Nonsmooth

Dynamics





- nonsmooth = lack of differentiability ($\not\in \mathcal{C}^1$),
- graphs with peaks, kinks, jumps.
- systems that evolves with time,
- branch of mechanics concerned with the motion of objects.

Where is nonsmoothness?

- nonsmooth solutions in time and space:
 - continuous, functions of bounded variations, measures and distributions.
- nonsmooth modeling of constitutive laws:
 - set-valued mapping, inequality constraints, complementarity, impact laws,
 - ODE with discontinuous r.h.s, differential inclusion, measure equation.

TRIPOP INRIA project team (LJK)

Research object:

Modeling, Simulation and Control of Nonsmooth Dynamics.

Main application:

Natural environmental risks in mountains.

- Research Scientists
 - Vincent Acary (INRIA, LJK)
 - Bernard Brogliato (INRIA, LJK)
 - Franck Bourrier (INRAE LJK & IGE)
 - Olivier Goury (INRIA, LJK)
 - Filippo Masi (INRIA, LJK)
 - Felix Miranda Villatoro (INRIA. LJK)
 - Arnaud Tonnelier (INRIA, LJK)
- Engineer
 - Franck Pérignon (CNRS, LJK)
 - Maurice Brémond (INRIA, SED)
 - Samuel Heidman (INRIA, SED)

- External collaborators
 - Thierry Faug (INRAE IGE)
 - Christophe Prieur (CNRS, Gipsa-Lab)
- PhD
 - Quang Hung Pham (UGA 2022-2025)
 - Louis Guillet (INRIA 2023–2025)
 - Florian Vincent (INRIA 2023–2026)
 - Mattéo Oziol (G-INP 2023-2026)
 - Chloé Gergely (UGA, 2024–2027)
- Post-doc
 - Ritesh Gupta (INRIA, 2023-2024)
 - Simon Le Berre (INRIA, 2024)



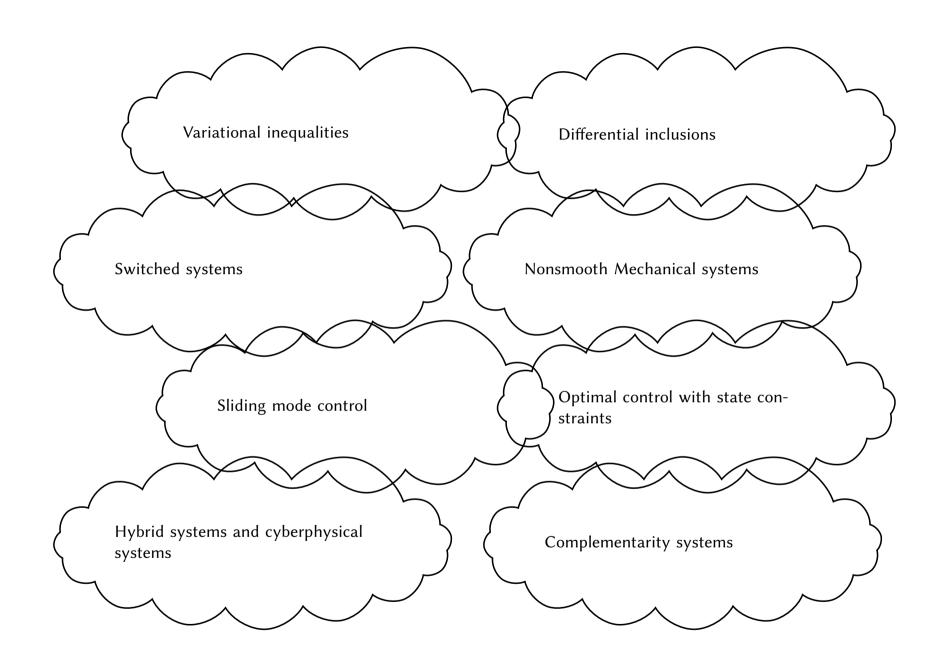




Scientific pairs: M. Jean, J.J. Moreau, M. Schatzman & C. Lemaréchal.

Our reference (bedside) books

- Moreau, J. J. (1966). Fonctionnelles convexes. Séminaire Jean Leray, Collège de France.
- Rockafellar, R.T. (1970). *Convex Analysis*. Princeton UP.
- Duvaut, G., & Lions, J.-L. (1972). Les inégalités variationnelles en mécanique et en physique. Dunod.
- ▶ Brézis, H. (1973). *Opérateurs maximaux monotones et semi−groupes de contractions dans les espaces de Hilbert.* Math. Studies.
- ► Germain, P. (1973) Cours de mécanique des milieux continus. Tome 1. Masson & Cie.
- Moreau, J. J. (1973) On Unilateral Constraints, Friction and Plasticity. Springer, 2011.
- Moreau, J. J., & Panagiotopoulos, P. D. (Eds.). (1988). Nonsmooth mechanics and applications. Springer.
- Cottle, R. W., Pang, J. S., & Stone, R. E. (1992). The linear complementarity problem. SIAM.
- Hiriart-Urruty, J.-B., & Lemaréchal, C. (1996). *Convex analysis and minimization algorithms.* Springer.
- ▶ Brogliato, B. (1999). *Nonsmooth mechanics*. Springer.
- Nguyen, Q. S. (2000). Stability and nonlinear solid mechanics. Wiley.
- Facchinei, F. & Pang J. S. (2003). Finite-dimensional variational inequalities and complementarity problems. Springer.
- Rockafellar, R. T., & Wets, R. J. B. (2009). Variational analysis (Vol. 317). Springer.



Elasto-dynamics with plasticity, contact and impact.

A second order sweeping process

$$\begin{cases} v^{+} = \dot{q}^{+} & \text{(velocity of bounded variations)} \\ M(q)dv + F(q, v^{+})dt + B^{\top}\sigma dt = \iota & \text{(differential measure)} \\ \dot{\sigma} = E(Bv - \dot{\varepsilon}^{p}) & \text{(elasticity)} \\ \dot{\varepsilon}^{p} \in N_{C}(\sigma) & \text{(plasticity)} \\ -\iota \in N_{T_{M}(q)}(v^{+} + ev^{-}) & \text{(impact and contact)} \end{cases}$$