Electrical Power Engineering Department

Electromagnetic Fields (EPE112&EPM2142)

Tutorial

Week(4)

Tutorial Objectives

- Differential Elements in different coordinate systems
- Electric fields due to continuous charge distributions
  - A Line Charge
  - A Surface Charge
  - A Circular Ring
  - A Volume Charge
- Gauss's law

DIFFERENTIAL ELEMENTS IN DIFFERENT COORDINATE SYSTEMS

Cartesian Coordinate System

Differential displacement is given by

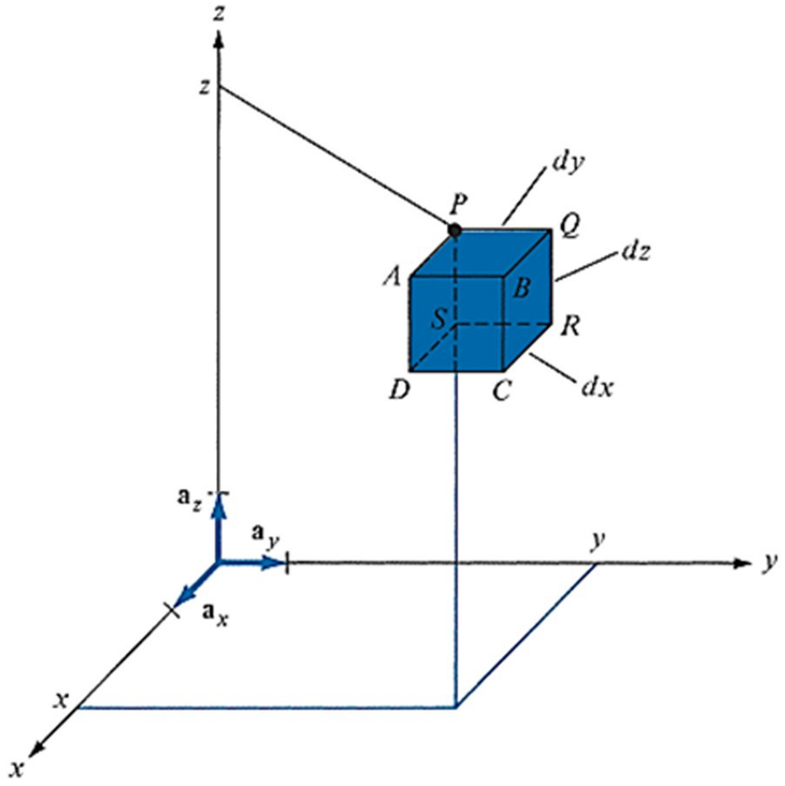
$$\boxed{d\mathbf{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z}$$

Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = {} & dy\,dz\,\mathbf{a}_x \\ & dx\,dz\,\mathbf{a}_y \\ & dx\,dy\,\mathbf{a}_z \end{aligned}}$$

Differential volume is given by

$$\boxed{dv = dx\,dy\,dz}$$



Cylindrical Coordinate System

Differential displacement is given by

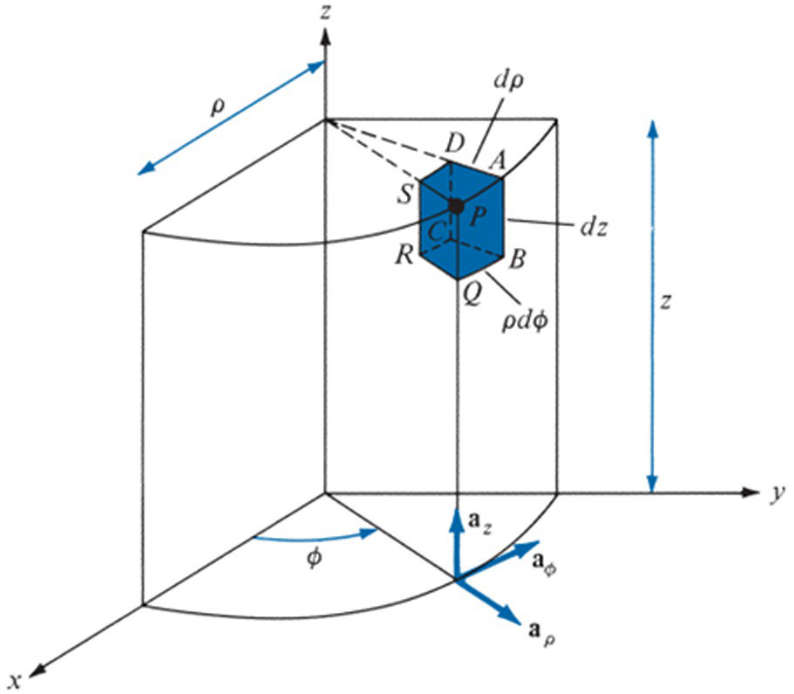
$$\boxed{d\mathbf{l} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z}$$

Differential normal surface area is given by

$$\boxed{\begin{aligned} d\mathbf{S} = &\ \rho\,d\phi\,dz\,\mathbf{a}_\rho \\ &\ d\rho\,dz\,\mathbf{a}_\phi \\ &\ \rho\,d\rho\,d\phi\,\mathbf{a}_z \end{aligned}}$$

Differential volume is given by

$$\boxed{dv = \rho\,d\rho\,d\phi\,dz}$$



Spherical Coordinate System

The differential displacement is

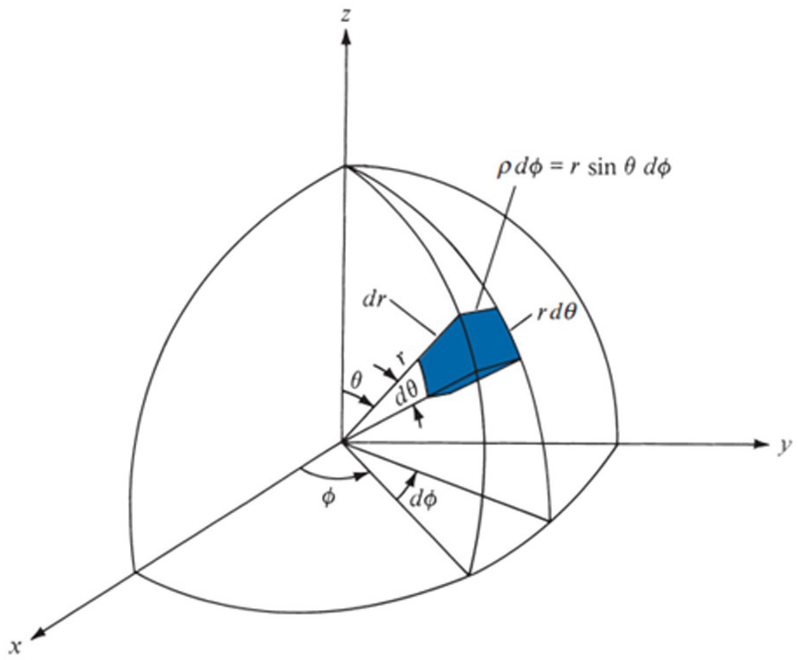
$$\boxed{d\mathbf{l} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi}$$

The differential normal surface area is

$$\boxed{\begin{aligned} d\mathbf{S} = {} & r^2\sin\theta\,d\theta\,d\phi\,\mathbf{a}_r \\ & r\sin\theta\,dr\,d\phi\,\mathbf{a}_\theta \\ & r\,dr\,d\theta\,\mathbf{a}_\phi \end{aligned}}$$

The differential volume is

$$\boxed{dv = r^2\sin\theta\,dr\,d\theta\,d\phi}$$



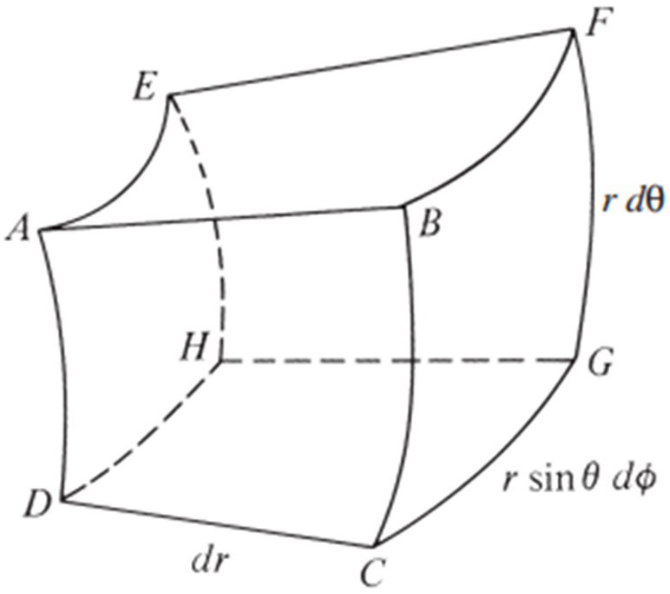
EXERCISES

9. The surfaces $r = 2$ and 4, $\theta = 30^\circ$ and 50°, and $\phi = 20^\circ$ and 60° identify a closed surface. Find:

(a) the enclosed volume;

(b) the total area of the enclosing surface;

(c) the total length of the twelve edges of the surface;



ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

Electric fields due to continuous charge distributions

$$\mathbf{E} = \int_L \frac{\rho_L \, dl}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(line charge)}$$

$$\mathbf{E} = \int_S \frac{\rho_S \, dS}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(surface charge)}$$

$$\mathbf{E} = \int_v \frac{\rho_v \, dv}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(volume charge)}$$

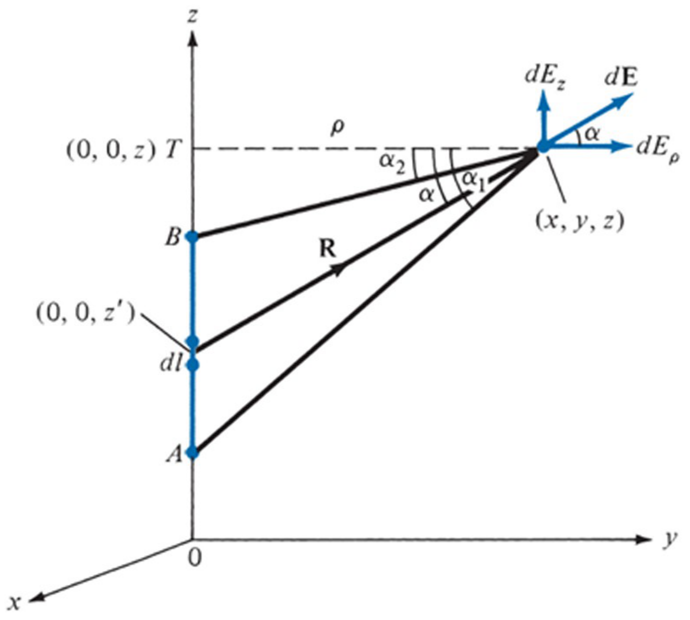
A Line Charge

Thus for a *finite line charge*,

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_o\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z\right]$$

for an *infinite line charge*,

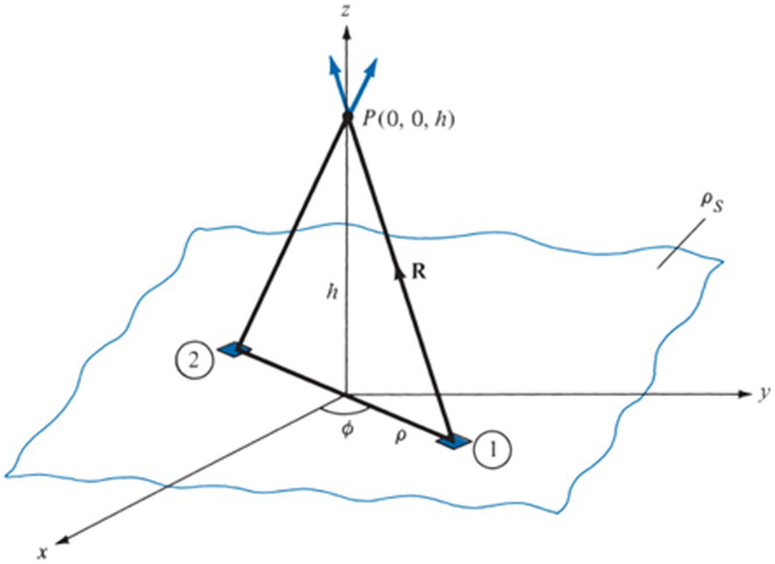
$$\boxed{\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\,\mathbf{a}_\rho}$$



A Surface Charge

for an *infinite sheet* of charge

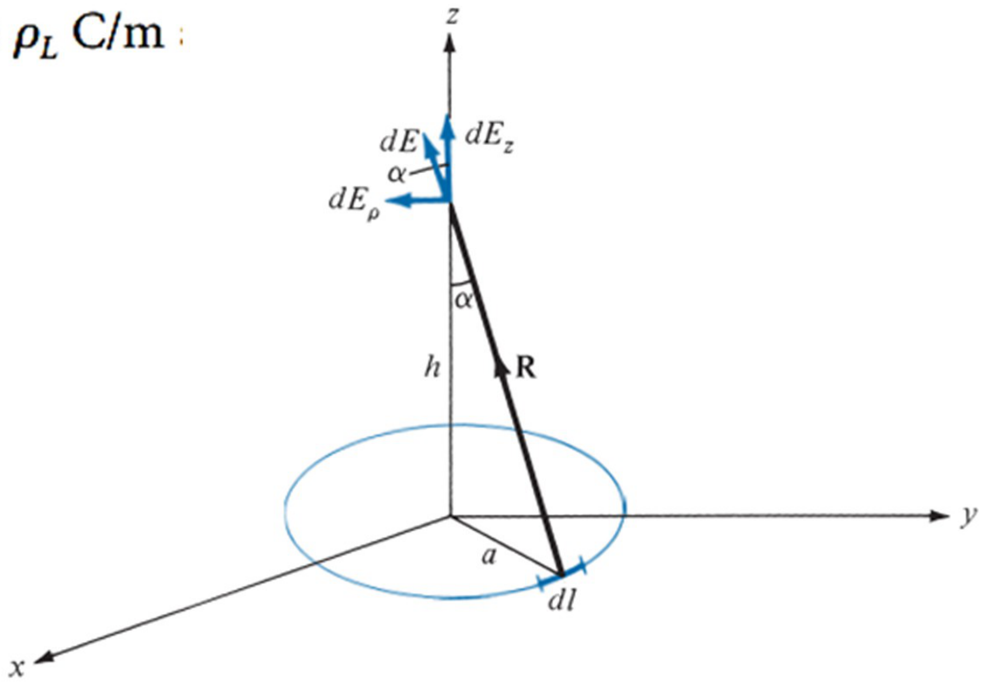
$$\boxed{\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n}$$



A Circular Ring Charge

A circular ring of radius a carries a uniform charge ρ_L C/m

$$\mathbf{E} = \frac{\rho_L a h}{2\varepsilon_o[h^2 + a^2]^{3/2}}\mathbf{a}_z$$



EXERCISES

8. Determine the total charge:

(a) On line $0 < x < 5$ m if $\rho_\ell = 12x^2$ mC/m

(b) On the cylinder $\rho = 3$, $0 < z < 4$ m if $\rho_s = \rho z^2$ nC/m^2.

(c) Within the sphere $r = 4$m if $\rho_v = \frac{10}{r \sin\theta}$ C/m^3.

9. A line charge density ρ_ℓ is uniformly distributed over a length of 2a with centre as origin along x-axis. Find $\overline{\mathbf{E}}$ at a point $\boldsymbol{P}$ which is on z-axis at a distance d.

4. Plane $z = -10$ m carries charge $-20\,\mathrm{nC/m^2}$. The electric field intensity at the origin is ——- V/m.

6. A sheet of charges $\rho_s = 2\ \mathrm{nC/m^2}$, is present at the plane $x = 3$ in free space, and a line charge $\rho_\ell = 2\ \mathrm{nC/m}$ is located at $x = 1$, $z = 4$. Find:

   (i) The magnitude of electric field intensity at the origin.

   (ii) The direction of $\overline{\mathbf{E}}$ at (4, 5, 6).

   (iii) What is the force per meter length on the line charge?

10. A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of $5\,\mu\text{C/m}$.

(a) Find $\overline{\mathbf{E}}$ at $P(3,0,0)$

(b) If two identical point charges Q are placed at $(0, -3, 0)$ and (0, 3, 0) in addition to the ring, Find the value of Q such that $\overline{\mathbf{E}} = 0$ at $\boldsymbol{P}$.

12. A point charge $100\,\mathrm{pC}$ is located at (4, 1, -3) while the x-axis carries charge $2\,\mathrm{nC/m}$. If the plane $z = 3$ also carries charge $5\,\mathrm{nC/m^2}$, Find $\overline{\mathbf{E}}$ at $(1, 1, 1)$.

GAUSS'S LAW

Gauss’s law

- ***Gauss’s law*** states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface.

$$\Psi = Q_{\text{enc}}$$

EXERCISES

2. Electric —— at a point may be defined as equal to lines of force passing normally through a unit cross-section at that point.

(a) field density

(b) field intensity

(c) flux density

(d) potential

3. Point charges $30\,\mathrm{nC}$, $-20\,\mathrm{nC}$, and $10\,\mathrm{nC}$ are located at $(-1, 0, 2)$, $(0, 0, 0)$, and $(1, 5, -1)$, respectively. The total flux leaving a cube of side 6 m centered at the origin is:

QUESTIONS ?

THANKS

References

- ELEMENTS OF ELECTROMAGNETICS, MATTHEW N. O. SADIKU