17/11/2021

MA5710 – MMI

Assignment 06

Preamble:

$$\text{Noisy image} \xrightarrow{\text{Input}} \boxed{\text{Diffusion Filter}} \longrightarrow \text{Cleaned image}$$

(Restored Image)

$$\Downarrow$$

I. Linear Diffusion

$$\frac{\partial u}{\partial t} = \nabla\cdot(\nabla u) = \nabla^2 u \quad \text{in } \Omega \times (0,T)$$

$$u(x,0) = u_0(x) \longleftarrow \text{Input image} \Big\} \text{ Initial Condition } \quad t=0$$

$$\frac{\partial u}{\partial \hat{n}} = 0 \text{ on } \partial\Omega \longleftarrow \text{Boundary Condition} \quad t>0$$

$$\Rightarrow \boxed{u(x,t) = K_\sigma * u_0(x)}$$

$$\underbrace{\text{Gaussian Kernel}}_{\text{Gaussian Convolution}}$$

Remember $u(x,t) = u(x,y,t)$: Intensity value.

and σ is the Smoothing parameter.

II Perona-Malik Diffusion

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(c(|\nabla u|^2)\, \nabla u \right) \quad \text{in } \Omega \times (0,T)$$

$$u(x,0) = u_0(x) \quad \text{in } \Omega \qquad t=0$$

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega \qquad t>0$$

where
$$\boxed{c(|\nabla u|^2) = \frac{1}{1+\frac{|\nabla u|^2}{\lambda^2}}}$$

λ is a Contrast Parameter.

III Catte et al Diffusion

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(c(|\nabla u_\sigma|^2)\, \nabla u \right) \quad \text{in } \Omega \times (0,T)$$

$$u(x,0) = u_0(x) \quad \text{in } \Omega \qquad t=0$$

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega \qquad t>0$$

where $c(|\nabla u_\sigma|^2)$ as given in Model II.

u_σ : Linear Diffusion

Question Input

(i) Use MATLAB for coding and appropriate MATLAB Tool Box.

(ii) Take the standard images given in MATLAB (for example, Cameraman image, Lena Image, vegitable Color image, ...)

(iii) Add noise Randomly (Random Noise) and treat it as Input Image.

(iv) For Color images do the analysis for Red, Blue and Green (RBG).

Question 1

$\boxed{(1)}$ Implement Linear Diffusion; $\boxed{(2)}$ Show the image reconstruction taking different σ; $\boxed{(3)}$ Compute PSNR; $\boxed{(4)}$ Compare the PSNR for $T = \frac{\sigma^2}{2}$ with $T > \frac{\sigma^2}{2}$ for a given σ; $\boxed{(5)}$ Check all the invariant properties

Question 2

(1) Implement Perona-Malik Diffusion taking different λ; (2) Compute PSNR and Compare with Question 1; (3) How do you compute λ?; (4) Demonstrate the result with the computed λ; (5) What is the Best Stopping Criteria (numerically)?

Question 3

(1) Implement Catte et al Diffusion taking different combinations of σ and λ;

(2) Compute PSNR and Compare with Question 1 and Question 2;

(3) What is the Best Stopping Criteria? (numerically).