Class Reading Summary

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1 Angrist, Imbens and Rubin: Causal Effects and Instrumental Variables

This paper reads more like a textbook chapter, so my response will be more of a summary than a "response". The authors' main goal for the paper appears to be reconciling the typical exclusion restrictions for an instrumental variables estimator within the Rubin Causal Model. To me it seems like a mildly helpful exercise, since it clarifies the local average treatment effect and how the exclusion restriction works to some degree. However, some of the formulae used in the exposition do not make sense to me, and it would be great to get a deeper explanation of them.

1.1 The Conventional Estimator

Sections 1 and 2 are an introduction and a rehash of the standard way the instrumental variables estimator is presented. Let Y be the outcome of interest, D be the endogenous treatment and Z be the instrument. Then we have the following system of equations:

$$Y_i = \beta_0 + \beta_1 D_i + \epsilon_i$$
$$D_i = \alpha_0 + \alpha_1 Z_i + \nu_i$$

We can derive the conventional estimator by plugging the second equation into the first:

$$Y_i = \beta_0 + \beta_1(\alpha_0 + \alpha_1 Z_i + \nu_i) + \epsilon_i$$

We can then solve for β_1:

$$\beta_1 = \frac{\frac{cov(Y,Z)}{var(Z)}}{\alpha_1} = \frac{\frac{cov(Y,Z)}{var(Z)}}{\frac{cov(D,Z)}{var(Z)}} = \frac{cov(Y,Z)}{cov(D,Z)}$$

In the case of a binary instrument and a binary outcome we have that $\frac{cov(Y,Z)}{var(Z)} = \bar{Y}_1 - \bar{Y}_0$, and $\frac{cov(D,Z)}{var(Z)} = \bar{D}_1 - \bar{D}_0$, since they are just the regression coefficients. This gives us the **Wald Estimator**:

$$\beta_1 = \frac{\bar{Y}_{Z=1} - \bar{Y}_{Z=0}}{\bar{D}_{Z=1} - \bar{D}_{Z=0}}$$

The main assumptions of the conventional model are:

1. $E[Z_i * \epsilon_i] = 0$, this is the **exclusion restriction**, which implies that Z affects Y only through D.

2. $E[Z_i * \eta_i] = 0$

3. $cov(D_i, Z_i) \neq 0$

One last thing: in case you want to add controls to the regressions above, you need to add them to both the first stage and second stage (reduced form) regressions.

1.2 IV and the Rubin Causal Model

In section 3, the authors go through the main assumptions of the Rubin Causal Model. Let Z_i be an instrument, $D_i(Z)$ be the treatment, and $Y_i(Z, D)$ be the outcome of interest. Then the **Causal Effect of Z on D** is $D_i(1) - D_i(0)$ and the **Causal Effect of Z on Y** is:

$$Y_i(1, D_i(1)) - Y_i(0, D_i(0))$$

The main **assumptions** to have a satisfactory instrumental variable Z are then:

1. - SUTVA (for D) If $Z_i = Z_i'$ then $D_i(Z) = D_i(Z')$
   - SUTVA (for Y) IF $Z_i = Z_i'$ and $D_i = D_i'$ then $Y_i(Z, D) = Y_i(Z', D')$

2. Random assignment: $P(Z = c) = P(Z = c')$ where c' has the same amount of subjects in T and C as c. This can be weakened to ignorable assignment, ie. $Y(0), Y(1) \perp\!\!\!\perp Z$ or conditional ignorability $Y(0), Y(1) \perp\!\!\!\perp Z|X$.

3. Exclusion restriction: $Y(Z, D) = Y(Z', D)$ for all Z, Z', D. This basically means that the effect of Z on Y is only through D.

4. Nonzero average causal effect of Z on D: $E[D_i(1) - D_i(0)] \neq 0$

5. Monotonicity: $D_i(1) \geq D_i(0)$ for all i.

Using SUTVA and random assignment, we can estimate the average causal effect of Z on Y with $E(Y|Z = 1) - E(Y|Z = 0)$ and the average causal effect of Z on D with $E(D|Z = 1) - E(D|Z = 0)$. The ratio of these two equals the conventional IV estimator derived above.

With all of these assumptions, we can then try to understand the IV estimand in more depth. First, we can simplify the numerator of the expression to:

$$\begin{aligned}
Y_i(1, D_i(1)) - Y_i(0, D_i(0)) &= Y_i(D_i(1)) - Y_i(D_i(0)) \\
&= [Y_i(1)D_i(1) + Y_i(0)(1 - D_i(1)] - [Y_i(1)D_i(0) + Y_i(0)(1 - D_i(0)] \\
&= Y_i(1)(D_i(1) - D_i(0)) - Y_i(0)(D_i(1) - D_i(0)) \\
&= (Y_i(1) - Y_i(0))(D_i(1) - D_i(0))
\end{aligned}$$

More succinctly:

$$Y_i(1, D_i(1)) - Y_i(0, D_i(0)) = (Y_i(1) - Y_i(0))(D_i(1) - D_i(0))$$

Ie. the numerator IV estimand is the product of the treatment effect of D on Y and that of Z on D. By expanding the expectation of the above out and using monotonicty we can write:

$$E[Y_i(D_i(1), 1) - Y_i(D_i(0), 0)] = E[(Y_i(1) - Y_i(0)|D_i(1) - D_i(0) = 1]P[D_i(1) - D_i(0) = 1]$$

Rearranging, we get the **causal interpretation of the IV estimand**:

$$\frac{E[Y_i(D_i(1), 1) - Y_i(D_i(0), 0)]}{E[D_i(1) - D_i(0) = 1]} = E[(Y_i(1) - Y_i(0)|D_i(1) - D_i(0) = 1]$$

This is also known as the **Local Average Treatment Effect**. We can explain this further in terms of $D_i(1)$, and $D_i(0)$. There are four categories of subjects:

- Never takers (Would never go to the military, regardless of draft status)
- Defiers (They go to the military if not drafted, don't if drafted)
- Compliers (They go to the military if drafted, don't if not drafted)
- Always Takers (Would always go to the military, regardless of draft status)

Our assumptions imply that the average causal effect of Z on Y is proportional to the average causal effect of D on Y for compliers. Never takers and always takers don't have much impact on IV, they comprise of subjects where Z has no impact on Y. However, we do not want never takers and always takers to be the whole sample (that would violate assumption 4, nonzero average causal effect of Z on D). On the other hand, defiers violate monotonicity and pose a fatal flaw to IV estimation if present.

The authors then compare this framework to the standard (structural equation framework) presentation of IV. I think that this was quite helpful in deepening my understanding of instrumental variables and how they connect to the Rubin Causal Model. The remainder of the paper goes through sensitivity of the model to violations of these assumptions and an application of the framework to the draft instrument on the effects of military service.

2 Lifetime Earnings and the Vietnam Era Draft Lottery

Since I am already at 2 pages, I will be short in my response to this paper. Overall, I think that using draft number as an instrument for military service is intuitive, and mostly satisfies the exclusion restrictions. One can make a case that SUTVA is violated if people want to serve with friends. One can also make the case that people with low lottery numbers changed their educational plans, which would violate the exclusion restriction. Nevertheless, I think that the instrument works, and makes sense.

My other concerns with this paper are that the results are often not statistically significant (especially table 4, the two stage IV estimates). One broader comment is that since the estimator essentially scales the treatment effect of being a veteran on compliers ($\alpha = (\bar{y}^e - \bar{y}^n)/(\hat{p}^e - \hat{p}^n)$), it appears to make the effects look quite large. Finally, I didn't fully understand a lot of what the authors were doing for "efficient IV estimation".

3 Other things to cover

Identifying a model: using variation of some sort in X to see how Y changes according to the variation in X.

3.1 Falsification exercise:

Apply to any kind of observational study where confounding is plausible but not possible to directly rule it out. You can:

- Examine outcomes vulnerable to the same confounders as treatment, but for which, according to theory, no effect of treatment is expected. Basically you find a similar Y, regress similar Y on D and see if it is significant. Example: Angrist and Krueger 1991 examine association between instrument (season of birth) and college graduation (vs. staying in school after compulsory age).

- Examine populations for whom, based on theory, no effect should be observed. Or populations for whom, based on theory, an effect should be observed. Eg. if marriage increases wages prior to marriage due to the causal effect of anticipation, no effect should be observed for 'shotgun marriages'.

In the IV case of Z is DEA budget, X is border patrol budget, and Y is some migrant decision making, there is a concern that there is some omitted time varying covariate that is driving both X and Y. For example a fear of migrants affects migratory decisions regardless of DEA budget. Good checks would be to:

- Include t, t^2 variables to see if the effect you are capturing is net of the time trend.

- Falsification exercise: instrument for something random, like military spending in Africa. It would be quite strange if you find a correlation between this and the outcome.

Reduced form: Y outcome, Z instrument, X treatment. The reduced form is then the regression of Y on Z.