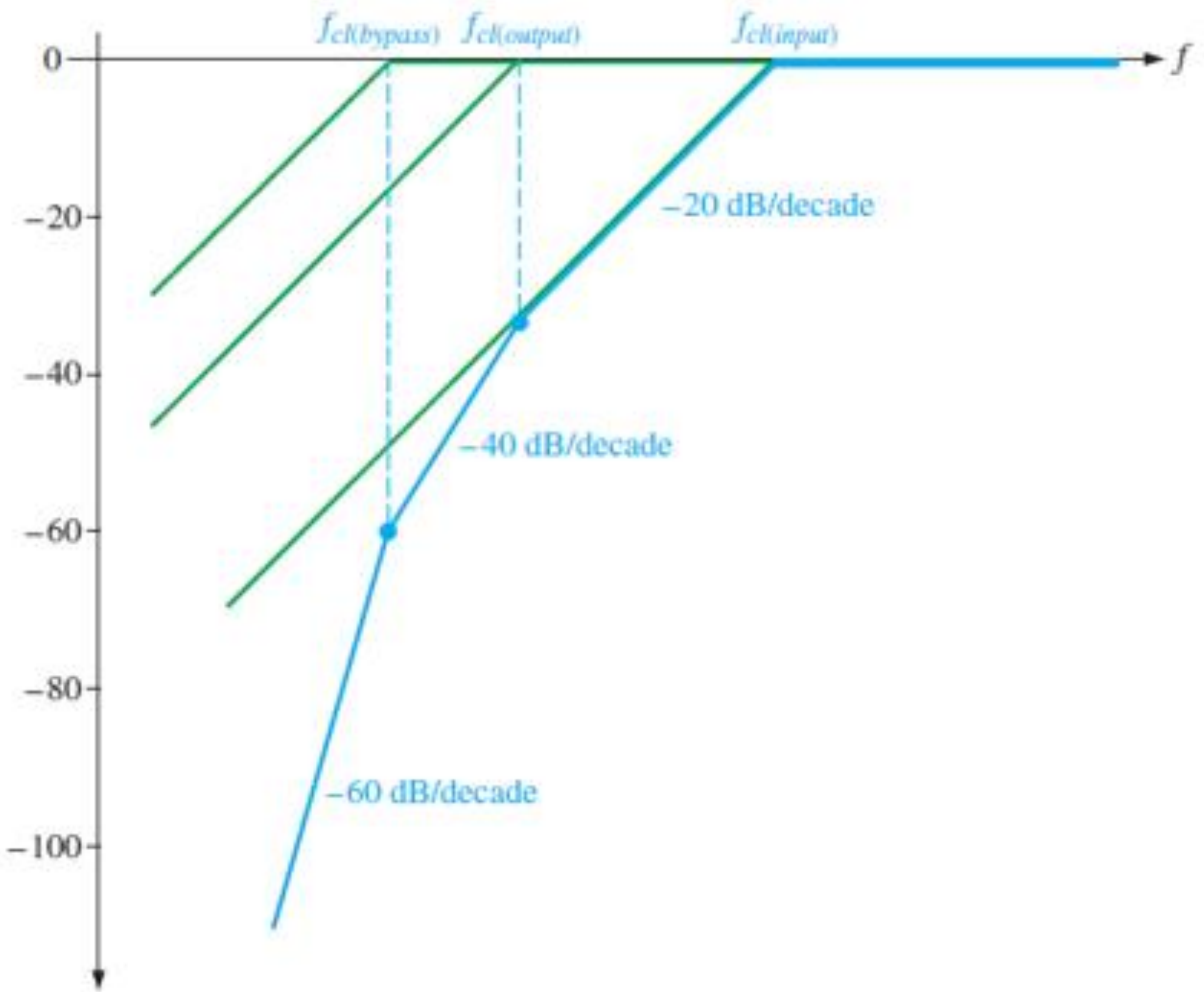
- **The combined effect of the three *RC* circuits (input, output, bypass) in a BJT amplifier**.
- Each circuit has a critical frequency determined by the *R* and *C* values.
- The critical frequencies of the three *RC* circuits are not necessarily all equal.
- If one of the *RC* circuits has a critical (break) frequency higher than the other two, then it is the ***dominant RC*** **circuit**.
- The dominant circuit determines the frequency at which the overall voltage gain of the amplifier begins to drop at -20dB/decade.
- The other circuits each cause an additional -20 dB/decade roll-off below their respective critical (break) frequencies.

# BJT Amplifier Low Frequency Response

## Total Low-Frequency Response of an Amplifier

▶ FIGURE 10–25

Composite Bode plot of a BJT amplifier response for three low-frequency *RC* circuits with different critical frequencies. Total response is shown by the blue curve.

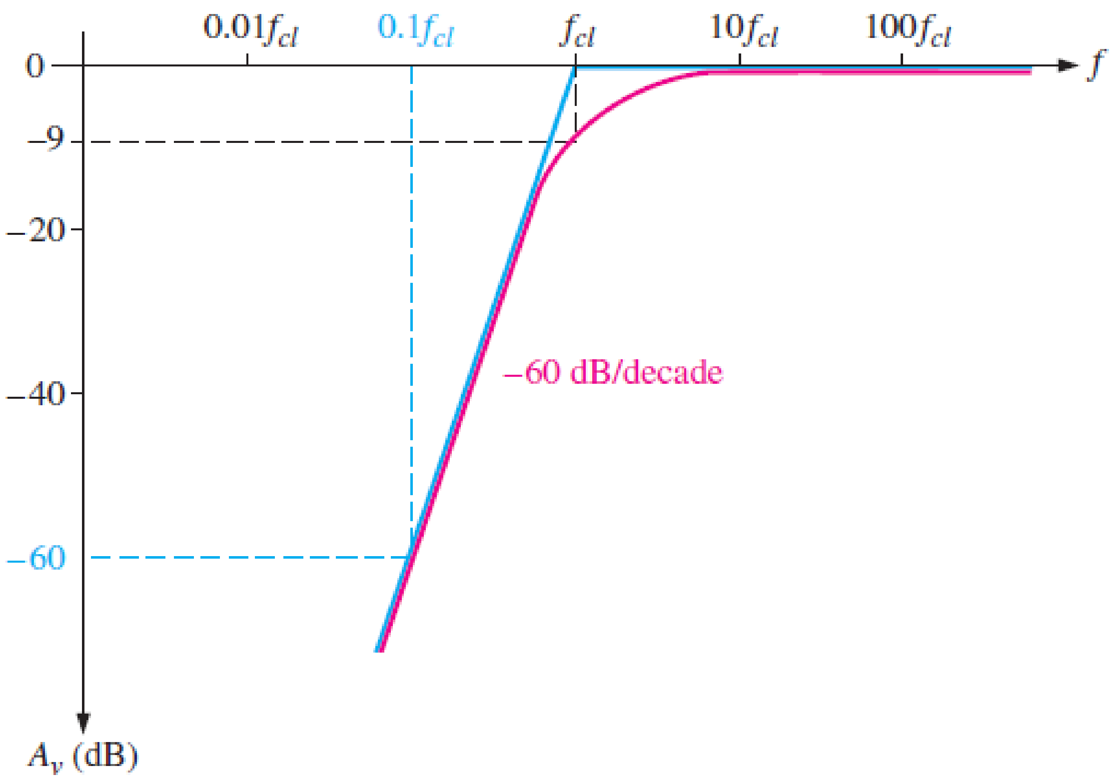


# BJT Amplifier Low Frequency Response

## Total Low-Frequency Response of an Amplifier

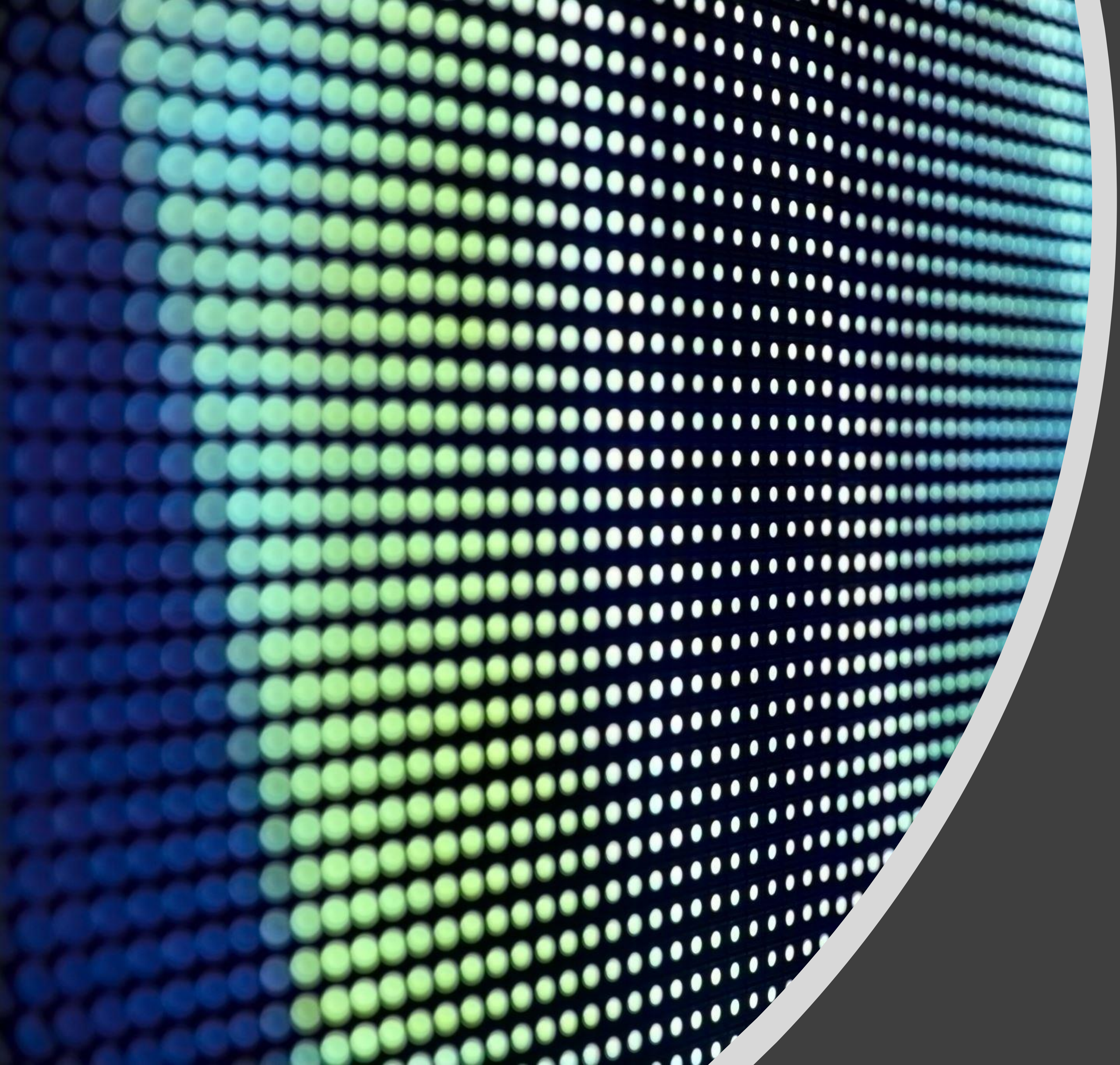
▶ FIGURE 10–26

Composite Bode plot of an amplifier response where all *RC* circuits have the same $f_{cl}$. (Blue is ideal; red is actual.)



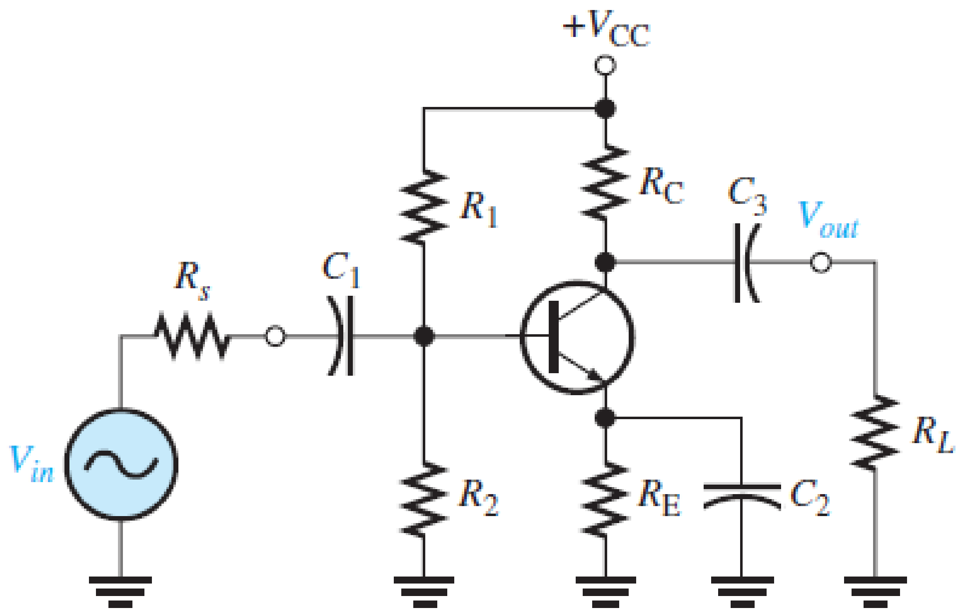

**Study Example 10-9 (Examples10-3+10-5+10-6)**

Draw the Bode Plot (calculate Av in dB)

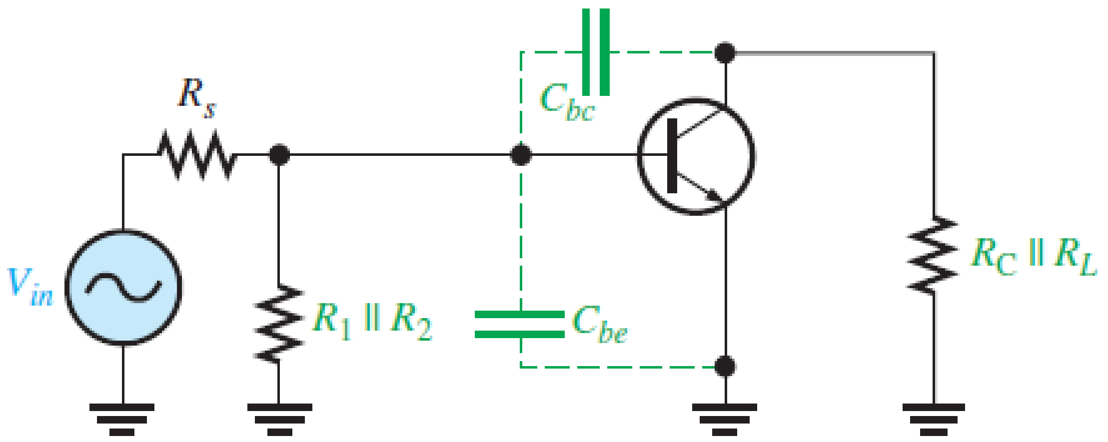


# High Frequency Amplifier Response

# BJT Amplifier High Frequency Response



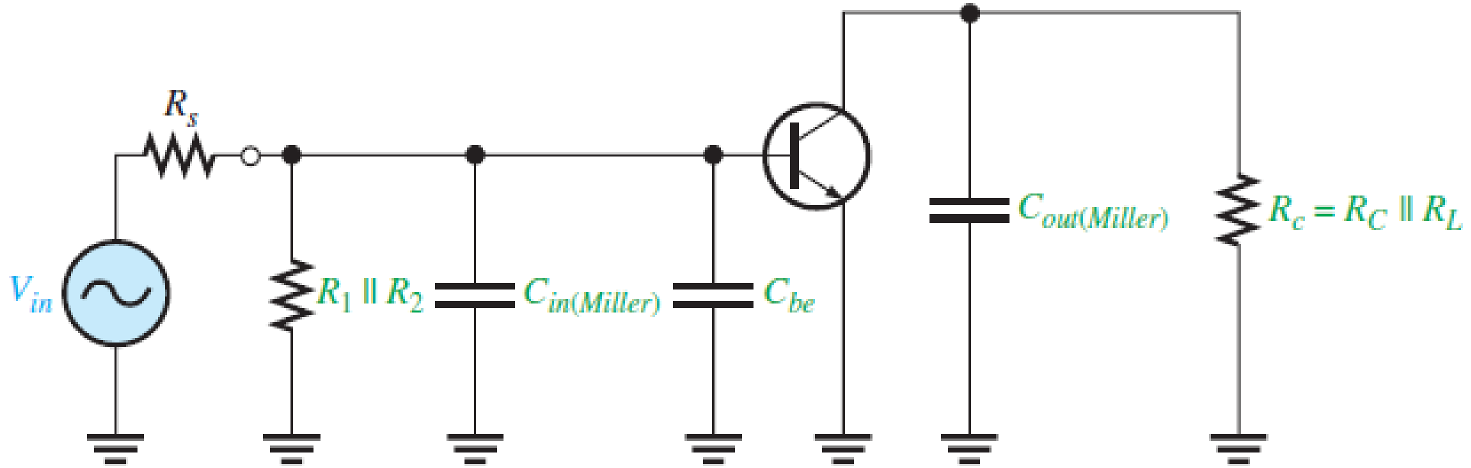

(a) Capacitively coupled amplifier




(b) High-frequency equivalent circuit

$$C_{in(Miller)} = C_{bc}(A_v + 1)$$

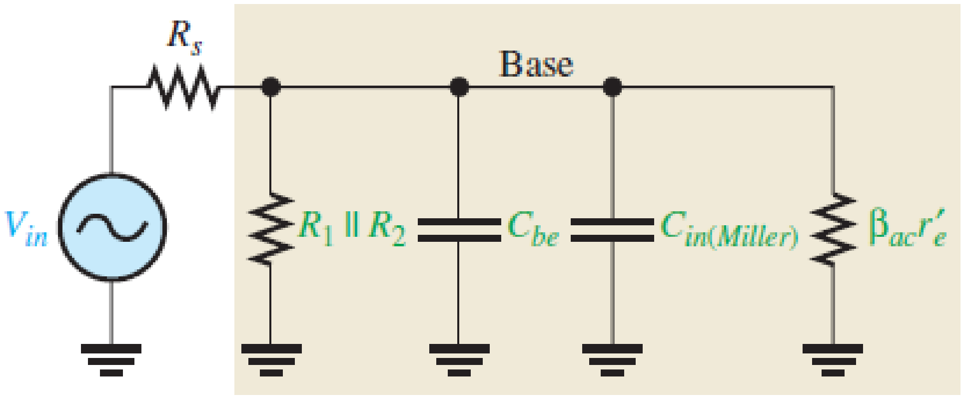
$$C_{out(Miller)} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$



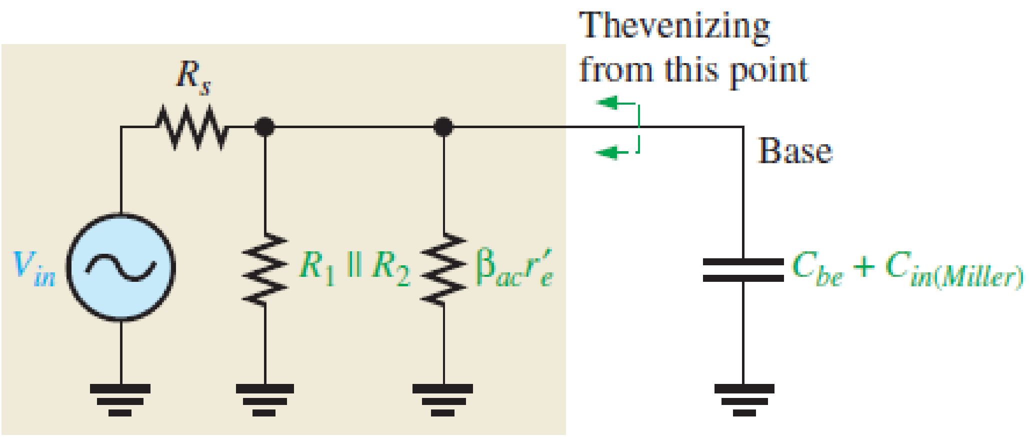
# BJT Amplifier High Frequency Response

## The Input *RC* Circuit

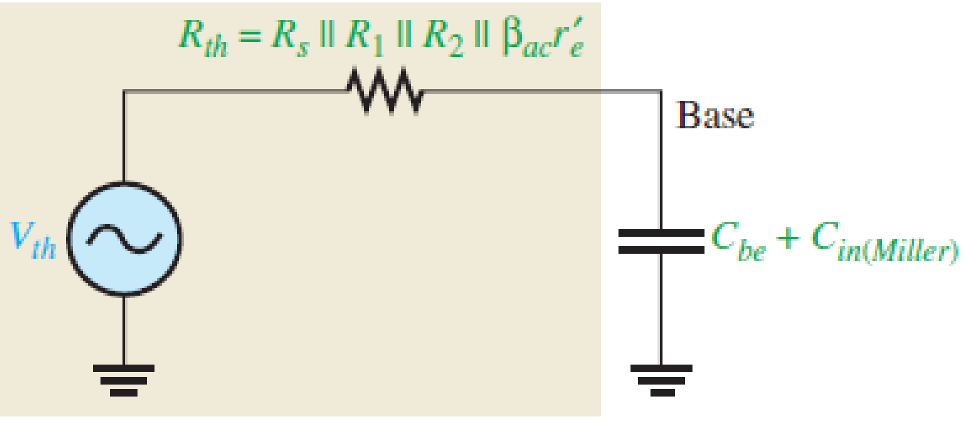
- $\beta\, r'_e$ is the input resistance at the base of the transistor because the bypass capacitor effectively shorts the emitter to ground.




(a)




(b)




(c)

# BJT Amplifier High Frequency Response

frequency of the input circuit, $f_{cu\,(input)}$, is the frequency at which the capacitive reactance is equal to the total resistance.

$$X_{C_{tot}} = R_s \| R_1 \| R_2 \| \beta_{ac} r'_e$$

Therefore,

$$\frac{1}{2\pi f_{cu\,(input)} C_{tot}} = R_s \| R_1 \| R_2 \| \beta_{ac} r'_e$$

and

$$f_{cu(input)} = \frac{1}{2\pi (R_s \| R_1 \| R_2 \| \beta_{ac} r'_e) C_{tot}}$$

where $R_s$ is the resistance of the signal source and $C_{tot} = C_{be} + C_{in(Miller)}$. As the frequency goes above $f_{cu\,(input)}$, the input $RC$ circuit causes the gain to roll off at a rate of $-20$ dB/decade just as with the low-frequency response.

# BJT Amplifier High Frequency Response

***Phase Shift of the Input RC Circuit*** Because the output voltage of a high-frequency input *RC* circuit is across the capacitor, the output of the circuit lags the input. The phase angle is expressed as

$$\theta = \tan^{-1}\left(\frac{R_s \| R_1 \| R_2 \| \beta_{ac} r'_e}{X_{C_{(tot)}}}\right)$$

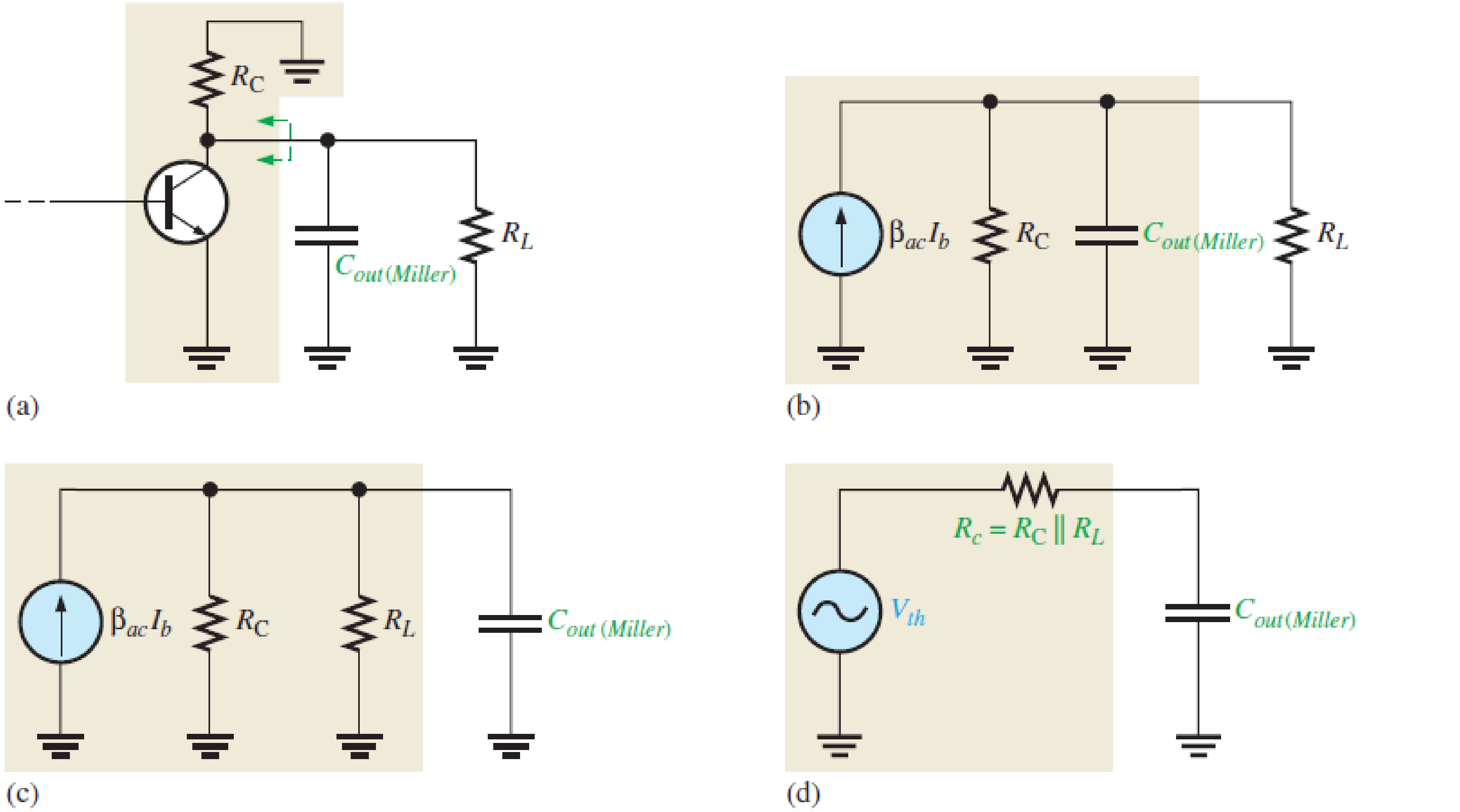
At the critical frequency, the phase angle is 45° with the signal voltage at the base of the transistor lagging the input signal. As the frequency increases above $f_c$, the phase angle increases above 45° and approaches 90° when the frequency is sufficiently high.

Study Example 10-11

$f_{cu(input)}$=1.62 MHz

# BJT Amplifier High Frequency Response

## The Output *RC* Circuit



# BJT Amplifier High Frequency Response

$$C_{out(Miller)} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$

If the voltage gain is at least 10, this formula can be approximated as

$$\boxed{C_{out(Miller)} \cong C_{bc}}$$

The upper critical frequency for the output circuit is determined with the following equation, where $R_c = R_C \parallel R_L$.

$$\boxed{f_{cu(output)} = \frac{1}{2\pi R_c C_{out(Miller)}}}$$

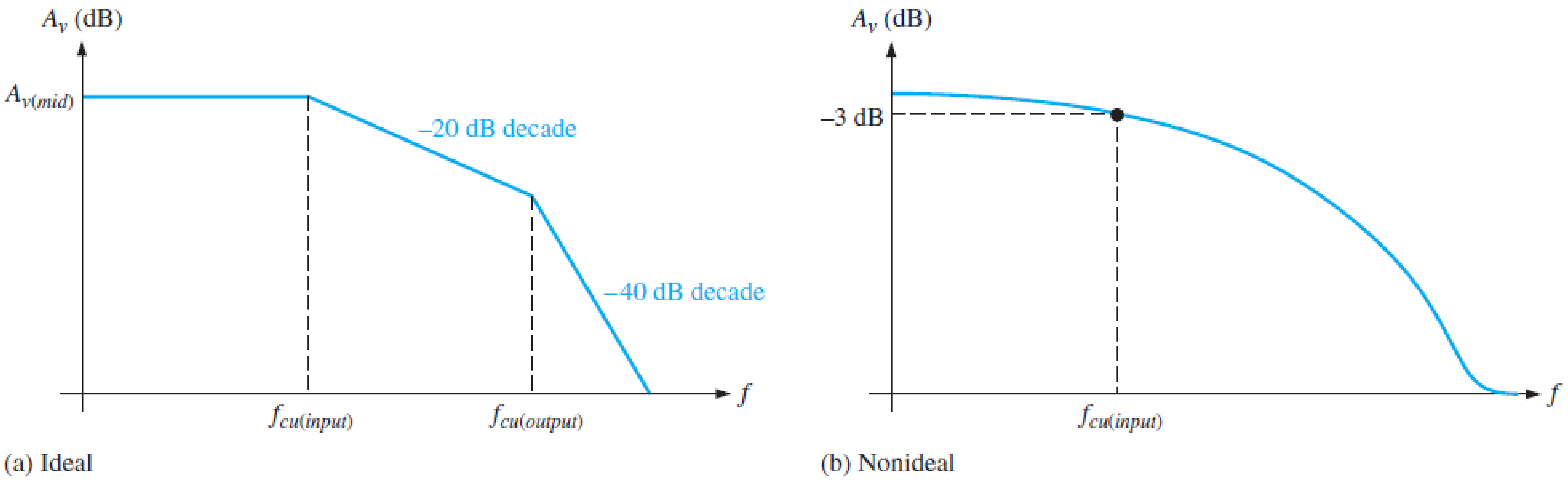
$$\boxed{\theta = \tan^{-1}\left(\frac{R_c}{X_{C_{out(Miller)}}}\right)}$$

Study Example 10-12

$f_{cu(output)}$= 60.3 MHz

# BJT Amplifier High Frequency Response

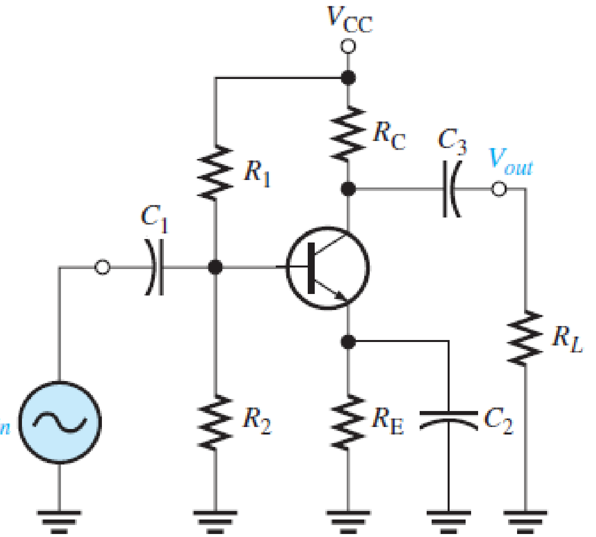
## Total High-Frequency Response of an Amplifier




▲ FIGURE 10–44

High-frequency Bode plots.

# BJT Amplifier **Total** Frequency Response

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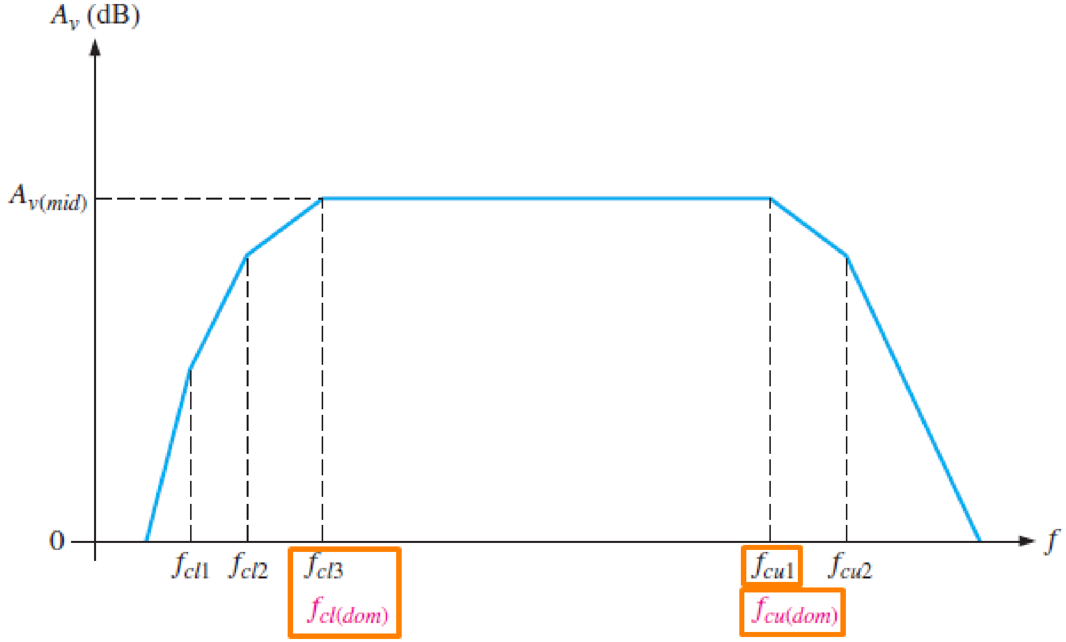
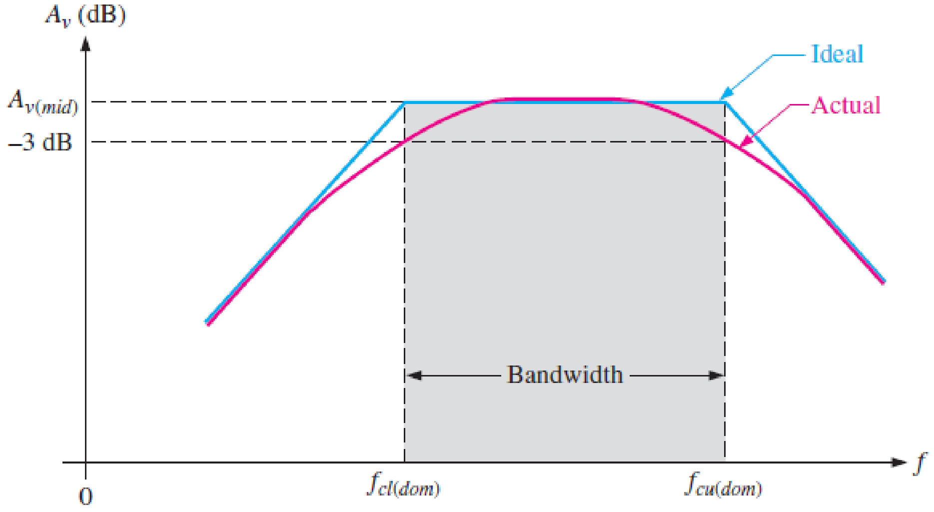



(b)




$$BW = f_{cu(dom)} - f_{cl(dom)}$$

$$f_{cl(dom)} \ll f_{cu(dom)}$$

$$BW = f_{cu(dom)} - f_{cl(dom)} \cong f_{cu}$$

- **The unity-gain frequency**

$$f_T = A_{v(mid)} BW$$

# Multistage Amplifiers Frequency Response

## Different Critical Frequencies

1. The overall dominant lower critical frequency $fcl'(dom)$, equals the dominant critical frequency of the stage with the highest $fcl(dom)$.

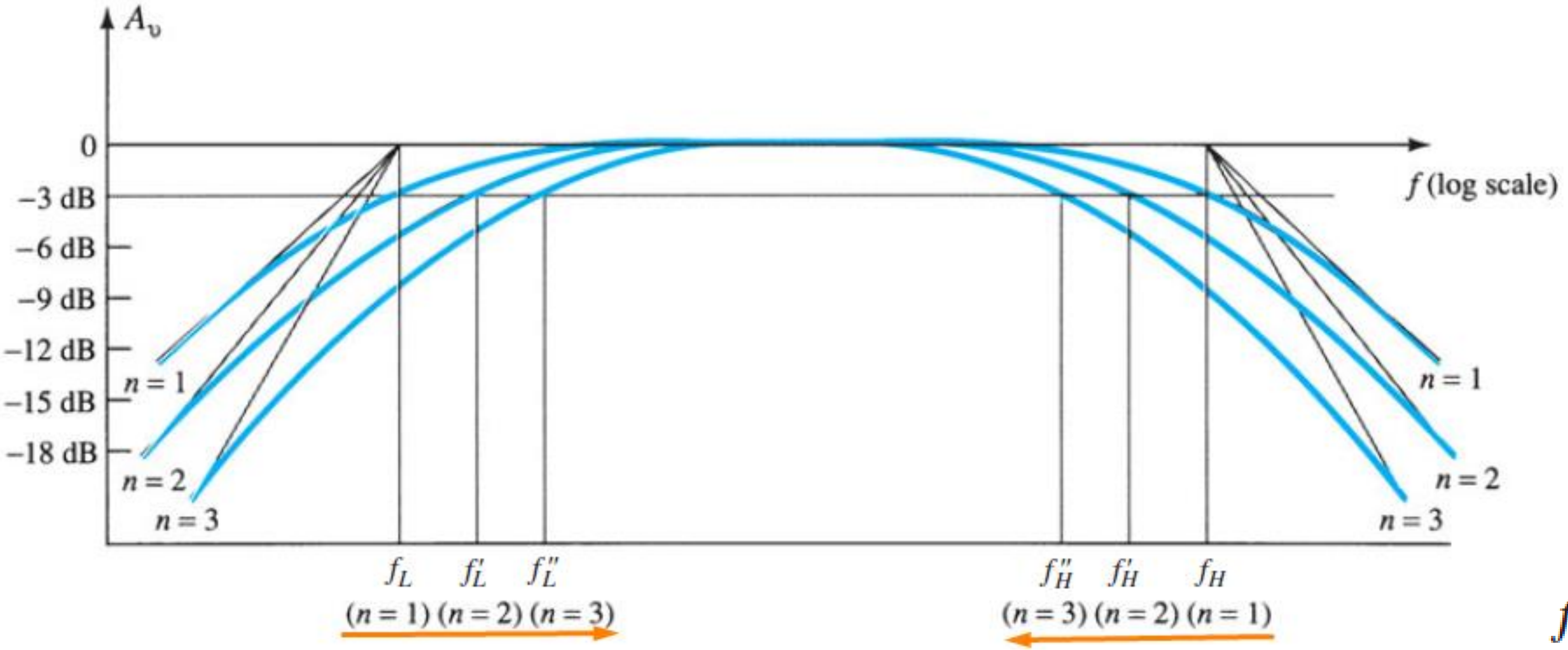
2. The overall dominant upper critical frequency $fcu'(dom)$, equals the dominant critical frequency of the stage with the lowest $fcu(dom)$.

***Overall Bandwidth*** The bandwidth of a multistage amplifier is the difference between the overall dominant lower critical frequency and the overall dominant upper critical frequency.

$$\boxed{BW = f'_{cu(dom)} - f'_{cl(dom)}}$$

# Multistage Amplifiers Frequency Response

## Equal Critical Frequencies

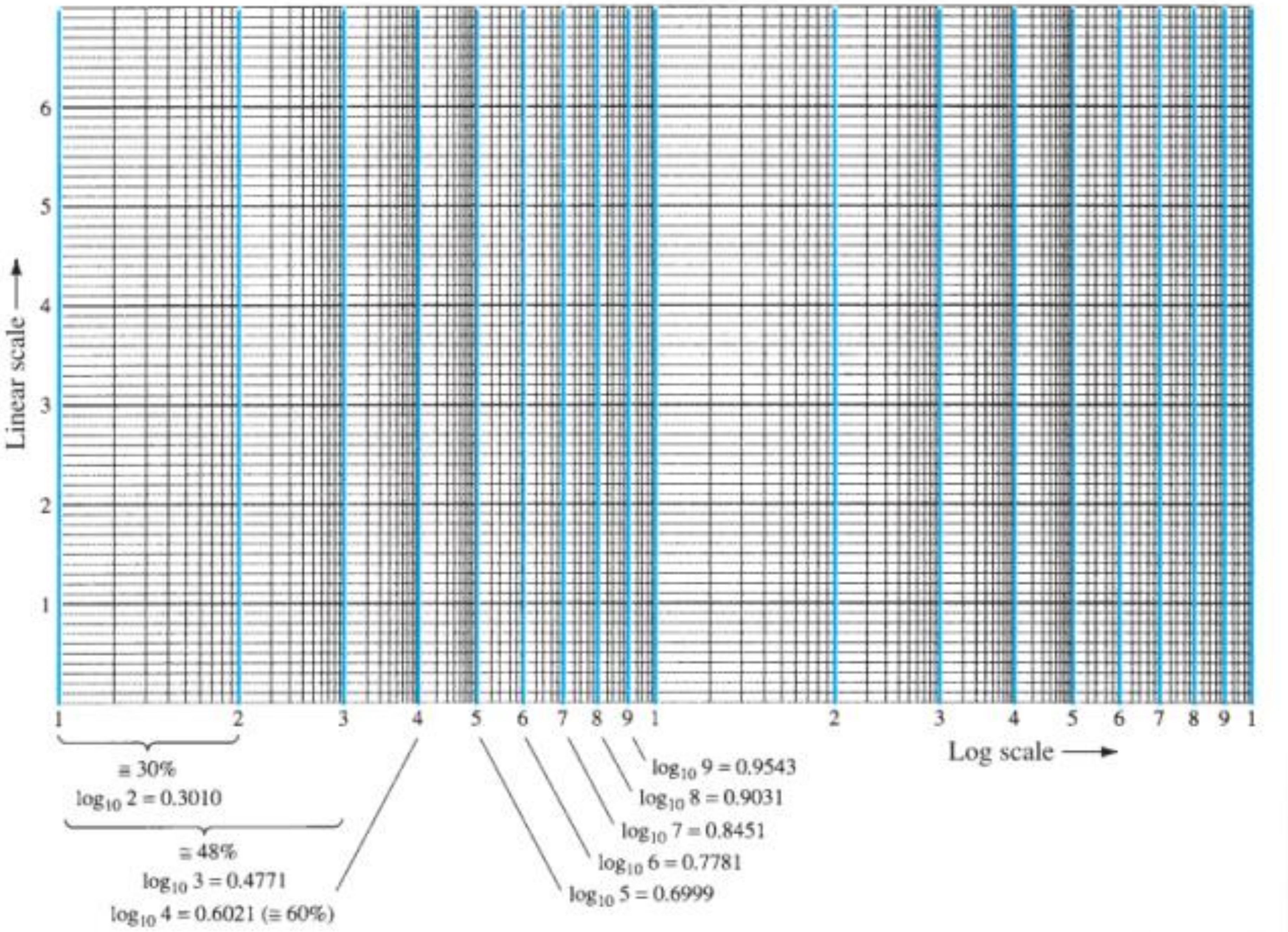



| $n$ | $\sqrt{2^{1/n} - 1}$ |
|---|---|
| 2 | 0.64 |
| 3 | 0.51 |
| 4 | 0.43 |
| 5 | 0.39 |

$$f'_{cl(dom)} = \frac{f_{cl(dom)}}{\sqrt{2^{1/n} - 1}}$$

$$f'_{cu(dom)} = f_{cu(dom)}\sqrt{2^{1/n} - 1}$$

Study Example 10-19



6
5
4
3
2
1
Linear scale
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1
Log scale
≅ 30%
log₁₀ 2 = 0.3010
≅ 48%
log₁₀ 3 = 0.4771
log₁₀ 4 = 0.6021 (≅ 60%)
log₁₀ 5 = 0.6999
log₁₀ 6 = 0.7781
log₁₀ 7 = 0.8451
log₁₀ 8 = 0.9031
log₁₀ 9 = 0.9543