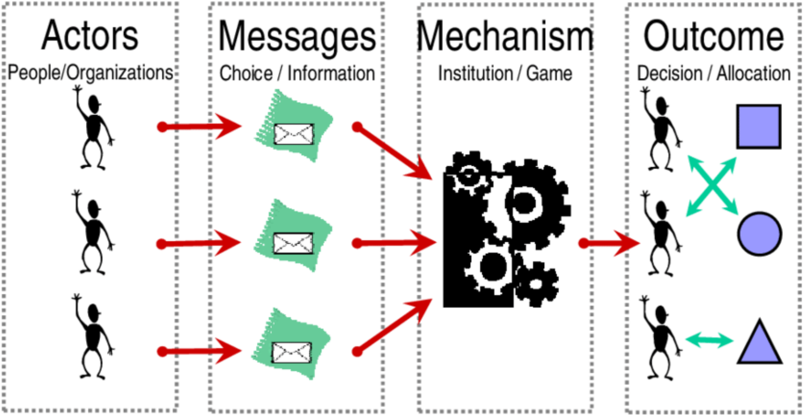
Topic 6: Mechanism Design

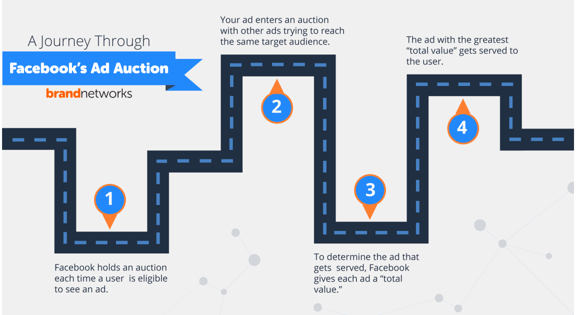


Outcomes & Objectives

- Be proficient in designing mechanisms with strategic players.
  - Revelation Principle
  - Impossibility Results
    - Voting
  - Vickrey-Clarke-Groves Mechanisms
    - Auctions

Mechanism Design in Computing Applications

- **Targeted Advertising and Recommendations:** Ads with max. bids are delivered to users to optimize user experience.

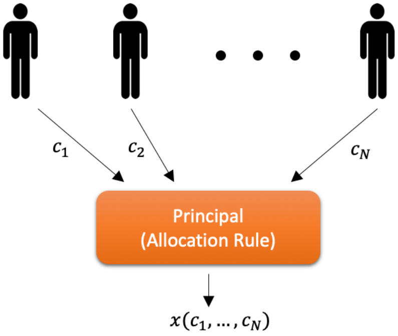


- **Opportunistic Spectrum Access:** Share/Allocate opportunistically available spectrum amongst users.
- **Dynamic Pricing in Electricity Markets:** Design electricity prices based on consumers' usage to improve market efficiency.

What is Mechanism Design?

Implement an optimal system-wide solution to a decentralized optimization problem with self-interested agents with private preferences for different outcomes.

In other words, this is the inverse problem to game theory!



Mechanisms can be implemented in centralized, or distributed form!

More formally...

Definition

A ***social choice function*** $f : \Theta_1 \times \cdots \times \Theta_N \to \mathcal{O}$ is a desired outcome $f(\boldsymbol{\theta})$ in the set of all outcomes $\mathcal{O}$, given the players' types $\boldsymbol{\theta} \in \Theta_1 \times \cdots \times \Theta_N$.

Definition

A ***mechanism*** $\mathcal{M} = (\mathcal{C}_1, \cdots, \mathcal{C}_N, x(\cdot))$ is a tuple that comprises of the set of choice strategies $\mathcal{C}_i$ available at i^{th} player, and an outcome rule $x : \mathcal{C}_1 \times \cdots \times \mathcal{C}_N \to \mathcal{O}$, such that $x(\boldsymbol{c})$ is the outcome implemented by the mechanism for choice profile $\boldsymbol{c} = \{c_1, \cdots, c_N\}$.

Definition

A mechanism $\mathcal{M} = (\mathcal{C}_1, \cdots, \mathcal{C}_N, x(\cdot))$ ***implements*** a social choice function f if

$$x\left(c_1^*(\theta_1), \cdots, c_N^*(\theta_N)\right) = f(\boldsymbol{\theta}),$$

for all $\boldsymbol{\theta} \in \Theta_1 \times \cdots \times \Theta_N$, where $c_1^*(\theta_1), \cdots, c_N^*(\theta_N)$ is the equilibrium of the game induced by $\mathcal{M}$.

Example: Mobile Crowdsensing

Crowd individuals having mobile devices capable of sensing/computing (e.g. smartphones, wearables) collectively share data and extract information to measure, map, analyze, estimate or infer (predict) any processes of common interest.

Example: Google's Waze (Transportation), 2013 Boston Marathon Bombing (Surveillance), BBC's Pandemic (Healthcare)

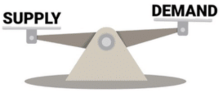
Auction (winner/all-pay):
- Allocation rule
- Payment rule

Lottery (Tullock contest):
- Contest success function
- Imperfectly discriminating

Trust & Reputation:
- Social recognition
- Peer pressure

Bargaining game:
- Rubinstein model
- Nash model

Contract:
- Adverse selection
- Moral hazard



Market-driven:
- Supply: data contributors
- Demand: service consumers

Individual Rationality and Direct Revelation

An agent should always achieve as much expected utility from participation as without participation, given prior beliefs about the preferences of other agent.

Definition

A mechanism $\mathcal{M} = (\mathcal{C}_1, \cdots, \mathcal{C}_N, x(\cdot))$ is ***individually rational*** if, for all agent types $\boldsymbol{\theta} \in \Theta_1 \times \cdots \times \Theta_N$, it implements a social choice function f such that

$$u_i\left(f(\boldsymbol{\theta})\right) \geq \bar{u}_i(\boldsymbol{\theta}),$$

where $u_i\left(f(\boldsymbol{\theta})\right)$ is the expected utility of i^{th} player averaged over a known distribution over other players' types $\boldsymbol{\theta}_{-i}$, and $\bar{u}_i(\boldsymbol{\theta})$ is the utility of the i^{th} player for not participating in $\mathcal{M}$.

Definition

$\mathcal{M} = (\mathcal{C}_1, \cdots, \mathcal{C}_N, x(\cdot))$ is a ***direct revelation*** mechanism if the choice set at every player is restricted to its own type set, i.e.,

$$\mathcal{C}_i = \Theta_i,$$

and has an outcome rule $x(\hat{\boldsymbol{\theta}})$ based on revealed (reported) types $\hat{\boldsymbol{\theta}} = \{\hat{\theta}_1, \cdots, \hat{\theta}_N\}$.

Note that the player in a direct revelation mechanism reveals $\hat{\theta}_i = c_i(\theta_i)$, based on

Examples: Auctions

Indirect-Revelation Auctions:

- ***English Auctions:*** In round $k = 1, 2, \cdots$, the auctioneer offers a price $A_k = k\epsilon$ to the item and asks if any bidder is interested.
  - If there is more than one interested bidder, auction continues.
  - If there is exactly one bidder, then he/she wins the item and pays A_k.
  - If no interested bidders, then a random bidder from the set of interested bidders in round $k-1$ wins, and pays A_{k-1}.
- ***Dutch Auctions:*** In round $k = 1, 2, \cdots$, the auctioneer offers a price $A_k = A - (k-1)\epsilon$ to the item and asks if any bidder is interested.
  - If there are no interested bidders, the auction continues.
  - If there is exactly one bidder, then he/she wins the item and pays A_k.
  - If we have more than one interested bidder, then a random bidder from the set of interested bidders wins, and pays A_k.

Direct-Revelation Auctions:

- ***First-Price Sealed-Bid Auctions:*** Bidders submit sealed bids to the auctioneer.
  - The bidder with max. bid wins the item and pays his/her own bid.
  - If there is a tie, choose a winner via picking a random bidder from the list of bidders with identical max. bids.

Incentive Compatibility

Definition

A strategy $c_i(\theta_i) \in \Theta_i$ is a ***truthful revealation*** if $c_i(\theta_i) = \theta_i$, for all $\theta_i \in \Theta_i$.

Definition

A mechanism $\mathcal{M} = (\mathcal{C}_1, \cdots, \mathcal{C}_N, x(\cdot))$ is ***incentive compatible*** if the equilibrium strategy profile $\boldsymbol{c}^* = \{c_1^*(\theta_1), \cdots, c_N^*(\theta_N)\}$ has every player reporting their true types (preferences) to $\mathcal{M}$.

Claim

A first-price sealed-bid auction is ***not*** incentive compatible.

Bidder with max. valuation may not win a first-price sealed-bid auction.

Revelation Principle

Theorem

Suppose that c^* was an equilibrium of the indirect mechanism $\mathcal{M}$. Then, there always exists a incentive-compatible direct-revelation mechanism $\mathcal{M}^*$ that is payoff-equivalent to $\mathcal{M}$.

Voting: Aggregating Social Preferences

- Set of voters: $\mathcal{N} = \{1, \cdots, N\}$.
- Set of alternatives (candidates): $\mathcal{A} = \{1, \cdots, M\}$.
- Preference: Ranking over all candidates
  - Example: Say, there are three candidates. A given voter's preference may be either $1 \succ 2 \succ 3$, $1 \succ 3 \succ 2$, $2 \succ 3 \succ 1$, $2 \succ 1 \succ 3$, $3 \succ 1 \succ 2$, or $3 \succ 2 \succ 1$.
- Set of preferences: $\mathcal{P}$ is the set of all permutations of $\{1, \cdots, M\}$.
- Preference profile $\boldsymbol{p} \in \mathcal{P}^N$.
- Voting rule: $f : \mathcal{P}^N \to A$.
- Example: Two candidates $\Rightarrow$ Majority Rule (Pairwise Elections)

Which candidate(s) should be chosen in a democracy?

Design f that aggregates voters' preferences in a democratic manner.

Voting: Examples

- **Plurality Vote[1]:** Each voter gives 1 point to the candidate she ranked first, and the winner is the candidate who receives the highest total number of points.

- **Borda Count[2]:** Each voter gives $M-1$ points to the candidate he/she ranked first, $M-2$ points to the candidate he/she ranked second, or in general $M-k$ points to the candidate he/she ranked k-th. The winner is the candidate who amasses the highest total number of points.

- **Condorcet's Method:** The Condorcet winner for a given preference profile is the candidate who beats every other candidate in pairwise elections.
  - Example: If $1: A \succ B \succ C$, $2: A \succ C \succ B$ and $3: B \succ C \succ A$, then $(A, B) = (2, 1)$, $(A, C) = (2, 1) \Rightarrow A$ is the winner.

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[1] Plurality vote disregards the remaining preference information, and works only with top choice.

[2] Borda count is used in the National Assembly of Slovenia, and is similar to that used in Eurovision song contest.

Desired Properties of Voting Rules

Basic Desires:

- The voting system treats each voter equally
- The voting system treats each candidate equally
- If there are only two candidates, the voting system chooses the majority choice.
- *Unanimity:* If all voters prefer $A \succ B$, then the social preference should reflect the same.

Ambitious Desires:

- *Transitivity:* If individual voters' preferences are transitive, aggregated social preferences should also be transitive (with ties allowed).
- *Independence of Irrelevant Alternatives (IIA):* Voting result is not affected by candidates entering or leaving the race (unless they win).
- *Strategy-Proof (Truthful):* Voters are not rewarded for exaggerating their vote.

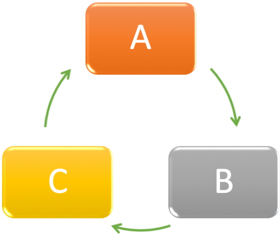
Condorcet's Paradox

Transitivity in Voter Preferences $\not\Rightarrow$ ***Transitivity in Aggregated Preferences***

Example:

- Let $\mathcal{N} = \{1, 2, 3\}$, and $\mathcal{A} = \{A, B, C\}$.
- Voter-1's preference: $A \succ B \succ C$
- Voter-2's preference: $B \succ C \succ A$
- Voter-3's preference: $C \succ A \succ B$
- What could be a reasonable aggregation (voting) rule?

In fact, a representative aggregation results in a cycle preference (intransitive).



Impossibility Results

Definition

A voting rule is a ***dictatorship*** if the social preference strictly prefers A over B, whenever a specific voter (dictator) strictly prefers A over B.

Theorem: Arrow-1951

For 3 or more alternatives, any social preference function that respects transitivity, unanimity and IIA is a dictatorship.

Theorem: Gibbard-1973, Satterthwaite-1975

An election mechanism for 3 or more alternatives which is unanimous and strategy-proof (truthful) is a dictatorship.

In other words,

Manipulation (not being truthful) is inevitable in any unanimous voting mechanism!

Mitigating Voter Manipulation...

- Idea 1: Make manipulation computationally difficult!
  - **Computational Social-Choice Theory**: *Combinatorial Voting*
- Idea 2: Make manipulation difficult via restricting information!
  - **Bayesian Voting:** Revealing other voters' preferences partially (or not revealing preferences) increases uncertainty at the manipulator!
- Idea 3: Make manipulation difficult via introducing restrictions in the domain!
  - **Conditional Admission of Preferences:** Admit only those preferences that satisfy certain conditions (that depend on the past decisions).
- Idea 4: Have desired outcomes, while preserving strategic choices!
  - **Sequential Voting:** Stackelberg games have first-mover's advantage!

A Way Out from the Impossibility Results...

Theorem: Arrow-1951

For 3 or more alternatives, any social preference function that respects transitivity, unanimity and IIA is a dictatorship.

Theorem: Gibbard-1973, Satterthwaite-1975

An election mechanism for 3 or more alternatives which is unanimous and strategy-proof (truthful) is a dictatorship.

Is it impossible to design mechanisms with multi-agent preferences?

Idea: *Introduce monetary payments/rewards to have quasi-linear preferences...*

Theorem

If there are two or more players, no social choice function in a quasi-linear mechanism is a dictatorship.

Groves Mechanism

- Set of Players $\mathcal{N} = \{1, \cdots, N\}$, and a principal $\mathcal{P}$.
- Type of players $\boldsymbol{\theta} = \{\theta_1, \cdots, \theta_N\} \in \Theta$
- Outcome: $(x, \boldsymbol{p})$, where $\boldsymbol{p}$ is the payment vector.
- (Quasi-Linear) Utility of i^{th} player: $u_i(x, \boldsymbol{p}, \boldsymbol{\theta}) = v_i(x, \theta_i) - p_i$
- Goal: Maximize the social welfare...

Definition

A ***Groves mechanism*** are direct mechanisms with allocation rule x^* and price $\boldsymbol{p}$ such that

$$x^* = \underset{x \in \mathcal{X}}{\text{arg max}} \sum_{i \in \mathcal{N}} v_i(x, \theta_i),$$

$$p_i = h_i(\boldsymbol{v}_{-i}) - \sum_{j \neq i} v_i(x^*, \theta_i)$$

An efficient (social-welfare maximizing) mechanism where players pay for the damage they impose to the society!

Vickery-Clarke-Groves Mechanisms

Definition

A ***Vickery-Clarke-Groves mechanism*** (or a pivotal mechanism) is a Groves mechanism such that

$$x^* = \underset{x \in \mathcal{X}}{\text{arg max}} \sum_{i \in \mathcal{N}} v_i(x, \theta_i),$$

$$p_i = \max_{x \in \mathcal{X}} \sum_{j \neq i} v_i(x, \theta_i) - \sum_{j \neq i} v_i(x^*, \theta_i)$$

Allocation rule: Maximizes social welfare.
Payment rule: Difference between

- Optimal welfare if the player is not participating
- Welfare of other players from chosen allocation rule.

Theorem

Truthful revelation is the dominant strategy under any Groves Mechanism (including the VCG mechanism).

VCG Mechanisms: Examples

- **Single-Item Auctions:** Then, we have the second-price auction!
  - Assume two agents in the mechanism with types θ_1 and θ_2.
  - Say, $\theta_1 > \theta_2$.
  - Then, $v_1 = x^*(\theta_1) \cdot \theta_1 = \theta_1$ and $v_2 = x^*(\theta_2) \cdot \theta_2 = 0$.
  - Consequently, $p_1 = 0 - (\theta_2) = -\theta_2$ and $p_2 = \theta_1 - \theta_1 = 0$.

- **Multi-Item Auctions:** Say, each bidder wants only one item...
  - VCG mechanism for 5-item auction: Highest-5 bids get one item each...
  - Say, there are 7 players with valuations $70,\ 30,\ 27,\ 25,\ 12,\ 5,\ 2$
  - Optimal welfare, if player i is not participating:
    $99,\ 139,\ 142,\ 144,\ 157,\ 164,\ 164$
  - Welfare of other players, if player i is participating:
    $94,\ 134,\ 137,\ 139,\ 157,\ 164,\ 164$
  - Payments: $5,\ 5,\ 5,\ 5,\ 0,\ 0,\ 0$ (winners pay $(5+1)^{th}$ bid)

Efficiency and Budget Balancing

Definition

An outcome $(x, \boldsymbol{p})$ is said to have an ***efficient allocation*** x^* if, for each $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, we have

$$x^*(\boldsymbol{\theta}) \in \arg\max_{x \in \mathcal{X}} \sum_{i \in \mathcal{N}} v_i(x, \theta_i).$$

Definition

Let p_0 is the reserved value of the allocated item(s) at the principal. Then, an outcome $(x, \boldsymbol{p})$ is said to be ***budget balanced*** if, for each $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, we have

$$\sum_{i \in \mathcal{N}} p_i(\boldsymbol{\theta}) = p_0.$$

VCG mechanisms are efficient, but not always budget-balanced,

i.e. it spends more than what it collects from the players...

Properties of Groves Mechanisms

Theorem

Groves mechanisms are allocatively efficient and strategy proof for agents with quasi-linear preferences.

Theorem

Groves mechanisms are the only allocatively efficient and strategy proof for agents with quasi-linear preferences and general valuation functions, amongst all direct-revelation mechanisms.

Theorem

VCG mechanisms are also individually rational.

Summary

- **Mechanism Design:** What are the desired properties of a mechanism with strategic players?
- **Revelation Principle:** Focus only on direct-revelation incentive-compatible mechanisms!
- **Voting:** How can we aggregate social preferences in a desired manner?
- **Impossibility Results:** What voting mechanisms are feasible, and what are not?
- **Groves (VCG) Mechanisms:** Direct mechanisms when agents have quasi-linear utilities.
- **Auctions:** Single-Item/Multi-Item Auctions