Chapter 2

Transfer of Signals

Lecture 5

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Contents

(1)

Deterministic and Random Signals

Deterministic Signal

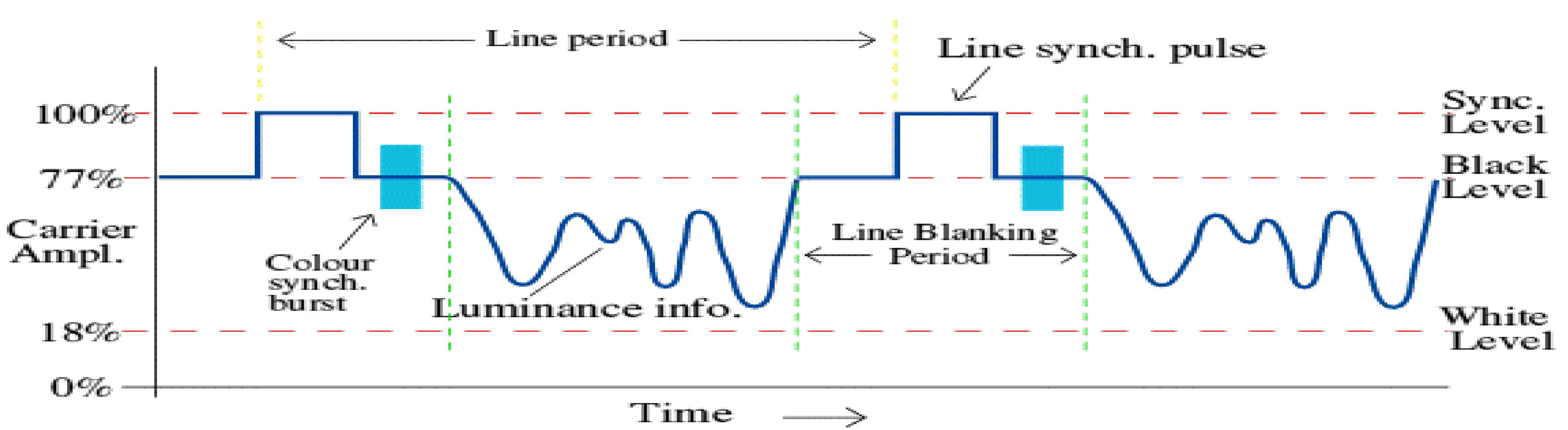
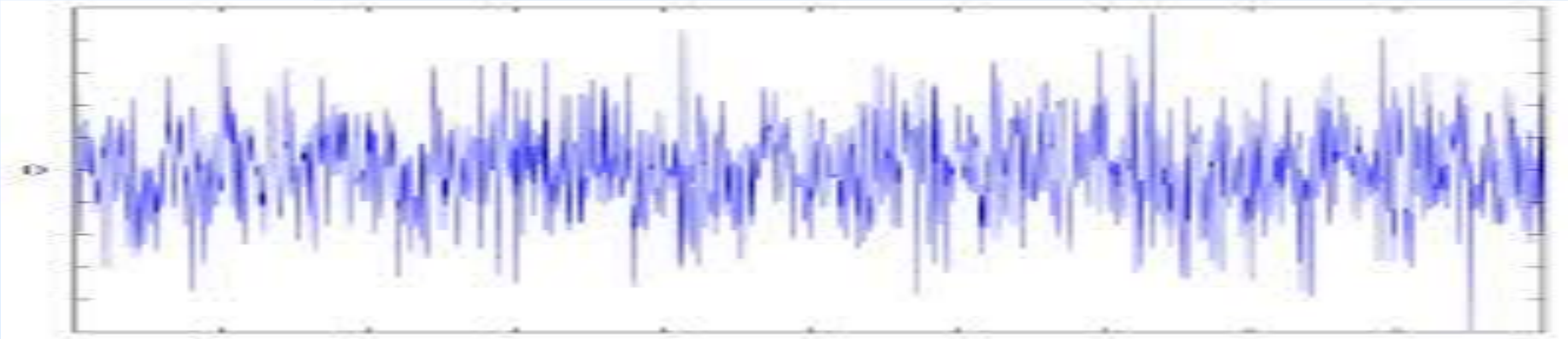


Figure 21.3 UK Video signal.

- Information to be Transmitted:
  - **Teletype Signal,**
  - **Video Signal**
  - **Speech Signal,**
- **Although speech has no determined rule, it has a certain band and certain level.**

Random Signals

- Undeterministic,



which have no rule in changing

- Examples: Noise signal.

(2)
Linear Time Invariant System

Linear System

- A system is said to be linear if its characteristics depends on itself and is independent on the cause.
- If $g_1(t)$ is the response of $f_1(t)$:

$$f_1(t) \rightarrow g_1(t)$$

$$\therefore \sum_{i=1}^{n} a_i f_i(t) \rightarrow \sum_{i=1}^{n} a_i g_i(t)$$

Time Invariant System

- The system is time invariant if its characteristics is independent on time.

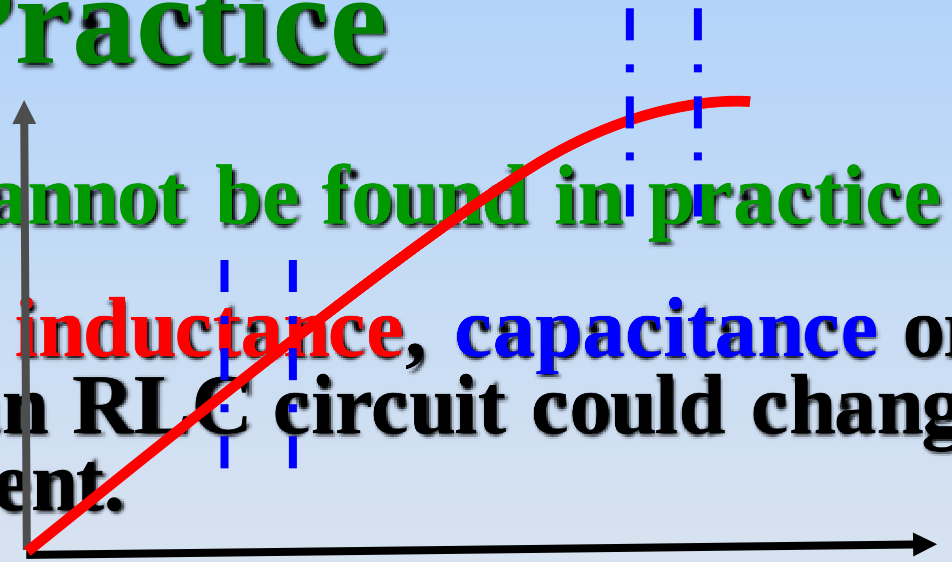
- Or system constants (coefficients) does not change with time or aging.

$$f_1(t) \to g_1(t)$$

$$\therefore f_1(t-\tau) \to g_1(t-\tau)$$

Linear Time Invariant System in Practice

- Ideal linearity cannot be found in practice



  - For example, the inductance, capacitance or even resistance of an RLC circuit could change after a certain current.
  - However, if the system operates in a certain small part of its characteristics one can assuming it as being nearly linear.

- Also, time invariant system is not practical
  - One can assume near time invariant for a system if its constants vary slowly with time.

(3)

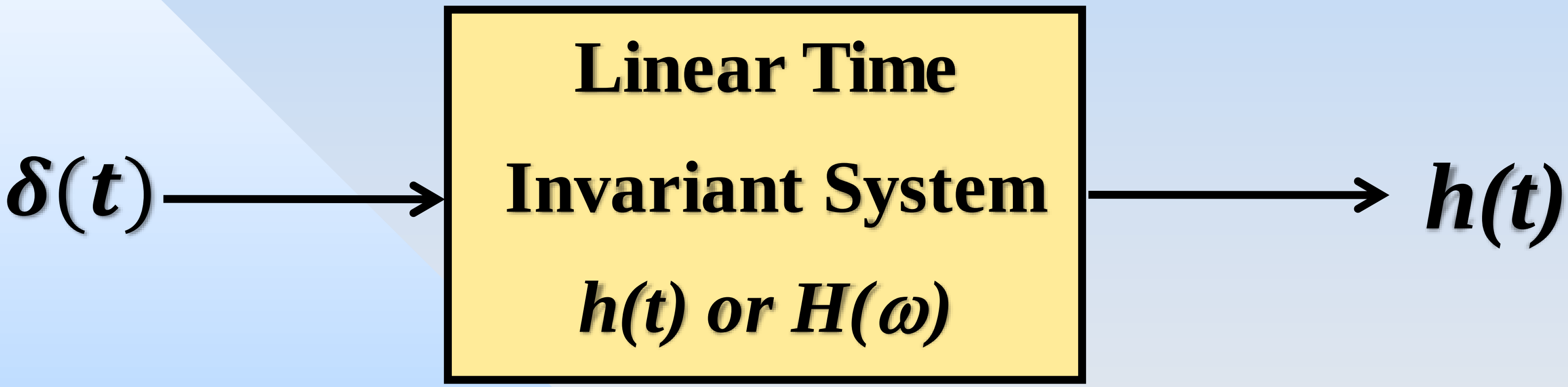
System

Characteristics

(a) Impulse Response

- How to estimate characteristics of a linear time invariant system $h(t)$.
- When applying an impulse voltage $\delta(t)$ to the system input.
- Output of an impulse input denoted as $h(t)$ is then a function of the system characteristics and is called the impulse response.

Free Running Impulse Response



Estimating the Characteristics $h(t)$ of a Linear Time Invariant System

Why $\delta(t)$

- On applying $\delta(t)$ to input, the system responds while its input $\delta(t)$ finished soon.
  - This is called free running response because the system that is excited by $\delta(t)$ responds (its characteristics moved) in the absence of $\delta(t)$.
  - Choice of $\delta(t)$ is very important since it is very rich of frequencies.
    - On exciting a particular system, it could resonate at some frequencies giving higher or lower amplitudes and may attenuate or distort other frequencies.

(b) Output Response

- On applying any signal $f(t)$ at the input of **linear time invariant system**
- Whose impulse response is $h(t)$,
- Its output response $g(t)$ is given by:

$$g(t) = f(t) * h(t)$$

Output Response

$$f(t) \longrightarrow \boxed{\begin{array}{c}\textbf{Linear Time} \\ \textbf{Invariant System} \\ h(t) \text{ or } H(\omega)\end{array}} \longrightarrow g(t)$$

Driver
Input Signal
Cause

Response
Output Signal
Effect

Proof of Output Response Rule

- On applying of input impulse:

$$\delta(t) \rightarrow h(t)$$

- Sense the system is a time invariant:

$$\delta(t-\tau) \rightarrow h(t-\tau)$$

- Multiplying both sides by $f(\tau)$:

$$f(\tau)\,\delta(t-\tau) \rightarrow f(\tau)\,h(t-\tau)$$

- Integrating both sides:

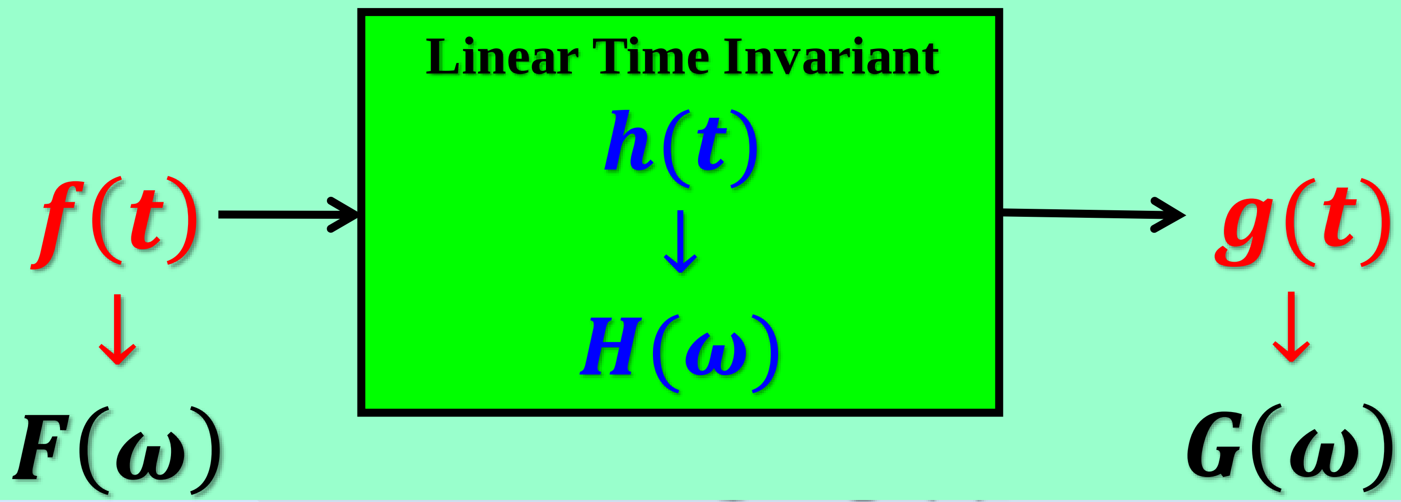
$$\int_{-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau \rightarrow \int_{-\infty}^{\infty} f(\tau)\,h(t-\tau)\,d\tau = f(t) * h(t)$$

- Since δ is an even function:

$$\therefore\ \delta(t-\tau) = \delta(\tau-t)$$

$$\therefore \int_{-\infty}^{\infty} f(\tau)\,\delta(\tau-t)\,d\tau = f(t) \rightarrow f(t) * h(t) = g(t)$$

Time and Frequency Domain Analysis



- First: $h(t)$ from $H(\omega)$: $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t} d\omega$
- Second is convolution: $g(t) = \int_{-\infty}^{\infty} f(t)\, h(t-\tau) d\tau$

- ii. In Frequency Domain: To get $g(t)$
  - First: $F(\omega)$ from $f(t)$: $F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$
  - $G(\omega) = F(\omega)\, H(\omega)$
  - Second: $g(t)$ of $G(\omega)$: $g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\, e^{j\omega t} d\omega$

(c) Transfer Function $H(\omega)$

- It describes system characteristics (impulse response) in frequency domain.

- Is the ratio between the output spectrum to the input spectrum as:

$$H(\omega) = \frac{Output\ Spectrum}{Input\ Spectrum} = \frac{G(\omega)}{F(\omega)}$$

Transfer Function per Harmonic

- On applying a harmonic signal $e^{j\omega_o t}$ to a linear time invariant system, the response will be the input but shifted by $e^{j\theta(\omega_o)}$ and its amplitude varied to $|H(\omega_o)|$:
- So, transfer function can be determined at a particular frequency ω_o as:

$$H(\omega_o) = \frac{Otput\ Time\ Response}{Input\ Harmonic} = \frac{g(t)}{e^{j\omega_o t}}$$

- It means, a linear time invariant system does not produce frequency components not existing at its input.

Time Domain Analysis

$$f(t) = e^{j\omega_o t}$$

$$g(t) = f(t) * h(t) = e^{j\omega_o t} * h(t)$$

$$g(t) = \int_{-\infty}^{\infty} e^{j\omega_o \tau}\, h(t-\tau)\, d\tau$$ **Multiply by:** $e^{\pm j\omega_o t}$

$$g(t) = e^{j\omega_o t} \int_{-\infty}^{\infty} e^{-j\omega_o (t-\tau)}\, h(t-\tau)\, d\tau$$

Put $x = t - \tau,\, dx = -d\tau, +\infty \le x \le -\infty$

$$g(t) = e^{j\omega_o t} \int_{+\infty}^{-\infty} h(x)\, e^{-j\omega_o x} (-dx) = e^{j\omega_o t} \int_{-\infty}^{+\infty} h(x)\, e^{-j\omega_o x}\, dx$$

$$g(t) = e^{j\omega_o t} H(\omega_o)$$

$$H(\omega_o) = \frac{g(t)}{e^{j\omega_o t}}$$

Frequency Domain Analysis

$$f(t) = e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega - \omega_o)$$

$$G(\omega) = F(\omega)\,H(\omega) = 2\pi\delta(\omega - \omega_o)H(\omega)$$

$$G(\omega) = H(\omega_o)\,2\pi\delta(\omega - \omega_o)$$

Taking the inverse Fourier Transform on both:

$$\Im^{-1}[G(\omega)] = \Im^{-1}[H(\omega_o)\,2\pi\delta(\omega - \omega_o)]$$

$$g(t) = H(\omega_o)\,\Im^{-1}[\,2\pi\delta(\omega - \omega_o)]$$

$$\therefore\ g(t) = H(\omega_o)\,e^{j\omega_o t}$$

$$H(\omega_o) = \frac{g(t)}{e^{j\omega_o t}}$$

How to Calculate the Transfer Function

1. **Apply a sinusoidal signal $\cos 2\pi f_0 t$ to input:**

$$\cos 2\pi f_0 t \longrightarrow \boxed{\text{System}} \longrightarrow Record: V_o, \theta_o$$

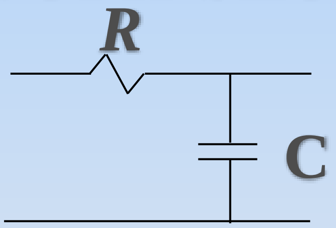
2. **Monitor input and output signals.**
3. **Record input and output voltages, calculate:**

$$|H(f_o)| = \frac{Ouput\ Voltage}{Input\ Voltage}$$

4. **Record phase between input and output θ_o.**
5. **Repeat steps 3 and 4 at $f_o, f_1, f_2, f_3, \ldots$**
6. **Trace both *amplitude and phase*.**

Transfer Function from Circuit

- Determine $H(\omega)$ for the network shown.



Answer

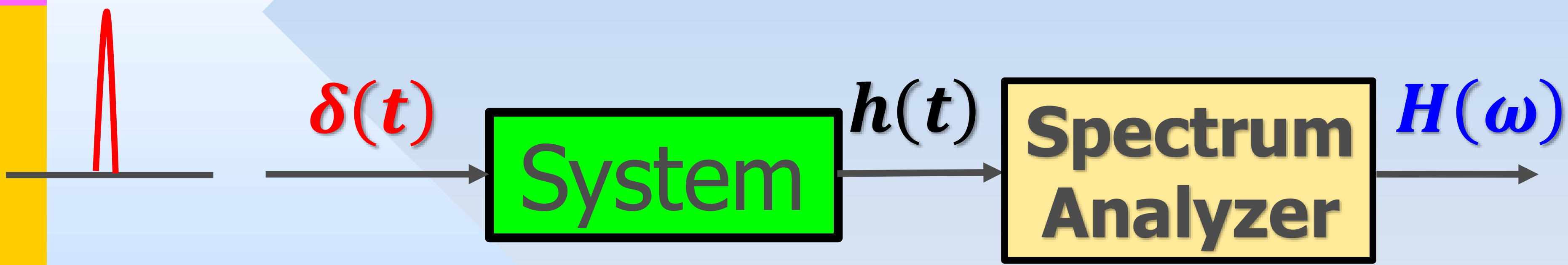
$$H(\omega) = \frac{Ouput\ Voltage}{Input\ Voltage} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$H(\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega\tau}$$

$$\tau = RC$$

How to Measure the Transfer Function

- Apply $\delta(t)$ for the system input:



- Then use spectrum analyzer to trace the output spectrum, transfer function $H(\omega)$.

- One could use inverse Fourier transform to estimate the impulse response, $h(t)$.

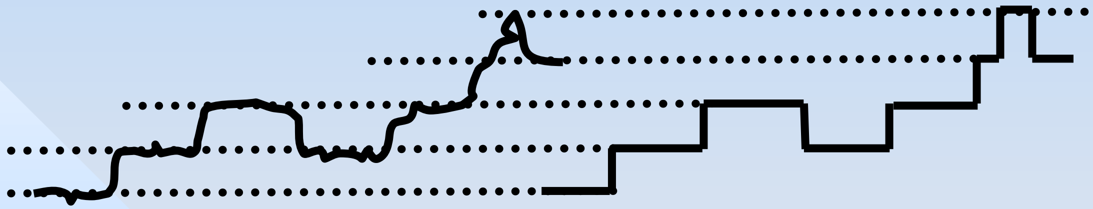
(4)

Distortion-Less

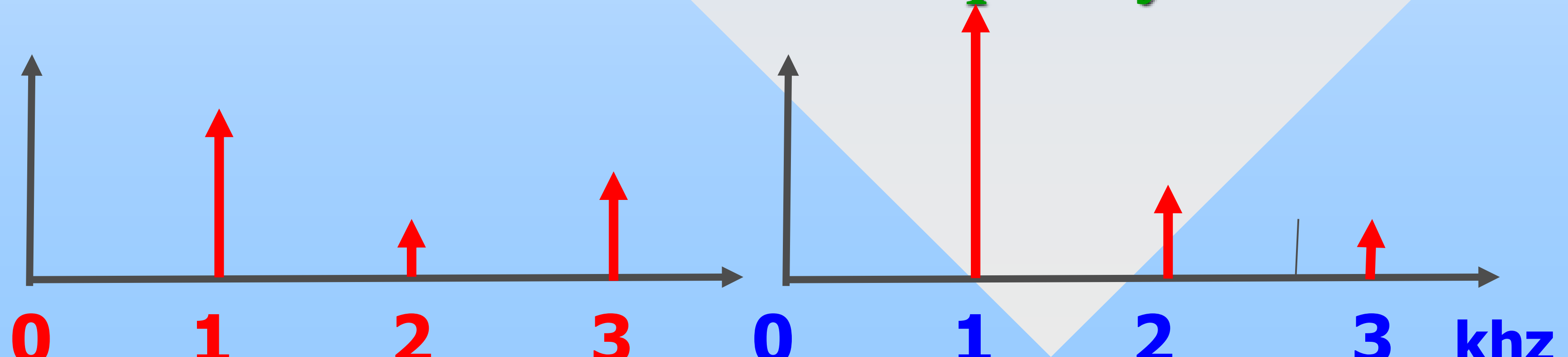
System

Distortion

- **In Time Domain**
  - Is the change in the shape of time signal.



- **In Frequency Domain**
  - The change in the relative weights of different harmonics in frequency domain.



Distortionless System

- **Distortionless** is a system whose output is an exact replica of input except for a time shift, amplification or attenuation:

$$f(t) \to g(t) = k\, f(t - t_o)$$

- So its **output response will be:**

$$F(\omega) \to G(\omega) = k\, F(\omega)\, e^{-j\omega t_o}$$

$$Since,\ G(\omega) = F(\omega)\, H(\omega)$$

$$H(\omega)_{Distortionless\ System} = k\ e^{-j\omega t_o}$$

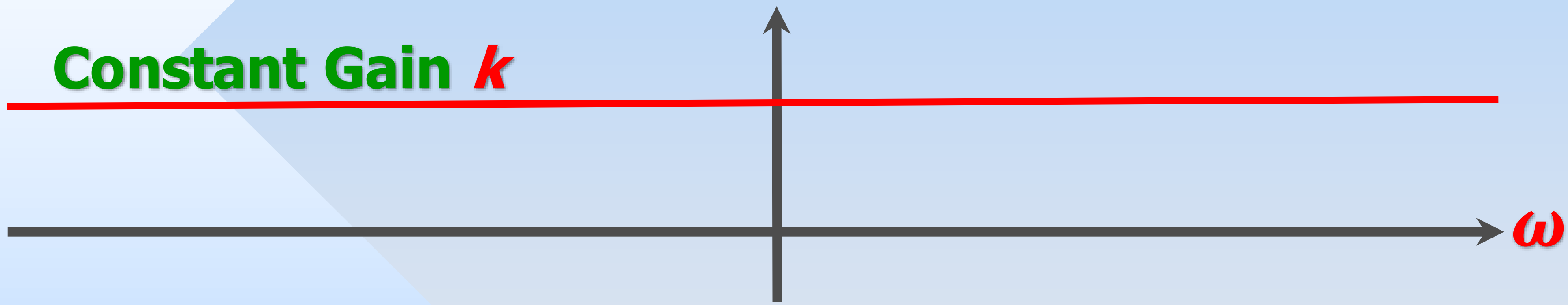
- **Transfer function of distortionless system** has a constant gain k and a phase proportion to ω.

Characteristics of Distortionless System

$$H(\omega) = k\, e^{-j\omega t_o}$$

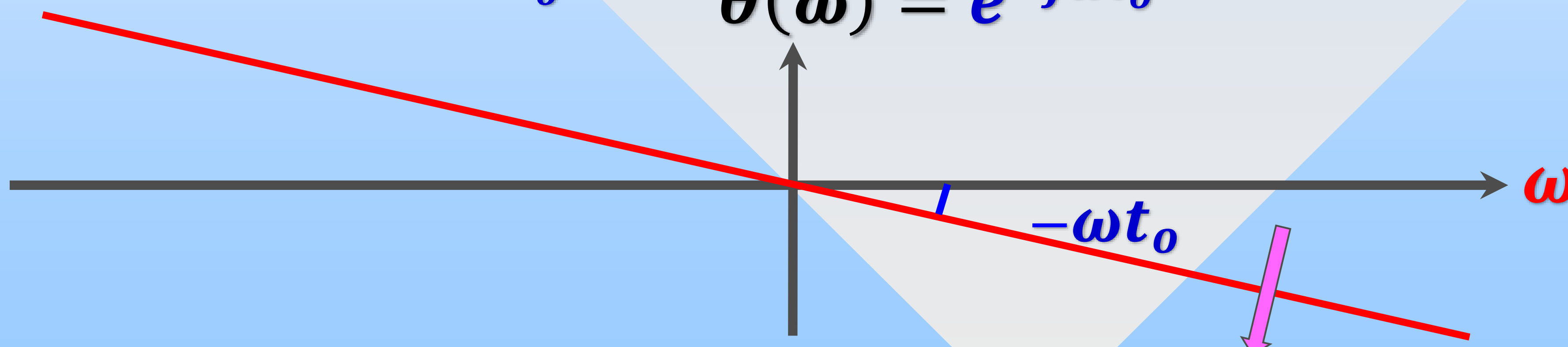
$$|H(\omega)| = k$$

Constant Gain k



Linear Phase $-\omega t_o$

$$\theta(\omega) = e^{-j\omega t_o}$$



- **Notice: Constant phase is distortion unless its 0.**

(5)

Filter

Circuits

Filter Circuits

- The filter is a frequency sensitive or selective circuit.
  - To bypass some frequency band and
  - Stop others.
- So, it does not deal equally to all frequencies.

Ideal

Low Pass

Filter

Transfer Function

- **Transfer Function of Ideal LPF:**

According to the definition of distortionless system, ideal LPF has the transfer function:

$$H(\omega) = Rect\left(\frac{\omega}{2w}\right)e^{-j\omega t_o}$$

- **Bandwidth of Ideal LPF:**

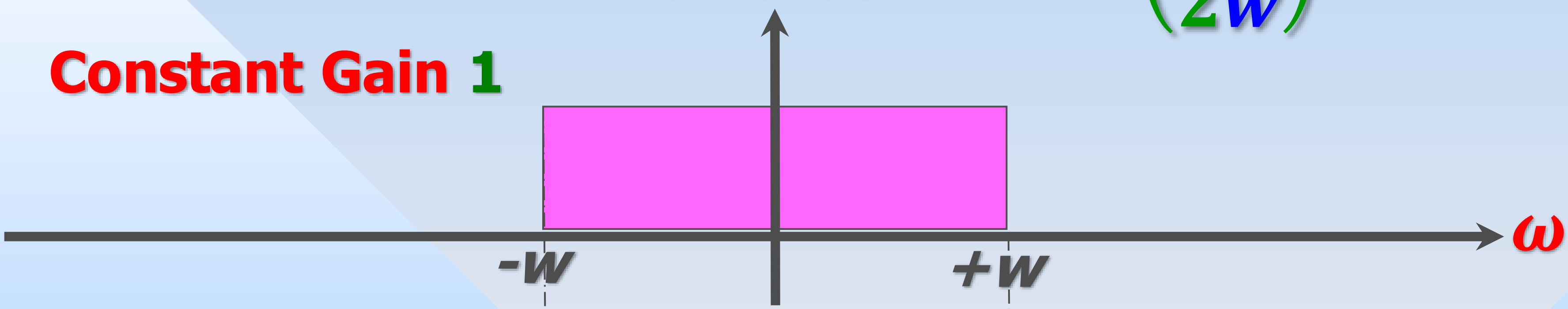
$$B = w \; rad/sec$$

Transfer Function of Distortionless Ideal Low Pass Filter

$$H(\omega) = Rect\left(\frac{\omega}{2w}\right)e^{-j\omega t_o}$$

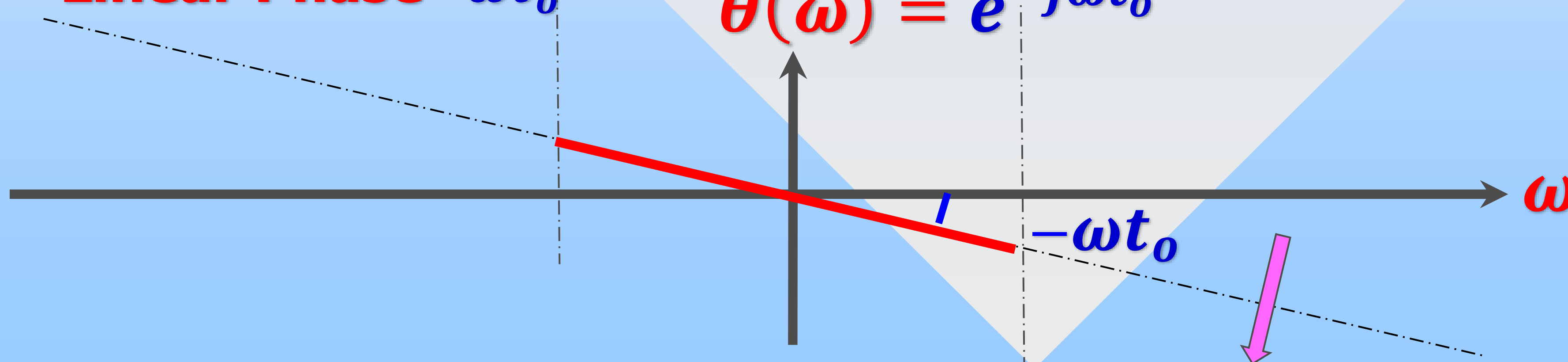
$$|H(\omega)| = Rect\left(\frac{\omega}{2w}\right)$$

Constant Gain 1



Linear Phase $-\omega t_o$

$$\theta(\omega) = e^{-j\omega t_o}$$

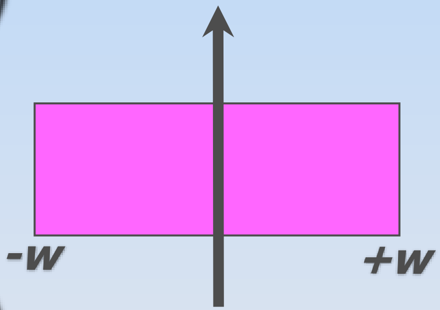


Impulse Response of Ideal LPF

$$Rect\left(\frac{t}{\tau}\right) \leftrightarrow \tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$\therefore w\, Sa\left(\frac{wt}{2}\right) \leftrightarrow 2\pi\, Rect\left(-\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi} Sa\left(\frac{wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{w}\right)$$



$$\therefore \frac{2w}{2\pi} Sa\left(\frac{2wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{2w}\right)$$

$$\therefore \frac{w}{\pi} Sa(wt) \leftrightarrow Rect\left(\frac{\omega}{2w}\right)$$

$$\therefore \frac{w}{\pi} Sa\,[w(t-t_o)] \leftrightarrow Rect\left(\frac{\omega}{2w}\right) e^{-j\omega t_o}$$

$$\therefore h(t) = \frac{w}{\pi} Sa\,[w(t-t_o)]$$

Non Causal System

The impulse response of a an ideal linear time invariant distortionless low pass filter is:

$$\therefore h(t) = \frac{w}{\pi}\, Sa\,[w(t - t_o)]$$

It shows that such an ideal filter is physically unrealizable since it responds to the impulse input before applying it.

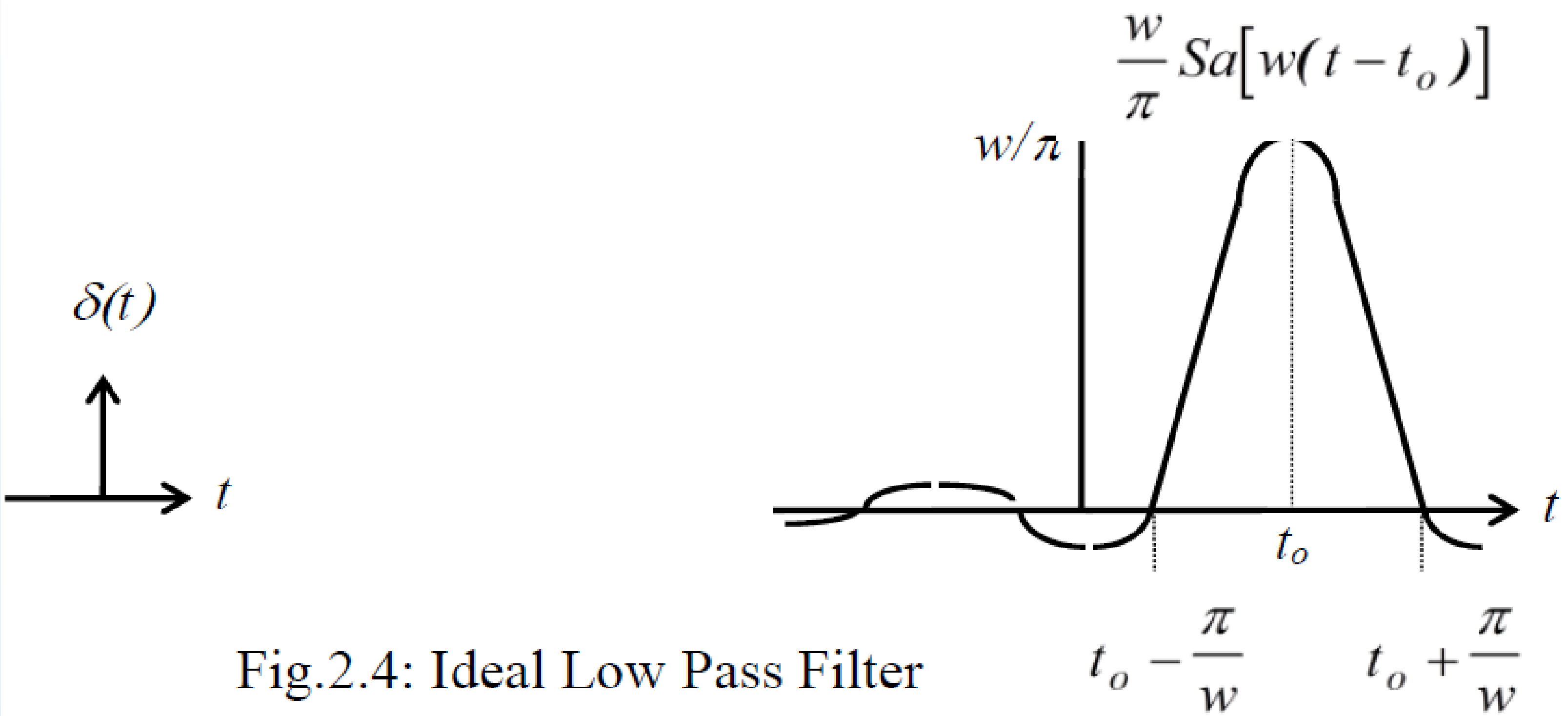


Fig.2.4: Ideal Low Pass Filter

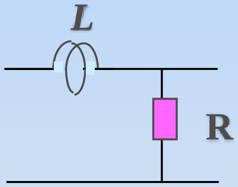
Practical

Low Pass

Filter

Transfer Function of Practical LPF

- ❑ Determine $H(\omega)$ for the circuit shown.



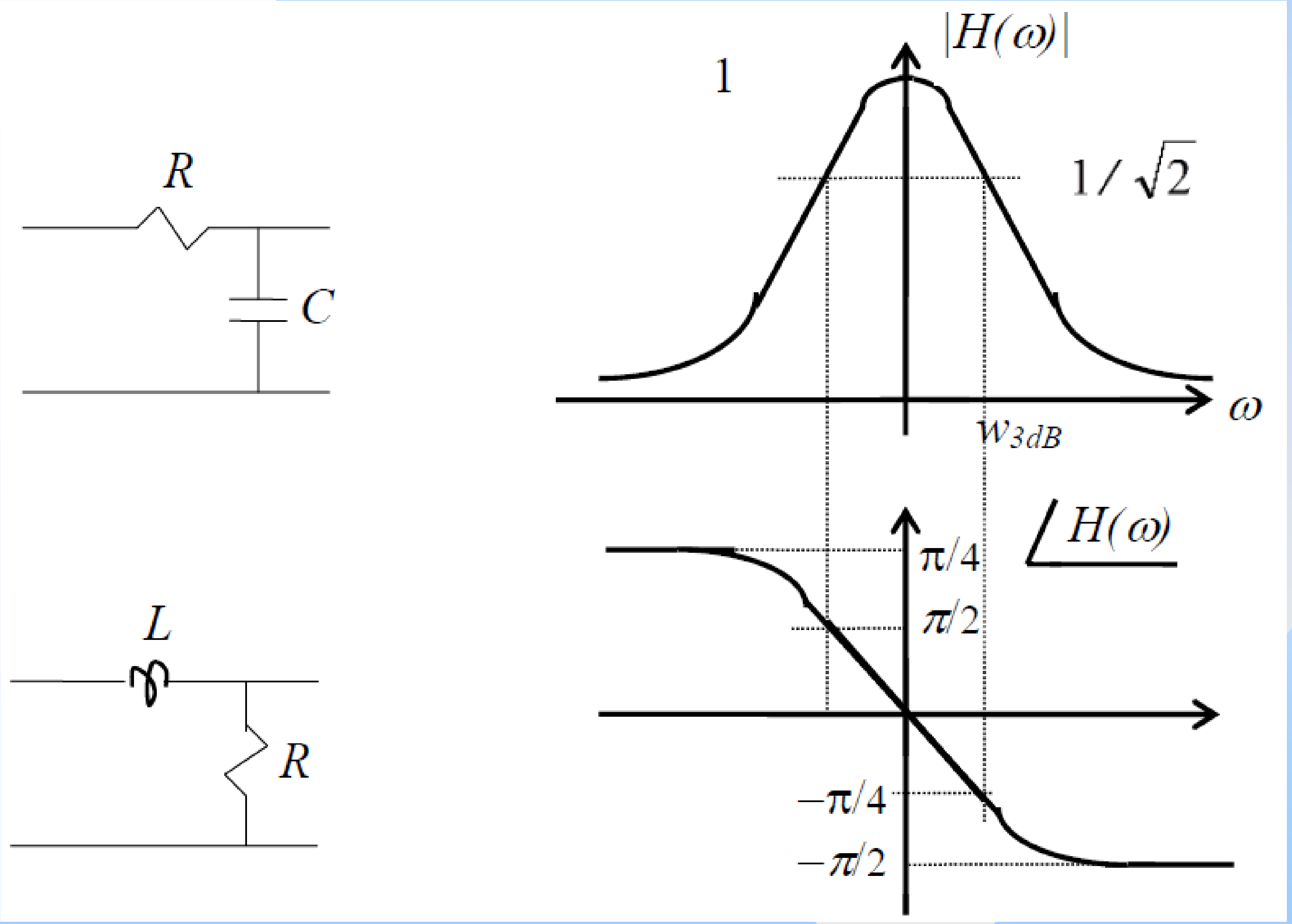
Answer

$$H(\omega) = \frac{Ouput\ Voltage}{Input\ Voltage} = \frac{R}{R + j\omega L}$$

$$H(\omega) = \frac{1}{1 + j\omega\left(\frac{L}{R}\right)} = \frac{1}{1 + j\omega\tau}, \tau = \frac{L}{R}$$

$$H(\omega) = \frac{1}{\sqrt{1 + (\omega\tau)^2}}\ e^{-j tan^{-1}\omega\tau}$$

Transfer Function of Practical LPF



Bandwidth of Practical LPF

- A 3 dB point bandwidth corresponds to:

$$|H(\omega)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega_{3dB}\tau)^2}}$$

$$\therefore 2 = 1 + (\omega_{3dB}\tau)^2$$

$$\therefore (\omega_{3dB}\tau)^2 = 1$$

$$\omega_{3dB} = \frac{1}{\tau} = \frac{R}{L} \quad rad/sec$$

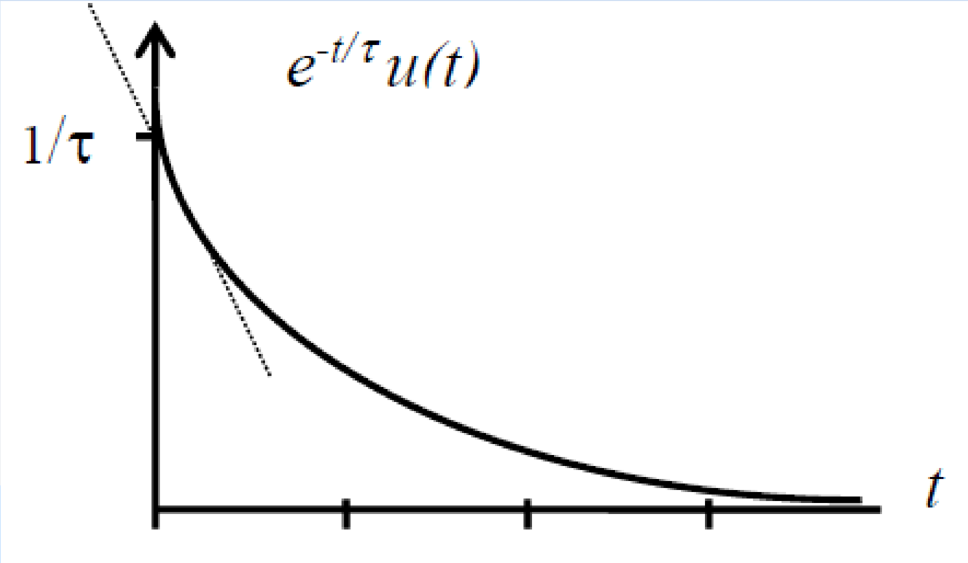
Impulse Response of Practical LPF

$$h(t) = \Im^{-1}\left[\frac{1}{1+j\omega\tau}\right]$$

$$\therefore h(t) = \Im^{-1}\left[\frac{\frac{1}{\tau}}{\frac{1}{\tau}+j\omega}\right]$$

$$\therefore h(t) = \frac{1}{\tau}\Im^{-1}\left[\frac{1}{\frac{1}{\tau}+j\omega}\right]$$

$$\therefore h(t) = \frac{1}{\tau}\left[e^{-\frac{t}{\tau}}\,u(t)\right]$$



Ideal

High Pass

Filter

Transfer Function

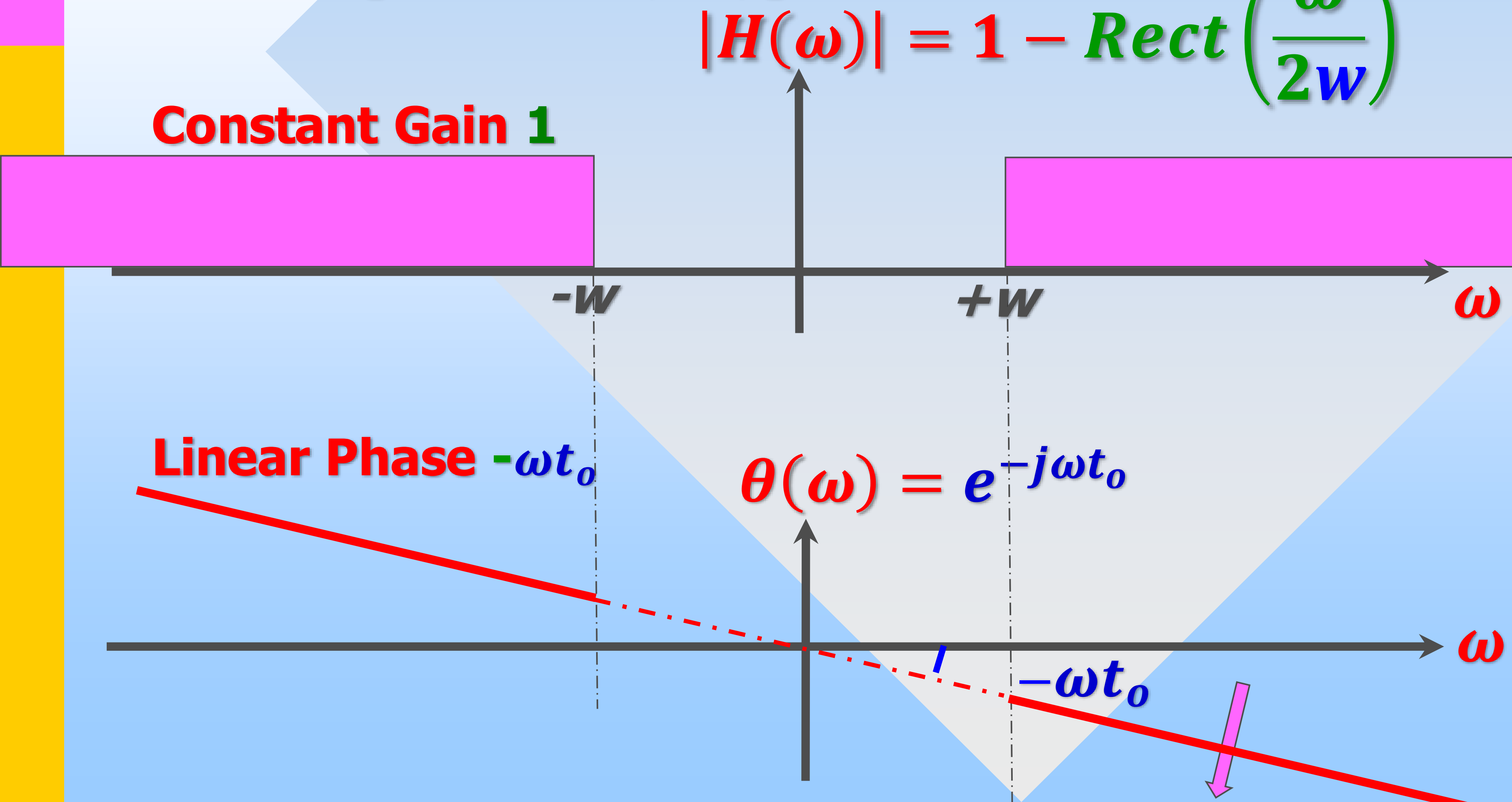
- The ideal high pass filter (HPF) passes the high frequencies while stops the dc and the lower frequencies:

- Hence, its characteristics is opposite to that of LPF so that its transfer function is readily as:

$$H(\omega) = \left[1 - Rect\left(\frac{\omega}{2w}\right)\right] e^{-j\omega t_o}$$

Transfer Function of Distortionless Ideal High Pass Filter

$$H(\omega) = \left[1 - Rect\left(\frac{\omega}{2w}\right)\right] e^{-j\omega t_o}$$



Impulse Response of Ideal HPF

$$Rect\left(\frac{t}{\tau}\right) \leftrightarrow \tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$\therefore w\, Sa\left(\frac{wt}{2}\right) \leftrightarrow 2\pi\, Rect\left(-\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi}\, Sa\left(\frac{wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{w}\right)$$

$$\therefore \frac{2w}{2\pi}\, Sa\left(\frac{2wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{2w}\right)$$

$$\therefore \frac{w}{\pi}\, Sa(wt) \leftrightarrow Rect\left(\frac{\omega}{2w}\right)$$

$$\therefore \delta(t) - \frac{w}{\pi}\, Sa(wt) \leftrightarrow 1 - Rect\left(\frac{\omega}{2w}\right)$$

$$\therefore \delta(t-t_o) - \frac{w}{\pi}\, Sa\left[w(t-t_o)\right] \leftrightarrow \left[1 - Rect\left(\frac{\omega}{2w}\right)\right] e^{-j\omega t_o}$$

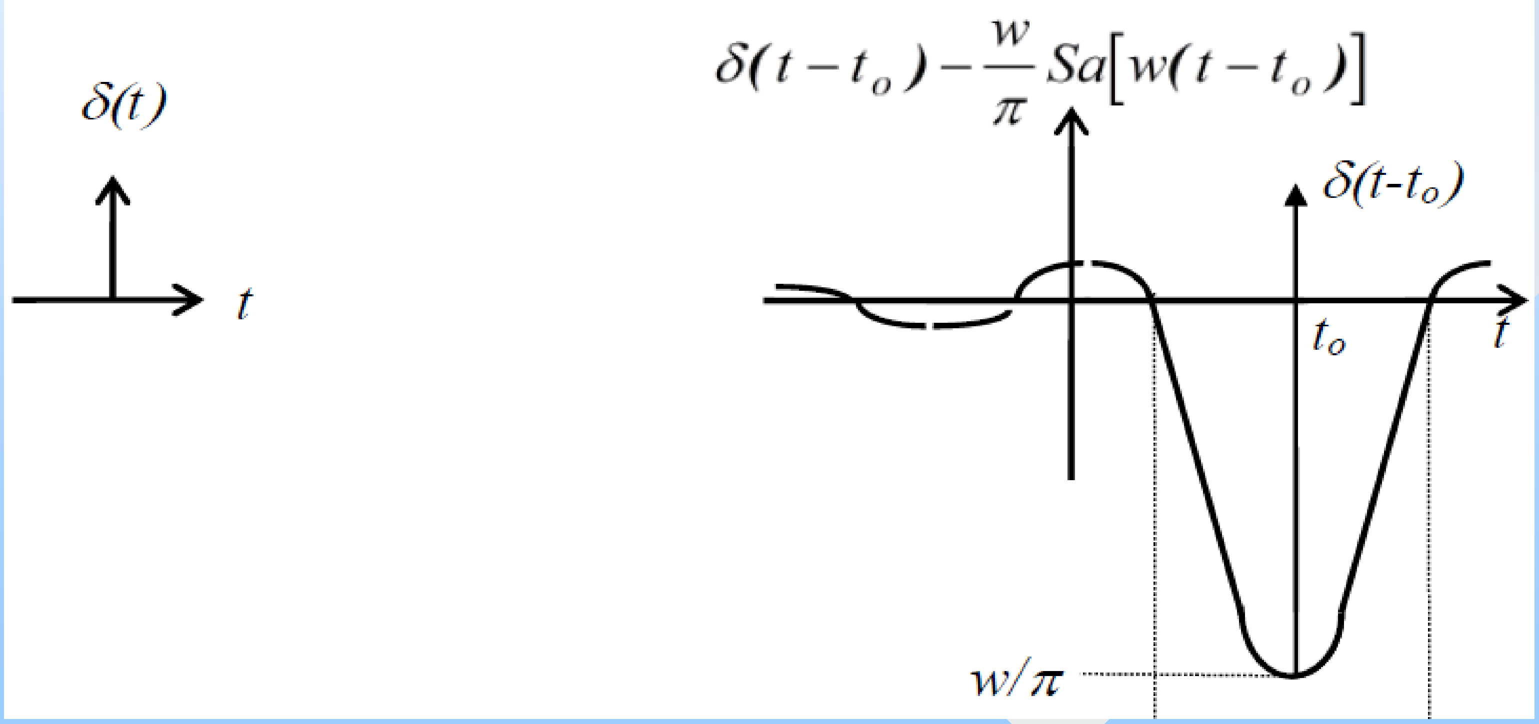
$$\therefore h(t) = \delta(t-t_o) - \frac{w}{\pi}\, Sa\left[w(t-t_o)\right]$$

Unrealizable Characteristics

The impulse response of a an ideal linear time invariant distortionless high pass filter is:

$$\therefore h(t) = \delta(t - t_o) - \frac{w}{\pi} Sa\,[w(t - t_o)]$$

It shows that such an ideal filter is physically unrealizable since it responds to the impulse input before applying it.

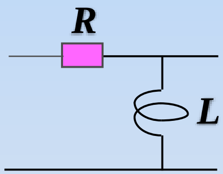


Practical High Pass Filter

Transfer Function of Practical HPF

- Determine $H(\omega)$ for HPF circuit shown.

Answer



$$H(\omega) = \frac{j\omega L}{R + j\omega L} = \frac{j\omega L(R - j\omega L)}{R^2 + \omega^2 L^2} = \frac{\omega^2 L^2 + j\omega LR}{R^2 + \omega^2 L^2}$$

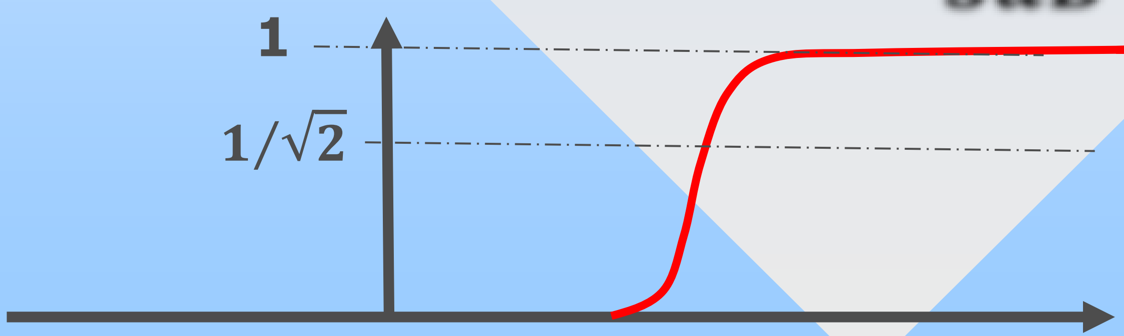
$$H(\omega) = \frac{1 + j\left(\frac{R}{\omega L}\right)}{\left(\frac{R}{\omega L}\right)^2 + 1} = \frac{1 + j\left(\frac{\tau}{\omega}\right)}{1 + \left(\frac{\tau}{\omega}\right)^2} = \frac{\sqrt{1 + \left(\frac{\tau}{\omega}\right)^2}}{1 + \left(\frac{\tau}{\omega}\right)^2}\, e^{j\tan^{-1}\left(\frac{\tau}{\omega}\right)}$$

$$\tau = {}^{R}/_{L}$$

Cutoff Frequency of Practical HPF

- A 3 dB point cutoff Frequency corresponds to:

$$|H(\omega_{3dB})| = \frac{1}{\sqrt{2}} = \frac{\sqrt{1+\left(\frac{\tau}{\omega_{3dB}}\right)^2}}{1+\left(\frac{\tau}{\omega_{3dB}}\right)^2}$$



Impulse Response of Practical HPF

$$H_{HPF}(\omega) = \frac{j\omega L}{R + j\omega L}$$

$$\frac{H_{HPF}(\omega)}{j\omega} = \frac{L}{R + j\omega L} = \frac{1}{\tau + j\omega} \quad , \qquad \tau = \frac{R}{L}$$

$$e^{-\tau t} u(t) \leftrightarrow \frac{1}{\tau + j\omega} = \frac{H_{HPF}(\omega)}{j\omega}$$

$$\int_{-\infty}^{t} f(t)\, dt \leftrightarrow \frac{F(\omega)}{j\omega}$$

$$h_{HPF}(t) = \int_{-\infty}^{t} e^{-\tau t} u(t)\; dt = \int_{0}^{t} e^{-\tau t}\; dt = \left. \frac{e^{-\tau t}}{-\tau} \right|_{0}^{t}$$

$$\therefore h_{HPF}(t) = \left. \frac{e^{-\tau t}}{-\tau} \right|_{0}^{t} = \frac{1}{-\tau}\left(e^{-\tau t} - 1\right) = \frac{1}{\tau}\left(1 - e^{-\tau t}\right)$$

Transfer Function of Ideal BPF

- Ideal band pass filter (BPF) passes a certain band of frequencies $(f_1 \ to \ f_2)$ while stops others:
- Hence, its characteristics is:

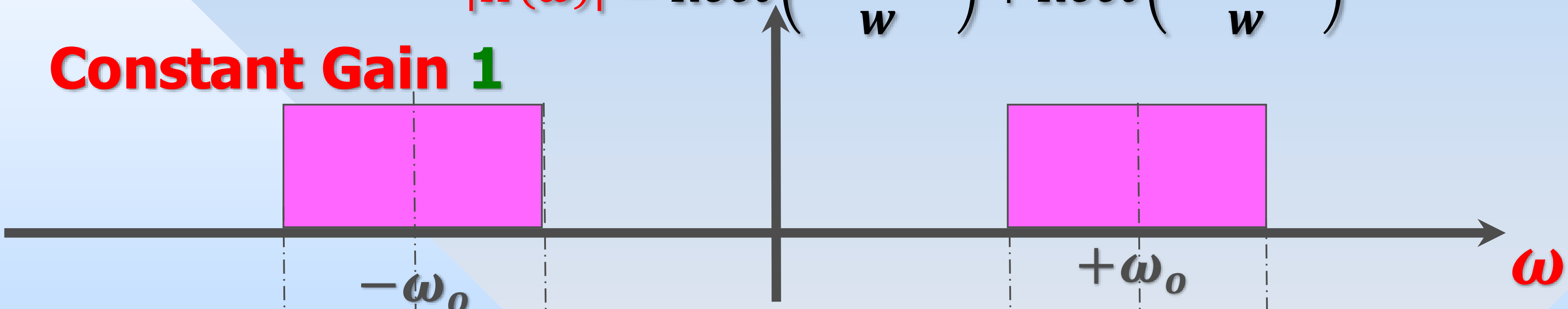
$$H(\omega) = \left[Rect\left(\frac{\omega - \omega_o}{w}\right) + Rect\left(\frac{\omega + \omega_o}{w}\right)\right] e^{-j\omega t_o}$$

Transfer Function of Distortionless Ideal Band Pass Filter

$$H(\omega) = \left[Rect\left(\frac{\omega - \omega_o}{w}\right) + Rect\left(\frac{\omega + \omega_o}{w}\right)\right] e^{-j\omega t_o}$$

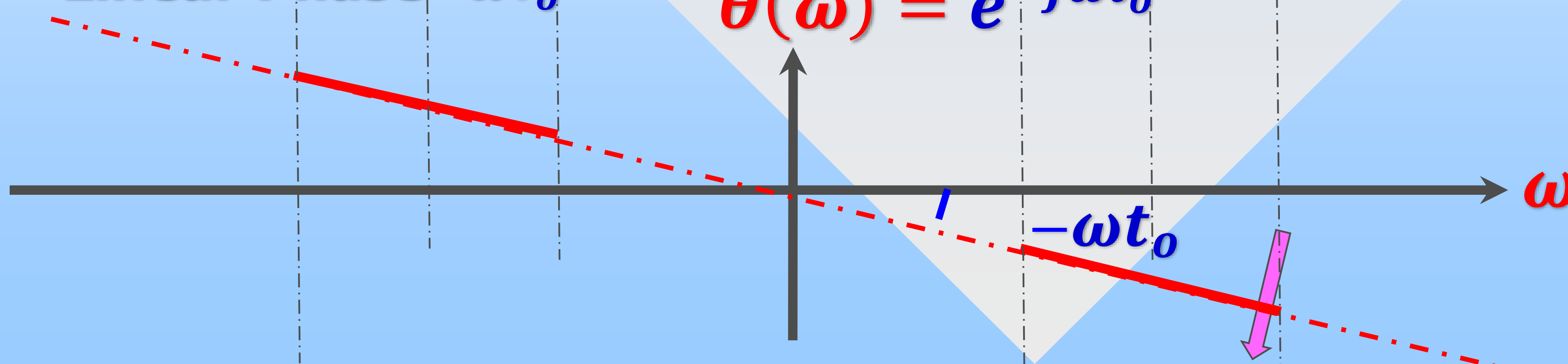
$$|H(\omega)| = Rect\left(\frac{\omega - \omega_o}{w}\right) + Rect\left(\frac{\omega + \omega_o}{w}\right)$$

Constant Gain 1



Linear Phase $-\omega t_o$

$$\theta(\omega) = e^{-j\omega t_o}$$



Impulse Response of Ideal BPF

$$Rect\left(\frac{t}{\tau}\right) \leftrightarrow \tau\, Sa\left(\frac{\omega\tau}{2}\right)$$

$$\therefore w\, Sa\left(\frac{wt}{2}\right) \leftrightarrow 2\pi\, Rect\left(-\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi}\, Sa\left(\frac{wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi}\, Sa\left(\frac{wt}{2}\right) e^{j\omega_o t} \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)$$

$$\therefore \frac{w}{2\pi}\, Sa\left(\frac{wt}{2}\right) e^{-j\omega_o t} \leftrightarrow Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \frac{w}{2\pi}\, Sa\left(\frac{wt}{2}\right)\left[e^{j\omega_o t}+e^{-j\omega_o t}\right] \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \frac{w}{\pi}\, Sa\left(\frac{wt}{2}\right)\cos\omega_o t \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \frac{w}{\pi}\, Sa\left(\frac{w(t-t_o)}{2}\right)\cos\omega_o(t-t_o) \leftrightarrow \left[Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)\right]e^{j\omega t_o}$$

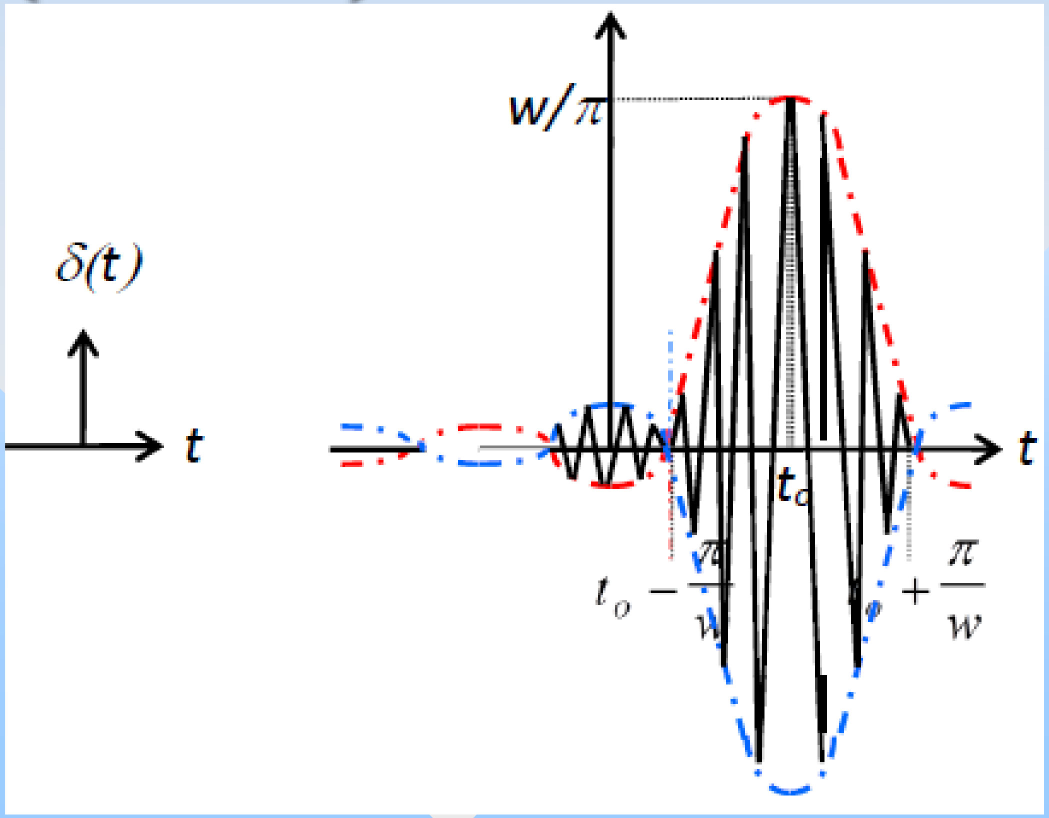
$$\therefore h(t) = \frac{w}{\pi}\, Sa\left(\frac{w(t-t_o)}{2}\right)\cos\omega_o(t-t_o)$$

Unrealizable Ch^s of BPF

The impulse response of a an ideal linear time invariant distortionless band pass filter is:

$$\therefore h(t) = \frac{w}{\pi} Sa\left(\frac{w(t-t_o)}{2}\right) \cos\omega_o(t-t_o)$$

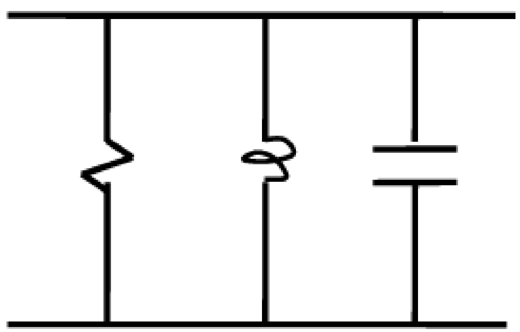
It shows that such an ideal filter physically unrealizable since it responds to impulse input before applying it.



Practical Band Pass Filter

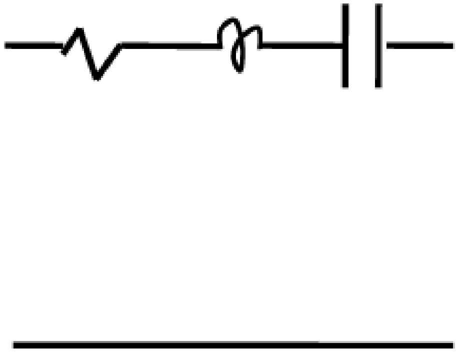
Band Pass Filter Circuits

Parallel Resonance in Chant:



i- Parallel RLC

Series Resonance in Series:



ii- Series RLC

BPF Circuits

- **Parallel Resonance Circuit in Chant:**

Its quality factor Q_o and bandwidth B are:

$$Q_o = \omega_o RC = \omega_o \tau$$

$$B = \frac{\omega_o}{Q_o} = \frac{1}{RC} = \frac{1}{\tau}$$

- **Series Resonance Circuit in Series:**

$$Q_o = \frac{\omega_o L}{R} = \omega_o \tau$$

$$B = \frac{\omega_o}{Q_o} = \frac{\omega_o R}{\omega_o L} = \frac{R}{L} = \frac{1}{\tau}$$

Ideal

Band Stop

Filter

Transfer Function

- Ideal band stop filter (BSF) rejects a certain band of frequencies $(f_1 \; to \; f_2)$ while passes others:

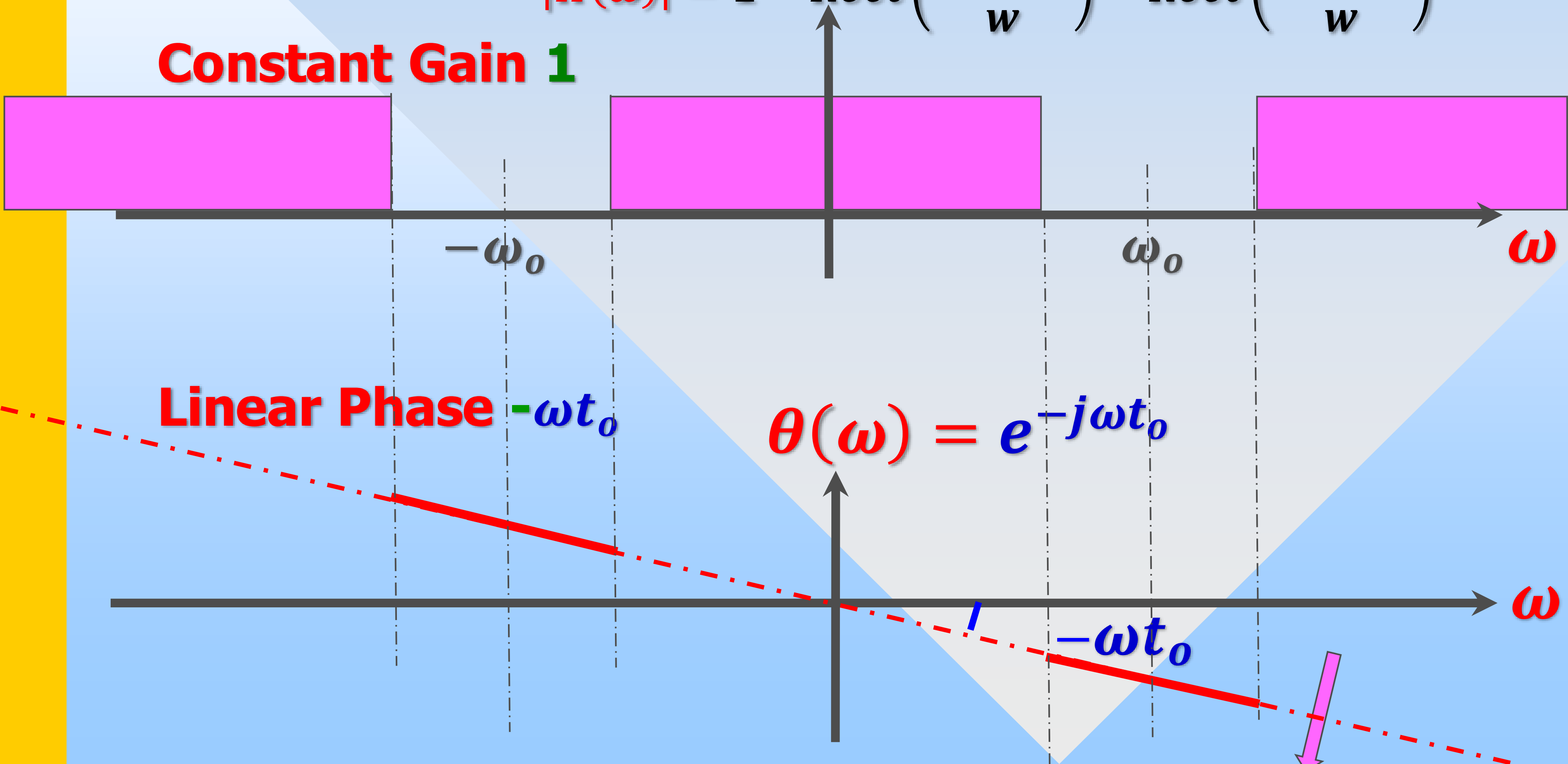
- Hence, its characteristics is the inverse of BPF as follows:

$$H(\omega) = \left[1 - Rect\left(\frac{\omega - \omega_o}{w}\right) - Rect\left(\frac{\omega + \omega_o}{w}\right)\right] e^{-j\omega t_o}$$

Transfer Function of Distortionless Ideal Band Stop Filter

$$H(\omega) = \left[1 - Rect\left(\frac{\omega - \omega_o}{w}\right) - Rect\left(\frac{\omega + \omega_o}{w}\right)\right] e^{-j\omega t_o}$$

$$|H(\omega)| = 1 - Rect\left(\frac{\omega - \omega_o}{w}\right) - Rect\left(\frac{\omega + \omega_o}{w}\right)$$



Impulse Response of Ideal BSF

$$Rect\left(\frac{t}{\tau}\right) \leftrightarrow \tau\, Sa\left(\frac{\omega\tau}{2}\right) \qquad \therefore w\, Sa\left(\frac{wt}{2}\right) \leftrightarrow 2\pi\, Rect\left(-\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi} Sa\left(\frac{wt}{2}\right) \leftrightarrow Rect\left(\frac{\omega}{w}\right)$$

$$\therefore \frac{w}{2\pi} Sa\left(\frac{wt}{2}\right) e^{j\omega_o t} \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)$$

$$\therefore \frac{w}{2\pi} Sa\left(\frac{wt}{2}\right) e^{-j\omega_o t} \leftrightarrow Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \frac{w}{2\pi} Sa\left(\frac{wt}{2}\right)\left[e^{j\omega_o t}+e^{-j\omega_o t}\right] \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \frac{w}{\pi} Sa\left(\frac{wt}{2}\right)\cos\omega_o t \leftrightarrow Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)$$

$$\therefore \delta(t)-\frac{w}{\pi} Sa\left(\frac{wt}{2}\right)\cos\omega_o t \leftrightarrow 1-\left[Rect\left(\frac{\omega-\omega_o}{w}\right)+Rect\left(\frac{\omega+\omega_o}{w}\right)\right]$$

$$\therefore \delta(t-t_o)-\frac{w}{\pi} Sa\left(\frac{w(t-t_o)}{2}\right)\cos\omega_o(t-t_o) \leftrightarrow \left[1-Rect\left(\frac{\omega-\omega_o}{w}\right)-Rect\left(\frac{\omega+\omega_o}{w}\right)\right]e^{j\omega t_o}$$

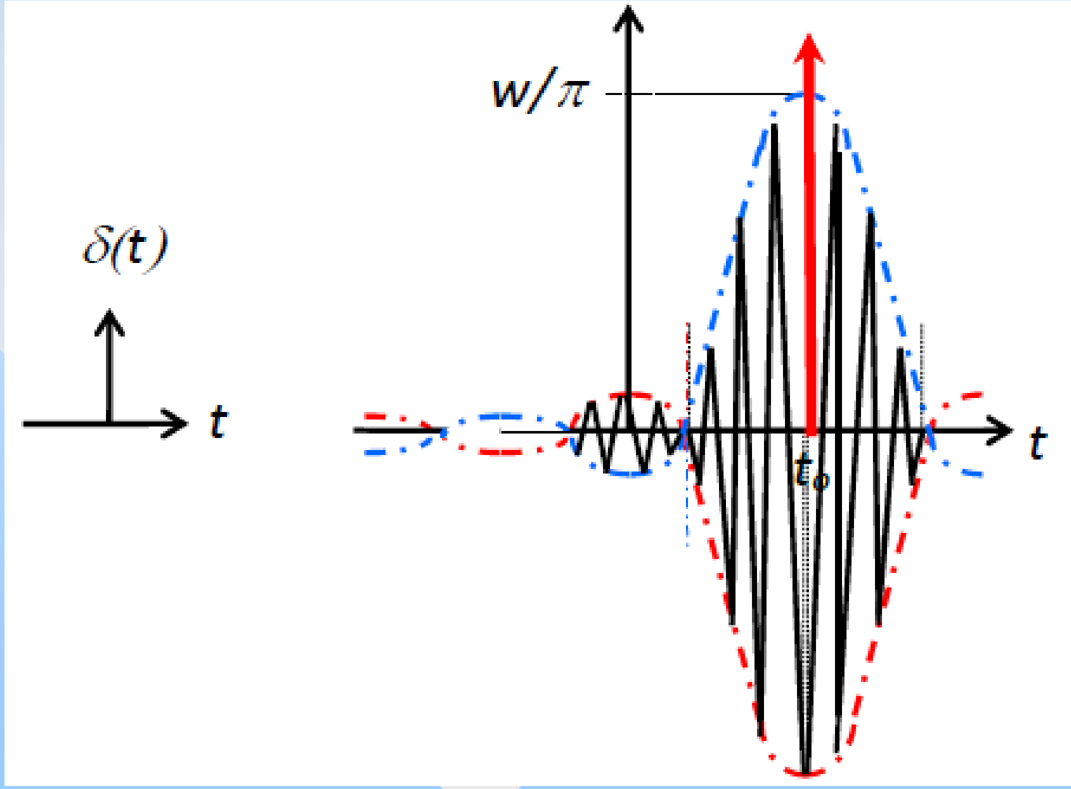
$$\therefore h(t)=\delta(t-t_o)-\frac{w}{\pi} Sa\left(\frac{w(t-t_o)}{2}\right)\cos\omega_o(t-t_o)$$

Unrealizable BSF Ch^s

The impulse response of a an ideal linear time invariant distortionless band stop filter is:

$$\therefore h(t) = \left[\delta(t - t_o) - \frac{w}{\pi} Sa\left(\frac{w(t - t_o)}{2}\right)\right] \cos\omega_o(t - t_o)$$

It shows that such an ideal filter physically unrealizable since it responds to impulse input before applying it.



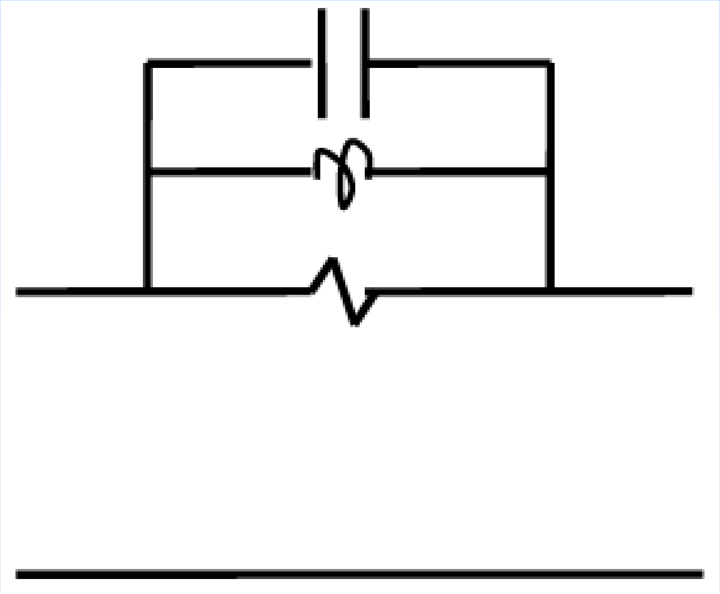
Practical

Band Stop

Filter

Band Stop Filter Circuits

Parallel Resonance in Series:



Series Resonance in Chant: