MAS710 – MM I

28/10/2021

Assigment 05

Preamble:

Lotka–Volterra Model for 2-Species Predator–Prey System

$$\Rightarrow \left\| \begin{aligned} N' &= N(a - bP) \\ P' &= P(-d + cN) \end{aligned} \right. \xrightarrow{\text{Re Scaling}} \boxed{\begin{aligned} u' &= u(1-v) \\ v' &= \alpha v(u-1) \end{aligned}}$$

$u := \frac{c}{d} N$, $v = \frac{b}{a} P$, $\tau = at$, $\alpha = \frac{d}{a}$.

Without Fishing, i.e. Without external interferance.

With Fishing with Common δ:

$$\Rightarrow \left\| \begin{aligned} N' &= N(a - bP - \delta) \\ P' &= P(cN - d - \delta) \end{aligned} \right\|$$

Catch is $\delta(N_e + P_e)$.

$\Rightarrow$ Equilibrium Point by Setting $f = 0$, $g = 0$ where f and g are Right Side functions.

Quality of the Equilibrium Point by Linearizing the Model around the eq. pt. and Shifting eq. pt. to $(0,0)$.

Classification of Equilibrium Point: $\Rightarrow$ Eigenvalues Complex.

Stable spiral, unstable spiral, Centre ... Stable Node, unstable Node and Saddle point. $\Downarrow$ Eigenvalues are Real.

Structurally Stable System (Modified Lotka-Volterra Model)

$$\Rightarrow \left\| \begin{aligned} N' &= N(a - bP - \epsilon a N) \\ P' &= P(-d + cN), \quad \epsilon \ll 1 \left(\ll \frac{c}{d}\right). \end{aligned} \right.$$

$\boxed{\text{Structured Stability:}}$

A non-linear system $\boxed{x' = f(x,y),\ y' = g(x,y)}$ is said to be structurally stable in a nhd of a steady state (x_e, y_e) if for bounded and smooth functions $s(x,y)$ and $w(x,y)$ and sufficiently small $\epsilon > 0$ the perturbed system

$$\Rightarrow \| \ x' = f(x,y) + \epsilon s(x,y), \quad y' = g(x,y) + \epsilon w(x,y)$$

will posseses a steady state $(\tilde{x}_e, \tilde{y}_e)$ near the steady state (x_e, y_e) and the type of the equilibrium is unchanged.

Question 1

Using MATLAB ODE65 Solver, show the trajectories of $N(t)$ and $P(t)$ for the Lotka-Volterra System (without fishing) taking $(N(0), P(0)) = (0,0) \rightarrow$ Initial Condition and setting the parameter $a = 4$, $b = 2$, $c = 3/2$ and $d = 3$.

Question 2

Repeat the Question 1 for with Fishing model taking $\delta = 0.2$.

Question 3

Plot the Phase Potrait for Question 1 and Question 2.

Question 4

Linearize the Lotka-Volterra Model (with fishing and without fishing) and apply the ODE solver for the Linearized Models. Demonstrate the trajectories (solution) and the Phase Potrait.

Question 5

Suppose the predator-prey species described by the Lotka-Volterra model are subject to selective fishing such that only the prey population is fished at the rate $\delta > 0$. Describe the effect of this fishing on the phase diagram.

Question 6

In the Revised Model of Lotka-Volterra System, introducing the Logistic Growth Law by incorporating Carrying capacity,

⇒ (i) take $\underline{\underline{\epsilon}} = 10^{-1}, 10^{-2}, 10^{-3}$ and 10^{-4}
(0.1) (0.001) (0.01), (0.0001)

with the data given in Question 1

Plot the trajectories of $N(t)$ and $P(t)$ and the Phase diagram.

⇒ (ii) Linearize the model about the equilibrium points and demonstrate the quality of equilibrium point and thus check whether the system has Structural Stability.

Question 7

Consider FOUR Species System: N_1, N_2, N_3 and N_4 such that N_2 preys on N_1; N_3 preys on both N_1 and N_2. Assume that N_4, in this instance, has a Symbolic relationship with N_2 and is neutral with respect to the other two. Take K as the carrying capacity for N_1.

⇒ (i) Derive the 4-Species Model.

⇒ (ii) Derive the Equilibrium points.

⇒ (iii) Demonstrate the Quality of the equilibrium points by Linearization.

(If needed you can use MATLAB or MATHEMATICA for Symbolic Computation).

Question 8

Consider the well known Lorenz Attractor System

$$\boxed{\begin{aligned} x' &= \sigma(y-x) \\ y' &= rx - y - xz \\ z' &= xy - bz \end{aligned}}$$

⇒ (i) Derive the equilibrium points including trivial equilibrium point.

⇒ (ii) Fix $\sigma = 10$, $b = 8/3$ and vary r from 1 to 28 (in steps of 1).

Take the initial Condition: $(x(0), y(0), z(0))$ as $(1,1,1)$.

Plot the trajectories and the phase diagram of this Lorenz System (Show it for $r = 28$).

⇒ (iii) Linearize the model about the equilibrium points and discuss the Quality of the equilibrium points.