Fairness in Matching under Uncertainty

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Motivation

Algorithmic decisions are often used in ***two-sided marketplaces***.
- e.g., assigning students to schools, users to advertisers, applicants to job interviews, etc.

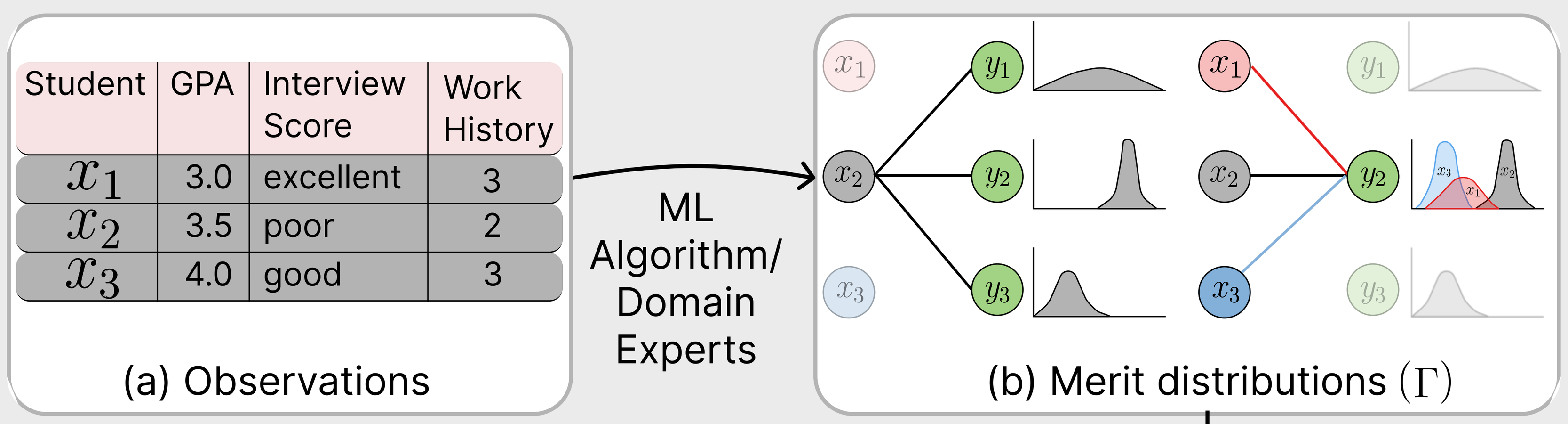
Decisions/assignments should respect the preferences of **individuals** and simultaneously be fair with respect to their ***merits***.
- Merit: how qualified is an individual for a position?

However, merit is often inherently ***noisy*** to estimate---merits conditioned on observable features are uncertain
- A single test score or interview is often not adequate to exactly determine preparedness/fit.

This is exacerbated by the ***widespread use of ML algorithms*** to infer merit from observable features.
- The outputs of ML algorithms are often uncertain.

Key Contributions

- We axiomatize a notion of ***individual fairness*** in two-sided marketplaces which respects the uncertainty in the merits.
- We design a linear programming framework to find fair utility-maximizing distributions over allocations.
- We prove that the linear program is robust to perturbations in the estimated parameters of the uncertain merit distributions, a key property in combining the approach with ML techniques.
- We verify our method empirically by designing and running an experiment in a two-sided market given data from a dating app.



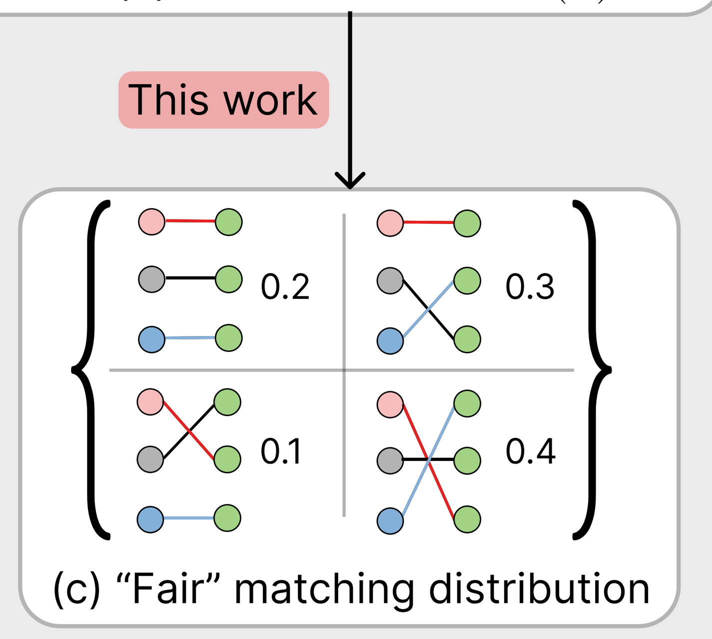


Figure 1*: A visual overview of our setup, using matching* ***students*** *and* ***jobs*** *as an example.*

Each ***job*** *has an ML algorithm or domain expert with access to observed features (a) for each* ***student****, and from those constructs a distribution Γ (b) which is an estimate of the merit or* ***fit*** *for the job/student pair.*

Importantly, these estimates could be based on different factors for different jobs (see heterogeneity of merit for student x_2).

Student preferences (not pictured) and Γ are given to the algorithm as input.

The algorithm seeks to output a distribution over matchings which is ***fair*** *(c) w.r.t. what each student is* ***contextually entitled*** *to, while simultaneously considering the preferences of* ***all*** *students and the* ***overall utility*** *of the solution.*

Problem Setting

Bipartite matching market with n **individuals** and n **resources**.

Each **individual** has deterministic preferences over all **resources**
- e.g., a certain student has an (ordered) ranked list over jobs.

Each **resource** has a ***noisy estimate*** of the merit of each **individual**.
- This takes the form of a distribution Γ over merit (merit $\in \mathbb{R}^{\ge 0}$).
- Merit is broadly construed: may be more than academic performance.

For example, a certain **job** may believe that a **candidate**, if hired, has a 90% chance of performing well and a 10% chance of performing poorly.
- Importantly, no assumptions on structure of Γ.

Our guiding notion is ***contextual entitlement*****:** what **resource** that an **individual** receives should depend not only on their qualifications and preferences, but also the preferences of ***other*** individuals.

References. [1]: A. Singh, D. Kempe, and T. Joachims. Fairness in ranking under uncertainty. In Proc. 35th Advances in Neural Information Processing Systems, pages 11896–11908, 2021. [2]: Lukáš Brožovský and Václav Petříček. Recommender system for online dating service. In *Proc. Znalosti*, pages 29–40, 2007.

Proposed Fairness Axioms

Axiom 1 (Meritocracy): If individual A has greater merit than individual B, then A should receive an outcome they weakly prefer over the outcome assigned to B.

Axiom 2 (Fairness with Uncertain Merit [1]): If A has merit greater than B with probability ρ, then individual A should receive an outcome they prefer over the outcome assigned to B with probability at least ρ**.**
- Captures that similar **individuals** should be treated similarly according to their own preferences: e.g., If two **candidates** are qualified with equal probability, they should each receive the **job** with equal probability.
- However, not yet precise enough to be operationalizable in matching.

Axiom 3 (Fairness in Decision Making under Uncertainty): Suppose that A_{det} is an algorithm which we consider fair when merits are fully observable and certain (e.g. ranking by exact merit). Let $s_1, s_2, \ldots, s_k : Outcomes \to \mathbb{R}^{\ge 0}$ be desired fairness statistics (e.g. top-k outcomes for all individuals). Then, a randomized algorithm A_{rand} is ϕ-fair with respect to a merit distribution Γ if it satisfies

$$\mathbb{E}_{z \sim A_{rand}}[s_j(z)] \ge \phi \mathbb{E}_{V\sim\Gamma}[s_j(A_{det}(V))]$$

for all statistics s_j and fairness parameter $\phi \in [0, 1]$.

Intuition: Uncertainty about an individuals' merit should not hurt the outcome they receive (in expectation, and w.r.t. "how we measure hurt" s_j).

Our proposed **Axiom 3** is a type of ***individual fairness*** generalizing **Axiom 2** of [1] which simultaneously accounts for:
1. the heterogeneous preferences of **individuals**;
2. inherent ***uncertainty in the estimation of merit***; and
3. potentially constrained **resources**.

Fair Bipartite Matching with Uncertainty

In matching, A_{det} is the celebrated Gale-Shapley stable marriage algorithm.

We choose n^2 fairness statistics $s_{i,k}$ corresponding to the probability that **individual** i receives a top k or better outcome (for all $i \in [n], k \in [n]$).
- This captures everything since the marginals fully define the output here.

In matching, a ϕ-fair A_{rand} (satisfying **Axiom 3**) ensures that the probability that any **individual** i receives a top k outcome (for all $k \in [n]$) is at least ϕ times the probability that if we randomly drew the merits for all individuals and then ran Gale-Shapley with the now deterministic preferences of both sides, i would indeed receive a top k outcome.

Why ϕ-fair instead of always 1-fair? Not all fair solutions provide the same utility to the mechanism designer, so we allow a parameter which trades of ***utility*** and ***fairness***.

Fair Utility Maximization, Empirical Results

Let $\mu_{i,j}$ be the utility that the mechanism designer receives for matching **individual** i and **resources** j, and $p_{i,j}$ be a variable representing the probability of this match. The following LP maximizes the utility subject to fairness (**Axiom 3**) and that the output is a distribution over matchings.

$$max \sum_{i=1}^{n}\sum_{i=1}^{n} \mu_{i,j} \cdot p_{i,j} \qquad \text{LP } (\star)$$

$$s.t. \underbrace{\sum_{k'=1}^{k} p_{i,r_i^{-1}(k')}}_{\text{Probability of top-}k\text{ or better outcome for individual } i \text{ under } A_{rand}} \ge \phi \mathbb{E}_{V\sim\Gamma}[s_{i,k}(A_{det}(V))] \;\; \forall i \in [n], \forall k \in [n]$$

$$\sum_{i=1}^{n} p_{i,j} = 1 \qquad \forall j \in [n]$$
$$\sum_{j=1}^{n} p_{i,j} = 1 \qquad \forall i \in [n]$$

$\mathbb{E}_{V\sim\Gamma}[s_{i,k}(A_{det}(V))]$ is difficult to compute *exactly,* so in practice we use an empirical estimate of it, motivating our main theoretical result.

Theorem (informal): $\mathbb{E}_{V\sim\Gamma}[s_{i,k}(A_{det}(V))]$ can be efficiently estimated within ϵ to find a $\left(\frac{\phi}{n\epsilon+1}\right)$-**fair**, $\left(\frac{1}{\phi n\epsilon+1}\right)$-**optimal** approximation to the solution of LP $(\star)$.

Figure 2: *We also verify our results empirically by showing that on the Libimseti dating dataset [2], our method achieves higher utility than a simpler ϕ-fair baseline. This demonstrates that LP $(\star)$ is* ***tractable*** *for fair utility maximization and can lead to non-trivial* ***utility increases****.*