Missouri University of Science & Technology — Department of Computer Science
Fall 2023 — **CS 5408: Game Theory for Computing**

Homework 4: Information Asymmetry and Dynamic Games

Instructor: *Sid Nadendla* — **Due:** *November 15, 2024*

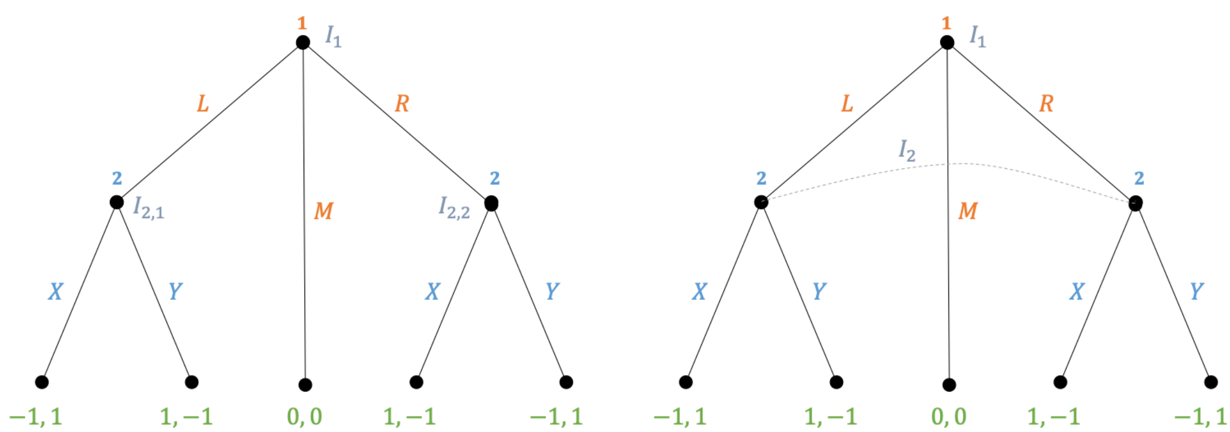
Problem 1 First-Price Auction *3 pts.*

Consider a first-price sealed-bid auction with N bidders. Assume each i^{th} bidder has a valuation $v_i \in [0,1]$ and has a belief $v_j \sim U[0,1]$ for all $j \neq i$, i.e. a uniform belief regarding the valuations of all the other players. Then, prove that the first-price sealed-bid auction has a Bayes-Nash equilibrium where every player adopts *strategic underbidding* by choosing

$$b_i^* = \left(\frac{N-1}{N}\right) v_i, \text{ for all } i = 1, \cdots, N.$$

Problem 2 Imperfect Extensive Games *4 pts.*

Consider the following modified matching pennies game, played in extensive form, where Prisoner 1 plays first, followed by Prisoner 2. The main difference from the traditional matching pennies is that Player 1 can decide whether to play this game, or not. If he decides not to play, both players get nothing.

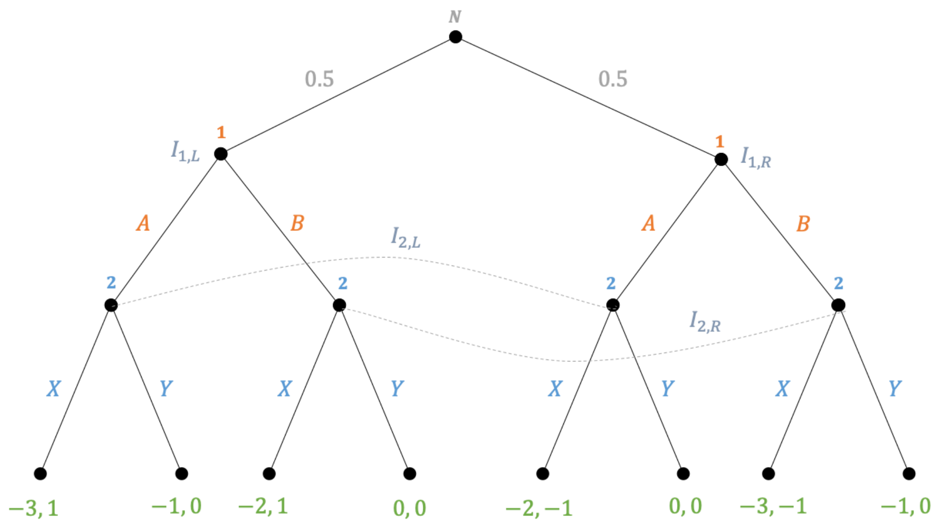


(a) Find the subgame perfect equilibrium for this game, when Player 2 can perfectly observe Player 1's choices as in the left figure.

(b) Find behavioral equilibria for this game, when Player 2 cannot observe Player 1's choices as in the right figure.

Problem 3 Perfect Bayesian Equilibrium *3 pts.*

Prove that there is no separating equilibrium in the following two-player signaling game (as depicted in the figure below), where the player set is $\mathcal{N} = \{1, 2\}$, the choice sets at the corresponding players are $\mathcal{C}_1 = \{A, B\}$ and $\mathcal{C}_2 = \{X, Y\}$ respectively. Assume that Player 1 can take two types $\{L, R\}$, and Player 2's belief about Player 1's type is uniformly distributed across types.



Problem 4 Repeated Games *3 pts.*

Consider the following repeated prisoner's dilemma game, where players play the game over an infinite time horizon. Prove that Tit-for-Tat strategy (given below) is a Nash equilibrium to this game, only when the discounting factor $\beta \geq \frac{1}{2}$.

Prisoner 1 \ Prisoner 2	C_2	D_2
C_1	2, 2	0, 3
D_1	3, 0	1, 1