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MA5710 MMI Assignment - 5

(1) a) ~~N~~ Without fishing

The nofishing model is given as:

$N' = N(a - bP) = f(N, P)$

$P' = P(-d + cN) = g(N, P)$

Let, $\bar{X} = \begin{pmatrix} N - N_0 \\ P - P_0 \end{pmatrix}$ $\quad \bar{X}' = \begin{pmatrix} N' \\ P' \end{pmatrix}$

(Based on shifting of origin & rescaled model)

Here, $(N_0, P_0) = \left(\frac{d}{c}, \frac{a}{b}\right)$.

We know that, for linearized models

$$\boxed{\bar{X}' = \begin{pmatrix} \left(\frac{\partial f}{\partial N}\right)_{N_0, P_0} & \left(\frac{\partial f}{\partial P}\right)_{N_0, P_0} \\ \left(\frac{\partial g}{\partial N}\right)_{N_0, P_0} & \left(\frac{\partial g}{\partial P}\right)_{N_0, P_0} \end{pmatrix} \bar{X}} \quad \text{--- (i)}$$

Now we calculate all the partial derivatives from eq^n (i)

$$\left(\frac{\partial f}{\partial N}\right)_{N_0, P_0} = (a - bP)\Big|_{N_0 = \frac{d}{c}, P_0 = a/b} = 0$$

$$\left(\frac{\partial f}{\partial P}\right)_{N_0, P_0} = -bN\Big|_{N_0 = \frac{d}{c}, P_0 = a/b} = -\frac{bd}{c}$$

$$\left(\frac{\partial g}{\partial N}\right)_{N_0, P_0} = cP \Big|_{N_0 = \frac{d}{c}, P_0 = \frac{a}{b}} = \frac{ca}{b}$$

$$\left(\frac{\partial g}{\partial P}\right)_{N_0, P_0} = (d + cN) \Big|_{N_0 = \frac{d}{c}, P_0 = \frac{a}{b}} = 0$$

Substituting these values back in eqn ①, we get the Linearized model of no fighing case as:

$$\boxed{\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & -\frac{bd}{c} \\ \frac{ca}{b} & 0 \end{pmatrix} \begin{pmatrix} N - d/c \\ P - a/b \end{pmatrix}}$$

For $a = 4$, $b = 2$, $c = 3/2$ & $d = 3$, we get,

$$\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} N - 8/3 \\ P - 2 \end{pmatrix}$$

$\Rightarrow N' = -4(P-2)$ & $P' = 3N - 8$

Some terms have been ignored while plotting because they don't make ~~any~~ lot of difference.

(b) With fishing:

$$\cancel{N' = N(a-\delta) - bP}$$

$$N' = N(a - bP - \delta) = N((a-\delta) - bP) = F(N, P)$$

$$P' = P(-d + (N - \delta) = P(-(d+\delta) + cN) = G(N, P)$$

Let, $\bar{X} = \begin{pmatrix} N - N_0 \\ P - P_0 \end{pmatrix}$ & $\bar{X}' = \begin{pmatrix} N' \\ P' \end{pmatrix}$

where, $N_0 = \frac{d+\delta}{c}$, $P_0 = \frac{a-\delta}{b}$

For linearized models, we find useful partial derivatives

$$\left(\frac{\partial F}{\partial N}\right)_{N_0,P_0} = (a-\delta) - bP\Big|_{N_0,P_0} = 0$$

$$\left(\frac{\partial F}{\partial P}\right)_{N_0,P_0} = -bN\Big|_{N_0,P_0} = -\frac{b(d+\delta)}{c}$$

$$\left(\frac{\partial G}{\partial N}\right)_{N_0,P_0} = cP\Big|_{N_0,P_0} = \frac{c(a-\delta)}{b}$$

$$\left(\frac{\partial G}{\partial P}\right)_{N_0,P_0} = (cN-d-\delta)\Big|_{N_0,P_0} = 0$$

Substituting these values in eqn (i), we get linearized model for fishing case as:

$$\boxed{\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & -\frac{b(d+\delta)}{c} \\ \frac{c(a-\delta)}{b} & 0 \end{pmatrix} \begin{pmatrix} N - \frac{(d+\delta)}{c} \\ P - \frac{(a-\delta)}{b} \end{pmatrix}} \quad \text{(ii)}$$

For $a=4$, $b=2$, $c=3/2$ & $d=3$, we get,

$$\cancel{\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & -4/3(3+\delta) \\ 3/4(a-\delta) \end{pmatrix}}$$

$$\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & \frac{-4}{3}(3+\delta) \\ \frac{3}{4}(4-\delta) & 0 \end{pmatrix} \begin{pmatrix} N - \frac{2}{3}(3+\delta) \\ P - \frac{(4-\delta)}{2} \end{pmatrix}$$

(5) Condⁿ: Selection fishing such that only prey population is fished at the rate $\delta > 0$.

So, the model becomes

$$\boxed{\begin{aligned} N' &= N(a-\delta-bP) \\ P' &= P(-d+cN) \end{aligned}}$$

Using eqⁿ (ii) from Q4, we get, the linearized model with selective fishing as:

$$\boxed{\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} 0 & \frac{-bd}{c} \\ \frac{c(a-\delta)}{b} & 0 \end{pmatrix} \begin{pmatrix} N - \frac{d}{c} \\ P - \frac{a-\delta}{b} \end{pmatrix}}$$

Observations & Comments:

(i) The eqᵐ point is at $(N_0, P_0) = \left(\frac{d}{c}, \frac{a-\delta}{b}\right)$ as seen from the phase diagram.

(ii) The predator population reaches lower levels and prey population reaches higher levels than in the case of no fishing.

(iii) Phase portrait & trajectory in selective fishing situation is similar to normal fishing situation as they are still oscillatory in nature.

(a)(b) As shown in class, the revised model of Lotka - Volterra system is given as:

$$\boxed{\begin{aligned} N' &= N(a - bP - \epsilon aN) = f(N,P) \\ P' &= P(-d + cN) = g(N,P) \end{aligned}}$$

Equilibrium Point:

$$\cancel{(N_0, P_0) = \left(\frac{d}{c}, \frac{d}{c}\right)}$$

$$(N_0, P_0) = \left(\frac{d}{c}, \frac{a}{b}\left(1 - \frac{\epsilon d}{c}\right)\right)$$

Linearization is done as follows:

$$\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial N} & \frac{\partial f}{\partial P} \\ \frac{\partial g}{\partial N} & \frac{\partial g}{\partial P} \end{pmatrix}_{N_0, P_0} \begin{pmatrix} N - N_0 \\ P - P_0 \end{pmatrix}.$$

$$\left.\frac{\partial f}{\partial N}\right|_{N_0,P_0} = (a - bP - 2\epsilon aN)\Big|_{N_0,P_0} = -\frac{\epsilon ad}{c}$$

$$\left.\frac{\partial f}{\partial P}\right|_{N_0,P_0} = -bN\Big|_{N_0,P_0} = \frac{-bd}{c}$$

$$\left.\frac{\partial g}{\partial N}\right|_{N_0,P_0} = cP\Big|_{N_0,P_0} = \frac{a}{b}(c - \epsilon d)$$

$$\left.\frac{\partial g}{\partial P}\right|_{N_0,P_0} = -d + cN\Big|_{N_0,P_0} = 0.$$

$$\begin{pmatrix} N' \\ P' \end{pmatrix} = \begin{pmatrix} -\frac{\epsilon ad}{c} & -\frac{bd}{c} \\ \frac{a}{b}(c - \epsilon d) & 0 \end{pmatrix} \begin{pmatrix} N - N_0 \\ P - P_0 \end{pmatrix}$$

$a = 4, b = 2, c = 3/2, d = 3$

Let Matrix,

$$M = \begin{pmatrix} -\frac{\epsilon a d}{c} & -\frac{b d}{c} \\ \frac{a}{b}(c - \epsilon d) & 0 \end{pmatrix}$$

The eigenvalues of M will give us a good idea about the quality of equilibrium.

$$|M - \lambda I| = 0$$

$$\Rightarrow \left| \begin{pmatrix} -\frac{\epsilon a d}{c} - \lambda & -\frac{b d}{c} \\ \frac{a}{b}(c - \epsilon d) & -\lambda \end{pmatrix} \right| = 0$$

$$\Rightarrow \lambda^2 + \lambda \frac{\epsilon a d}{c} + \frac{a d}{c}(c - \epsilon d) = 0$$

$$\lambda_{1,2} = -\frac{\epsilon a d}{2c} \pm \frac{1}{2}\sqrt{\left(\frac{\epsilon a d}{c}\right)^2 - 4\frac{a d}{c}(c - \epsilon d)}$$

$$= -\frac{\epsilon a d}{2c} \pm \sqrt{\left(\frac{\epsilon a d}{2c}\right)^2 - \frac{a d}{c}(c - \epsilon d)}$$

The real part of λ is always < 0
We need to see how ϵ values affect the term under the square root (D)

(i) $\epsilon = 0.1$

$$D = \left(\frac{0.1(4)(3)}{3}\right)^2 - \frac{(4)(3)}{(3/2)}\left(3/2 - 0.3\right)$$

$\Rightarrow$ $D = 0.16 - (4)(3)(2)\, 8(1.2)$

$\Rightarrow$ $D = -9.44 < 0$.

$\because$ $\lambda_{1,2}$ have $Re(\lambda) < 0$ & are complex conjugates

$\therefore$ The system is stable (spiral)

This is in accordance to the plot.

(ii) $\epsilon = 0.01$

$$D = \left(\frac{0.01 (4)(3)}{(3)} \right)^2 - \frac{(4)(3)}{(3/2)} \left(\frac{3}{2} - 3(0.01) \right).$$

$= 0.0016 - 8(1.47)$

$\doteq -11.744 < 0$

$\because$ $\lambda_{1,2}$ have $Re(\lambda) < 0$ & are complex conjugates

$\therefore$ The system is stable (spiral)

This is in accordance with the plot.

(iii) For $\epsilon = 0.001$ & $\epsilon = 0.0001$,

It is seen that the magnitude of D keeps increasing & $Re(\lambda)$ keeps decreasing (almost tending to 0) as the order of ϵ is decreased.
For such small values of $Re(\lambda)$, the system will approximately have eigenvalues of the form
$\lambda = \pm i\alpha$
& hence, its solutions in the form of concentric ellipses
These results are in accordance with the plot obtained.
For $\epsilon = 0.0001$, there is some degree of oscillating in the system

Summary:

System is stable for $\epsilon = 0.1, 0.01$
System is unstable for $\epsilon = 0.001, 0.0001$
for eqm pts $\left(\frac{d}{c}, \frac{a}{b}\left(1 - \frac{\epsilon d}{c}\right)\right)$
Eqm pt. $(0, 0)$ is trivial as for no population, the system won't exist.

No.

For small perturbations, the qualitative behaviour of the trajectories of the system are affected, that is for small perturbations ϵ at eqm points, the type of equilibrium changes. Hence the system is structurally unstable.

Reference: Structural Stability | Wikipedia.

(7) Given that,
N_2 preys on N_1
N_3 preys on both N_1 & N_2
N_4 has symbiotic relationship with N_2 & is neutral w.r.t others.

The system can be represented by the equations:

$$N_1' = a_1 N_1 \left(1 - \frac{N_1}{K}\right) - b_1 N_1 N_2 - b_2 N_1 N_3 \quad \text{(i)}$$

K = carrying capacity of N_1

NaP)

$$\boxed{N_2' = c_1 N_1 N_2 - b_3 N_2 N_3 - a_2 N_2 + d_1 N_2 N_4}$$ —— (ii)

$$\boxed{N_3' = c_2 N_1 N_3 + c_4 N_3 N_2 - a_3 N_3}$$ —— (iii)

$$\boxed{N_4' = d_2 N_2 N_4 - a_4 N_4}$$ —— (iv)

For eqm pt.s,

$N_1' = N_2' = N_3' = N_4' = 0$

$\boxed{\text{Soln (I)} \quad N_1 = N_2 = N_3 = N_4 = 0}$

(a) $N_1 = 0$

$N_2' = 0 \Rightarrow N_2 (d_1 N_4 - b_3 N_3 - a_2) = 0$

$N_3' = 0 \Rightarrow N_3 (c_4 N_2 - a_3) = 0$

$N_4' = 0 \Rightarrow N_4 (d_2 N_2 - a_4) = 0.$

Case-1

$N_2 = \dfrac{a_3}{c_4}$

$\Rightarrow N_4 = 0$

$\Rightarrow N_3 = \dfrac{-a_2}{b_3}$

(Not Possible)

Case-2

$N_2 = \dfrac{a_4}{d_2}$

$\Rightarrow N_3 = 0$

$\Rightarrow N_4 = \dfrac{a_2}{d_1}$ ✓

$$\boxed{\text{Soln (II)}: \begin{array}{cccc} N_1 & N_2 & N_3 & N_4 \\ \end{array} \left(0, \frac{a_4}{d_2}, 0, \frac{a_2}{d_1}\right)}$$

If $N_2 = \dfrac{a_3}{c_4} = \dfrac{a_4}{d_2}$,

then $N_1 = 0$ & $d_1 N_4 - b_3 N_3 = a_2$ } Soln (III)

(b) Let $N_2 = 0$

$$N_1' = 0 \Rightarrow a_1 N_1 \left(1 - \frac{N_1}{K}\right) - b_2 N_1 N_3 = 0.$$

$$\Rightarrow N_1 \left(a_1 \left(1 - \frac{N_1}{K}\right) - b_2 N_3\right) = 0$$

$$N_3' = 0 \Rightarrow N_3 (c_2 N_1 - a_3) = 0$$

$$N_4' = 0 \Rightarrow a_4 N_4 = 0$$

$$\Rightarrow N_4 = 0.$$

Case: $N_3 = 0$ or Case: $N_1 = 0$

$$N_1 a_1 \left(1 - \frac{N_1}{K}\right) = 0$$

$$\Rightarrow N_1 = K$$

$$a_1 - b_2 N_3 = 0$$

$$\Rightarrow N_3 = \frac{a_1}{b_2}.$$

$$\boxed{\text{Sol}^n \text{ IV}: (N_1, N_2, N_3, N_4) = \left(K, 0, \frac{a_1}{b_2}, 0\right).}$$

(c) Let $N_3 = 0$

$$N_1' = a_1 N_1 \left(1 - \frac{N_1}{K}\right) - b_1 N_1 N_2 = 0$$

$$N_2' = c_1 N_1 N_2 - a_2 N_2 + d_1 N_2 N_4 = 0$$

$$N_4' = N_4 (d_2 N_2 - a_4) = 0. \Rightarrow 2 \text{ sol}^n\text{s } N_4 = 0 \text{ or } N_2 = \frac{a_4}{d_2}$$

$$\boxed{\text{Sol}^n \text{ V}: (N_1, N_2, N_3, N_4) = \left(\frac{a_2}{c_1}, \frac{a_1}{b_1}\left(1 - \frac{a_2}{c_1 K}\right), 0, 0\right)}$$

Q6

$N_2 = \frac{a_4}{d_2}$

$N_1' = 0 \Rightarrow N_1\left[a_1\left(1-\frac{N_1}{K}\right) - b_1 N_2\right] = 0.$

$\Rightarrow a_1\left(1-\frac{N_1}{K}\right) - \frac{b_1 a_4}{d_2} = 0.$

$\Rightarrow 1 - \frac{N_1}{K} = \frac{b_1 a_4}{a_1 d_2}$

$\Rightarrow N_1 = K\left(1 - \frac{b_1 a_4}{a_1 d_2}\right).$

$\Rightarrow N_1 = \frac{K}{a_1}\left(a_1 - \frac{b_1 a_4}{d_2}\right).$

$N_2' = 0$

$\Rightarrow \frac{c_1 K}{a_1}\left(a_1 - \frac{b_1 a_4}{d_2}\right)\frac{a_4}{d_2} - \frac{a_2 a_4}{d_2} + \frac{d_1 a_4}{d_2} N_4 = 0.$

$\Rightarrow N_4 = \frac{1}{d_1}\left(a_2 - \frac{c_1 K}{a_1}\left(a_1 - b_1 N_2\right)\right).$

$$\boxed{\begin{aligned} &\text{Soln (V)}: N_1 = \frac{K_1}{a_1}\left(a_1 - \frac{b_1 a_4}{d_2}\right), N_2 = \frac{a_4}{d_2},\\ &N_3 = 0, N_4 = \frac{1}{d_1}\left(a_2 - \frac{c_1 K}{a_1}\left(a_1 - b_1\frac{a_4}{d_2}\right)\right)\end{aligned}}$$

Note: I had discussed Q.1 with 2 of my classmates for better understanding

(ii) let $N_4 = 0$

$N_1' = 0 \Rightarrow N_1\left(r\left(1 - \frac{N_1}{K}\right) - b_1 N_2 - b_2 N_3\right) = 0.$

$N_2' = 0 \Rightarrow N_2(c_1 N_1 - b_3 N_3 - a_2) = 0.$

$N_3' = 0 \Rightarrow N_3(c_2 N_1 + c_4 N_2 - a_3) = 0.$

$$\boxed{\text{Sol}^n\ (\text{VII}): \quad N_1 = \frac{a_3}{c_2},\ N_2 = 0,\ N_3 = \frac{a_1}{b_2}\left(1 - \frac{a_3}{c_2 K}\right), \quad N_4 = 0}$$

Sol^m (VIII)

The same eq^m points & corresponding eigenvalue are found in the attached MATLAB code.
Since the values of constants pertaining to this model weren't provided, diff. values can be input to find comment on the quality of equilibrium.

(3) Given that,

$x' = \sigma(y - x)$

$y' = rx - y - xz$

$z' = xy - bz$

(a) At eq^m pt.s,

$x' = y' = z' = 0.$

$x' = 0 \Rightarrow \sigma(y - x) = 0 \Rightarrow \boxed{y = x.}$

let this value of x be x^*

$y' = 0 \Rightarrow rx - y - xz = 0$

$\Rightarrow rx - x - xz = 0$

$\Rightarrow x^*(r - 1 - z) = 0 \Rightarrow \boxed{z = r - 1}$

$z' = 0$

$\Rightarrow xy - bz = 0$

$\Rightarrow x^2 = bz$

$\Rightarrow \boxed{x^2 = b(r-1)}$

→ ## Note that pt.s A & B aren't eqm pt.s for $r < 1$.

Solns:

A: $(x, y, z) = (\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1)$

B: $(x, y, z) = (-\sqrt{b(r-1)}, -\sqrt{b(r-1)}, r-1)$

C: $(x, y, z) = (0, 0, 0)$

(b) In code

(c) For pt. C, $(x_0, y_0, z_0) = (0, 0, 0)$

The linearized model is given as:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} -10 & 10 & 0 \\ r & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}}_{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Eigenvalues of Matrix M:

$$\left| \begin{pmatrix} -10-\lambda & 10 & 0 \\ r & -1-\lambda & 0 \\ 0 & 0 & -8/3-\lambda \end{pmatrix} \right| = 0$$

$$\Rightarrow -\left(\frac{8}{3}+\lambda\right)\left[(10+\lambda)(1+\lambda) - 10r\right] = 0$$

$$\lambda_1 = -\frac{8}{3}, \quad \lambda^2 + 11\lambda + 10(1-r) = 0$$

So, we get,

$$\lambda_1 = -\frac{8}{3}, \quad \lambda_{2,3} = \frac{-11 \pm \sqrt{121 - 40(1-r)}}{2}$$

~~→ For $r < 1$, all eigenvalues: λ_1, λ_2 & λ_3 are negative & real. So, the system is stable for $r < 1$~~

~~→ For $r > 1$, λ_1 & λ_3 are always negative & λ_2 is always > 0~~

→ For $-\frac{81}{40} \le r \le 1$, all eigenvalues are: λ_1, λ_2 & λ_3 are negative & real. So, the system is stable.

→ For $r < -\frac{81}{40}$, $\lambda_1 < 0$, λ_2 & λ_3 are complex conjugate with $Re(\lambda) < 0$. So, the system is stable.

→ For $r > 1$, λ_1 & λ_3 are always negative & λ_2 is always > 0. So, the equilibrium of the system is saddle-like.

~~For eq^m pt: A: $(x, y, z) = (\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1)$~~

For eq^m pt. A, $(x_0, y_0, z_0) = (\sqrt{b(r-1)}, \sqrt{b(r-1)}, r-1)$

The linearized model is: The linearization is

$$\boxed{\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 \\ 1 & -1 & \sqrt{8/3(r-1)} \\ \sqrt{8/3(r-1)} & \sqrt{8/3(r-1)} & -8/3 \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}} \quad (A)$$

Linearization:

$x' = r(y - x)$

$y' = rx - y - xz$

$z' = xy - \frac{8}{3} z$

The linearization process is similar for eq^m pt. B also.

Say, $X = x - \sqrt{b(r-1)}$

$Y = y - \sqrt{b(r-1)}$

$Z = z - (r-1)$

X', Y', Z' become:

$X' = 10\left(Y + \sqrt{b(r-1)} - X - \sqrt{b(r-1)}\right)$

$\Rightarrow \boxed{X' = 10(Y - X)}$

$Y' = r\left(X + \sqrt{b(r-1)}\right) - \left(Y + \sqrt{b(r-1)}\right) - \left(X + \sqrt{b(r-1)}\right)(Z + r - 1)$

$\Rightarrow Y' = rX + r\sqrt{b(r-1)} - Y - \sqrt{b(r-1)} - XZ - X(r-1) - Z\sqrt{b(r-1)} - \sqrt{b(r-1)}(r-1)$

$\Rightarrow Y' = rX - Y - XZ - X(r-1) - Z\sqrt{b(r-1)}$

$\Rightarrow \boxed{Y' = -Y + X + Z\sqrt{\frac{8}{3}(r-1)}}$ (Ignoring XZ term as it is negligible)

$Z' = (X + \sqrt{b(r-1)})(Y + \sqrt{b(r-1)}) - \frac{8}{3}(Z + r - 1)$

$= XY + X\sqrt{b(r-1)} + Y(\sqrt{b(r-1)}) + b(r-1) - \frac{8}{3}Z - \frac{8}{3}(r-1)$

$$\boxed{Z' = (X+Y)(\sqrt{b(r-1)}) - \frac{8}{3}Z}$$

(Ignoring XY term as it is negligible)

X', Y', Z' have been derived ~~along~~ by ignoring higher degree terms & constants (as they don't lead to a huge difference)

We get the linearized model as:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 \\ 1 & -1 & \sqrt{8/3(r-1)} \\ \sqrt{8/3(r-1)} & \sqrt{8/3(r-1)} & -8/3 \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}.$$

Using Mathematica for calculation of eigen values, we get

$$p(\lambda) = 3\lambda^3 + 41\lambda^2 + 8(r+10)\lambda + 160(r-1)$$

→ $\lambda_1 \in R$ for $1 < r < 28$ & $\lambda_1 < 0$.

→ We get 2 relevant values $r_1 \approx 1.34$ & $r_2 \approx 24.74$

→ for $1 < r < r_1$
~~& all~~ $\lambda_1, \lambda_2, \lambda_3 \in R$ & $\lambda_1, \lambda_2, \lambda_3 < 0$.
So, the system is stable.
However, this region is small & can be neglected.

→ for $r_1 < r < r_2$
$\lambda_1 \in \mathbb{R}$ & $\lambda_1 < 0$, λ_2 & λ_3 are complex conjugates with $Re(\lambda_2)$ & $Re(\lambda_3) < 0$. So, the system is stable (spiral)

→ for $r_2 < r < 28$
$\lambda_1 \in \mathbb{R}$ & $\lambda_1 < 0$, λ_2 & λ_3 are complex conjugate but $Re(\lambda_2)$ & $Re(\lambda_3) > 0$. So, the system is unstable.

For eqm pt. B, $(x_0, y_0, z_0) = (-\sqrt{b(r-1)}, -\sqrt{b(r-1)}, r-1)$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -10 & 10 & 0 \\ 1 & -1 & \sqrt{8/3(r-1)} \\ -\sqrt{8/3(r-1)} & -\sqrt{8/3(r-1)} & -8/3 \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}.$$

For pt. B, the results for different r values are similar to those for pt. A.
This is all in agreement with the phase diagram in Q 8 (ii).