Electrical Power Engineering Department

Electromagnetic Fields (EPE112&EPM2142)

Tutorial

Week(2)

VECTOR ALGEBRA (Part. II)

Tutorial Objectives

- Vector Multiplication
  - Scalar
  - Vector
- Coordinate Systems and Transformation.
  - Cartesian coordinates (x, y, z)
  - Circular cylindrical coordinates (ρ, ϕ, z)
  - Spherical coordinates (r, θ, ϕ)

Scalar (Dot) Multiplication

- The dot product of two vectors **A** and **B**, written as (**A . B**), is defined geometrically as the product of the magnitudes of **A** and **B** and the cosine of the smaller angle between them when they are drawn tail to tail.

$$\boxed{\boldsymbol{A}\,.\,\boldsymbol{B} = |\boldsymbol{A}||\boldsymbol{B}|\cos(\theta_{AB}) = A_x B_x + A_y B_y + A_z B_z}$$

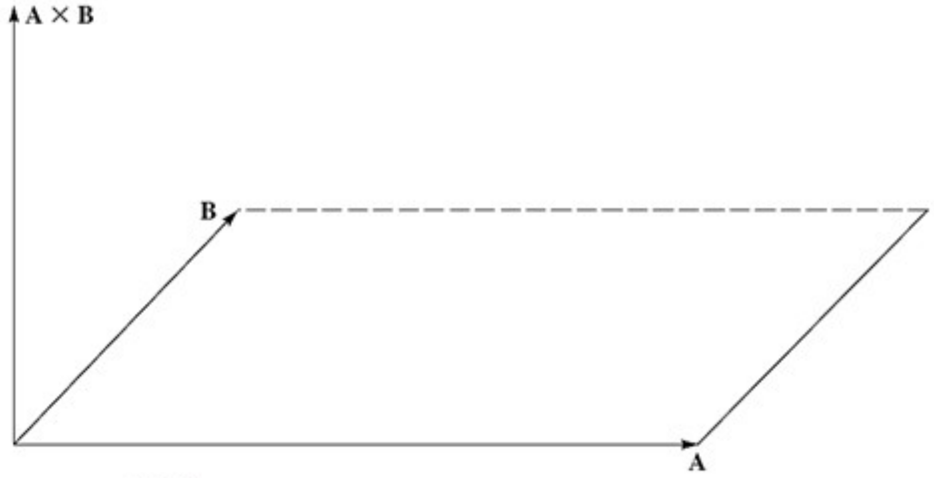
Scalar (Dot) Multiplication

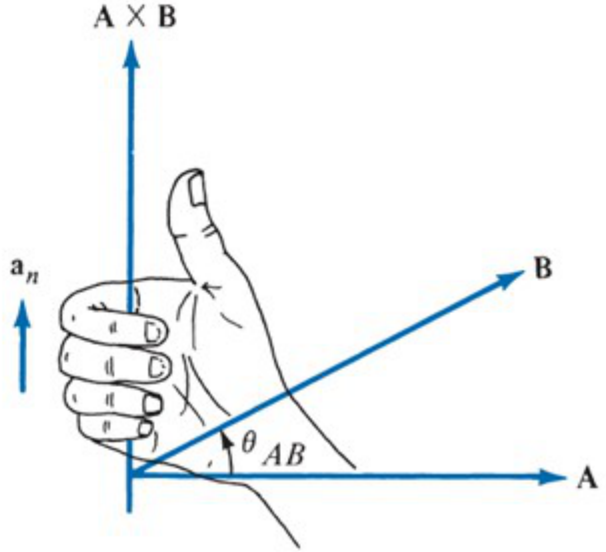
- **Notes**
  - Two vectors **A** and **B** are said to be <u>orthogonal</u> (or perpendicular) with each other if ($\boldsymbol{A} \cdot \boldsymbol{B} = 0$).
  - **A . A** = $|A|^2$

Law	Dot Product
Commutative	$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
Distributive	$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Cross Product

- The cross product of two vectors **A** and **B**, written as (**A** X **B**), is a <u>vector quantity</u> whose magnitude is the <u>area of the parallelogram</u> formed by **A** and **B** and <u>is in the direction of advance of a right-handed screw as A is turned into B.</u>





Cross Product

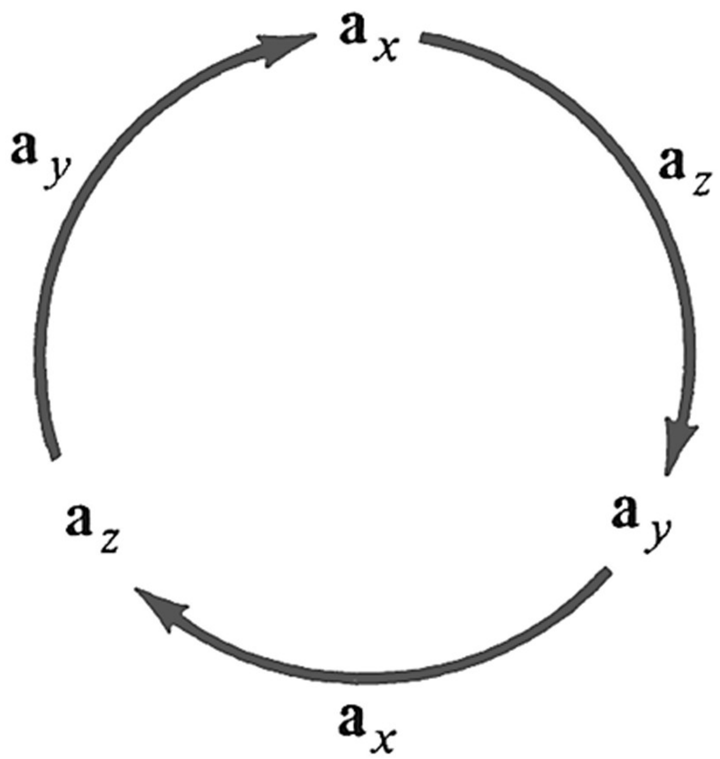
$$\boldsymbol{A}\ x\ \boldsymbol{B} = |\boldsymbol{A}||\boldsymbol{B}|\sin(\theta_{AB}) = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

Law	Cross Product
<u>NOT</u> Commutative	$\vec{A} \times \vec{B} \neq \vec{B} \,.\!\times \vec{A}$ but $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
<u>NOT</u> Associative	$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$
Distributive	$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
Scaling	$k\vec{A} \times \vec{B} = \vec{A} \times k\vec{B} = k(\vec{A} \times \vec{B})$

Cross Product

- **Notes**
  - $\boldsymbol{A \times A = 0}$
  - $\boldsymbol{a_x \times a_y = a_z}$
  - $\boldsymbol{a_y \times a_z = a_x}$
  - $\boldsymbol{a_z \times a_x = a_y}$



Triple Product

Scalar Triple Product	Vector Triple Product
$\vec{A}\,.(\vec{B}\times\vec{C})=\vec{B}\,.(\vec{C}\times\vec{A})=\vec{C}\,.(\vec{A}\times\vec{B})$ $\vec{A}\,.(\vec{B}\times\vec{C})=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$	$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}\times(\vec{A}.\vec{C})-\vec{C}\times(\vec{A}.\vec{B})$

EXERCISES

2. Find the actual angle between the two vectors $\overline{\mathbf{A}} = 2\overline{\mathbf{a}}_{\mathrm{x}} + \overline{\mathbf{a}}_{\mathrm{y}} + 3\overline{\mathbf{a}}_{\mathrm{z}}$ and $\overline{\mathbf{B}} = \overline{\mathbf{a}}_{\mathrm{x}} - 3\overline{\mathbf{a}}_{\mathrm{y}} + 2\overline{\mathbf{a}}_{\mathrm{z}}$.

2. Find the actual angle between the two vectors $\overline{\mathbf{A}} = 2\overline{\mathbf{a}}_{\mathrm{x}} + \overline{\mathbf{a}}_{\mathrm{y}} + 3\overline{\mathbf{a}}_{\mathrm{z}}$ and $\overline{\mathbf{B}} = \overline{\mathbf{a}}_{\mathrm{x}} - 3\overline{\mathbf{a}}_{\mathrm{y}} + 2\overline{\mathbf{a}}_{\mathrm{z}}$.

$$\boxed{\theta_{AB} = 69.075^o}$$

6. Given the two vector $\overline{\mathbf{F}} = 10\overline{\mathbf{a}}_x - 6\overline{\mathbf{a}}_y + 5\overline{\mathbf{a}}_z$ and $\overline{\mathbf{G}} = 0.1\overline{\mathbf{a}}_x + 0.2\overline{\mathbf{a}}_y + 0.3\overline{\mathbf{a}}_z$, find:

(a) the vector component of $\overline{\mathbf{F}}$ that is parallel to $\overline{\mathbf{G}}$;

(b) the vector component of $\overline{\mathbf{F}}$ that is perpendicular to $\overline{\mathbf{G}}$;

(c) the vector component of $\overline{\mathbf{G}}$ that is perpendicular to $\overline{\mathbf{F}}$.

6. Given the two vector $\overline{\mathbf{F}} = 10\overline{\mathbf{a}}_x - 6\overline{\mathbf{a}}_y + 5\overline{\mathbf{a}}_z$ and $\overline{\mathbf{G}} = 0.1\overline{\mathbf{a}}_x + 0.2\overline{\mathbf{a}}_y + 0.3\overline{\mathbf{a}}_z$, find:

(a) the vector component of $\overline{\mathbf{F}}$ that is parallel to $\overline{\mathbf{G}}$;

(b) the vector component of $\overline{\mathbf{F}}$ that is perpendicular to $\overline{\mathbf{G}}$;

(c) the vector component of $\overline{\mathbf{G}}$ that is perpendicular to $\overline{\mathbf{F}}$.

$$a)\ (0.9284, 1.8569, 2.7853)$$
$$b)\ (9.0715, -7.856, 2.2146)$$
$$c)\ (0.0192, 0.2484, 0.2596)$$

7. Three vectors extending from the origin are given as $\overline{\mathbf{R}}_1 = 7\overline{\mathbf{a}}_\mathbf{x} + 3\overline{\mathbf{a}}_\mathbf{y} - 2\overline{\mathbf{a}}_\mathbf{z}$, $\vec{R}_2 = -2\overline{\mathbf{a}}_\mathbf{x} + 7\overline{\mathbf{a}}_\mathbf{y} - 3\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{R}}_3 = 2\overline{\mathbf{a}}_\mathbf{y} + 3\overline{\mathbf{a}}_\mathbf{z}$. Find:

   (a) a unit vector perpendicular to $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

   (b) a unit vector perpendicular to the vectors $\overline{\mathbf{R}}_1 - \overline{\mathbf{R}}_2$ and $\overline{\mathbf{R}}_2 - \overline{\mathbf{R}}_3$;

   (c) the area of the triangle defined by $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

   (d) the area of the triangle defined by the heads of $\overline{\mathbf{R}}_1$, $\overline{\mathbf{R}}_2$, and $\overline{\mathbf{R}}_3$.

7. Three vectors extending from the origin are given as $\overline{\mathbf{R}}_1 = 7\overline{\mathbf{a}}_\mathbf{x} + 3\overline{\mathbf{a}}_\mathbf{y} - 2\overline{\mathbf{a}}_\mathbf{z}$, $\vec{R}_2 = -2\overline{\mathbf{a}}_\mathbf{x} + 7\overline{\mathbf{a}}_\mathbf{y} - 3\overline{\mathbf{a}}_\mathbf{z}$ and $\overline{\mathbf{R}}_3 = 2\overline{\mathbf{a}}_\mathbf{y} + 3\overline{\mathbf{a}}_\mathbf{z}$. Find:

(a) a unit vector perpendicular to $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

(b) a unit vector perpendicular to the vectors $\overline{\mathbf{R}}_1 - \overline{\mathbf{R}}_2$ and $\overline{\mathbf{R}}_2 - \overline{\mathbf{R}}_3$;

(c) the area of the triangle defined by $\overline{\mathbf{R}}_1$ and $\overline{\mathbf{R}}_2$;

(d) the area of the triangle defined by the heads of $\overline{\mathbf{R}}_1$, $\overline{\mathbf{R}}_2$, and $\overline{\mathbf{R}}_3$.

$$a)\ (\, 0.0824\, , 0.4123\, , 0.9072\,)$$
$$b)\ (\, 0.2853\, , 0.7809\, , 0.5556\,)$$
$$c)\ 30.311\ unit\ area$$
$$d)\ 33.294\ unit\ area$$

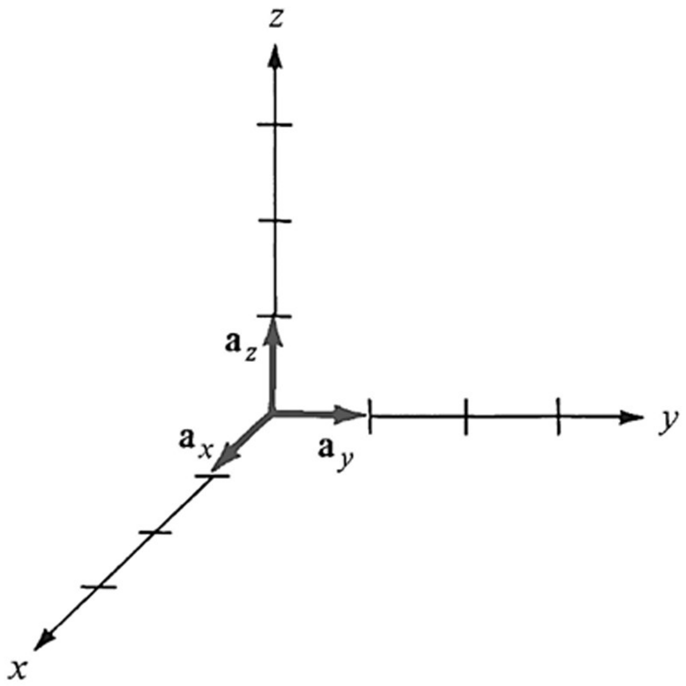
Coordinate Systems and Transformation

- In general, the physical quantities we shall be dealing with in EM are functions of space and time. In order to describe the spatial variations of the quantities, we must be able to define all points uniquely in space in a suitable manner. **This requires using an appropriate coordinate system.**
- **Orthogonal system**
  - Cartesian (or rectangular).
  - the circular cylindrical.
  - the spherical.
  - the elliptic cylindrical.
  - the parabolic cylindrical.
  - the conical.
  - ... ETC

Cartesian Coordinates (x, y, z)

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

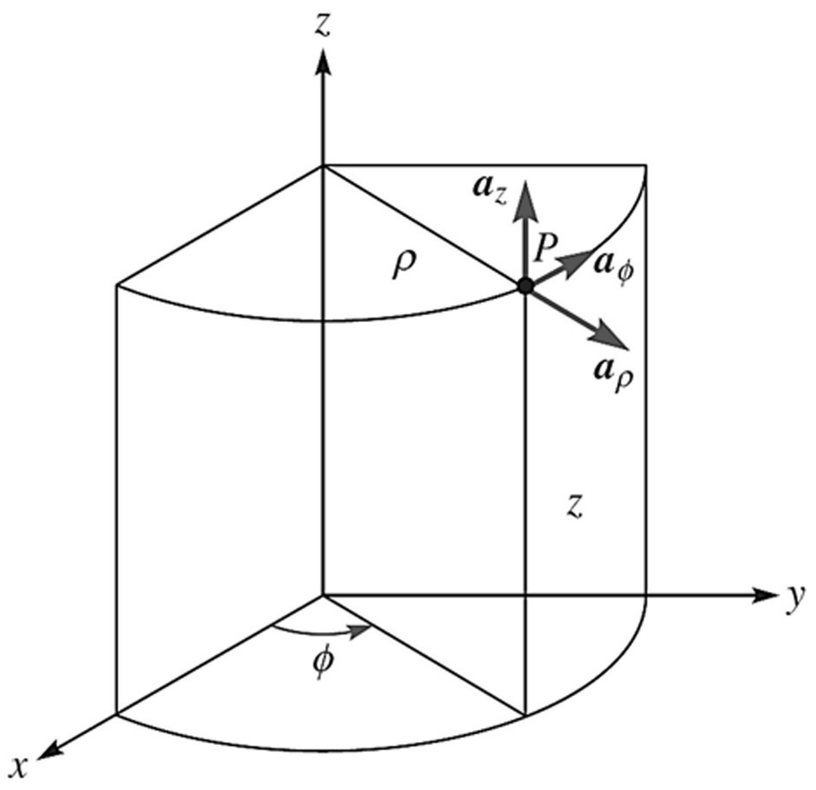
$$(A_x, A_y, A_z) \quad \text{or} \quad A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$



Circular Cylindrical Coordinates (ρ, ϕ, z)

$$0 < \rho < \infty$$
$$0 < \phi < 2\pi$$
$$-\infty < z < \infty$$

$$(A_\rho, A_\phi, A_z) \quad \text{or} \quad A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z$$



Circular Cylindrical Coordinates (ρ, ϕ, z)

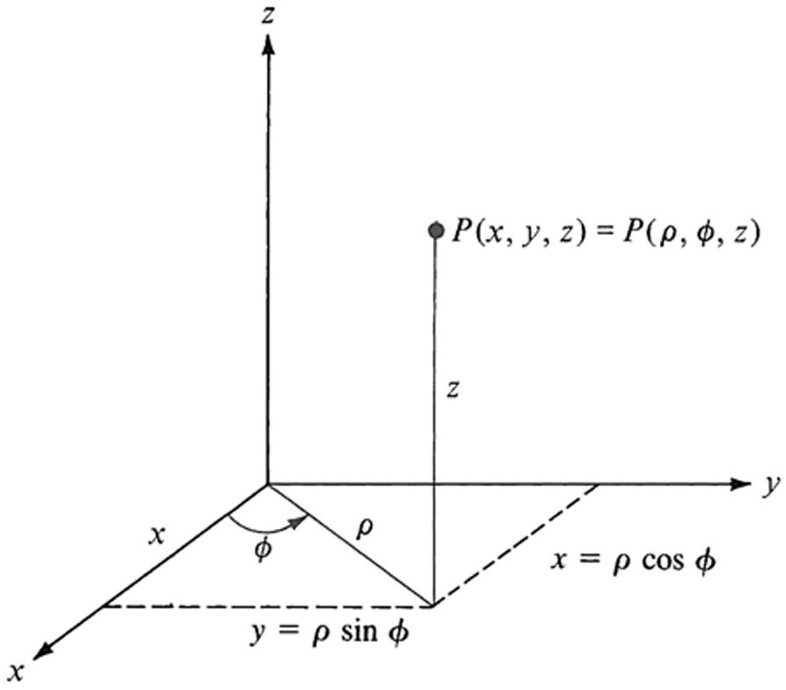
Point Coordinate Transformation

- From Cartesian

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

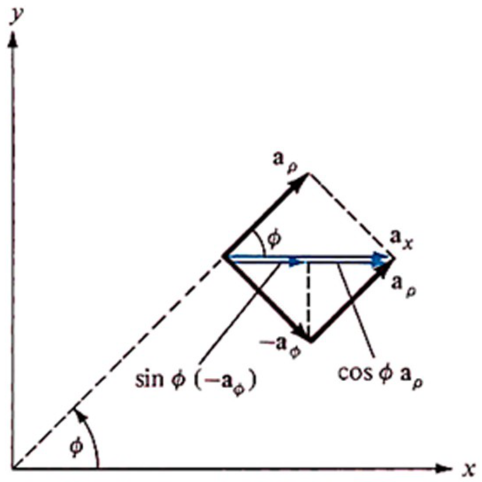
- To Cartesian

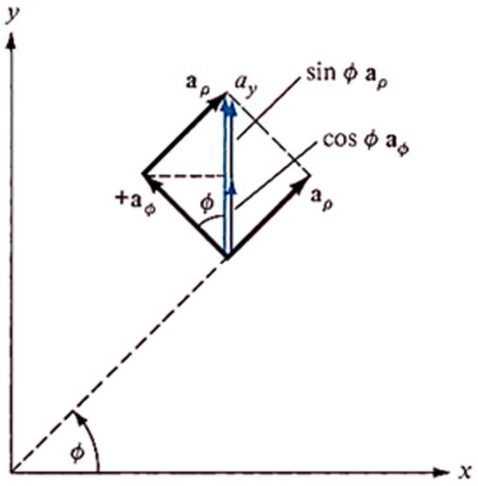
$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad z = z$$



Circular Cylindrical Coordinates (ρ, ϕ, z)

Vector Transformation





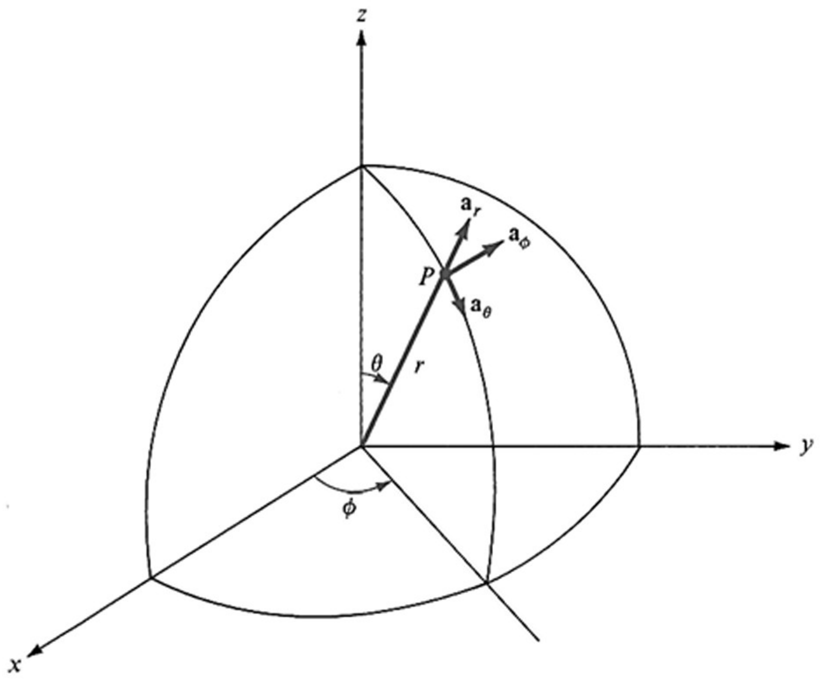
$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Spherical Coordinates (r, θ, ϕ)

$$0 < r < \infty$$
$$0 < \theta < \pi$$
$$0 < \phi < 2\pi$$

$$(A_r, A_\theta, A_\phi) \quad \text{or} \quad A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$



Spherical Coordinates (r, θ, ϕ)

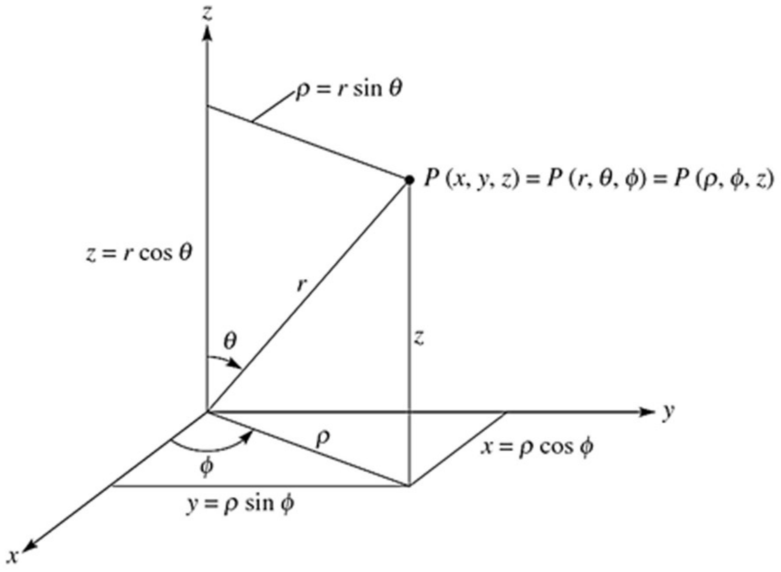
Point Coordinate Transformation

- From Cartesian

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}$$

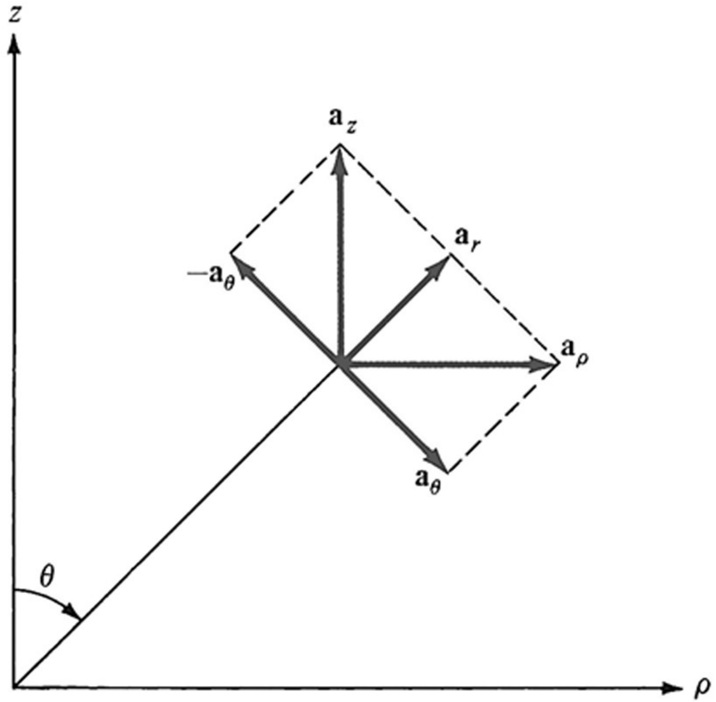
- To Cartesian

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$



Spherical Coordinates (r, θ, ϕ)

Vector Transformation



$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

8. Express in cylindrical components:

(a) the vector from $\mathbf{C}(3, 2, -7)$ to $\mathbf{D}(-1, -4, 2)$;

(b) a unit vector at $\mathbf{D}$ directed toward $\mathbf{C}$;

(c) a unit vector at $\mathbf{D}$ directed toward the origin.

8. Express in cylindrical components:

(a) the vector from $\mathbf{C}(3, 2, -7)$ to $\mathbf{D}(-1, -4, 2)$;

(b) a unit vector at $\mathbf{D}$ directed toward $\mathbf{C}$;

(c) a unit vector at $\mathbf{D}$ directed toward the origin.

$$a)\overline{CD} = -(4cos\varphi + 6sin\varphi)\bar{a}_\rho + (4sin\varphi - 6cos\varphi)\bar{a}_\varphi + 9\bar{a}_z$$

$$b) - 0.5889\bar{a}_\rho + 0.2103\bar{a}_\varphi - 0.7803\bar{a}_z$$

$$c) - 0.8997\bar{a}_\rho + 0\bar{a}_\varphi - 0.4364\bar{a}_z$$

10. Express the unit vector $\overline{\mathbf{a}}_x$ in spherical components at the point:

(a) $r = 2\text{m}, \theta = 1\text{rad}, \phi = 0.8\text{rad};$

(b) $x = 3\text{m}, y = 2\text{m}, z = -1\text{m};$

(c) $\rho = 2.5\text{m}, \phi = 0.7\text{rad}, z = 1.5\text{m}.$

10. Express the unit vector $\overline{\mathbf{a}}_x$ in spherical components at the point:

(a) $r = 2\text{m}, \theta = 1\text{rad}, \phi = 0.8\text{rad};$

(b) $x = 3\text{m}, y = 2\text{m}, z = -1\text{m};$

(c) $\rho = 2.5\text{m}, \phi = 0.7\text{rad}, z = 1.5\text{m}.$

$$a)\ 0.5862\ \boldsymbol{a}_r + 0.3764\ \boldsymbol{a}_\theta - 0.7173\ \boldsymbol{a}_\phi$$
$$b)\ 0.8017\ \boldsymbol{a}_r - 0.2223\ \boldsymbol{a}_\theta - 0.5547\ \boldsymbol{a}_\phi$$
$$c)\ 0.6558\ \boldsymbol{a}_r + 0.3935\ \boldsymbol{a}_\theta - 0.6442\ \boldsymbol{a}_\phi$$

11. Express the field vector

$$\overline{\mathbf{H}} = xy^2z\overline{\mathbf{a}}_{\mathbf{x}} + x^2yz\overline{\mathbf{a}}_{\mathbf{y}} + xyz^2\overline{\mathbf{a}}_{\mathbf{z}}$$

(a) In cylindrical and spherical coordinates.

(b) In spherical coordinates

11. Express the field vector

$$\overline{\mathbf{H}} = xy^2z\overline{\mathbf{a}}_{\mathbf{x}} + x^2yz\overline{\mathbf{a}}_{\mathbf{y}} + xyz^2\overline{\mathbf{a}}_{\mathbf{z}}$$

(a) In cylindrical and spherical coordinates.

(b) In spherical coordinates

$$a)\ \boldsymbol{H} = \frac{1}{2}p^3zsin(2\varphi)^2\boldsymbol{a_p} + \frac{1}{4}p^3zsin(4\varphi)\boldsymbol{a_\phi} + \frac{1}{2}p^2z^2sin(2\varphi)\boldsymbol{a_z}$$

$b)$ ***Exercise***

12. Given $\overline{\mathbf{A}} = 2\overline{\mathbf{a}}_{\mathbf{x}} + 4\overline{\mathbf{a}}_{\mathbf{y}} + 10\overline{\mathbf{a}}_{\mathbf{z}}$ and $\overline{\mathbf{B}} = -5\overline{\mathbf{a}}_{\rho} + \overline{\mathbf{a}}_{\phi} - 3\overline{\mathbf{a}}_{\mathbf{z}}$, find:

(a) $\overline{\mathbf{A}} + \overline{\mathbf{B}}$ at $\mathbf{P}(0, +2, -5)$.

(b) The angle between $\overline{\mathbf{A}}$ and $\overline{\mathbf{B}}$ at $\mathbf{P}$.

(c) The scalar component of $\overline{\mathbf{A}}$ along $\overline{\mathbf{B}}$ at $\mathbf{P}$.

EXERCISE

13. A vector field in mixed coordinate variables is given by:

$$\overline{\mathbf{G}} = \frac{x\cos\phi}{\rho}\overline{\mathbf{a}}_{\mathbf{x}} + \frac{2yz}{\rho^2}\overline{\mathbf{a}}_{\mathbf{y}} + \left(1 - \frac{x^2}{\rho^2}\right)\overline{\mathbf{a}}_{\mathbf{z}}$$

Express $\overline{\mathbf{G}}$ completely in spherical system.

EXERCISE

QUESTIONS ?

THANKS

References

- ELEMENTS OF ELECTROMAGNETICS, MATTHEW N. O. SADIKU