Quantitative Methods for Logistics

ME44206

Assignment 1

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October 26, 2020

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Chapter 1

Mathematical model

1.0.1 Problem formulation

Let us consider 3 products which is produced across 12 months. There are two operations which needs to be completed, to produce one product completely. After finishing the first operation, the product becomes a semi-finished product. The semi-finished product becomes a fully finished product after the second operation is completed. For each month, the demand should be met for all the products. We can also produce more than the demand in a month and carry it over to the next month.

The cost of performing specific operations varies across the months. Each product needs different set of operations for completion. The cost for producing all the 3 products across 12 months is our production cost. The products which are overproduced and should be carried over to the next month and they must be stored. And there is a separate holding cost for semi-finished and fully finished products for all the 3 products.

The total cost for production system would be the cost of production combined with the holding cost. This total cost should be minimized

1.0.2 Given data

1) Cost of production

Operation	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	12	11	11	11	10	8	9	12	16	18	18	14
2	18	25	25	20	15	15	17	19	19	18	18	18
3	10	12	11	12	11	12	12	12	12	11	11	12
4	8	9	12	13	15	15	18	15	10	8	8	8

2) Holding cost

Product	SemiFinished HC	Finished HC
1	2	4
2	1	3
3	1	4

3) Operation Time

Product	1	2	3	4
1	0.6	0.8	0	0
2	0	0.3	0.9	0
3	0.4	0	0	0.7

4) Demand

Product	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	650	600	600	550	500	500	450	600	650	600	600	550
2	200	250	250	300	350	400	400	500	500	400	300	250
3	300	350	400	350	300	400	400	300	350	350	300	350

1.0.3 Assumptions

- Initial inventory is zero.
- Each of the 4 operations are done in four separate machines.
- As the operations are done in separate machines, we need not consider scheduling or sequencing.
- Demand should be met for every month.
- Overproduction is allowed.

1.0.4 Indices

Set	**Index**	**Range**
Months	i $\in I$	[1,2,3,4,5,6,7,8,9,10,11,12]
Products	j $\in J$	[1,2,3]
State of Product	k $\in K$	[1,2]
Operation 1	fo $\in FO$	[1,2,1]
Operation 2	so $\in SO$	[2,3,4]
Operations	o $\in O$	[1,2,3,4]

1.0.5 Parameters

1) $D_{i,j}$ - Demand across 12 months for 3 products.

$$i \in I, j \in J$$

2) $PC_{i,o}$ - Production cost across 12 months for 4 operations.

$$i \in I, o \in O$$

3) $HC_{j,k}$ - Holding cost for the products on 2 states (Semi finished, Fully finished)

$$j \in J, k \in K$$

4) $OT_{j,o}$ - Operation time for 3 products across 4 operations.

$$j \in J, o \in O$$

1.0.6 Decision variables

1) $SP_{i,j}$ - Semi finished products produced for the month i and product j.

$$(i \in I, j \in J)$$

2) $SC_{i,j}$ - Semi finished products which are carried over for the month i and product j.

$$(i \in I, j \in J)$$

3) $FP_{i,j}$ - Fully finished products produced for the month i and product j.

$$(i \in I, j \in J)$$

4) $FC_{i,j}$ - Fully finished products carried over for for the month i and product j.

$$(i \in I, j \in J)$$

5) $SW_{i,j}$ - Semi finished products which are carried over and needs to completed(made to fully finished product) for the month i and product j.

$$(i \in I, j \in J)$$

1.0.7 Objective Function

Holding Cost:

$\sum_{i \in I} \sum_{j \in J} \Big[(SC_{i,j}.HC_{j,1}) + (FC_{i,j}.HC_{j,2}) \Big]$

Production Cost:

$\sum_{i \in I} \sum_{j \in J} \Big[(SW_{i,j}.PC_{i,so(j)}) + (SP_{i,j}.PC_{i,fo(j)}) + (FP_{i,j}.(PC_{i,fo(j)} + PC_{i,so(j)})) \Big]$

Total Cost = Holding cost + Production cost

Minimize Z =

$$\sum_{i\in I}\sum_{j\in J}\Big[(SC_{i,j}.HC_{j,1}) + (FC_{i,j}.HC_{j,2})\Big] +$$
$$\sum_{i\in I}\sum_{j\in J}\Big[((SW_{i,j} + FP_{i,j}).PC_{i,so(j)}) + ((SP_{i,j} + FP_{i,j}).PC_{i,fo(j)})\Big]$$

The objective of the problem is to minimize the total cost of the production for the 3 products across 4 operations. The total cost of our system is the sum of production cost and the holding cost. Since, over production is allowed we can produce more than the demand and carry over the extra products to the next months.

The holding cost for our problem is the summation across the 12 months and 3 products of the number of semi finished products carried over times the semi finished holding cost plus the number of fully finished products carried over times the fully finished holding cost.

The production cost of our problem is the the summation across the 12 months and 3 products for - $\Big[$ number of semi finished carried over products which needs to completed to fully finished times the cost of the second operation, plus the number of semi finished products produced times the cost of the first operation plus the of number of fully finished products produced times the cost of the first as well the second operation.$\Big]$

1.0.8 Constraints

Constraint 1:

$$\mathrm{D}_{i,j} \leq FP_{i,j} + FC_{i,j} + SW_{i,j} \quad \forall\, i \in I, j \in J$$

Constraint 2:

$$\sum_{i\in I} D_{i,j} = \sum_{i\in I} FP_{i,j} + SP_{i,j} \quad \forall\, i \in I, j \in J$$

Constraint 3:

$$\mathrm{FC}_{1,j} + SC_{1,j} = 0 \quad \forall\, j \in J$$

Constraint 4:

$$\mathrm{FC}_{i,j} = (FP_{i-1,j} + SW_{i-1,j} + FC_{i-1,j}) - D_{i-1,j} \quad \forall\, i \in I, j \in J$$

Constraint 5:

$$\mathrm{SC}_{i,j} = SP_{i-1,j} + SC_{i-1,j} - SW_{i-1,j} \quad \forall\, i \in I, j \in J$$

Constraint 6:

$$\mathrm{SP}_{12,j} + SC_{12,j} = SW_{12,j} \quad \forall\, j \in J$$

Constraint Non-negativity:

$$\mathrm{FP}_{i,j}, SP_{i,j}, FC_{i,j}, SC_{i,j}, SW_{i,j} \geq 0 \quad \forall\, i \in I, j \in J$$

Constraint Real-Valued:

$$\mathrm{FP}_{i,j}, SP_{i,j}, FC_{i,j}, SC_{i,j}, SW_{i,j} \in R \quad \forall\, i \in I, j \in J$$

Chapter 2

Implementation and Capacity Problem

2.0.1 Implementation

After implementing the model in Python using Gurobi solver, the following result was obtained
Objective function (Z) = Minimum Total Cost = **424500.0 euros**
Production Cost = **399700.0 euros**
Holding Cost = **24800.0 euros**

2.0.2 Capacity Problem

Now we are considering a monthly capacity limit for all the four operations throughout the year. The capacity limit for four operations are [550h, 750h, 450h, 400h] for operation 1,2,3,4 respectively.
In order to consider the capacity the capacity limit, we need to add more constraints to our existing mathematical model. They are listed below

Constraint 2.1:

$$[(\mathrm{SP}_{i,1} + FP_{i,1}).OT_{1,1}] + [(SP_{i,3} + FP_{i,3}).OT_{3,1}] \leq 550 \quad \forall\, i \in I$$

Constraint 2.2:

$$[(\mathrm{SP}_{i,2} + FP_{i,2}).OT_{2,2}] + [(SW_{i,1} + FP_{i,1}).OT_{1,2}] \leq 750 \quad \forall\, i \in I$$

Constraint 2.3:

$$[(\mathrm{SW}_{i,2} + FP_{i,2}).OT_{2,3}] \leq 450 \quad \forall\, i \in I$$

Constraint 2.4:

$$[(\mathrm{SW}_{i,3} + FP_{i,3}).OT_{3,4}] \leq 400 \quad \forall\, i \in I$$

2.0.3 Implementation - Capacity Problem

After implementing the new model in Python using Gurobi solver, the following result was obtained
Objective function (Z) = Minimum Total Cost = **432822.2 euros**
Production Cost = **424263.714 euros**
Holding Cost = **8557.0 euros**

Compared to the previous model, our total cost has increased as a result of increase in production costs. Also, our holding cost has reduced.
This is because of the capacity limit which we set for operation in a month. The production can happen only till the capacity limit. Because of the capacity limit set on production there will be decrease in overproduced products produced in the month when the production cost is low, owing to higher production costs. And since the number of overproduced products decreases, the holding costs decreases.

Chapter 3

Verification and Experimentation

3.0.1 Verification

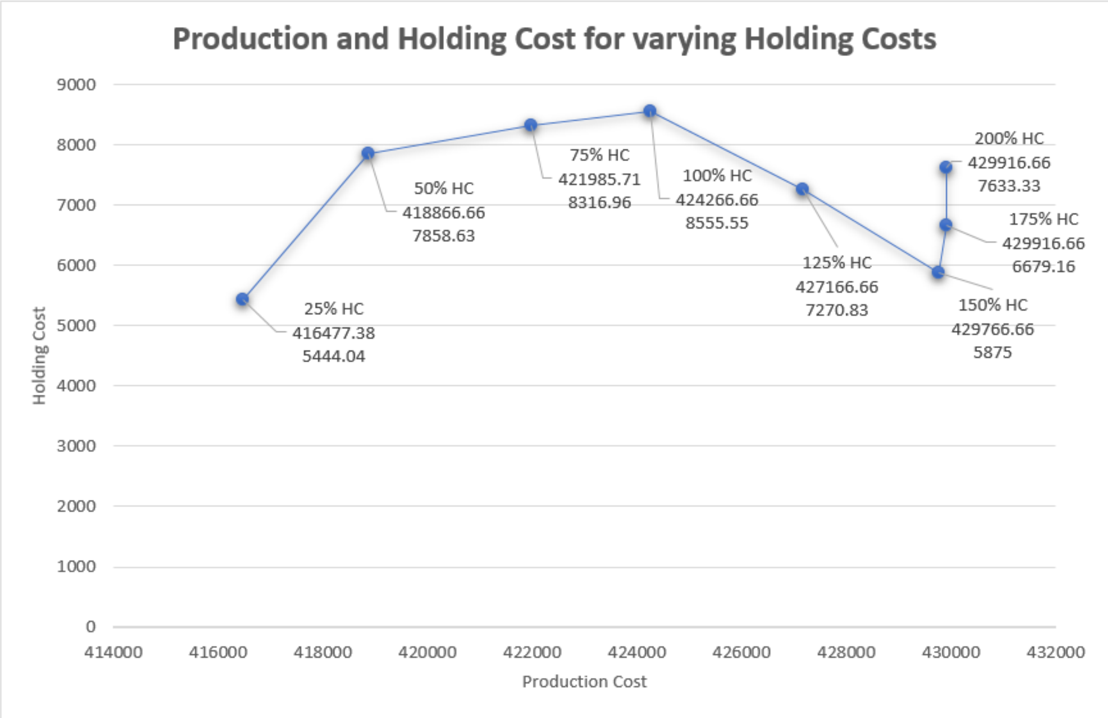
Description	Expected	Result	OK
Increase the capacity for all operations by 100 hours.	Z will decrease	Z=430341.6 euros	Pass
Decrease demand by 10 times for all months and products.	Z will decrease	Z=42540.0 euros	Pass
Increase the holding cost by 10 times for both semi-finished and fully finished	Z will increase	Z=441250.0 euros	Pass
Make the capacity zero for all the operations	Infeasible solution (Demand can't be met)	Infeasible solution	Pass
Increase demand by 10 times for all months and products	Infeasible solution (Demand can't be met)	Infeasible solution	Pass
Make holding cost zero for both semi-finished and fully finished	Z will decrease	Z=415908.3 euros	Pass
Decrease production cost by 50%	Z will decrease	Z=218775.0 euros	Pass
Decrease Operation time by 2 times	Z will decrease	Z=425633.3 euros	Pass
Increase Operation time by 2 times	Z will increase	Infeasible solution (Demand is not met)	Fail

- When the capacity limit is increased, the cost of production will go down, as the system can overproduce more in the cheaper month. Hence the total cost goes down.
- When the demand decreases, the production and holding costs go down. Hence the total cost goes down
- When the holding prices are increased by 10 times, the holding cost of the system increases. Hence the total cost goes up.

- When the capacity limit is set to zero for each month and operation, the system can't produce. Hence, the solution is infeasible
- When the demand is increased by 10 times, the system can't meet the demand. Hence, the solution is infeasbible.
- When the holding prices are set to zero, the system will overproduce more in the cheap month and the hold the products for free of cost. Hence, the total cost goes down
- When the cost of performing operations reduce, the production costs reduces. Hence the total cost goes down.
- When the time to perform an operation is reduced by 2 times, the system can produce more in the cheaper month within the capacity limit. Hence, the total cost goes down
- When the time to perform an operation is increased by 2 times, I expected the system the total cost to increase. But the demand itself was not being met with that high operation time.

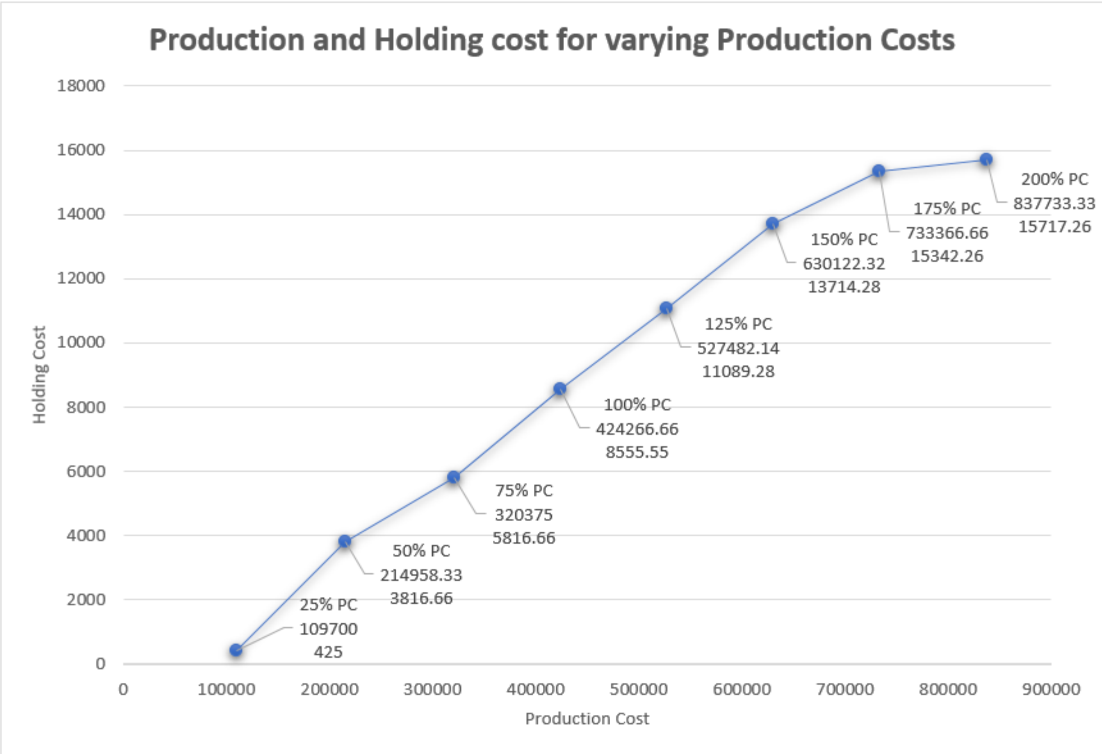
3.0.2 Experimentation

After verification, different values of production and holding costs were experimented with the model. First, the holding cost was varied by 25 percent starting 25 percent of the holding cost to 200 percent of the holding cost. The results from the experiment was plotted in the graph, Production cost vs Holding cost.



As the holding cost started to increase, the system decided to carry over less products to each, to save on holding costs. The system decides to meet the demand for every month with minimum carry over as the carry over cost keeps increasing.

For the second experiment, the production cost was varied by 25 percent, starting 25 percent of production cost per operation to 200 percent of production cost per operation. The results were plotted in a Production Cost vs Holding Cost graph.



As the production cost increased, the holding cost also increased as the system tries to carry over more products from a month when the cost of production is low and tries to carry over the products to the other months where productions are high in order to meet the demand. Therefore, to holding cost increases as the system holds the products more to carry over to subsequent months.

Chapter 4

Revision Problems

4.0.1 Revision in one month

In this problem, each of the machines used for the 4 operations goes through revision in only one month. Each machine undergoes revision in a different month and during the month of revision, that specific operation done by the machine is stopped.
The mathematical model is updated to account for the revision and it is then implemented in Python using Gurobi solver.

4.0.1.1 Updating the Mathematical model

An additional binary decision variable needs to be added.

$S_{i,o}$ - Binary decision variable for the month i and operation o.

$$(i \in I,\ O \in O)$$

Additional constraints to be added.

Constraint 4.1

$$\sum_{i \in I} S_{i,o} = 11 \quad \forall\, i \in I, o \in O$$

Constraint 4.2

$$\mathrm{SP}_{i,j} + FP_{i,j} = (SP_{i,j} + FP_{i,j}).S_{i,fo(j)} \quad \forall\, i \in I, j \in J$$

Constraint 4.3

$$\mathrm{SW}_{i,j} + FP_{i,j} = (SW_{i,j} + FP_{i,j}).S_{i,so(j)} \quad \forall\, i \in I, j \in J$$

4.0.1.2 Implementation

After implementing the new model in Python using Gurobi solver, the following result was obtained
Objective function (Z) = Minimum Total Cost = **435669.64 euros**
Production Cost = **421090.47 euros**
Holding Cost = **14579.16 euros**

Since the production variables are multiplied by the binary variable and we have declared the sum of the binary variable values across all the months to be 11, there will be one month in which the binary variable value would be zero. That month, would be the month of revision

The system decides the revision of month for each operation in a way that it uses the lowest cost for producing the products and meeting the demand for all the months. The holding cost also increases because there is no production in a month and to meet the demand for that month, all products must be carried over to it. So, there is an obvious increase in the holding cost of the system.

4.0.2 Operation Cost Reduction after Revision

In this problem, each of the machines used for the 4 operations goes through revision in only one month. Each machine undergoes revision in a different month and during the month of revision, that specific operation done by the machine is

stopped.After the revision, the production cost for performing that specific operation reduces by 10 percent for all the subsequent months.
The mathematical model is updated to account for the revision and it is then implemented in Python using Gurobi solver.

4.0.2.1 Updating the Mathematical model

One more binary decision variable needs to be added.
$S2_{i,o}$ - Binary decision variable for the month i and operation o.

$$(i \in I,\ O \in O)$$

Objective function needs to be updated :

Minimize Z =

$$\sum_{i \in I} \sum_{j \in J} [(SC_{i,j} * HC_{j,1}) + (FC_{i,j} * HC_{j,2})] +$$

$$\sum_{i \in I} \sum_{j \in J} [(SW_{i,j} + FP_{i,j}) * [(0.9PC_{i,so[j]}) + (0.1S2_{i,so[j]}PC_{i,so[j]})]] +$$

$$\sum_{i \in I} \sum_{j \in J} [(SP_{i,j} + FP_{i,j}) * [(0.9PC_{i,fo[j]}) + (0.1S2_{i,fo[j]}PC_{i,fo[j]})]] +$$

Additional constraints to be added.
Constraint 4.4

$$\mathrm{S2}_{1,j} = 1 \quad \forall\, j \in J$$

Constraint 4.5

$$\mathrm{S2}_{i,j} = S2_{i-1,j}.S_{i-1,j} \quad \forall\, i \in I, j \in J$$

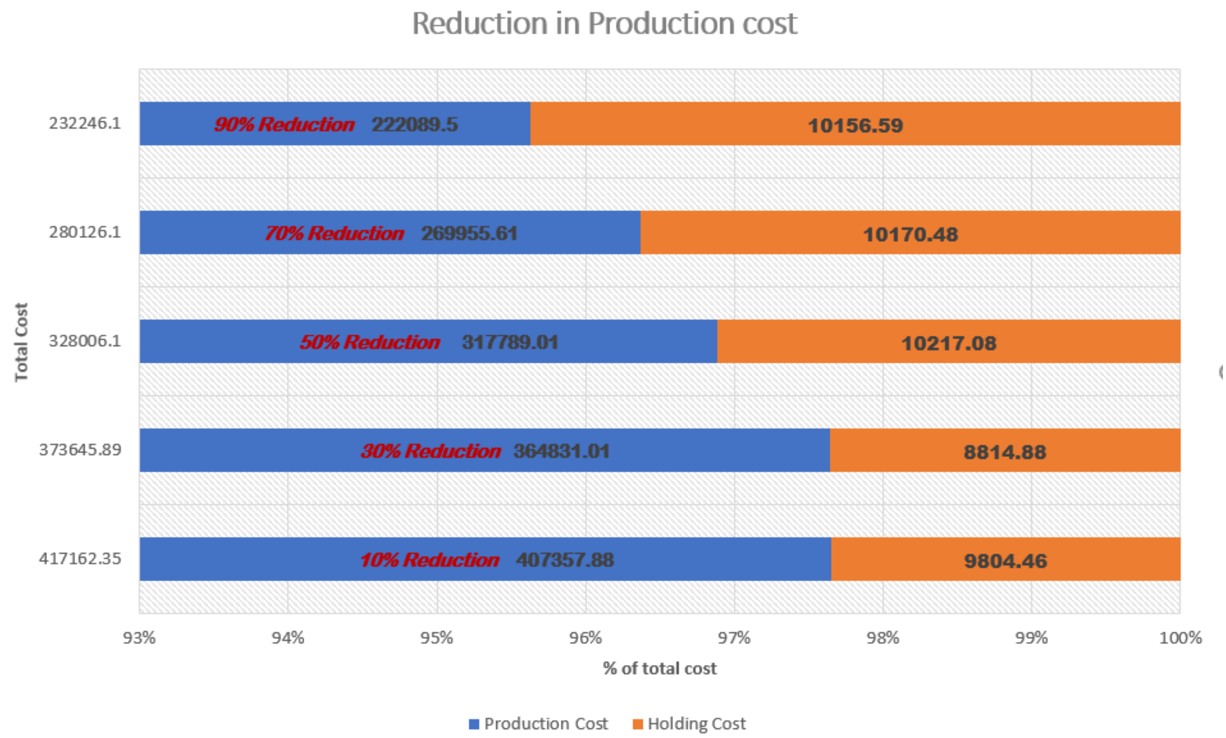
4.0.2.2 Implementation

After implementing the new model in Python using Gurobi solver, the following result was obtained
Objective function (Z) = Minimum Total Cost = **417162.35 euros**
Production Cost = **407357.88 euros**
Holding Cost = **9804.46 euros**

Since there was a reduction in the cost to perform the operations after the month of revision, the overall production cost came down, giving a lower total cost for the system.

4.0.3 Experimenting with Reduction Cost

Further, various percentages of reduction in production cost was experimented on the model starting from 10 percent to 90 percent reduction. The values of production and holding cost varied like this



As we reduce the cost of production, the percentage of production of production cost in the total cost tends to decrease and the percentage of holding cost in the total cost, tends to increase. This happens because as the cost of production decreases, the system tends to overproduce in the month where the cost is low and carry over those overproduced products in order to meet the demand. So, for those carried over products, the holding cost comes into account and thus it increases.