Electromagnetics

Week (2)

Sheet - DRAFT

[5] Given : $\vec{A} = (2, 1, 3)$, $\vec{B} = (1, -3, 2)$

Req. : Find: θ_{AB}

Sol

$\because \vec{A} \cdot \vec{B} = AB \cos(\theta_{AB}) \longrightarrow \theta_{AB} = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{AB}\right)$

$A = \sqrt{2^2 + 1^2 + 3^2} = 3,742$, $B = \sqrt{1+9+4} = 3,742 \longrightarrow \therefore AB = 14$

$\vec{A} \cdot \vec{B} = 1 \times 2 + 1 \cdot x - 3 + 3 \times 2 = 5 \longrightarrow \therefore \theta_{AB} = \cos^{-1}\left(\frac{5}{14}\right) \simeq 69^\circ$ #

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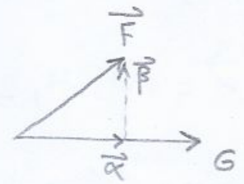
[6] Given $\vec{F} = (10, -6, 5)$, $\vec{G} = (0,1, 0.2, 0.3)$

Req. :



Sol

(a) $\alpha = \frac{\vec{F} \cdot \vec{G}}{G} = \frac{1,3}{0,374} = 3,474$, $\vec{a}_G = \frac{\vec{G}}{G} = (0.2672, 0.5345, 0.8017)$

$\therefore \vec{\alpha} = \alpha . \vec{a}_G = (0.9284, 1.8569, 2.7853)$ #a

(b) $\because \vec{\alpha} + \vec{\beta} = \vec{F} \longrightarrow \therefore \vec{\beta} = \vec{F} - \vec{\alpha} = (9.0715, -7.856, 2.2146)$ #b

(c) $\vec{G}_{\parallel \vec{F}} = \frac{\vec{G} \cdot \vec{F}}{F} \vec{a}_F = (0.0807, -0.048, 0.0403)$

$\vec{G}_{\perp \vec{F}} = \vec{G} - \vec{G}_{\parallel \vec{F}} = (0.0192, 0.2484, 0.2596)$ #c

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[7] Given : $\vec{R}_1 = (7, 3, -2)$, $\vec{R}_2 = (-2, 7, -3)$, $\vec{R}_3 = (0, 2, 3)$

Req :

Sol.

(a) $\vec{R}_1 \times \vec{R}_2 = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 7 & 3 & -2 \\ -2 & 7 & -3 \end{vmatrix} = (-9+14)\vec{a}_x + (4+21)\vec{a}_y + (49+6)\vec{a}_z = (5, 25, 55)$

$\vec{a}_{\vec{R}_1 \times \vec{R}_2} = \frac{\vec{R}_1 \times \vec{R}_2}{|\vec{R}_1 \times \vec{R}_2|} = (0.0824, 0.4123, 0.9072)$ #a

ⓑ $\vec{R_1}-\vec{R_2}=(9,-4,1)$, $\vec{R_2}-\vec{R_3}=(-2,5,-6)$

$$\vec{a}_b=\frac{(\vec{R_1}-\vec{R_2})\times(\vec{R_2}-\vec{R_3})}{|(\vec{R_1}-\vec{R_2})\times(\vec{R_2}-\vec{R_3})|}=\frac{(19,52,37)}{66,588}=(0.2853,0.7809,0.5556)\ \#b$$

ⓒ Area of triangle $=\frac{1}{2}|\vec{R_1}\times\vec{R_2}|=\frac{1}{2}\times(60,6217)=\underline{30.311}$ Unit area #c

ⓓ $\vec{R_1}-\vec{R_2}=(9,-4,1)$, $\vec{R_3}-\vec{R_2}=(2,-5,6)$

#d

$\therefore$ area of triangle $=\frac{1}{2}\cdot|(\vec{R_1}-\vec{R_2})\times(\vec{R_3}-\vec{R_2})|=\frac{1}{2}\times 66,5883=\underline{33.294}$ unit area

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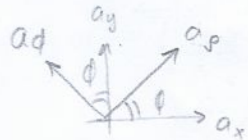

ⓐ $\vec{CD}=\vec{\alpha}=(-1,-4,2)-(3,2,-7)=(-4,-6,9)$

$$\begin{bmatrix}A_\rho\\A_\phi\\A_z\end{bmatrix}=\begin{bmatrix}\cos(\phi)&\sin(\phi)&0\\-\sin(\phi)&\cos(\phi)&0\\0&0&1\end{bmatrix}\begin{bmatrix}-4\\-6\\9\end{bmatrix}\quad\therefore\ \vec{CD}=\begin{pmatrix}-4\cos(\phi)-6\sin(\phi)\\4\sin(\phi)-6\cos(\phi)\\9\end{pmatrix}\begin{matrix}\vec{a}_\rho\\\vec{a}_\phi\\\vec{a}_z\end{matrix}\quad\#$$

ⓑ $|\vec{DC}|=|\vec{CD}|=\sqrt{16+36+81}=\sqrt{133}$

$$\vec{DC}=\begin{pmatrix}4\cos(\phi)+6\sin(\phi)\\-4\sin(\phi)+6\cos(\phi)\\-9\end{pmatrix}\begin{matrix}\vec{a}_\rho\\\vec{a}_\phi\\\vec{a}_z\end{matrix}\qquad\therefore\ \vec{a}_{\vec{DC}}=\begin{matrix}(0,3468\cos(\phi)+0.5202\sin(\phi))\,\vec{a}_\rho\\+(0,3468\sin(\phi)+0.5202\cos(\phi))\,\vec{a}_\phi\\-(\quad 0,7803\quad)\,\vec{a}_z\end{matrix}$$

at $D\longrightarrow\rho=\sqrt{1^2+4^2}=\sqrt{17}$, $\phi=180+\tan^{-1}(4)=255.9637^\circ$

$$\therefore\ \vec{a}_{\vec{DC}}\Big|_{\text{at }D}=-0.5889\,\vec{a}_\rho+0,2103\,\vec{a}_\phi-0,7803\,\vec{a}_z\quad\#$$

ⓒ $\vec{DO}=(1,4,-2)$, $|\vec{DO}|=\sqrt{21}$, $\vec{DO}=(\cos(\phi)+4\sin(\phi))\,\vec{a}_\rho$

$+(-\sin(\phi)+4\cos(\phi))\,\vec{a}_\phi$

$\phi=180+\tan^{-1}\left(\frac{4}{1}\right)=255,964^\circ$

$+(\quad -2\quad)\,\vec{a}_z$

$$\therefore \vec{a}_{\vec{D_0}} @ D = \frac{1}{\sqrt{21}}\left(-4{,}1231\vec{a}_\rho + 0\vec{a}_\phi - 2\vec{a}_z\right) \quad \#$$

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[10]

	A_x	A_y	A_z
A_r	$\sin(\theta)\cos(\phi)$	$\sin(\theta)\sin(\phi)$	$\cos(\theta)$
A_θ	$\cos(\theta)\cos(\phi)$	$\cos(\theta)\cos(\phi)$	$-\sin(\theta)$
A_ϕ	$-\sin(\phi)$	$\cos(\phi)$	0

$$\vec{a}_x = \sin(\theta)\cos(\phi)\vec{a}_r + \cos(\theta)\cos(\phi)\vec{a}_\theta + (-\sin(\phi))\vec{a}_\phi$$

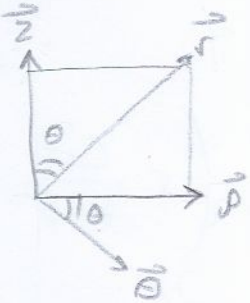
ⓐ at $\theta = 1^r$, $\phi = 0.8^r$

$$\therefore \vec{a}_x = 0.5862\vec{a}_r + 0.3764\vec{a}_\theta - 0.7173\vec{a}_\phi \quad \#$$

ⓑ at $(x, y, z) = (3, 2, -1)$

$$\theta = 180 - \tan^{-1}\left(\frac{\sqrt{9+4}}{1}\right) = 105{,}5013^\circ \quad , \phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$$

$$\vec{a}_x = 0.8017\vec{a}_r - 0.2223\vec{a}_\theta - 0.5547\vec{a}_\phi \quad \#$$



ⓒ $(\rho, \phi, z) = (2.5, 0.7^r, 1.5)$

$$r = \sqrt{\rho^2 + z^2} = 2.9155\,m$$

$$\theta = \tan^{-1}\left(\frac{\rho}{z}\right) = 59.0362^\circ$$

$$\phi = 40.107^\circ$$

$$\therefore \vec{a}_x = 0.6558\vec{a}_r + 0.3935\vec{a}_\theta - 0.6442\vec{a}_\phi \quad \#$$

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[11]

	A_x (xy^2z)	A_y (x^2yz)	A_z (xyz^2)
A_ρ	$\cos(\phi)$	$\sin(\phi)$	0
A_ϕ	$-\sin(\phi)$	$\cos(\phi)$	0
A_z	0	0	1

$$\therefore \vec{H}_\rho = xyz\left(y\cos(\phi) + x\sin(\phi)\right)\vec{a}_\rho$$

$$= \rho\cos(\phi) \cdot \rho\sin(\phi) \cdot 2z\rho\sin(\phi)\cos(\phi)$$

$$= 2\rho^3 z\left(\frac{1}{2}\sin^2(2\phi)\right) = \frac{1}{2}\rho^3 z\sin^2(2\phi)\,\vec{a}_\rho$$

$$\therefore \vec{H}_\phi = xyz\left(-y\sin(\phi) + x\cos(\phi)\right)\vec{a}_\phi$$

$$= \rho^2\sin\phi\cos\phi\, z\left(\rho\cos^2\phi - \rho\sin^2\phi\right)\vec{a}_\phi$$

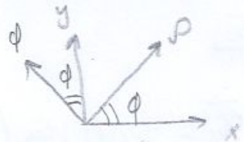
$$= \rho^3\frac{1}{2}\sin(2\phi) \cdot z \cdot \cos(2\phi)\,\vec{a}_\phi$$

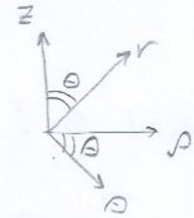
$$= \frac{1}{4}\rho^3 z\sin(4\phi)\,\vec{a}_\phi$$

$$\vec{H}_z = \rho\cos\phi \cdot \rho\sin\phi \cdot z^2\,\vec{a}_z$$

$$= \frac{1}{2}\rho^2 z^2\sin(2\phi)\,\vec{a}_z$$

$$\therefore \vec{H} = \frac{1}{2}\rho^3 z \sin^2(2\phi)\,\vec{a}_\rho + \frac{1}{4}\rho^3 z \sin(4\phi)\,\vec{a}_\phi + \frac{1}{2}\rho^2 z^2 \sin(2\phi)\,\vec{a}_z \quad \text{※a}$$





	xy^2z A_x	x^2yz A_y	xyz^2 A_z
A_r	$\cos(\phi)\sin(\theta)$	$\sin(\phi)\sin(\theta)$	$\cos(\theta)$
A_θ	$\cos(\phi)\cos(\theta)$	$\sin(\phi)\cos(\theta)$	$-\sin(\theta)$
A_ϕ	$-\sin(\phi)$	$\cos(\phi)$	0

$$\vec{H}_r = xyz\left(y\cos(\phi)\sin(\theta) + x\sin(\phi)\sin(\theta) + z\cos(\theta)\right)\vec{a}_r$$

$r = x\cos(\phi)\sin(\theta)$

$r = y\sin\phi\,\sin(\theta)$

$r = z\cos(\theta)$

$$\therefore \vec{H}_r = \frac{r^3}{\cos(\phi)\sin(\phi)\sin^2(\theta)\cos(\theta)}\left(\frac{r\cdot\cos(\phi)\cancel{\sin(\theta)}}{\sin(\phi)\cancel{\sin(\theta)}} + \frac{r\cdot\sin\phi\,\cancel{\sin\theta}}{\cos\phi\,\cancel{\sin\theta}} + r\right)$$

$$\therefore \vec{H}_r = \frac{r^3}{\square}\left(r\frac{\cos\phi}{\sin\phi} + r\frac{\sin\phi}{\cos\phi} + r\right)\vec{a}_r \quad , \square = \frac{1}{4}\sin(2\phi)\sin(2\theta)\sin(\theta)$$

⋮