MA5710: Mathematical Modelling in Industry

Assignment-2 Q8

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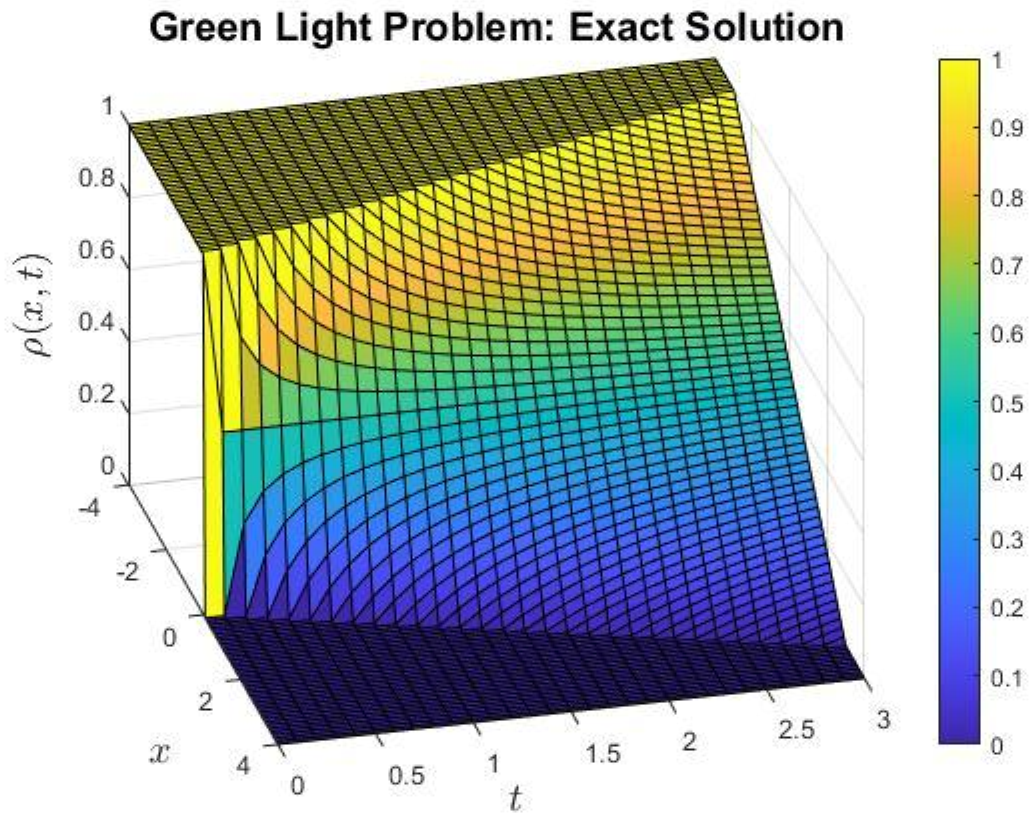
BE19B032

References Used

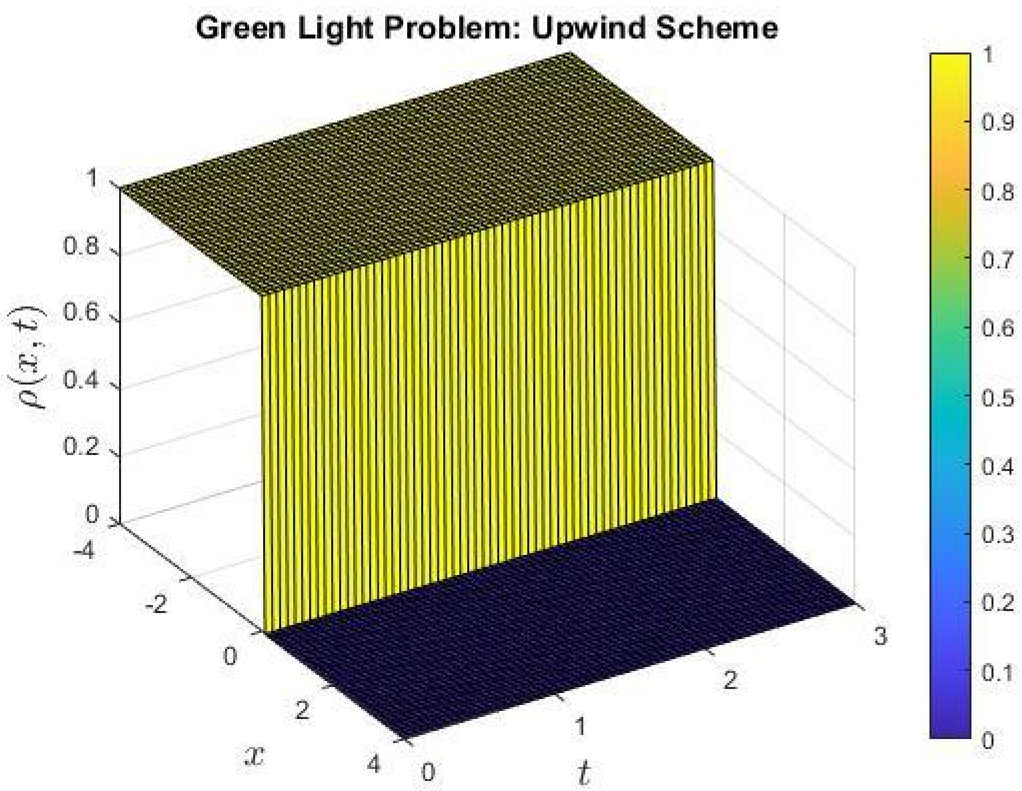
1. Professor Sundar's Classnotes
2. *Mathematical Modelling: A Case Studies Approach Volume 27* by Reinhard Illner, C.Sean Bohun, Samantha McCollum, Thea van Roode **(Chapter-9: Traffic Dynamics: Macroscopic Modelling)**
3. *Principles of Mathematical Modeling Second Edition* by Clive L.Dym **(Chapter-6: Traffic Flow Models)**

Green Light Problem Plots

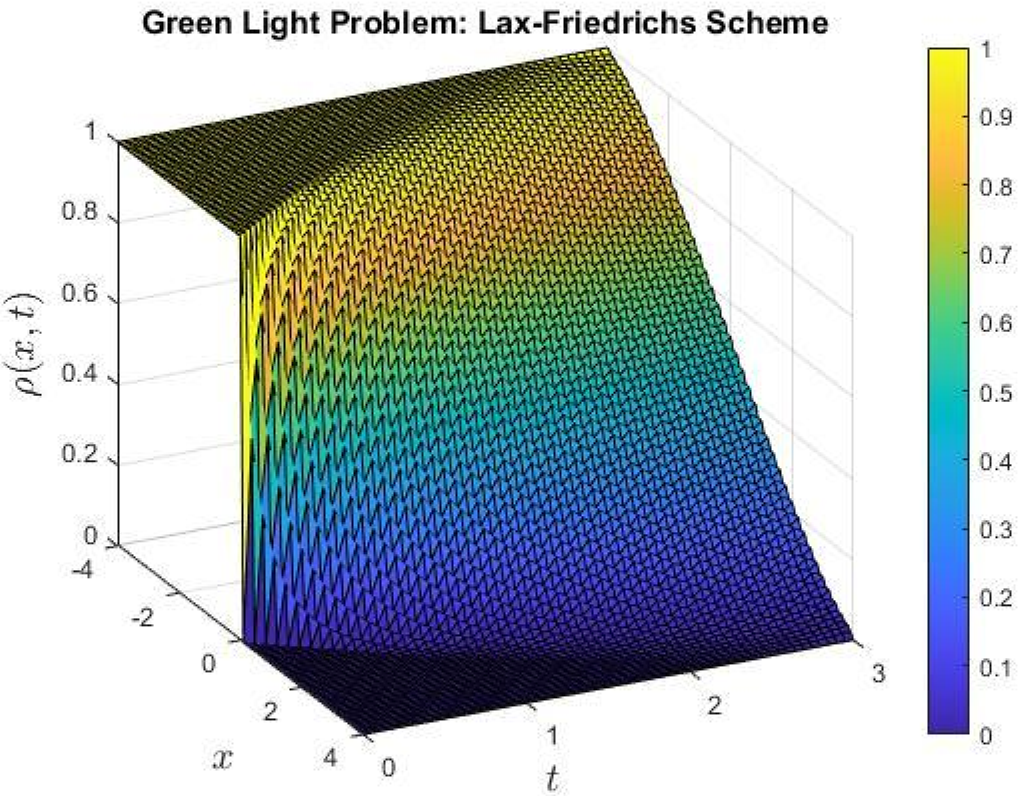
Exact Solution:



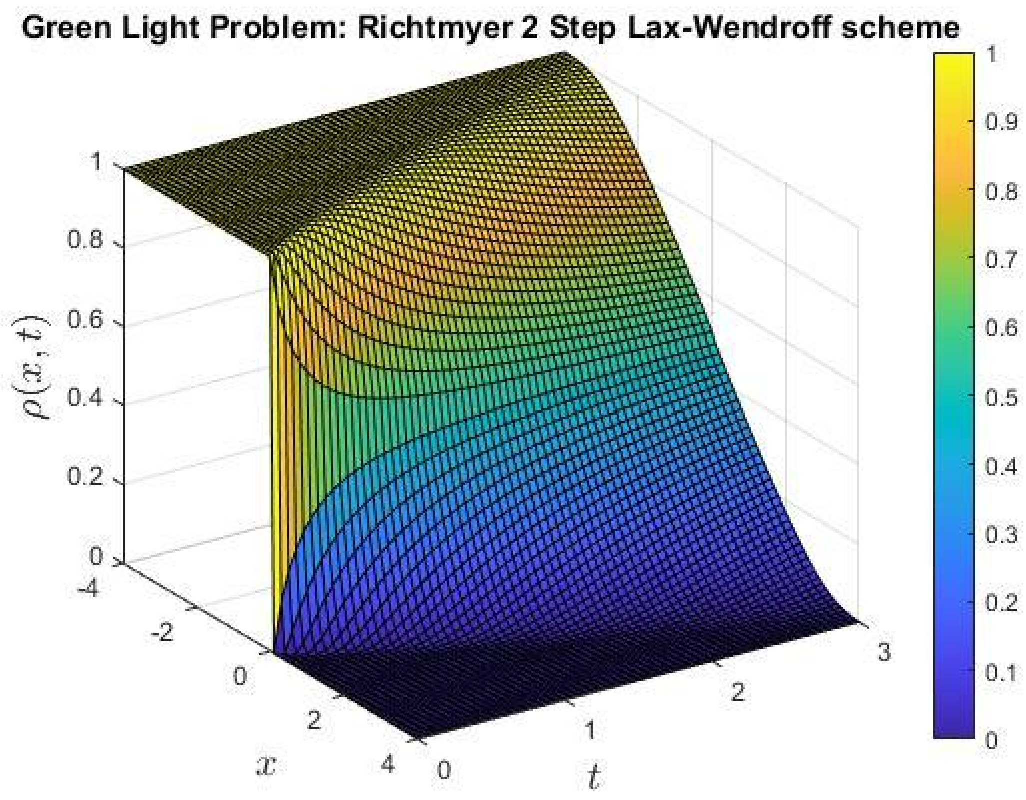
Upwind Scheme Solution:



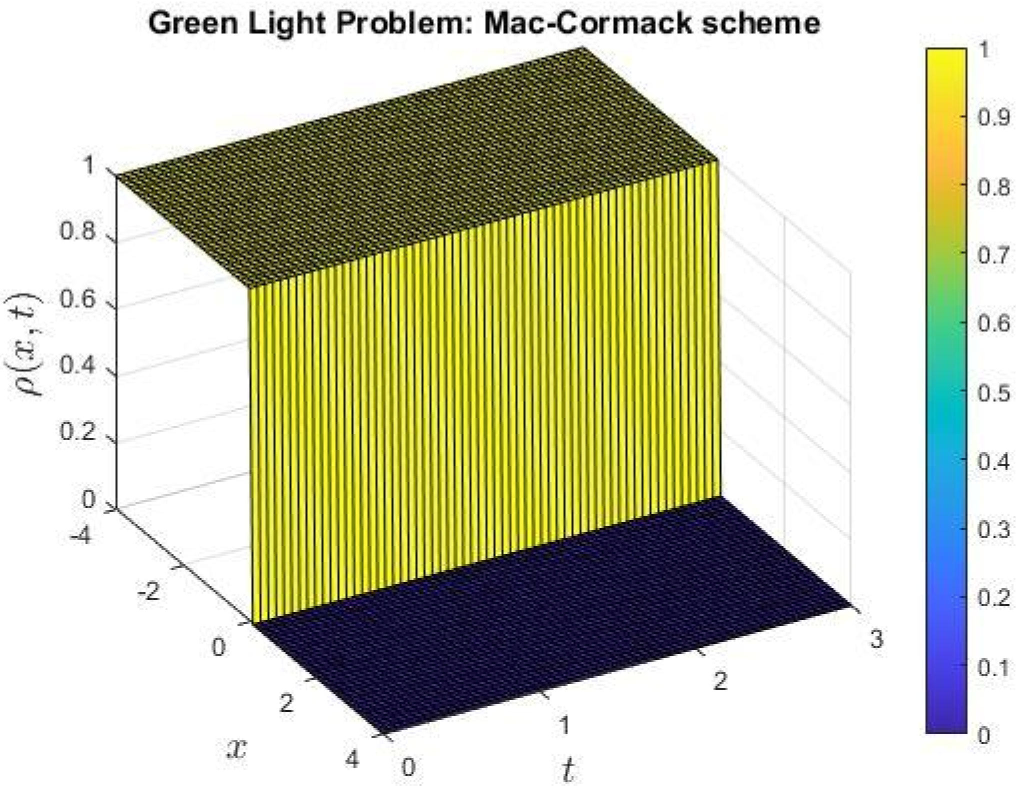
Lax-Friedrichs Scheme Solution:



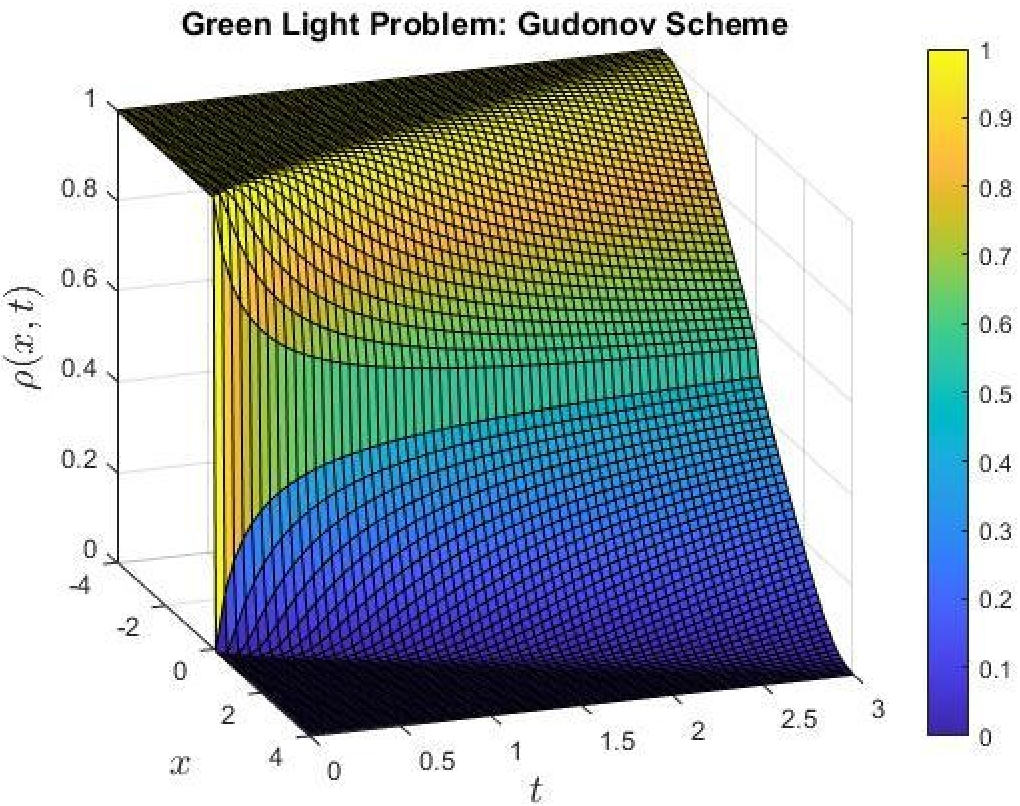
Richtmyer Two-Step Lax-Wendroff Scheme Solution:



Mac-Cormack Scheme Solution:



Gudonov Scheme Solution:



Conclusions:

1) The **Upwind Method and Mac-Cormack Scheme** do not yield the correct solutions. Given that, $j(\rho) = \rho(1 - \rho)$. So, flux is 0 for both $\rho = 0$ and $\rho = 1$. This means that the contibution of $j(\rho)$ is 0 for the first iteration of the time loop. This means that,

   $$\boxed{\rho_i^{n=2} = \rho_i^{n=1} \text{ for the first iteration in schemes 1 and 4}}$$

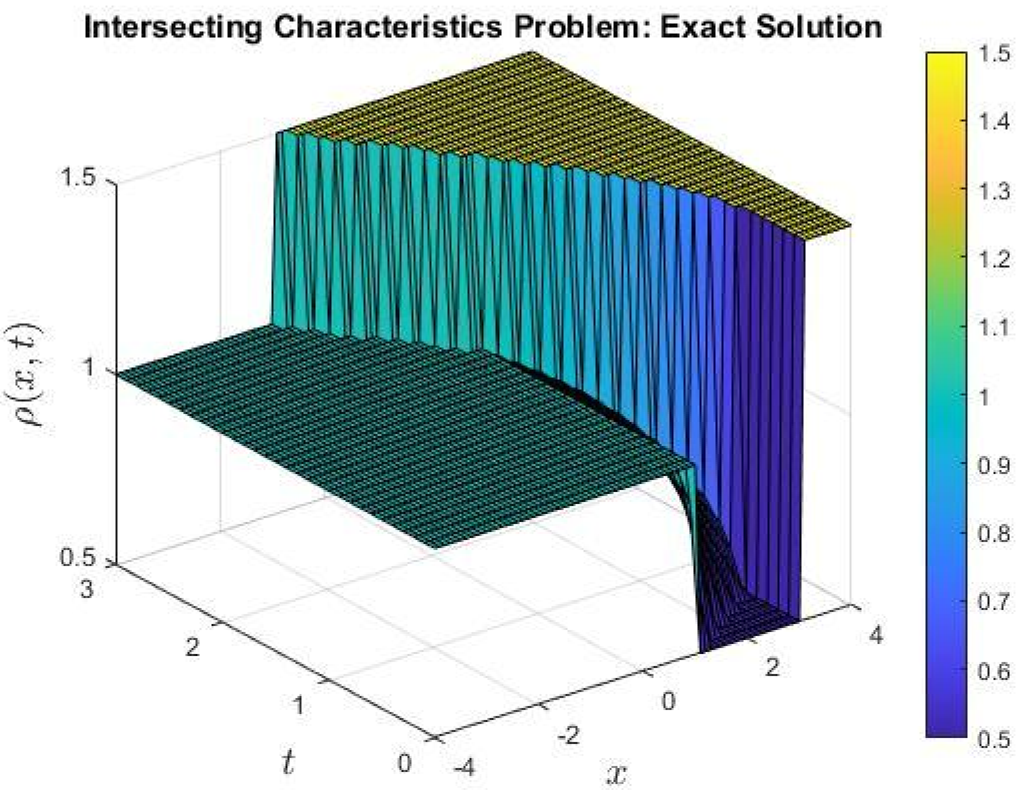
   Meaning that ρ does not change for the net iteration also and so on. Hence, for methods 1 and 4,

   $$\boxed{\rho_i^{n+1} = \rho_i^{n}}$$

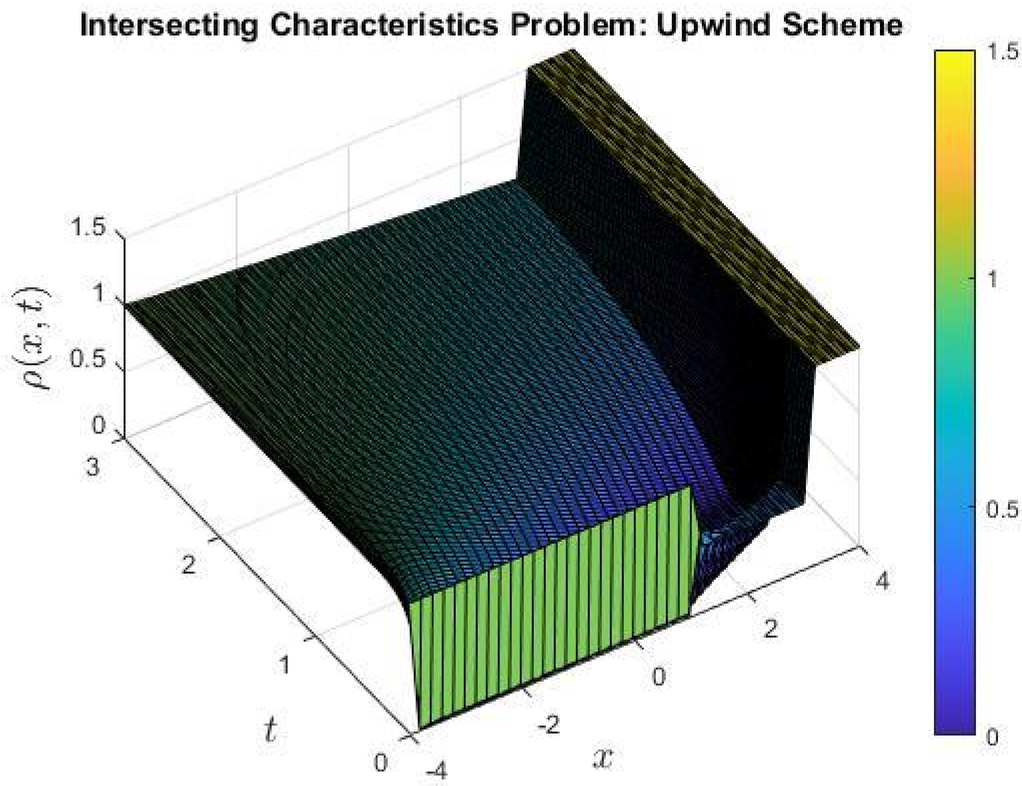
2) The **Richtmyer Two-Step Lax-Wendroff and Gudonov schemes** yield the plots closest to the actual solution. It must be noted that these plots are smoother compared to the actual solution.

Intersecting Characteristics Problem Plots

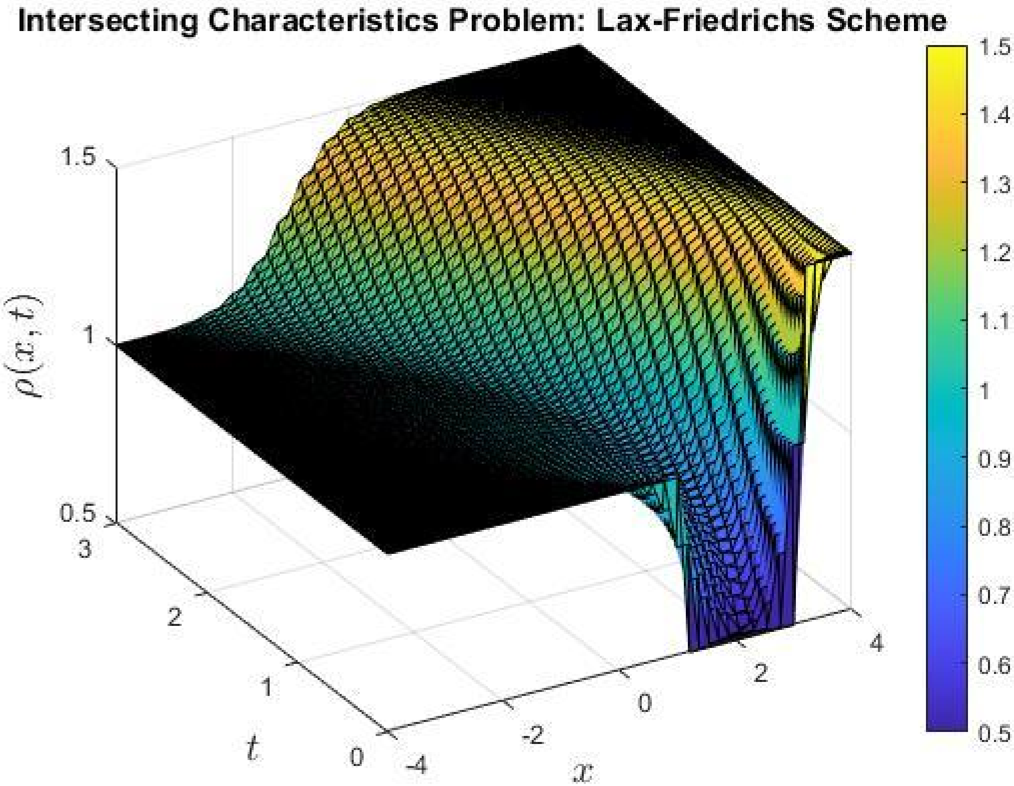
Exact Solution:



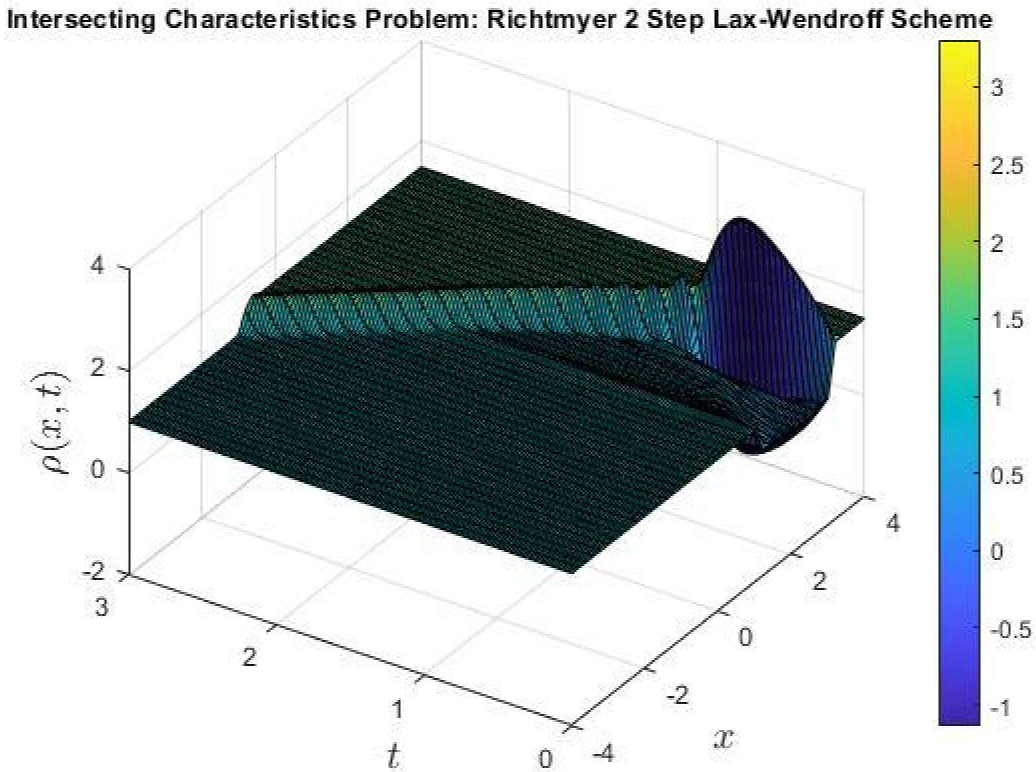
Upwind Scheme Solution:



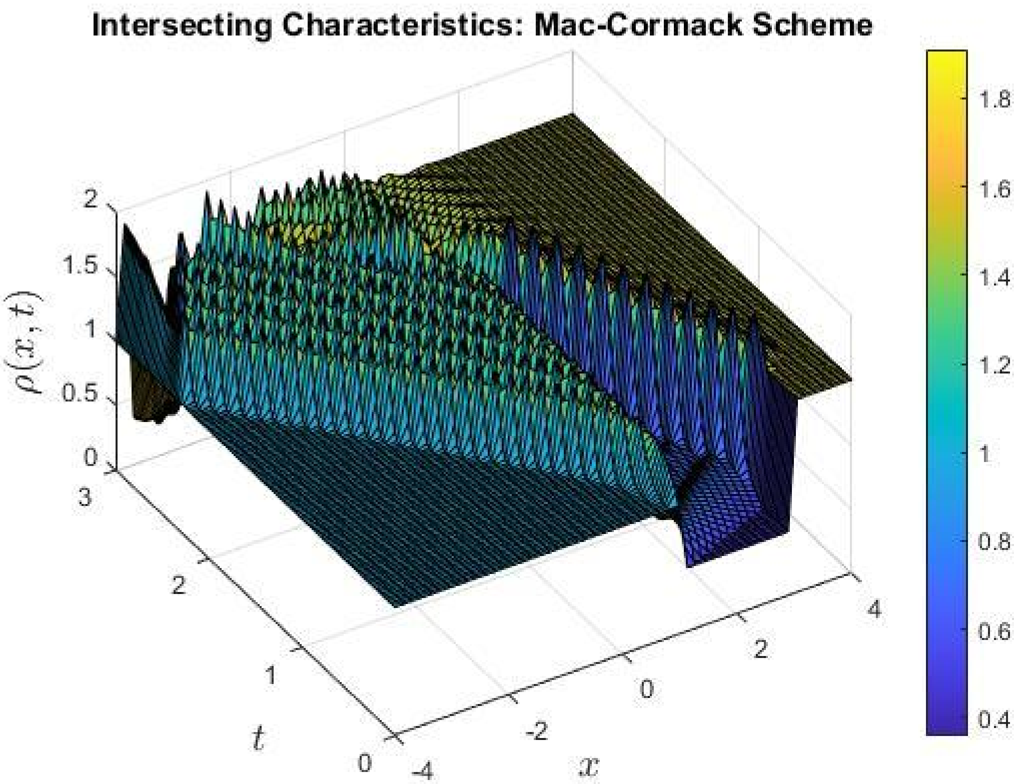
Lax-Friedrichs Scheme Solution:



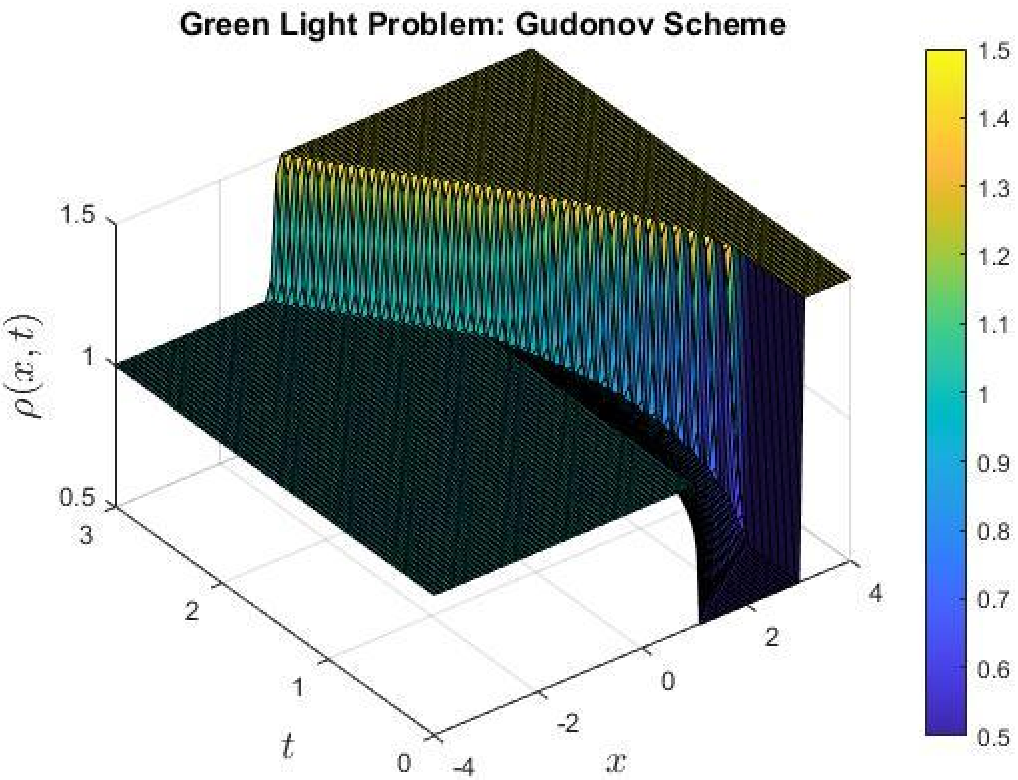
Richtmyer Two-Step Lax-Wendroff Scheme Solution:



Mac-Cormack Scheme Solution:



Gudonov Scheme Solution:



Conclusions:

1) The **Upwind Scheme** does not yield the desired result as the rarefaction wave isn't represented accurately.
2) **Richtmyer Two-Step Lax-Wendroff and Mac-Cormack** schemes lead to plots with regions where the ρ(x, t) function blows up. These regions are close to the shock wave, where the meaning of the partial differntial equation is lost.
3) Even though the **Lax-Friedrichs** scheme yields a smooth distribution, the shockwave isn't aptly represented by its plot.
4) In the plot obtained by using the **Gudonov scheme**, the shockwave is perceptible and the values of ρ(x, t) are within the range set by the exact solution. This is because, this method has a provision to account for the shockwave and the values of ρ are assigned accordingly. Hence, the **Gudonov scheme gives the closest result to the actual solution** among all the schemes.

Which Numerical Scheme is the best?

For both the problems, we see that the Gudonov scheme gives a very good approximation of $\rho(x, t)$. Hence, it is the best numerical scheme amongst the five mentioned in the problem.

Please check the accompanying MATLAB files and plots to see how the aforementioned solutions and plots were generated.