BT6270: Computational Neuroscience

Assignment-1: FitzHugh-Nagumo Model

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The FitzHugh–Nagumo model is a simplified 2D version of the Hodgkin–Huxley model, which models in a detailed manner activation and deactivation dynamics of a spiking neuron. This simplification is obtained by condensing the ion channel activations into a w variable. The FHN Model is an example of a relaxation oscillator because, if the external current I_{ext} exceeds a certain threshold value, the system will exhibit a characteristic excursion in phase space, before the variables v and w relax back to their rest values.

The two variable FitzHugh-Nagumo model can be simulated using the following equations:

$$\frac{dv}{dt} = v(a-v)(v-1) - w + I_m$$

$$\frac{dw}{dt} = bv - rw$$

The parameters for the first 3 cases are given as: $a = 0.5,\ b = r = 0.1$

Note: Please go through the attached colab notebook for a better understanding of how these equations were modeled.

Case-1: $I_{ext} = 0$

Phase Plot:

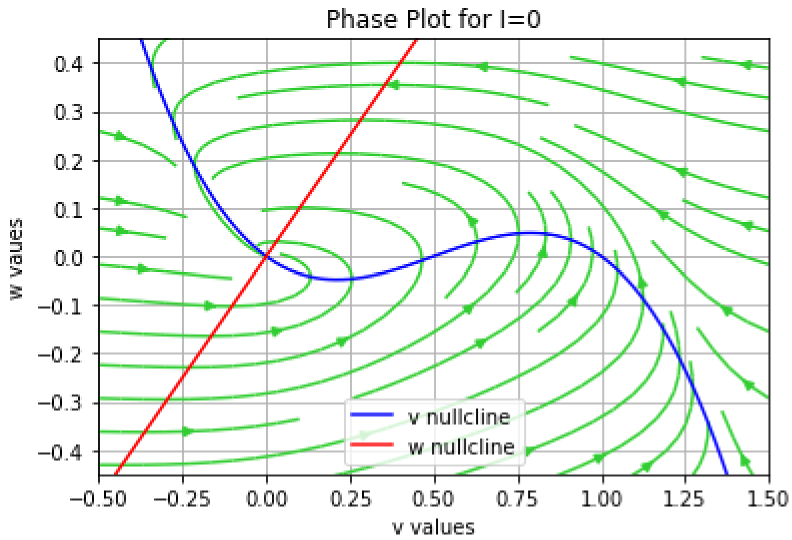


Figure-1: Phase Plot of the System at $I_{ext} = 0$. The obtained stationary point is a stable point.

We can see from analyzing the trajectories using the initial locations v = [0, 0.4, 0.6, 1], w = 0, that even if the initial start point is perturbed, we still approach the equilibrium position at [0, 0]. The point [0, 0] is a stable fixed point as a result.

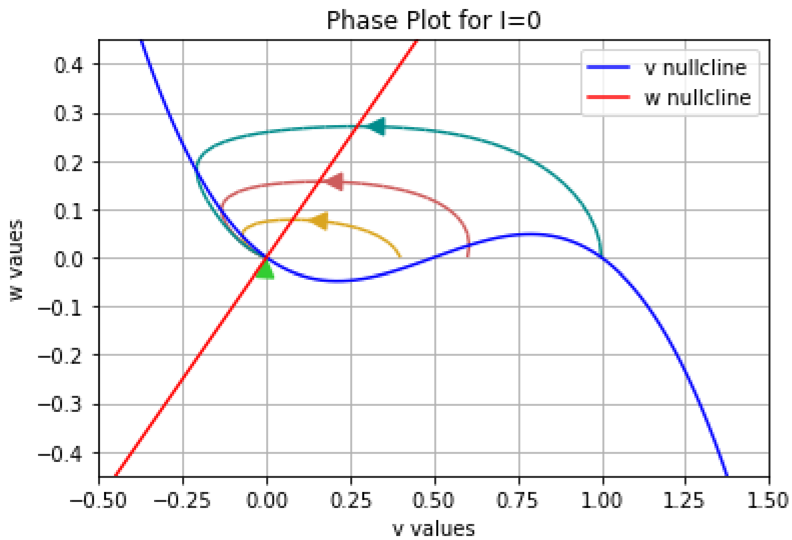


Figure-2: Analysis of the equilibrium point's stability. Regardless of the original conditions, the model moves to the equilibrium point. Therefore, the equilibrium point is a fixed point that is stable.

V(t), W(t) v/s t

For an I_{ext} value of 0, no action potentials are observed.

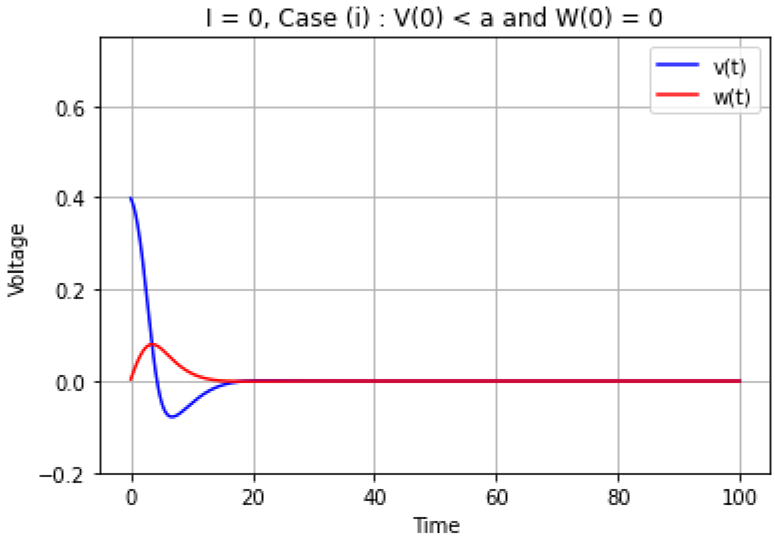


Figure-3: $V(t)$, $W(t)$ v/s t for $V(0) < a$ and $W(0) = 0$. No action potential is observed for sub-threshold pulses.

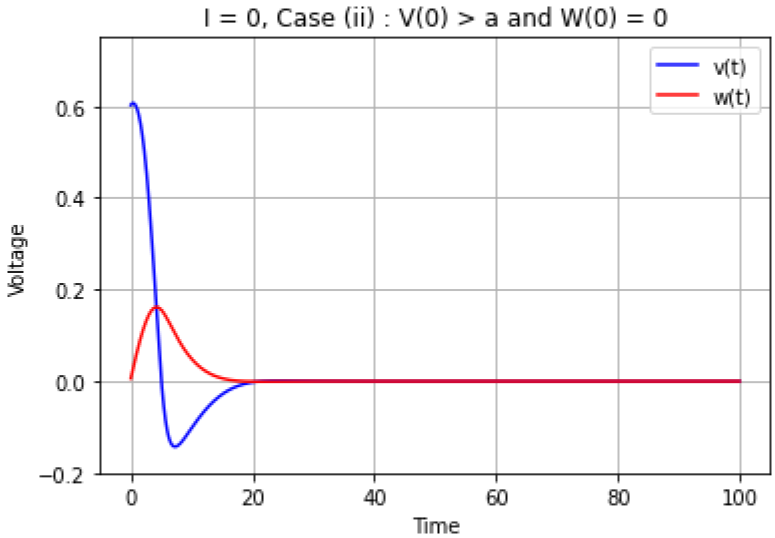
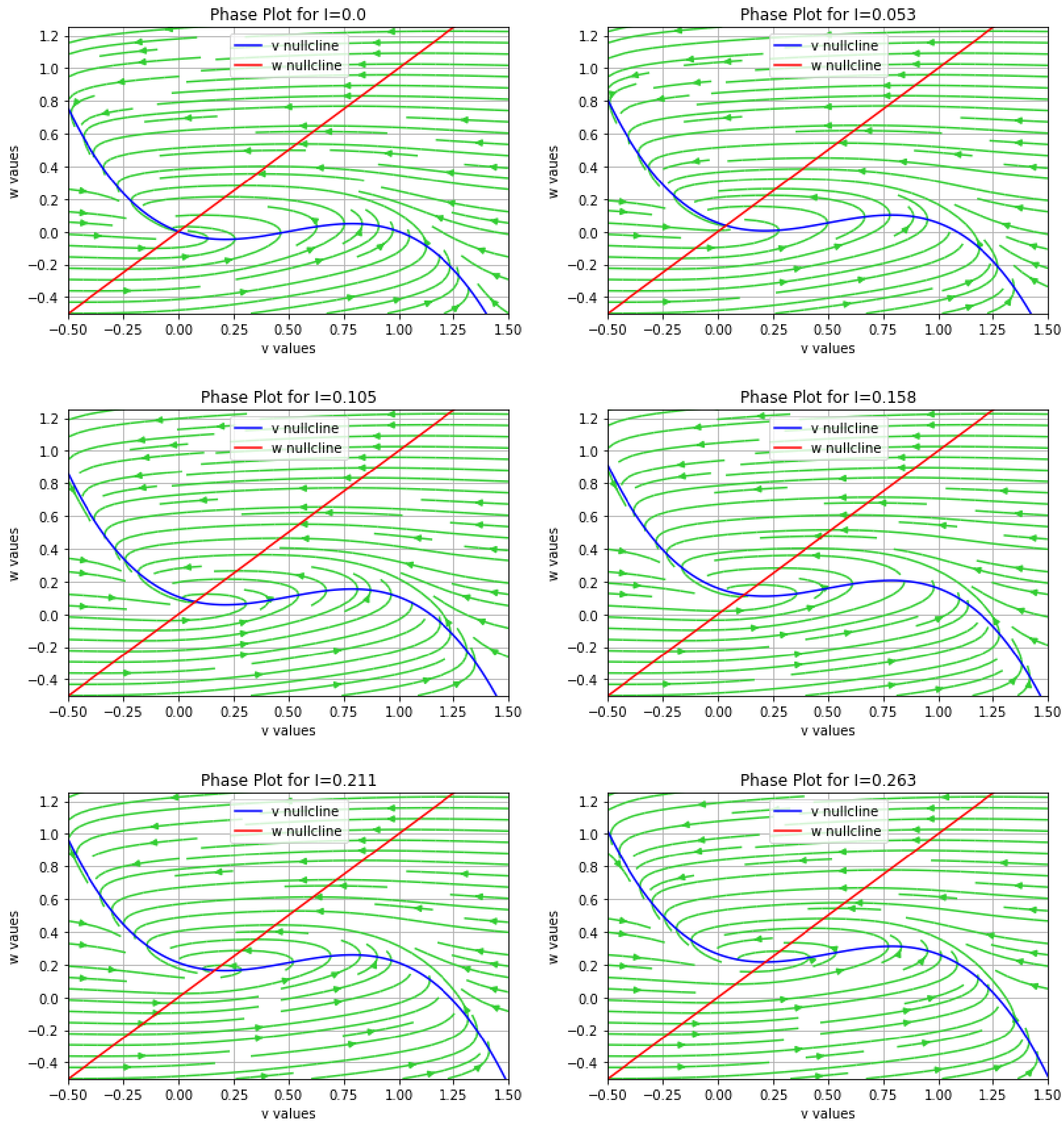
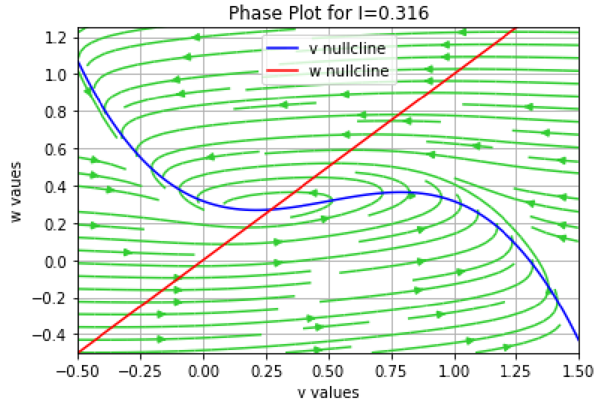


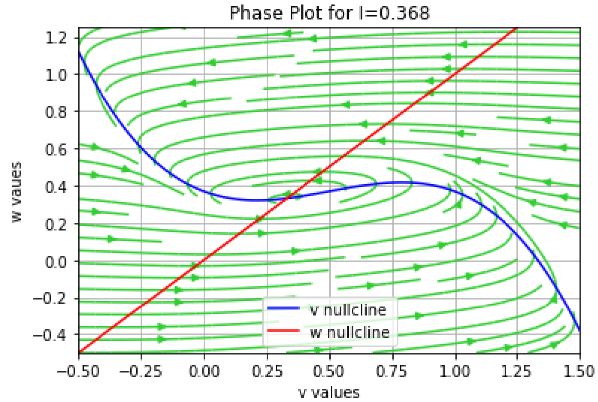
Figure-4: $V(t)$, $W(t)$ v/s t for $V(0) > a$ and $W(0) = 0$. No action potential is observed for sub-threshold pulses.

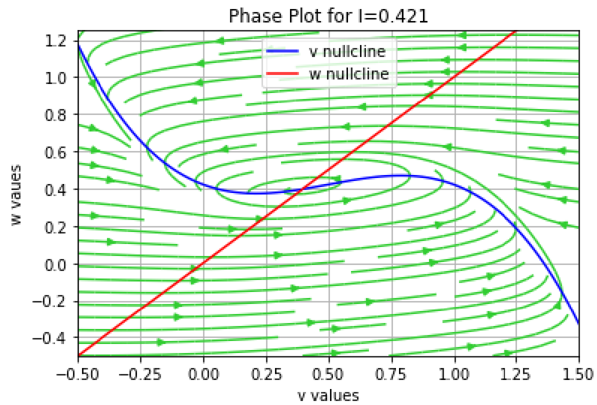
Finding I1 and I2 for the subsequent cases

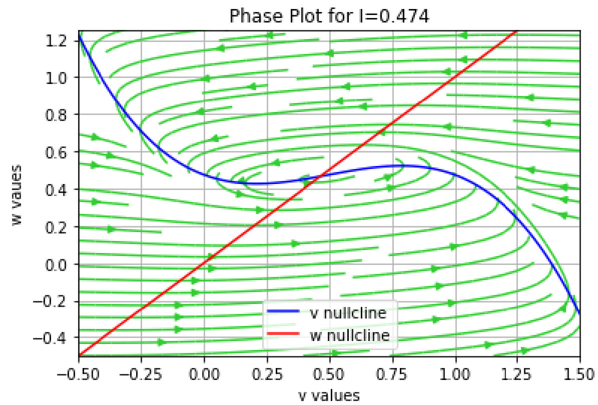
I_1 and I_2 are the boundary parameters between which we observe oscillations. In order to find the value of I_1 and I_2, we gradually change the value of I_{ext} from 0 to 1 and observe the phase plane.

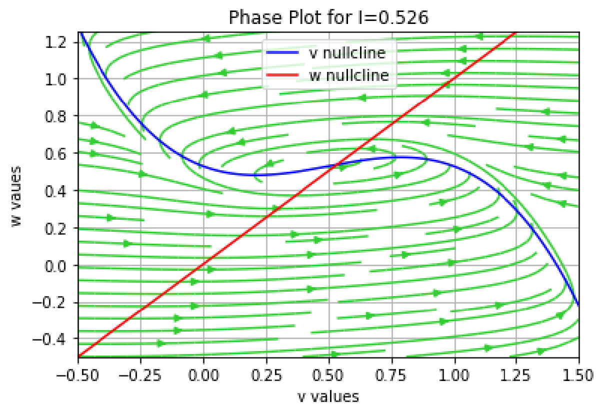


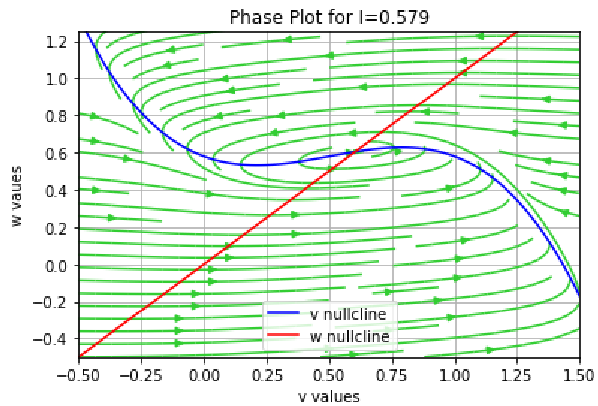

Phase Plot for I=0.316
v nullcline
w nullcline
w vaues
v values

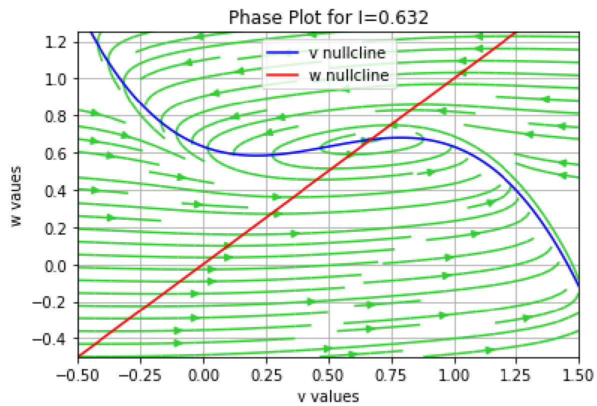

Phase Plot for I=0.368
v nullcline
w nullcline
w vaues
v values

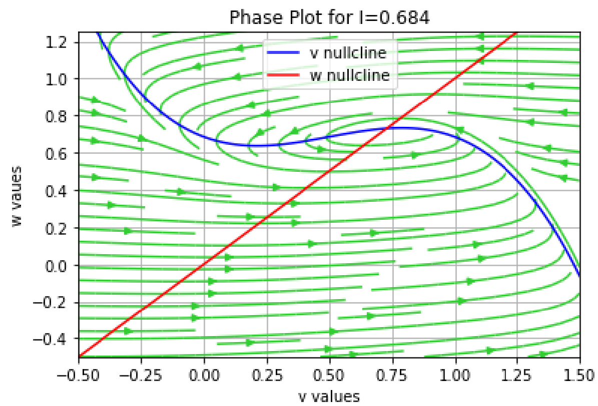

Phase Plot for I=0.421
v nullcline
w nullcline
w vaues
v values

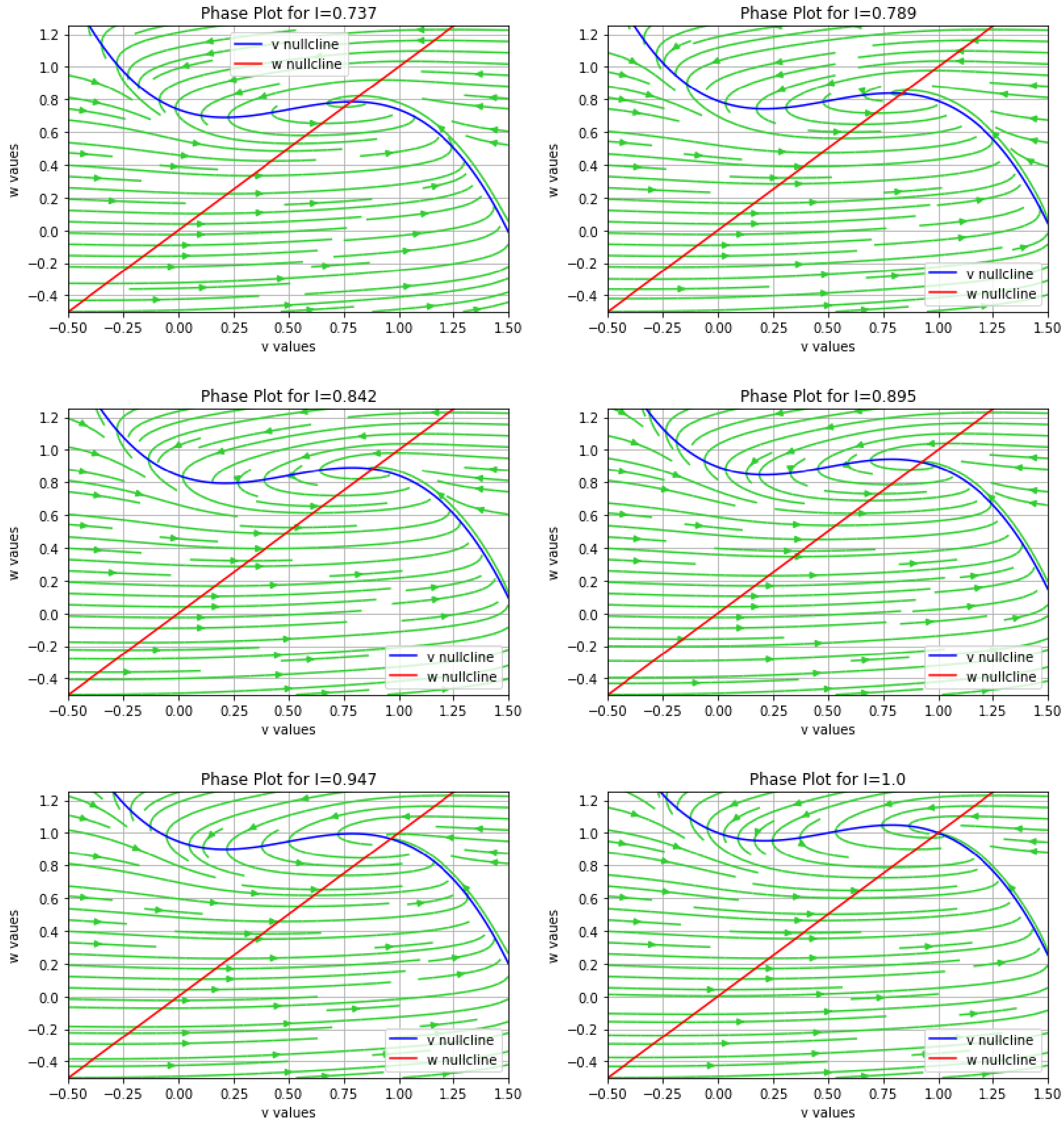

Phase Plot for I=0.474
v nullcline
w nullcline
w vaues
v values


Phase Plot for I=0.526
v nullcline
w nullcline
w vaues
v values


Phase Plot for I=0.579
v nullcline
w nullcline
w vaues
v values


Phase Plot for I=0.632
v nullcline
w nullcline
w vaues
v values


Phase Plot for I=0.684
v nullcline
w nullcline
w vaues
v values



As it can be seen from the plots above, oscillations start happening at around $I_{ext} = 0.368$ and end at around $I_{ext} = 0.684$.

Hence, we assign $I_1 = 0.368$ and $I_2 = 0.684$.

For Case-2, we select $I_{ext} = 0.5$ for further analysis. This value satisfies the given condition of $I_1 < I_{ext} < I_2$.

For Case-3, we select $I_{ext} = 1$ for further analysis. This value satisfies the given condition of $I_{ext} > I_2$.

Case-2: $I_{ext} = 0.5$

Phase Plot

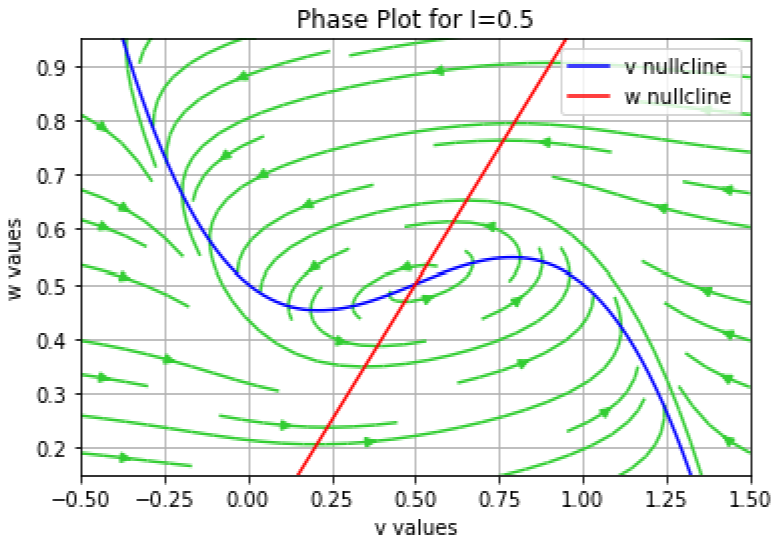


Figure-5: Phase Plot of the System at $I_{ext} = 0.5$. The stationary point is found to be unstable.

Initial locations v = [0, 0.4, 0.6, 1], w = 0, were used to examine the trajectories. We can see that there are circulating fields surrounding the unstable stationary point at the intersection of the nullclines. In addition, a limit cycle can be seen encircling the stationary point.

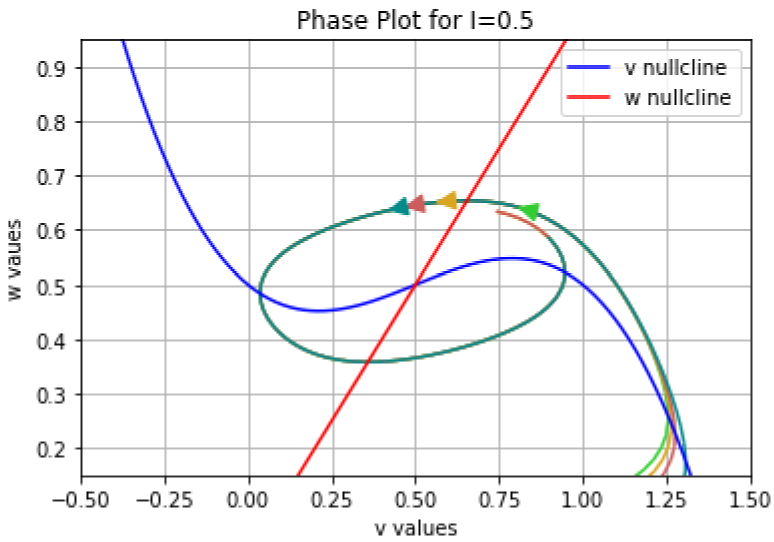


Figure-6: Analysis of the stationary point's stability. The stationary point is observed to be unstable and exhibits limit cycle behavior.

V(t), W(t) v/s t

For $I_{ext} = 0.5$, we observe an oscillatory membrane potential in the limit cycle region.

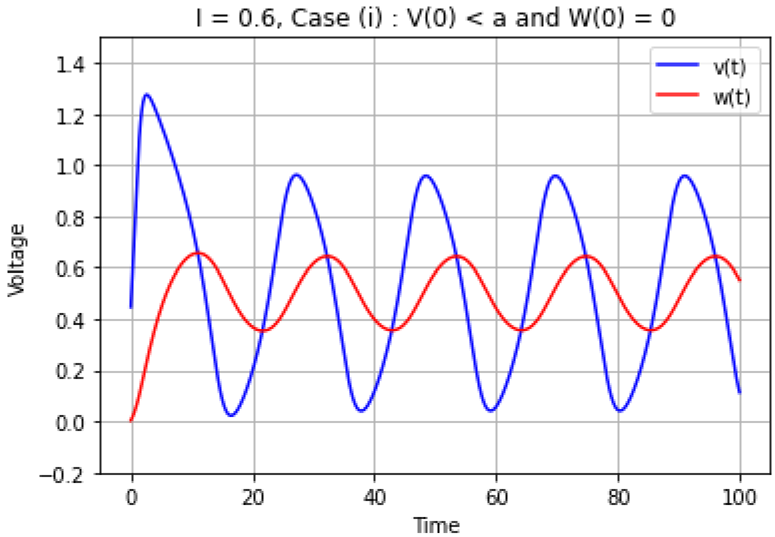


Figure-7: $V(t)$, $W(t)$ v/s t for $V(0) < a$ and $W(0) = 0$. Sustained oscillations are seen in this case.

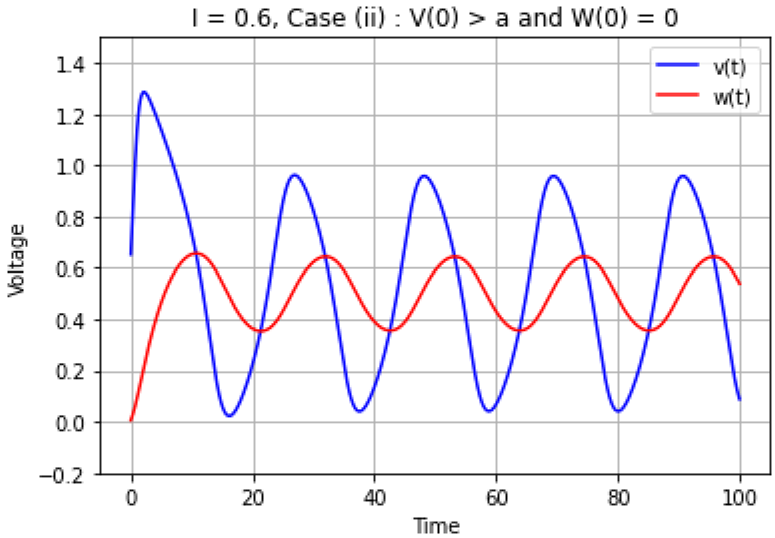


Figure-8: $V(t)$, $W(t)$ v/s t for $V(0) > a$ and $W(0) = 0$. Sustained oscillations are seen in this case.

Case-3: $I_{ext} = 1$

Phase Plot

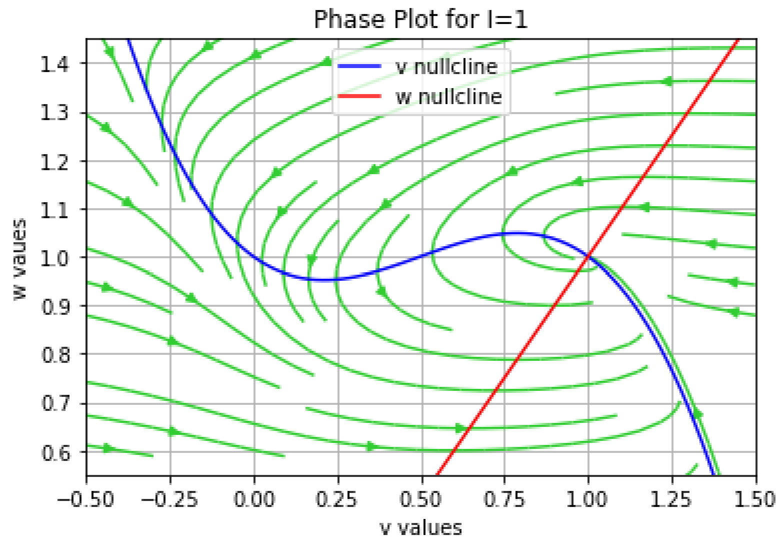


Figure-9: Phase Plot of the System at $I_{ext} = 1$. The stationary point is found to be stable.

The initial locations v = [0, 0.4, 0.6, 1], w = 0.6, and [0, 0.4, 0.6, 1], w = 1.4 were used to study the trajectories. As we can see, even with significant deviations from the initial start point, we are getting closer to the equilibrium point at [1, 1]. The point [1, 1] is a stable fixed point as a result.

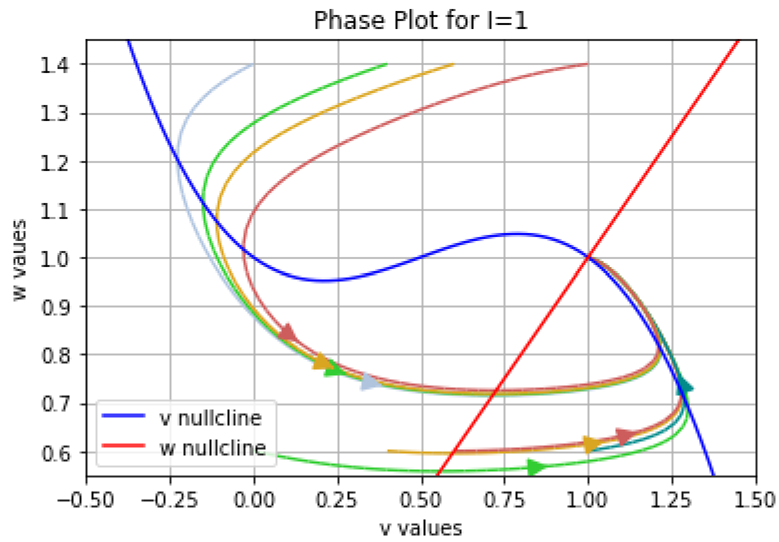


Figure-10: Analysis of the stationary point's stability. The stationary point is found to be stable.

V(t), W(t) v/s t

Depolarization of the membrane potential occurs when I_{ext} = 1. The voltage increases at first and then remains high.

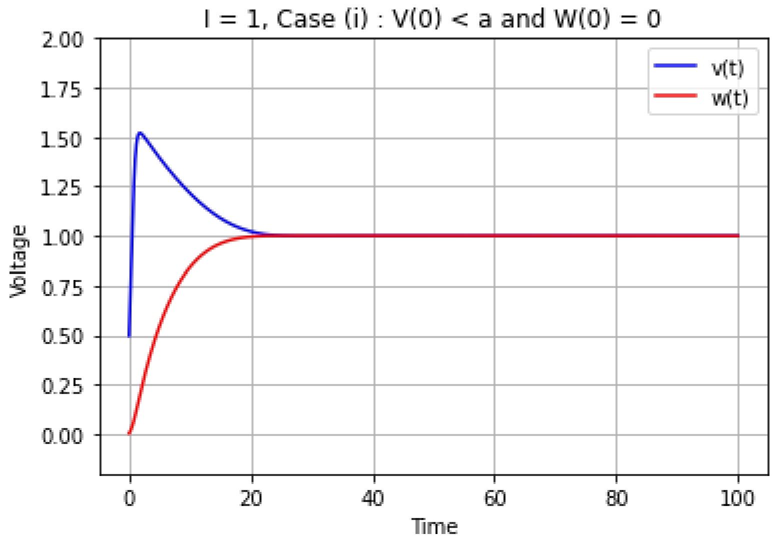


Figure-11: $V(t)$, $W(t)$ v/s t for $V(0) < a$ and $W(0) = 0$. Depolarization in action potential is observed for sub-threshold pulses.

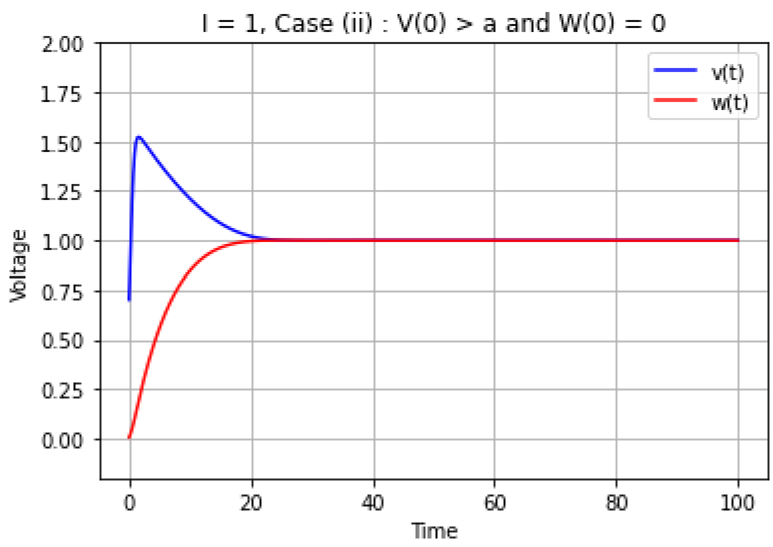


Figure-12: $V(t)$, $W(t)$ v/s t for $V(0) > a$ and $W(0) = 0$. Depolarization in action potential is observed for sub-threshold pulses.

Case-4

I_{ext} and $\frac{b}{r}$ have to be selected such that w nullcline intersects v nullcline at three distinct points. In order to find the values that provided such a system, the value of I_{ext} is fixed and the values of b and r are varied until such a situation arises.

The values obtained are $I_{ext} = 0.02$**,** $b = 0.01$ **and** $r = 0.8$**. Hence,**

$$\frac{b}{r} = 0.0125$$

Phase Plot

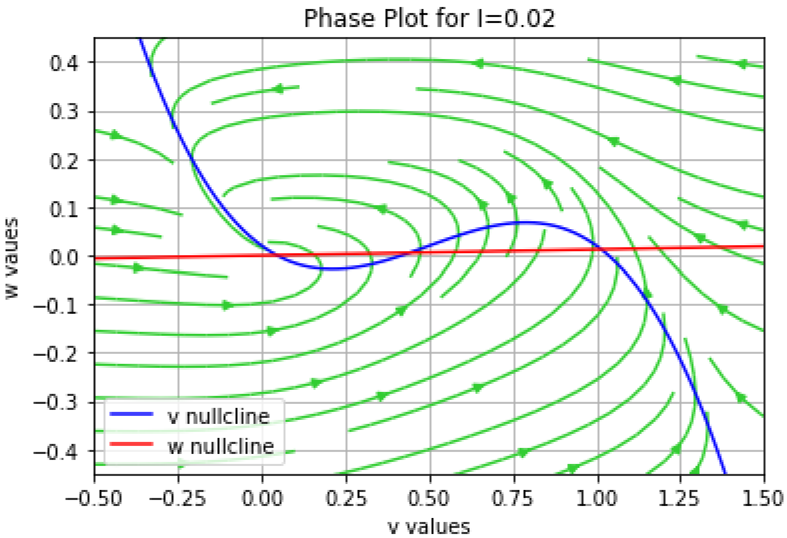


Figure-13: Phase Plot of the System at $I_{ext} = 0.02$. From left to right, the points exhibit stable, saddle, and stable behavior respectively.

The initial locations v = [0, 0.4, 0.6, 1], w = 0.6, and [0, 0.4, 0.6, 1], w = 1.4 were used to study the trajectories.

Small and intermediate perturbations around the first and third points, respectively, return to point 1 and point 3. Hence, they are stable.

For the second point, slight deviations along one direction/axis result in significant modifications to the end point. Hence, it is a saddle node.

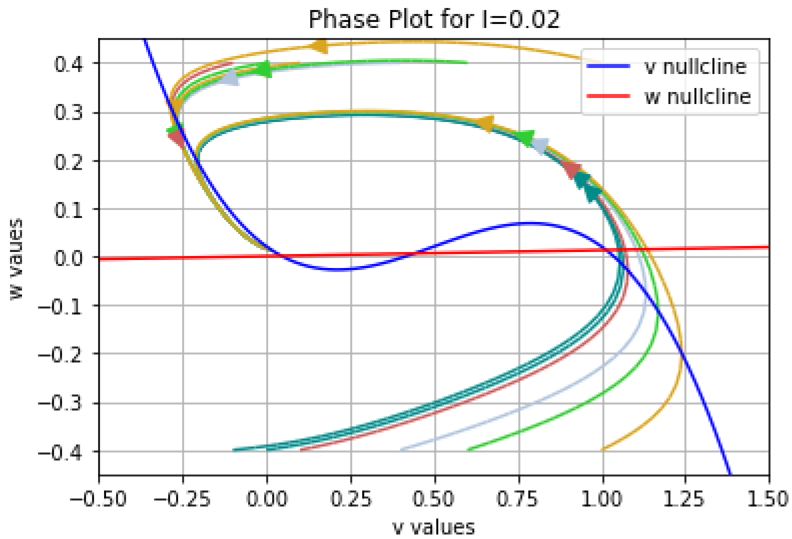


Figure-14: Analysis of the stationary points' stability.

V(t), W(t) v/s t

For $I_{ext} = 0.02$, $b = 0.01$, and $r = 0.8$, bi-stability is observed.

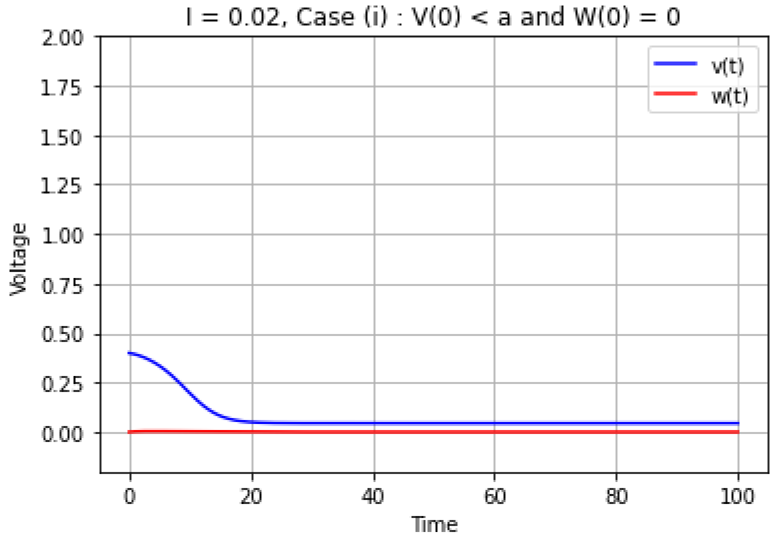


Figure-15: $V(t)$, $W(t)$ v/s t for $V(0) < a$ and $W(0) = 0$. Neuron is constantly active in a low state.

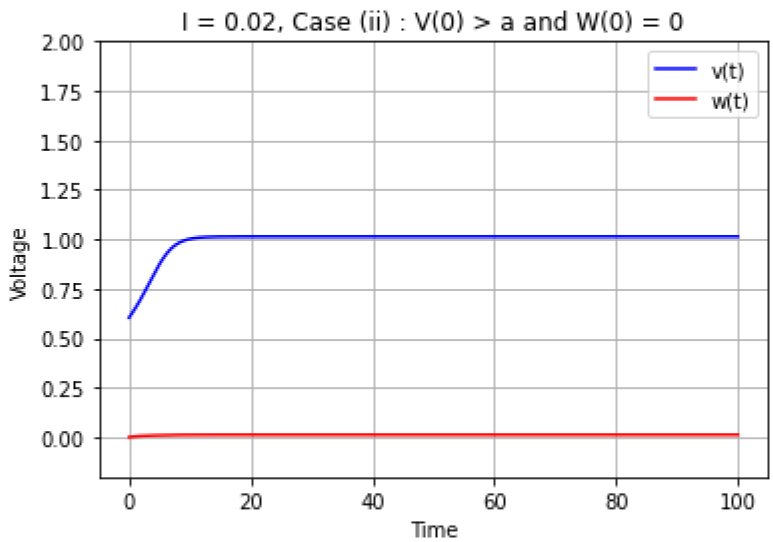


Figure-16: $V(t)$, $W(t)$ v/s t for $V(0) > a$ and $W(0) = 0$. Neuron is constantly active in a high state.