ch.3 في الكتاب   ch.1 : interpolation   والاستكمال

يستخدم في ؟؟ (1) ما

لو عندي جدول تجريبي قيمة فيه مفقودة
فينا نعوضها بعد معادلة Polynomial
أجيب منها قيمة القيمة المفقودة

X	x_0	x_1	--	x_{n-1}	x_n
y	y_0	y_1	--	y_{n-1}	y_n

$P_n(x) = $ ___   معادلة تمثل القراءات الموجودة

* note :- n = num of reading - 1

inter polation

un equal interval   equal interval
$x_1 - x_0 \neq x_2 - x_1 \neq x_3 - x_2$ $x_1 - x_0 = x_2 - x_1 = h$

طرق الحل   →   تختلف في equal و un equal

① lagrange interpolation
② newton's divided difference

*

① lagrange interpolation

$$P_n(x) = \frac{(x-x_1)(x-x_2)\_(x-x_n)}{(x_0-x_1)(x_0-x_2)\_(x_0-x_n)} y_0 \underset{\to L_0(x)}{+} \frac{(x-x_0)(x-x_2)(x-x_3)\_(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\_(x_1-x_n)} y_1$$
$\to L_1(x)$

$$+ \frac{(x-x_0)(x-x_1)\_(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\_(x_n-x_{n-1})} y_n$$
$\to L_n(x)$

$$P_n(x) = \sum_{i=0}^{i=n} L_i(x) \, y_i \quad F(x_i)$$

Error term :- (حد الخطأ)

$$En(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(n+1)!} F^{n+1}(c)$$

Error Bound :-

$$|En(x)| = \frac{|(x-x_0)(x-x_1)\cdots(x-x_n)|}{(n+1)!} |F^{n+1}(c)|$$

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notes :-

* $n+1 \longrightarrow$ المشتقة.

* $M_{n+1} = |F^{n+1}(c)|$

* $x_0 \leq c \leq x_n$

EX:-

lagrange polynomial

x	x_0	x_1	x_2
x	-1	0	3
y=F(x)	8	-2	4
	y_0	y_1	y_2

$n=2 \longrightarrow$ درجة ثانية

$$P_2(x) = \frac{(x-0)(x-3)}{(-1-0)(-1-3)} \times (8) + \frac{(x+1)(x-3)}{(0+1)(0-3)}(-2) + \frac{(x+1)(x-0)}{(3+1)(3-0)} \times 4$$

$$P_2(x) = 2x^2 - 6x + \frac{2}{3}(x^2-2x-3) + \frac{1}{3}(x^2+x)$$

$P_2(1) = \text{---}$ ↰ نعوض بالـ x وظلع قيمة y

+ EX$_2$ @ في الأقئات

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equal interval   **Error Bound :-**

* $F_1(x) \leq \frac{h^2}{8} M_2$ $*$ $h \longrightarrow$ عرض الفترة

* $E_2(x) \leq \frac{h^3}{9\sqrt{3}} M_3$

* $E_3(x) \leq \frac{h^4}{24} M_4$

② Newton divided difference

X	x_0	x_1	x_2	x_3	x_4	
F(x)=y	y_0	y_1	y_2	y_3	y_4	

X	y	S_0	S_0^2	S_0^3	S_0^4
x_0	y_0				
x_1	y_1	$\dfrac{y_1-y_0}{x_1-x_0}=S_0$			
x_2	y_2	$\dfrac{y_2-y_1}{x_2-x_1}=S_1$	$\dfrac{S_1-S_0}{x_2-x_0}=S_0^2$		
x_3	y_3	$\dfrac{y_3-y_2}{x_3-x_2}=S_2$	$\dfrac{S_2-S_1}{x_3-x_1}=S_1^2$	$\dfrac{S_1^2-S_0^2}{x_3-x_0}=S_0^3$	
x_4	y_4	$\dfrac{y_4-y_3}{x_4-x_3}=S_3$	$\dfrac{S_3-S_2}{x_4-x_2}=S_2^2$	$\dfrac{S_2^2-S_1^2}{x_4-x_1}=S_1^3$	$\dfrac{S_1^3-S_0^3}{x_4-x_0}=S_0^4$

* $P_n(X) = y_0 + (X-X_0)S_0 + (X-X_0)(X-X_1)S_0^2 + (X-X_0)(X-X_1)(X-X_2)S_0^3$
  $(X-X_0)(X-X_1)(X-X_2)(X-X_3)S_0^4$

EX.4   using newton

X	0	4	6	8	
y	4	8	14	16	

X	y	S_0	S_0^2	S_0^3
0	4			
4	8	$\dfrac{8-4}{4-0}=1$	$\dfrac{3-1}{6-0}=\dfrac{1}{3}$	
6	14	$\dfrac{14-8}{6-4}=3$	$\dfrac{1-3}{8-4}=-\dfrac{1}{2}$	$\dfrac{\frac{1}{2}-\frac{1}{3}}{8-0}=\dfrac{-5}{48}$
8	16	$\dfrac{16-14}{8-6}=1$		

$P_3(X) = 4 + (X-0)\,1$
$+ (X-0)(X-4)\,\dfrac{1}{3}$
$+ (X-0)(X-4)(X-6)$
$\cdot\dfrac{-5}{48}$

$+ EX 5$ قانون كذا