Dynamics on Networks and Network Dynamics

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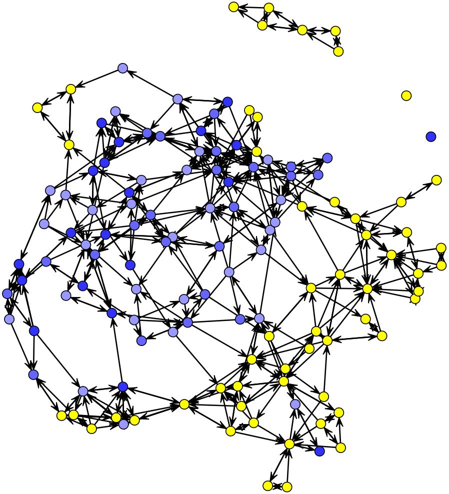
Networks are especially interesting
in terms of their association with individual behavior.



Potential questions

"What factors play a role in tie formation?"

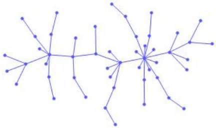
"Why do attitudes/behaviors/performance change?"

Examples

- Weapon carrying of adolescents in US high schools
  (Dijkstra, Lindenberg, Veenstra, Steglich, Isaacs, Card & Hodges, 2010)
- Interpersonal trust and job satisfaction in organizations
  (Agneessens & Wittek, 2008)
- Online echo chambers, opinion polarization

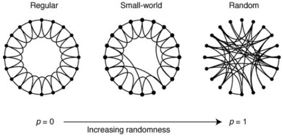
In order to study these questions we need to model the evolution of social networks and attributes of network actors simultaneously.

Modeling dynamics



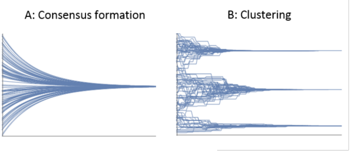
preferential attachment

Barabási-Albert



small worlds

Watts-Strogatz



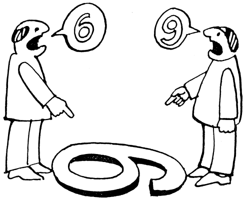
social influence by averaging

Abelson, DeGroot, ...

Today, we will discuss a *statistical* model for dynamics of/on networks, which allows us to assess the strength of the various social mechanisms driving network and actor attribute change.

Today

- Dynamics on networks:
  modeling the co-evolution of networks and continuous behavior
- Example: stress among Master's students
- Network dynamics:
  taking into account the role of the perceiver



Co-evolution model for networks and actor attributes

Network evolution

Network evolution is a continuous-time process, which we model based on discrete-time observations:

observations at	$t_1, \ldots, t_M$
actors	$\mathcal{I} = \{1, \ldots, n\}$
network data	$x(t_1), \ldots, x(t_M)$

Stochastic actor-oriented model for directed networks [Snijders, 2001]:

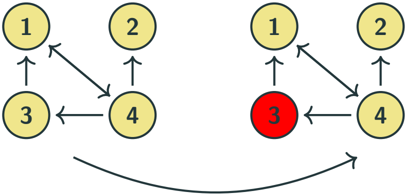
- Continuous-time Markov model chain on $\{0,1\}^{n \times n}$
- Actor-oriented: actors control their outgoing relations
  - Rate of change for actor i: $\lambda_i(\gamma, x) = \lambda_m$ in period m
  - Change based on evaluation of the consequent network state
- Network changes in smallest possible steps, choice set: $\mathcal{A}_i(x) = \{x^{(\pm ij)} \,|\, j \in \mathcal{I}\}$

Time t

(including behavior)

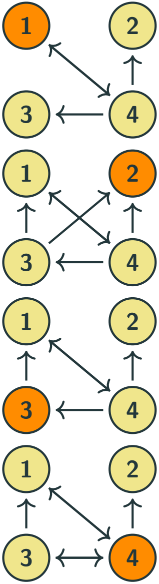
who will make a change?

$\lambda_i(\gamma, x)/\lambda_+$



$\Delta t \sim \mathrm{Exp}(\lambda_+)$

$\lambda_+ = \sum_{k=1}^{4} \lambda_k(\gamma, x)$



and to whom?

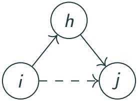
$$p_i(\tilde{x} \mid x) = \begin{cases} \dfrac{e^{\sum_k \beta_k \, s_{ik}(\tilde{x})}}{\sum_{x' \in \mathcal{A}_i(x)} e^{\sum_k \beta_k \, s_{ik}(x')}} & \text{if } \tilde{x} \in \mathcal{A}_i(x), \\ 0 & \text{if } \tilde{x} \notin \mathcal{A}_i(x). \end{cases}$$

outdegree effect

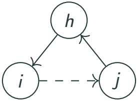
$s_{i1}(x) = \sum_j x_{ij}$

reciprocity effect

$s_{i2}(x) = \sum_j x_{ij}\, x_{ji}$



transitivivity effect



cyclicity effect

covariate sender

covariate receiver

covariate similarity

Attribute evolution

Stochastic differential equation: for period $m = 1, ..., M-1$, we have

$$dZ_i(t) = \tau_m \left[a\, Z_i(t) + b^\top u_i(t)\right] dt + \sqrt{\tau_m}\, dW_i(t)$$

$u_i(t)$: covariate effects, *network effects*

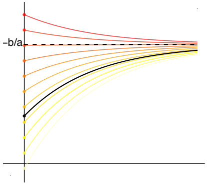
a : feedback parameter

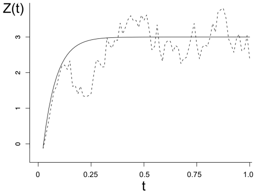
b : intercept and covariate parameters

τ_m : accounts for time scaling

Explicit transition density (Bergstrom, 1984)

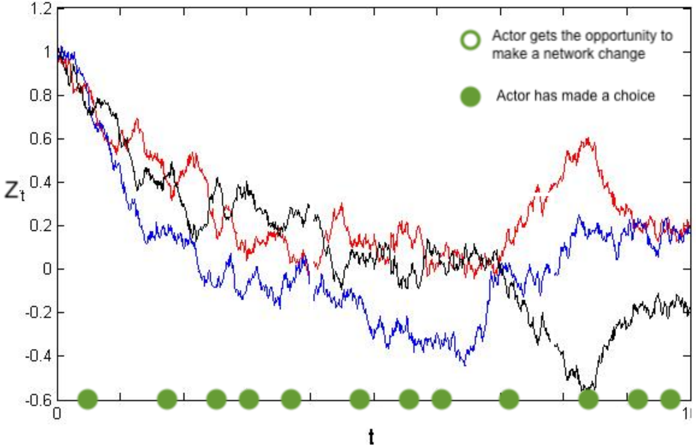
$$z_{i,t+\Delta t} = A_{\Delta t} z_{i,t} + B_{\Delta t} u_{i,t} + w_{i,\Delta t}, \quad w_{i,\Delta t} \sim \mathcal{N}(0, Q_{\Delta t})$$





Integrating the SDE in the SAOM (NN & Snijders, 2018)

Actors' behavior variables are updated at each potential network change.



Parameter estimation: simulated method of moments

- Method of moment estimation with moment equations solved by stochastic approximation, implemented in R package `RSiena`.

$\theta = (\theta_k)$: the vector of all parameters in the model
$S = (S_k)$: a corresponding set of statistics
$Y(t)$: all the data at time t

$$\mathrm{E}_\theta\{S(Y)\} = S(y)$$

Markov assumption and longitudinal data structure:

$$\mathrm{E}_\theta\{S_k(Y(t_{m+1})) \mid Y(t_m) = y(t_m)\} = S_k(y(t_{m+1}))$$

and for parameters constant across periods:

$$\sum_{m=1}^{M-1} \mathrm{E}_\theta\{S_k(Y(t_{m+1})) \mid Y(t_m) = y(t_m)\} = \sum_{m=1}^{M-1} S_k(y(t_{m+1}))$$

Parameter estimation: stochastic approximation

(A variation on) Robbins-Monro stochastic approximation:

$$\hat{\theta}_{N+1} = \hat{\theta} - a_N \tilde{D}^{-1}(S_N - s_{obs})$$

- S_N : simulate Y with parameter $\hat{\theta}_N$, compute $S(Y)$
- $\tilde{D}$: (partially) diagonalized estimate of $D_\theta = \partial E_\theta[S]/\partial\theta$
- a_N : smaller with increasing N

Phase 1: Estimate $\tilde{D}$ based on a small number n_1 of simulations
Phase 2: Robbins-Monro approximation
Phase 3: Estimate $\mathrm{cov}(\hat{\theta})$ based on a larger number n_3 of simulations

$$\mathrm{cov}(\hat{\theta}) \approx D_\theta^{-1}\ \mathrm{cov}_\theta(S)\,(D_\theta^{-1})^\top$$

using that $D_\theta = E[SJ^\top]$, for complete-data score $J = \partial \log p_\theta(Y)/\partial\theta$.

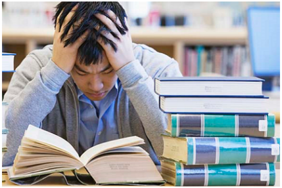
Example: Student stress

Research context and data

Question: How do social networks play a role in stress development among students in higher education?

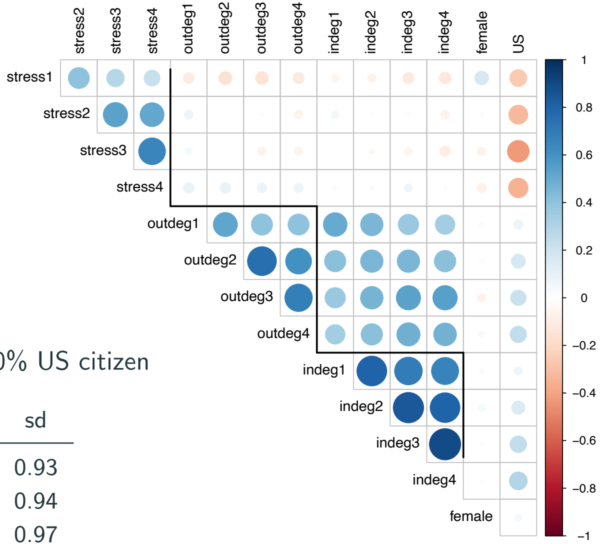
Data: 6-month, 4-wave longitudinal study of 315 early- to mid-career adults in professional Master's programs (Li, Krackhardt, Niezink, 2023)

- friendship network
- perceived stress scale (PSS-4)
- gender (female = 1)
- citizenship (US = 1)
- same program



PSS-4 item: In the last month, how often have you felt that you were unable to control the important things in your life? (5-point Likert item, score 1–5)

Descriptives



54.6% female, 40% US citizen

stress	mean	sd
time 1	2.99	0.93
time 2	3.07	0.94
time 3	3.21	0.97
time 4	3.34	1.03

Preliminary regression modeling

We are interested in whether there is social influence on stress and how the number of friends one has affects stress.

We control for gender and citizenship.

Preliminary (incorrect) model, for data collected at time $1, ..., T$:

1. Stack dependent behavior data at time $1, ..., T-1$ in a vector (pre)
2. Stack dependent behavior data at time $2, ..., T$ in a vector (post)
3. Stack covariate data at time $1, ..., T-1$ in a vector (pre)
4. Run a linear model with (post) as the dependent variable and (pre) as the independent variables

Preliminary (incorrect) model results

Effect	estimate	(s.e.)	p-value
Intercept	-0.094	(0.397)	
Stress$_{t-1}$	0.341	(0.055)	< 0.001
Indegree$_{t-1}$	0.010	(0.014)	
Outdegree$_{t-1}$	0.028	(0.015)	≈ 0.05
Average alter$_{t-1}$	0.685	(0.113)	< 0.001
Female	-0.091	(0.097)	
US citizen	-0.206	(0.107)	≈ 0.05

Co-evolution model

Friendship evolution:

- Network mechanisms: density, reciprocity, transitivity, ...
- Covariate effects: same program, same gender, both US citizens
- Stress effects: stress ego, stress alter, stress similarity
- Rate parameters

Stress evolution:

- Average alter effect (social influence)
- Indegree, outdegree
- Female, US citizen
- Scale parameters, feedback, intercept

Subset of the results

		estimate	(s.e.)	p-value
Friendship	Stress ego	-0.064	(0.021)	< 0.01
	Stress alter	-0.063	(0.018)	< 0.001
	Stress similarity	0.219	(0.154)	
Stress	Indegree	-0.007	(0.010)	
	Outdegree	0.023	(0.008)	< 0.01
	Average alter	0.718	(0.148)	< 0.001
	Female	-0.087	(0.068)	
	US citizen	-0.223	(0.090)	< 0.05

Interpretation

- Interpreting the direction and hypothesis testing of effects is simple.
- Interpreting effect sizes is harder.

In period m, the estimated model is given by

$$\begin{aligned} \mathrm{d}Z_i(t) =& \hat{\tau}_m \left[-0.70\, Z_i(t) - 0.05 + 0.72\, \mathrm{avAlt}_i(t) + \right. \\ & \left. 0.024\, \mathrm{outdeg}_i(t) + \ldots\right] \mathrm{d}t + \sqrt{\hat{\tau}_m}\, \mathrm{d}W_i(t) \end{aligned}$$

- Continuous-time parameters are most easily interpreted in terms of their discrete-time consequences.
- In a SAOM, each observation period has simulation time $[0, 1]$.

Interpretation: outdegree example

```
                              Estimate  Standard Error
22. stress_beh outdegree       0.0235   ( 0.0094 )
```

Suppose outdegree x_{i+} is constant. The expectation of the trajectory is

$$\mathrm{E}(Z_i(t) \mid z_i(0)) = e^{a\,\tau_m t}\, z_i(0) + \frac{1}{a}(e^{a\,\tau_m t} - 1)(b_0 + b_{\text{outdeg}} x_{i+} + \ldots)$$

After $t = 1$, an increase in outdegree by 1 leads to an increase in stress by

$$\frac{1}{a}(e^{a\,\tau_m} - 1) b_{\text{outdeg}}.$$

Plugging in estimates yields 0.022, 0.019, and 0.016 for the three periods.

Question: Is this large? (stress mean=3.14, sd=0.97)

Interpretation: US citizen example

```
                                 Estimate   Standard Error
  22. stress_beh outdegree        0.0235    ( 0.0094 )
  24. effect from US_cov         -0.2233    ( 0.3581 )
```

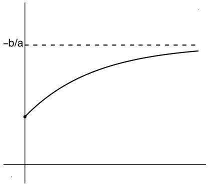
Being a US citizen (`US_cov = 1`) leads to an increase in stress of

$$\frac{1}{a}(e^{a\,\tau_m} - 1)b_{\mathrm{US}}.$$

We estimate this to be -0.21, -0.18, and -0.15 for the three periods.

The values differ per period:

Effect	Period 1	Period 2	Period 3
Outdegree	0.022	0.019	0.016
US	-0.21	-0.18	-0.15
Scale $\hat{\tau}_m$	1.53	1.17	0.91



A higher scale parameter means moving away more from $z_i(0)$.

Note: $z(t) = e^{a\tau_m t} \cdot z_i(0) + (1 - e^{a\tau_m t}) \cdot -\frac{b}{a}$ (feedback+intercept model)

Looking back

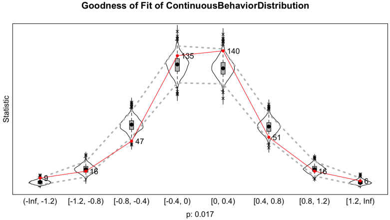
Remember our (incorrect) preliminary model results?

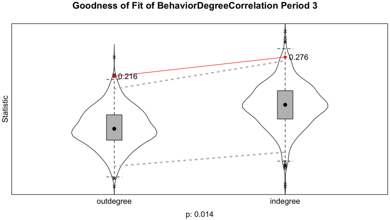
Effect	estimate	(s.e.)	*p*-value
Intercept	-0.094	(0.397)	
Stress_{t-1}	0.341	(0.055)	< 0.001
Indegree_{t-1}	0.010	(0.014)	
Outdegree_{t-1}	0.028	(0.015)	≈ 0.05
$\text{Average alter}_{t-1}$	0.685	(0.113)	< 0.001
Female	-0.091	(0.097)	
US citizen	-0.206	(0.107)	≈ 0.05

These are similar to the discrete-time (t=1) results of our SDE analysis!

Effect	Period 1	Period 2	Period 3
Outdegree	0.022	0.019	0.016
US	-0.21	-0.18	-0.15

Next steps: methodology





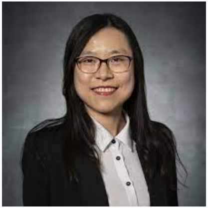
1. Make goodness of fit functions available
2. Implement multi-group model
3. Investigate effect size computation: explained variance
4. ...

Next steps: applications

With Shihan Li @ University of Kentucky

Question: How does the balance in advice out- and indegree in organizations affect one's performance?

Data: R&D units in company with ±300 employees, 4 waves, supervisor evaluations.

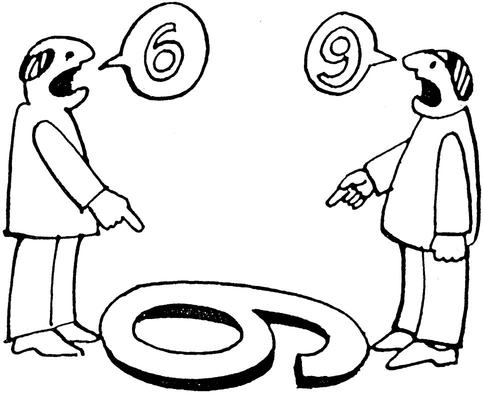


With Daniel Redhead @ University of Groningen

Question: how does how one's friendship and advice indegree affect one's prestige and dominance?

Data: Three classrooms of ±250 students each, 4 waves.

Network dynamics: Perceptions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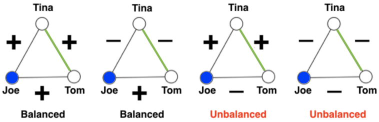
Network perceptions

Individuals perceive the networks of relationships around them differently.

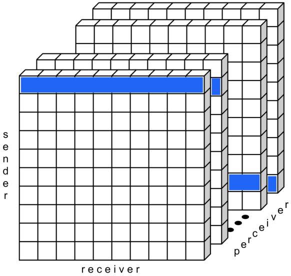
- Originally: focus on accuracy [Bernard, Killworth & Sailer, 1977,1979,1982]
  - Determinants of accuracy variability [Casciaro, 1998; Lee et al, 2022]
- More recently: focus on intrinsic interest
  - Balance theory [Heider, 1958; Cartwright & Harary, 1956]
  - Effects on individual and group outcomes



Imbalance is unstable: The psychological tension produced by being in an unbalanced state "becomes relieved only when change within the situation takes place in such a way that a state of balance is achieved." [Heider, 1958, p.180]

Cognitive social structures

Network perception data: for a social system among n actors, $X = (x_{ij}^{(p)}) \in \{0,1\}^{n \times n \times n}$ with $x_{ij}^{(p)} = 1$ if actor p perceives that actor i has a tie to actor j. [Krackhardt, 1987]



Methodology

Inferring the *'true'* network:

- focusing on informants' true and false positive rates [Kumbasar et al., 1996; Batchelder et al., 1997; Koskinen, 2002a]
- solving the network inference/informant accuracy problem using a hierarchical model [Butts, 2000]
- considering the true network as latent variable [Koskinen, 2002b]

Focusing on perceptions:

- including covariates and extending the social relations model [Kenny, 1994] to analyze triadic relations [Bond et al., 1997, 2000]
- latent space models [Sosa and Rodríguez, 2021; Sewell, 2019]
- stochastic block models

Studies on perception dynamics are mostly descriptive. [Ertan et al., 2019]

First step: SAOM for perceived ties of others

perceiver 1 perceiver 2 perceiver n

$x^{(1)}(t_1)$ — $x^{(1)}(t_2)$ — $x^{(1)}(t_3)$ $\cdots$ $x^{(1)}(t_M)$ $x^{(2)}(t_2)$ — $x^{(2)}(t_2)$ $\cdots$ $x^{(2)}(t_M)$ $\cdots$ $x^{(n)}(t_M)$

$\lambda_1^{(1)}$ $\lambda_2^{(1)}$ ... $\lambda_{M-1}^{(1)}$ $\lambda_1^{(2)}$ ... $\lambda_{M-1}^{(2)}$ ... $\lambda_{M-1}^{(n)}$

- We model p's perceived relations between others; not ties $x_{pj}^{(p)}$
- We assume heterogeneous rates $\lambda_m^{(p)}$ and constant choice mechanism parameters β_k
- This allows us to study perceiver effects through network-level covariates

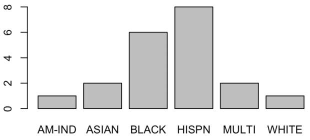
Illustration

Women were found to demonstrate better network perception accuracy than men. [Cappella et al., 2012; Neal et al., 2016]

When considering network perception change,

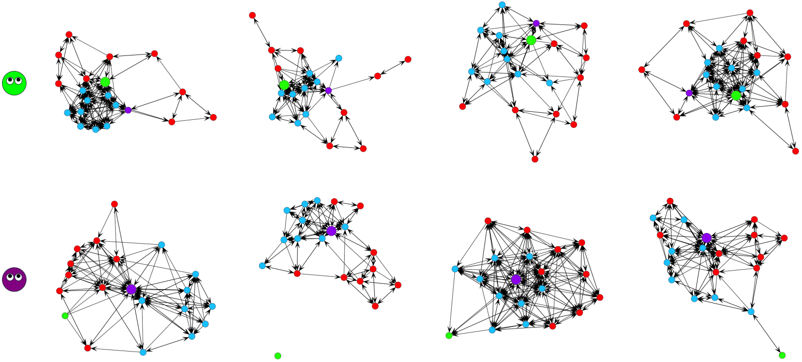
1. Are women more likely than men to form perceptions of relations that were self-reported?
2. Are men differentially likely to perceive men as active or popular?

- 20 undergraduates at a 6-week summer program for prospective graduate students[1]
- 10 female, 10 male
- friendship perception data

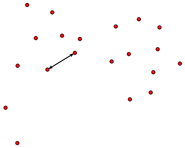


[1] Data courtesy: Keith Hunter

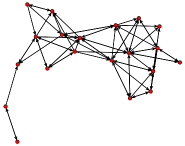
week 1 | week 2 | week 3 | week 4



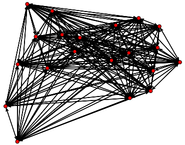
Consensus?



In week 1, everyone thinks these 2 friendship ties exist.

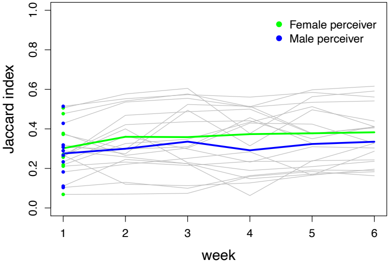


At least 10 people think these 73 ties exist.



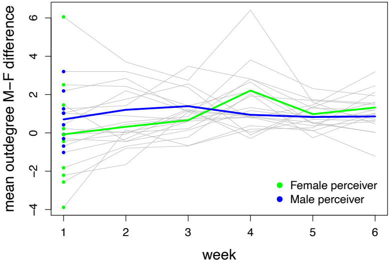
Every friendship tie is perceived by at least one person.

Women better accuracy?

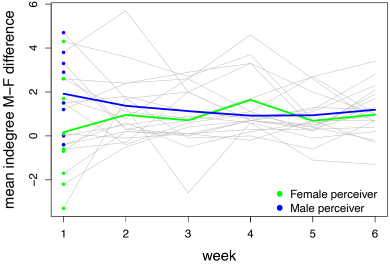


Distance to self-reported network

Men deem men active?



...or popular?



SAOM specification

- structural effects (reciprocity, transitivity, etc.)
- male sender (activity), male receiver (popularity), same gender effect
- same ethnicity effect
- self-reported network ties at start of period (perception accuracy)

And the effects of being a male perceiver on the strength of:

density	perception accuracy
$v_p \sum_j x_{ij}^{(p)}$	$v_p \sum_j x_{ij}^{(p)} w_{ij}$

male activity	male popularity
$v_p\, v_i \sum_j x_{ij}^{(p)}$	$v_p \sum_j x_{ij}^{(p)} v_j$

$v_i = 1$ if i is male

w_{ij}: self-reported tie from i to j

	Self-reported		*Perception*	
	$\hat{\beta}$	(s.e.)	$\hat{\beta}$	(s.e.)
rates periods 1–5	–	–	–	–
outdegree (density)	−2.49	(0.37)	−2.20	(0.055)
reciprocity	1.84	(0.34)*	1.71	(0.042)*
transitivity	0.17	(0.048)*	0.13	(0.009)*
reciprocity × transitivity	−0.21	(0.055)*	−0.12	(0.007)*
indegree popularity	0.01	(0.030)	0.07	(0.005)*
outdegree activity	0.04	(0.013)*	−0.02	(0.004)*
sender male	0.06	(0.097)	0.22	(0.030)*
receiver male	0.25	(0.10)*	−0.05	(0.027)
same gender	0.09	(0.095)	0.22	(0.021)*
same ethnicity	0.24	(0.12)*	0.18	(0.023)*
self-reported network (accuracy)			0.48	(0.030)*
male perceiver			−0.09	(0.045)
male perceiver × male sender			0.03	(0.044)
male perceiver × male receiver			0.08	(0.040)*
male perceiver × self-reported network			0.02	(0.046)

$^{*}p < 0.05$

Next steps

- Many open questions in the context of network perceptions.
  - How does the assumption that all actors' network perceptions are equal affect our network analyses?
  - How do perception differences affect individual or group outcomes?

Towards: modeling the dynamics of three-way social network data.

Meaning: modeling the joint evolution of 'own' ties and perceived ties.

The availability of software catalyzes the feedback loop between methodological developments and advances in the social sciences.

With Tobias Stark @ Utrecht University:
perceived roles of Dutch people with a migration background: dual identity $\rightarrow$ bridge?

Discussion

- Model framework for dynamics on networks and network dynamics
- Widely used for dynamic network analysis in the social sciences
- Limitations:
  - not suitable for large networks
  - lacks formal causal interpretation (ongoing work)
- Three-way network analysis (e.g., for network perceptions) is an exciting open research area

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Appendix

References

Niezink, N.M.D., M.A.J. van Duijn and T.A.B. Snijders (2019). No longer discrete: Modeling the dynamics of social networks and continuous actor attributes. Sociological Methodology, 49(1).

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Niezink, N.M.D. and T.A.B. Snijders (2017). Co-evolution of social networks and continuous actor attributes. The Annals of Applied Statistics, 11(4), 1948–1973.

RSiena user's group: `https://groups.io/g/RSiena`

Explicit transition density (Bergstrom, 1984)

$$z_{i,t+\Delta t} = A_{\Delta t} z_{i,t} + B_{\Delta t} u_{i,t} + w_{i,\Delta t}, \quad w_{i,\Delta t} \sim \mathcal{N}(0, Q_{\Delta t})$$

$$\begin{aligned}
A_{\Delta t} &= e^{\tau_m A \Delta t} \\
B_{\Delta t} &= A^{-1}(A_{\Delta t} - I_p) B \\
Q_{\Delta t} &= \mathrm{ivec}[(A \otimes I_p + I_p \otimes A)^{-1}(A_{\Delta t} \otimes A_{\Delta t} - I_p \otimes I_p)\,\mathrm{vec}(GG^\top)].
\end{aligned}$$