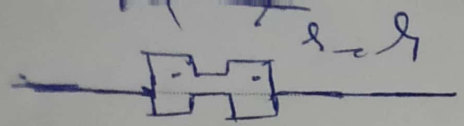


(Ch-2-) [Equal intervals]

Numerical differentiation and Integration

الحلول التقريبية للمشتقة الاولى

① Approximation for first derivative

ايجاد الحلول التقريبية للمشتقات المختلفة

كل ما بزود الـ order كل ما الحل كان احسن والخطأ اقل

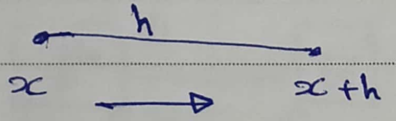
3 طرق

① Taylor Expansion

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x)$$

a) forward difference

هحتاج و تعطيني ? عاملين المشتقة الاولي



(x+h) النقطة x والنقطة اللي بعدها (forward)

$$\cancel{h} f'(x) = \frac{1}{h}\left[f(x+h) - f(x) - \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x) - \dots\right]$$

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2!} f''(x) - \frac{h^2}{3!} f'''(x)$$

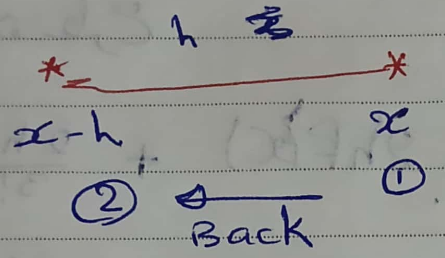
cut T. Error

لما احسب الحلول التقريبية هههمل الحدود دي

$$T.E \leq \frac{h}{2} f''(x) - \frac{h^2}{3!} f'''(x)$$

$$f'(x) = \frac{f(x+h) - f(x)}{h} \simeq O(h)$$

b) Backward difference



النقطة اللي قبلها

$$f(x-h) = f(x) - \underline{h f'(x)} + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x)$$

$$h f'(x) = f(x) - f(x-h) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x)$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \frac{h}{2!} f''(x) - \frac{h^2}{3!} f'''(x)$$

الصيغة التقريبية هي حاصل المطروح

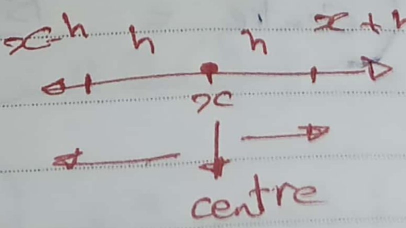
$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$T.E \leq \frac{h}{2} |f''(x)|$$ الحدود مختلفة عن ال Forward

لها $\simeq O(h^2)$

c) central difference :-

(x+h , x-h)



① $f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3!} f'''(x)$

② $f(x-h) = f(x) - h f'(x) + \frac{h^2}{2!} f''(x) - \frac{h^3}{3!} f'''(x)$

(1 − 2) لو عايز $f'(x)$ هتظهر
ف هنطرح

$$f(x+h) - f(x-h) = 2h f'(x) + \frac{2h^3}{3!} f'''(x)$$

$$f'(x) = \frac{f(x+h) - f(x-h) - \frac{2h^3}{3!} f'''(x)}{2h}$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^3}{3} f'''(x)$$

$T.E = \leq + \frac{h^3}{3} |f'''(x)|$ مـ $= O(h)$

اكبر قيمة للمشتقة التالتة

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

ال centre احسن من ال Forward وال Backward

$$\boxed{Centre = \frac{Back + Forward}{2}}$$

عايز ارفع ال order بتاع T.E من 2 → 4

$$F'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{3!} F'''(x) \mp \frac{h^4}{5!} f^{(5)}(\xi)$$

$$f'(x) = \phi(h) + C_2 h^2 + C_4 h^4 \rightarrow (1)$$

عايزين نحذف الحد ده

بداية T.E

$$C_2 = -\frac{F'''(x)}{3!}$$

$$C_4 = -\frac{F^{(5)}}{5!}$$

$$\left(\frac{h}{2}\right)^4$$

replace $h \rightarrow \frac{h}{2}$

$$F'(x) = \phi\left(\frac{h}{2}\right) + C_2 \frac{h^2}{4} + C_4 \frac{h^4}{16} + \dots \rightarrow (2)$$

$$\boxed{-4(2) + (1)}$$

$\frac{1}{4}$

$$f'(x) - 4f'(x) = \cancel{-3\phi\left(\frac{h}{2}\right) - C_4\frac{h^4}{4}}$$

$$= \phi(h) - 4\phi\left(\frac{h}{2}\right) + C_4 h^4 - \frac{1}{4} C_4 h^4 + \dots$$

$$-3f'(x) = \phi(h) - 4\phi\left(\frac{h}{2}\right) + \frac{3}{4} C_4 h^4 +$$

$$f'(x) = \underbrace{-\frac{1}{3}\phi(h) + \frac{4}{3}\phi\left(\frac{h}{2}\right)}_{F'(x)} \underbrace{- \frac{1}{4} C_4 h^4}_{T.E}$$

$$T.E = \frac{1}{4}\frac{h^4}{5!}\,|f^{(5)}(c)| \simeq O(h^4)$$

E(x)

$f(x) = e^x$ $\qquad h = 0.1$ $\qquad f'(x) = e^x$

approximate $f'(1.5)$

Solution

using
1 — Forward
2 — Back
3 — Cent
4 — RichCharsen

ⓐ F.d

$h = 0.1$

x $\quad$ $x+h$

1.5 $\quad$ 1.6

$$f'(x) = \frac{f(x+h) - f(x)}{h} = \frac{e^{x+h} - e^{x}}{h}$$

$$= \frac{e^{1.6} - e^{1.5}}{0.1} \quad \#$$

$$T.E \le \frac{h}{2}\,|f^{(2)}(c)| \rightarrow \frac{0.1}{2}\,|e^{1.6}|$$

ⓑ B.d

$$f'(x) = \frac{f(x) - f(x-h)}{h} = \frac{e^{x} - e^{x-h}}{h}$$

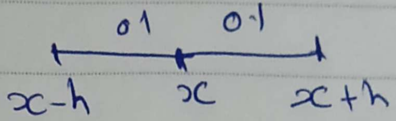
$h = 0.1$

1.4 $\quad$ x $\quad$ 1.5

$$= \frac{e^{1.5} - e^{1.4}}{h}$$

$$T.E \le \frac{0.1}{2}\,e^{1.5}$$

[5] central difference



$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} = \frac{e^{1.6} - e^{1.4}}{2*(0.1)}$$

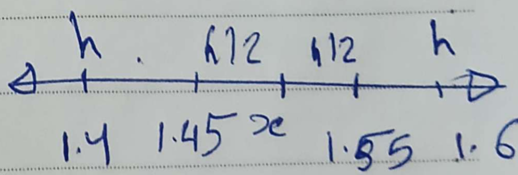
$$T.E \leq \frac{h^2}{3!} f'''(x) \leq \frac{0.1^2}{3!} \left| e^{(1.6)} \right|$$

$$f'(x) = \frac{4}{3} \phi \left(\frac{h}{2}\right) - \frac{1}{3} \phi(h)$$

$$\phi(h) = \frac{f(x+h) - f(x-h)}{2h} = \frac{e^{1.6} - e^{1.4}}{2*0.1}$$

$$\phi(h/2) = \frac{f(x+h/2) - f(x-h/2)}{h} = \frac{e^{1.5+\frac{0.1}{2}} - e^{1.5-\frac{0.1}{2}}}{0.1}$$

$$T.E \leq \frac{h^4}{5!*4} * \left| f^{(5)} \right|$$



$$= \frac{(0.1)^4}{5!*4} * e^{1.6}$$

بسم الله الرحمن الرحيم

• Ch2

Numerical differentiation and Integration

Ex

$f(x) = e^{x}$

Dem

$f'(1.5)$, $h = 0.1$

Exact Solution $f'(1.5) = e^{1.5}$

Solution

F.D

$h = 0.1$, $x = 1.5$, $x+h = 1.6$

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h} = \frac{e^{x+h} - e^{x}}{h}$$

$$= \frac{e^{1.6} - e^{1.5}}{0.1} = \checkmark$$

$$T.E \le \frac{h}{2} |f''(c)| \le \frac{0.1}{2} e^{1.6} = \checkmark$$

B.D

$h = 0.1$, $x-h = 1.4$, $x = 1.5$

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$= \frac{e^{x} - e^{x-h}}{h}$$

$$= \frac{e^{1.5} - e^{1.4}}{0.1} = \checkmark$$

$$T.E \le \frac{h}{2} |f''(c)|$$

$$\le \frac{0.1}{2} e^{1.5} = \checkmark$$

C.D

$h = 0.1$, $h = 0.1$, $x-h = 1.4$, $x = 1.5$, $x+h = 1.6$

$$f'(x) \simeq \frac{f(x+h) - f(x-h)}{2h} = \frac{e^{1.6} - e^{1.4}}{2(0.1)} = \checkmark$$

$$T.E \le \frac{h^2}{3!} |f'''(c)| \le \frac{(0.1)^2}{3!} e^{1.6} = \checkmark$$

Richardson Extrapolation

$$f'(x) \simeq \frac{4}{3} \Phi\left(\frac{h}{2}\right) - \frac{1}{3} \Phi(h)$$

$$\Phi(h) = \frac{f(x+h) - f(x-h)}{2h} = \frac{e^{1.6} - e^{1.4}}{2(0.1)} = \checkmark$$

$$\Phi\left(\frac{h}{2}\right) = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{2\,\frac{h}{2}}$$

$\frac{h}{2} = 0.05$, $\frac{h}{2} = 0.05$; 1.4, $x - \frac{h}{2} = 1.45$, $x = 1.5$, $x + \frac{h}{2} = 1.55$, 1.6

$$= \frac{e^{1.55} - e^{1.45}}{0.1} = \checkmark$$

$$f'(x) = \checkmark$$

$$T.E \le \frac{h^4}{5! \cdot 4} |f^{(5)}(c)|$$

$$\le \frac{(0.1)^4}{4 \times 5!} e^{1.6} = \checkmark$$

بسم الله الرحمن الرحيم

. Ch2

Numerical differentiation and Integration

Ex:

$f(x) = \sin x$

$h = 0.1$

Req

$f''(0.5)$

Exact Solution

$f''(0.5) = -0.4794255$

Solution

F.D

$x = 0.5$, $x+h = 0.6$, $x+2h = 0.7$ ($h = 0.1$, $h = 0.1$)

$$f''(x) \simeq \frac{\sin(0.7) - 2\sin(0.6) + \sin(0.5)}{(0.1)^2} = \checkmark$$

Error = Exact − app. = ✓

B.D

$x-2h = 0.3$, $x-h = 0.4$, $x = 0.5$ ($h = 0.1$, $h = 0.1$)

$$f''(x) \simeq \frac{\sin(0.3) - 2(\sin(0.4)) + \sin 0.5}{(0.1)^2} = \checkmark$$

Error

Error = | Exact − app. | = ✓

C.D

$x-h = 0.4$, $x = 0.5$, $x+h = 0.6$ ($h = 0.1$, $h = 0.1$)

$$f''(x) \simeq \frac{\sin 0.6 - 2\sin 0.5 + \sin 0.4}{(0.1)^2} = \checkmark$$

Error = | Exact − app. | = ✓

$$f''(x) \simeq \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

$$T.E \le h\,|f'''(c)| \simeq O(h)$$

B.C

$x-2h$, $x-h$, x (h, h)

$$f''(x) \simeq \frac{f(x-2h) - 2f(x-h) + f(x)}{h^2}$$

$$T.E \le h\,|f'''(c)| \simeq O(h)$$

Central difference:-

$x-h$, x, $x+h$

$$f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$T.E \le \frac{h^2}{12}\,|f^{(4)}(c)| \simeq O(h^2)$$