



# Comparing the Impact of Learning in Bidding Decision-Making Processes Using Algorithmic Game Theory

Rayan Assaad, S.M.ASCE<sup>1</sup>; Muaz O. Ahmed, S.M.ASCE<sup>2</sup>; Islam H. El-adaway, F.ASCE<sup>3</sup>; Amr Elsayegh, S.M.ASCE<sup>4</sup>; and Venkata Sriram Siddhardh Nadendla<sup>5</sup>

**Abstract:** Although previous research efforts have developed models to assist contractors in different bidding decisions, there is a lack of research work that investigates the impact of integrating learning algorithms into the construction bidding decision-making process. As such, this paper develops a simulation framework to determine the bid decision that would result in the optimal outcomes in the long run. To this end, the authors used a research methodology based on an algorithmic game theory approach. First, data was collected for 982 US public construction projects. Second, a framework was formulated to represent the bidding decision-making process. Third, a comparison between three learning algorithms was performed, including the multiplicative weights, the exponential weights, and the Roth-Erev. Fourth, two bidding strategies were simulated: the first strategy aims to win more projects while the second strategy aims to reduce the cases the contractor might fall prey to negative profits (known as the winner's curse). The outcomes of this study demonstrated that integrating learning into construction bidding decision-making process (1) gives contractors competitive advantage over their competitors by either doubling their chance of winning more projects or reducing losses in the long run, and (2) benefits owners by ending-up paying less for their projects in the long run. Ultimately, this study adds to the body of knowledge by equipping contractors with a practical bidding framework that can be used in their bidding decision-making process to overcome the inherent complexities and uncertainties in the competitive construction bidding environment. DOI: [10.1061/\(ASCE\)ME.1943-5479.0000867](https://doi.org/10.1061/(ASCE)ME.1943-5479.0000867). © 2020 American Society of Civil Engineers.

## Introduction

The construction industry is ripe for many disruptions (Assaad et al. 2020c) and is considered a highly competitive environment for contractors (Assaad et al. 2020b; Assaad and El-adaway 2020b). One of the aspects of this competitive environment is the contractor selection process. While different bid allocation methods are used in construction procurement, the lowest bid approach is the most commonly used in the US competitive bidding environment (Ioannou and Awwad 2010). The lowest bid means that contracts are allocated based on a competitive bidding process (Seydel 2003).

Competitive bidding is considered a legal requirement in the public sector (USGSA 2016). For instance, in public US construction projects, such as those funded by the DOTs, public bidding and procurement laws require the project to be allocated to the lowest responsible and responsive bidder. This is because state and local government public works shall seek to protect the public against the squandering of public funds and shall prevent abuses such as fraud, waste, and favoritism (Rowles and Cahalan 2020).

Contractors reflect their desires to carry out a construction project by submitting their proposals for an agreed price (Park and Chapin 1992). That said, one of the hardest decisions that contractors face is the bid amount to submit for a given construction project. This critical decision has a significant influence on the company because it needs considerable time and expenses to be prepared, requires huge efforts, and affects the financial status of the firm (Leśniak and Plebankiewicz 2015). As such, contractors aim to design and submit their bids so that they are the lowest responsive and responsible bidder. The lowest responsive and responsible bidder is the contractor “who fully complied with all of the bid requirements and whose past performance, reputation, and financial capability is deemed acceptable, and who has offered the most advantageous pricing or cost benefit, based on the criteria stipulated in the bid documents” (Navigate 2017).

According to Gates (1967), contractors aim to gain projects as much as their capabilities allow for many reasons, including (1) increasing earned profits, (2) minimizing losses because they need to keep the firm intact even during recession periods, and (3) minimizing the profits of competitors to have a long-term good competitive position within the construction market. As such, contractors need to take different important bidding decisions (Bagies and Fortune 2006), including (1) the bid or no-bid decision where the contractor weighs many factors that determine the expected benefits from a

<sup>1</sup>Ph.D. Candidate, Dept. of Civil, Architectural, and Environmental Engineering, Missouri Univ. of Science and Technology, Rolla, MO 65409. Email: [rayan.assaad@mst.edu](mailto:rayan.assaad@mst.edu)

<sup>2</sup>Ph.D. Graduate Student, Dept. of Civil, Architectural, and Environmental Engineering, Missouri Univ. of Science and Technology, Rolla, MO 65409. Email: [muaz.ahmed@mst.edu](mailto:muaz.ahmed@mst.edu)

<sup>3</sup>Hurst-McCarthy Professor of Construction Engineering and Management, Professor of Civil Engineering, and Founding Director of Missouri Consortium for Construction Innovation, Dept. of Civil, Architectural, and Environmental Engineering and Dept. of Engineering Management and Systems Engineering, Missouri Univ. of Science and Technology, Rolla, MO 65409 (corresponding author). ORCID: <https://orcid.org/0000-0002-7306-6380>. Email: [eladaway@mst.edu](mailto:eladaway@mst.edu)

<sup>4</sup>Ph.D. Candidate, Dept. of Civil, Architectural, and Environmental Engineering, Missouri Univ. of Science and Technology, Rolla, MO 65409. Email: [amr.elsayegh@mst.edu](mailto:amr.elsayegh@mst.edu)

<sup>5</sup>Assistant Professor, Dept. of Computer Science, Missouri Univ. of Science and Technology, Rolla, MO 65409. Email: [nadendla@mst.edu](mailto:nadendla@mst.edu)

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construction project; and (2) the bid value or the markup decision, which is determined based on the bidding strategy.

Contractors quote a price before the project starts; that is, when all the costs are not yet known (Awwad et al. 2015). As such, a contractor should figure out what bid values the competitors will submit so that they can adjust their bid price to win the project, which is difficult and even impossible to certainly know in the competing marketplace (Wu and Lo 2009). This desire to be the lowest responsible and responsive bidder creates the so called the *winner's curse*. The winner's curse is the situation when the bidder with the most optimistic (low) project cost estimate wins the project due to a submitted bid less than the true project cost, and thus will most likely earn negative or, at least, below-normal profits (Ahmed et al. 2015, 2016). The winner's curse is very hard to avoid; however, it can be reduced (Dyer et al. 1989). That said, when choosing their bidding strategy(ies), contractors weight two objectives: (1) winning more projects than the competitors by submitting lower bids; and (2) reducing the winner's curse, but this could mean increasing the bid value and thus lowering the probability of winning the project.

Based on the previously mentioned points, a practical and reasonable bidding model is needed to be used in the contractor's bidding decision making process to overcome the inherent complexities and uncertainties in the competitive construction bidding environment. One of such practical and reasonable approaches is determining the bid price based on the historical bidding patterns of competitors (Abotaleb and El-adaway 2017). Therefore, this paper provides a practical algorithmic game theoretical framework that helps contractors in their bidding decision-making process by learning from historical bids.

## Goal and Objectives

The goal of this paper is to study the impact of using different learning algorithms in the construction bidding decision-making process. The associated objectives are (1) to formulate and simulate a learning model that can be used in the competitive bidding environment; (2) to exploit historical bid sequences to decide the best bidding value; (3) to compare the performance of three learning algorithms; and (4) to simulate two bidding strategies—the first aims to win more projects while the second one aims to reduce the winner's curse.

## Background Information

### Algorithmic Game Theory and Construction Bidding

Game theory is defined as “the study of mathematical models of conflict and cooperation between intelligent rational decision-makers” (Myerson 1991). A special area of game theory is algorithmic game theory, which is a field at the intersection of game theory and computer science, with the objective of understanding and designing algorithms in strategic environments (Nisan et al. 2007). In relation to the benefits of this integration between game theory and computer science, Miltersen (2019) states that “On the one hand, game theory and solution concepts such as equilibrium and rationalizability give us the appropriate framework in which to understand, design, and analyze multiagent protocols. On the other hand, the classical theory of algorithms provides the necessary tools for efficiently analyzing models of economics, such as games and markets.” That said, algorithmic game theory is the fertile marriage between theoretical computer science, mathematics, and economics that deals with the optimal strategic behavior in interactive situations (Brandt 2019). In fact, algorithmic game theory is

considered a rapidly developing area of research due to its wide applications and useful concepts (Czumaj 2007).

Since the publication of *Theory of Games and Economic Behavior* in 1944 by John von Neumann and Oskar Morgenstern, game theory concepts have had substantial contributions to social and behavioral sciences through providing a tool for studying different decision-making processes. Since then, game theory has been applied to different fields, such as battlefield decisions, nuclear war deterrence, political problems, economics studies, sociology, psychology, biology, and engineering, among many others. According to Turocy and Stengel (2001), the awarding of the Nobel Prize in economic sciences to John Nash, John Harsanyi, and Reinhard Selten in 1994 was a substantial breakthrough in recognition and spread of game theory concepts. Auction theory is a subdiscipline of game theory. Historically and over hundreds of years, auctions have been used for selling and allocation of various types of goods and services to customers. In today's world, auctions are of great practical importance in both the public and private sectors. In the public sector, governments usually use auctions to sell assets, purchase services, and fund their national debt. In the private sector, auctions are used widely in many areas, such as the utility market and the selling of items through internet auctions (Kagel and Levin 2002). There are two major types of auctions: (1) private value auctions, and (2) common value auctions. In private value auctions, the bidders know, with certainty, their own valuation of the auctioned item. Conversely, in common value auctions, all bidders have the same value of the auctioned item, but no bidder knows it with certainty.

According to Dyer and Kagel (1996), construction bidding is considered a common value auction. In construction bidding, the project cost is considered a common variable for the different bidders. Basically, contractors have two sources of incomplete information at the time of submitting their bids: (1) actual (realized) project cost, and (2) their competitors' estimates of the project's cost. In common value auctions, bidders are subject to the winner's curse. The term “winner's curse” was first introduced by Capen et al. (1971) who discussed its corresponding relation to outer continental shelf (OCS) oil lease auctions. Thereafter, many researchers have studied its effect on different domains, such as publications rights (Dessauer 1981), corporate takeover battles (Roll 1986), real-estate auctions (Ashenfelter and Genesore 1992), and cattle auctions (Coatney et al. 2012).

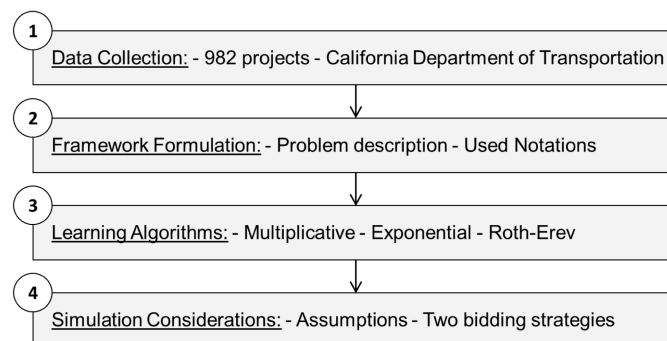
### Previous Related Research Work

Because new research should build on previous efforts to convey prospective findings that add to the body of knowledge (Assaad and Abdul-Malak 2020a, b), the authors reviewed previous related research work. In relation to that, game theory has been applied to many construction- and management-related aspects to explain, model, and predict outcomes of different decision-making challenges. For instance, Dyer et al. (1989) conducted laboratory experiments and presented the symmetric risk-neutral Nash equilibrium function for first price sealed-bid common value auctions to study the behavior of experienced executives and inexperienced students in relation to the winner's curse in construction bidding. Ho (2001) employed a game-theoretic model to study build-operate-transfer (BOT) project procurement process in case of asymmetric information. Ho and Liu (2004) presented an analytical framework based on game theory for analyzing the claims and opportunistic bidding in the construction industry. Ho (2005) implemented game theory in analyzing the effect of bid compensation on the construction bidding process. Drew and Skitmore (2006) performed a bidding experiment to test the applicability of Vickery's revenue equivalence theory in construction auctions. Furthermore, Unsal and Taylor (2011)

applied game theory concepts to study and examine different strategies for subcontractor(s) selection process. Tan and Suranga (2008) presented a study on the size of the winner's curse in the Sri Lankan construction industry and found the existence of large winner's curses in the studied sector. Karl (2014) developed and tested a module-oriented modeling approach that can be used in the simulation of multicausal and dynamic relationships on different levels in the construction industry and in educational games employed in academic and further education. Moreover, Ho and Hsu (2014) used game theoretic analysis to study the strategic interactions among heterogeneous project bidders, and showed that—under certain conditions—the offering of bid compensation can effectively encourage the stronger bidders to make extra efforts in the early stage. Awwad et al. (2015) presented an agent-based construction bidding model that allows modeling different bidding strategies of contractors and detects emergent market behavior and bidding patterns. Eid et al. (2015) used an evolutionary game theory approach to find an equilibrium profile of postdisaster insurance plans purchased by resident families and sold by insurance companies, as well as ex post disaster relief implemented by a government agency. Leśniak and Plebankiewicz (2015) developed a model based on the fuzzy set theory to study the decision-making process concerning participation in construction bidding as related to the bid/no bid decision. In addition, Ahmed et al. (2016) implemented the symmetric risk neutral Nash equilibrium function to study the effect of the winner's curse in single-stage and multistage construction bidding environment. De Clerck and Demeulemeester (2016) built a game-theoretic bidding model that represents the key peculiarities of the public-private partnerships (PPP) competitive tender. Asgari et al. (2016) investigated the importance of risk attitude on business success of contractors and determined the optimal relative range of risk-averseness that helps contractors outperform their competitors in the long run. Jeong et al. (2018) proposed a framework for producing coastal adaptation under multilateral participation that minimizes conflict between stakeholders based on a game theory analysis. Nichols (2018) studied the reverse auction bidding process in order to understand the player behavior. Kalan and Ozbek (2020) developed a decision-making tool to assist decision makers in making construction bidding decisions and selecting the most appropriate projects to bid on.

### Knowledge Gap

Based on the aforementioned information, previous research efforts have provided important knowledge and have developed good models that help contractors in different bidding decision-making processes such as bid/no-bid decision, the selection of the projects to bid on, determination of the optimal level of risk-averseness, and modeling of different bidding strategies. Nevertheless, little research work has been conducted to help contractors decide their bid values based on learning from competitors' historical bid sequences. As such, this paper takes previous research works a step further by developing a simulation framework that can be adopted by construction bidders to help in determining the bid decision that would result in the optimal outcomes in the long run. In fact, many previous research works have highlighted the need for practical bidding models. For instance, Awwad et al. (2015) stated that there is a need for developing a model that helps in understanding the dynamics of the construction bidding environment because existing research studies have methodological restrictions and limited applicability. Ahmed et al. (2016) highlighted the need for a more efficient construction bidding model that matches the realistic situation of the construction bidding process. In addition, Abotaleb and El-adaway (2017) stated that although many game theoretical works were beneficial to the construction bidding's body of



**Fig. 1.** Followed research methodology.

knowledge, they do not directly assist bidders in their selection of optimum bid prices. To this end, despite many research publications related to game theory and construction bidding, there is a lack of research work that investigates the effect of integrating learning algorithms into the construction bidding decision-making process by exploiting historical bid sequences. In fact, equipping decision makers with effective tools is crucial in ensuring successful results (Assaad and El-adaway 2020c). That said, this paper tackles this research gap by following an algorithmic game theoretical approach.

## Methodology

To achieve the research goal and objectives, the authors followed a research methodology based on algorithmic game theory concepts. The followed steps include data collection, framework formulation, incorporation of three different learning algorithms from algorithmic game theory literature, simulation considerations in terms of identification of simulation assumptions, and development of two bidding strategies. A summary of the followed methodology is shown in Fig. 1. Additional details pertaining to each one of the followed steps are presented in the subsequent subsections.

### Data Collection

The dataset utilized in this paper consists 982 public construction projects funded and managed by California DOT (Caltrans). The data includes the projects' names/description, the winning bid for each project, the name of the winning contractor, and the actual cost of the project. The collected data includes transportation and infrastructure construction projects because they are considered one of the most important systems that affect the economies of nations (Assaad et al. 2020a; Assaad and El-adaway 2020a). These projects include bridge widening, freeway and interchange construction, light rail structures, roadway rehabilitation, seismic retrofit, pavement rehabilitation and replacement, among others. The considered projects are multimillion projects with monetary values ranging from USD 1 to 5 million. These actual projects were used to simulate the learning process in construction bidding.

### Framework Formulation, Problem Description, and Notations

This subsection provides all details pertaining to the framework formulation. In relation to that, the problem description and the used notations in this paper are as follows:

- Let  $n$  be the total number of contractors/bidders submitting a bid for a certain construction project, such that  $n = 1, 2, \dots, i$ .



- Let  $k$  be the total number of projects that the  $n$  contractors bid on, such that  $k = 1, 2, \dots, j$ .
- Because the actual cost of a construction project is not known at the time of bidding, it can be represented or modeled as a random variable. That said, let  $C_j$  be a random variable that represents the actual cost of the construction project  $j$ .
- Each contractor  $i$  has their own estimate of the actual cost of the construction project at the time of bidding. This estimate differs from one contractor to the other as well as from one project to the other depending on the nature, type, and scope of the work. That said, let  $S_{ij}$  denote the estimate of contractor  $i$  for the actual cost of project  $j$  at the time of bidding. In other words,  $S_{ij}$  is the  $i$ th contractor's expected value of  $C_j$ .
- To account for the estimation error of the contractor's estimate of the actual cost of the project, let  $\delta$  be the contractor's maximum error or standard deviation around the actual cost of the project. Contractors could possess different estimation errors  $\delta$  compared to their competitors depending on the nature of the project that they are submitting a bid for, their experience in the industry sector, the competency of their estimation team, and the complexity of the project. The estimation error  $\delta$  can be expressed as a percentage such as 0.5%, 1%, 2%, 5%, 10%, or any other possible percentage value. Thus, the value of the submitted bid by the contractor is dependent on  $\delta$ , which could be divided into equal intervals. Thus, the action set includes possible bid values belonging to the interval  $[S_{ij} - \delta, S_{ij} + \delta]$ .
- Each contractor submits their bid based on their estimate of the actual cost of the project. That is, let  $B_{ij}$  be the bid submitted by contractor  $i$  for project  $j$ , which is dependent on  $S_{ij}$  and  $\delta$ , thus  $B_{ij}(S_{ij}, \delta) \in [S_{ij} - \delta, S_{ij} + \delta]$ .
- Let  $A$  be the set of all possible actions  $a$  (that is, bid values) that the contractor  $i$  can take.
- Let  $R_j(a)$  be the reward that contractor  $i$  receives for choosing action  $a$  for project  $j$ . The reward is usually considered to be within the  $[0, 1]$  interval or the  $[-1, 1]$  interval. The reward function depends on the bidding strategy that the contractor wants to follow. More details on the construction of the reward function are found in next subsections.
- Contractor  $i$  wins project  $j$  if they submit the lowest bid among the participating  $n$  contractors. Let  $P_{ij}$  be the profit earned by the winning contractor  $i$  for project  $j$ . This profit is only known when the construction project is completed and is calculated using Eq. (1)

$$P_{ij} = B_{ij}(S_{ij}, \delta) - C_j \quad (1)$$

where  $B_{ij}(S_{ij}, \delta)$  = submitted bid; and  $C_j$  = actual cost of the project.

The previously described construction bidding problem could be simulated and solved as a *regret-minimization* problem where the contractor tries to minimize their regret—or equivalently maximize their reward. In this setting, the agent/bidder is presented with a choice of actions (set of bid values), each with a different outcome (winning or losing the project and the occurrence of the winner's curse). The inherent complexities are related to the fact that this outcome is not known at the time of bid submission but rather after the bid is submitted and the winning and losing contractors are determined. That said, a reasonable objective for an agent would be to maximize their average rewards (Greenwald and Jafari 2003). In the cases where each outcome is deterministic, the agent needs only to undertake a linear search for an action that yields the maximum reward and choose that action forever in their future decision making (Greenwald and Jafari 2003). Nevertheless, this is not the case in the construction competitive bidding because each outcome is

not deterministic (it is not known at the time of bid submission), and it differs from one project to the other. In this case, a more complex strategy or learning algorithm is needed for the agent who seeks to maximize average rewards (Greenwald and Jafari 2003). Different learning algorithms are present in the literature of algorithmic game theory, but the more optimal learning algorithms are those that fall under the umbrella of *no-regret* because they are geared toward maximizing rewards (or equivalently minimizing regret) in nondeterministic settings (Greenwald and Jafari 2003) such as those present in construction competitive bidding environment. To this end, this paper uses three learning algorithms. Details on each one of those algorithms are shown in the next subsection.

## Learning Algorithms

This subsection presents all details pertaining to each one of the used learning algorithms in this paper. These algorithms are the multiplicative weights, the exponential weights, and the Roth-Erev.

### Multiplicative Weights Learning Algorithm

The multiplicative weights learning algorithm is one of the most commonly used methods in decision-making processes and prediction applications, and it is widely deployed in game theory and algorithm design (Laux 2019). This algorithm was first discovered in the context of online learning and has been rediscovered several times since then (Vazirani and Rao 2012). The earliest version of this learning algorithm was in an algorithm called fictitious play, which was proposed in game theory in the early 1950s. The multiplicative weights algorithm could be applied to wide variety of learning and optimization problems, which makes this algorithm a state-of-the-art method for learning. The multiplicative weight algorithm assigns initial weights to all possible actions (usually identical initial weights) and updates these weights multiplicatively and iteratively according to the feedback of how well the taken action performed, reducing them in case of poor performance, and increasing them otherwise (Laux 2019). Arora et al. (2012) best described the usefulness of this algorithm by stating that:

A decision maker has a choice of  $n$  decisions, and needs to repeatedly make a decision and obtain an associated payoff. The decision maker's goal, in the long run, is to achieve a total payoff that is comparable to the payoff of that fixed decision that maximizes the total payoff with the benefit of hindsight. While this best decision may not be known a priori, it is still possible to achieve this goal by maintaining weights on the decisions, and choosing the decisions randomly with probability proportional to the weights. In each successive round, the weights are updated by multiplying them with factors which depend on the payoff of the associated decision in that round. Intuitively, this scheme works because it tends to focus higher weight on higher payoff decisions in the long run.

To this end, the multiplicative weights learning algorithm was used in this paper by implementing the following pseudo-code:

1. Choose a learning rate  $\eta \in (0, 1)$ .
2. Initialize the weights  $W(a)$  for each possible action  $a$  (bid value) to 1.
3. For each construction project  $j$ :
  - a. Pick a bid value proportional to  $\mathbf{p}_j = \{w_j(a) / [\sum_{a \in A} w_j(a)]\}$ . Note that  $\mathbf{p}_j$  is a vector.
  - b. Observe the outcome/reward vector  $R_j$  based on the chosen bid value.

- c. For each action  $a$ , increase the weights if they resulted in a desired outcome, and decrease them otherwise as follows:  
 $w_{j+1}(a) = w_j(a) \times (1 + \eta R_j(a))$ .

Note that a higher learning rate does not necessarily mean more preferred outcomes. The reason behind this is attributed to the so-called *exploration-exploitation dilemma or tradeoff*. This dilemma is a fundamental problem in learning algorithms when one has to choose between different options (Jing 2019). Exploitation is defined as choosing an action that was selected before and provided a desirable outcome; whereas exploration is defined as choosing an action that was not selected before which may (or may not) result in an even more desirable outcome. Therefore a balance between both exploitation and exploration is needed. That said, for the multiplicative weights algorithm, small values of the learning rate encourage exploration (the algorithm explores more frequently), whereas large values of the learning rate encourage exploitation (the algorithm exploits aggressively) (Roughgarden 2016). Hence, a higher learning rate does not always guarantee the best outcomes because it does not provide enough exploration. In summary, the best value for the learning rate is data dependent and shall be determined for the particular dataset at hand through assessing the outcomes for different incremental values for the learning rate. To this end, different values of the learning rate ranging from (0, 1) were used in the simulation with a step of 0.1. Note that a learning rate ranging from (0, 1) was considered in this paper because it (1) ensures appropriate bounds for the algorithm's regret (Chastain 2017), (2) results in more stable outcomes (MCAI 2020), and (3) is the most commonly used interval (Sahai and Muthukumar 2018; Bubeck et al. 2017).

### Exponential Weights Learning Algorithm

The exponential weights algorithm is considered as a universal method used for learning, and thus it is perceived as a state-of-the-art learning algorithm (Anderson and Leith 2020). As compared with the multiplicative weights learning algorithm, the exponential weights algorithm uses a different update rule for the individual weights. In simple terms, given a set of possible actions, the exponential weights algorithm begins with equal weights for each one of the actions. For each iteration, the algorithm chooses an action proportional to the weights assigned to all actions. Afterward, the outcome of each iteration is realized, and the algorithm updates the weights of the actions through multiplying their previous weights by an exponential factor, thus it is known as the exponential weights algorithm. To this end, the exponential weights learning algorithm was used in this paper by implementing the following pseudo-code:

1. Choose a learning rate  $\eta \in (0, 1)$ .
2. Initialize the weights  $W(a)$  for each possible action  $a$  (bid value) to 1.
3. For each construction project  $j$ :
  - a. Pick a bid value or action  $a$  proportional to  $\mathbf{p}_j = \{[w_j(a)] / [\sum_{a \in A} w_j(a)]\}$ . Note that  $\mathbf{p}_j$  is a vector.
  - b. Observe the outcome/reward vector  $R_j$  based on the chosen bid value.
  - c. Update the individual weights for each action  $a$  as follows:  $w_{j+1}(a) = w_j(a) \times e^{\eta R_j(a)}$ .

Similar to the multiplicative weights algorithm, different values of the learning rate ranging from (0, 1) were used in the simulation with a step of 0.1.

### Roth-Erev Learning Algorithm

The Roth-Erev learning algorithm was introduced in the 1990s utilizing a game theory approach to facilitate the learning of players based on experiments and observations (Erev and Roth 1998), and since then it has been considered a common algorithm for different learning applications. The algorithm first observes the taken action

$a^*$  and the associated reward  $R(a^*)$ . Accordingly, the reward for each possible action  $a$  is calculated using Eq. (2)

$$E(a) = \begin{cases} R(a^*) \times (1 - \varepsilon) & \text{if } a = a^* \\ \frac{R(a^*) \times \varepsilon}{M - 1} & \text{otherwise} \end{cases} \quad (2)$$

where  $E(a)$  = algorithm's calculated reward associated with each possible action  $a$ ;  $R(a^*)$  = actual reward received for taking action  $a^*$ ; and  $M$  = total number of possible actions that could be taken by the bidder.

The Roth-Erev algorithm updates a propensity (that is, weight) for each one of the possible actions using Eq. (3) and calculates the probability distribution over the entire set of actions using Eq. (4)

$$w_{j+1}(a) = w_j(a) \times (1 - \emptyset) + E_j(a) \times (1 - \varepsilon) \quad (3)$$

$$\mathbf{p}_j = \frac{w_j(a)}{\sum_{a \in A} w_j(a)} \quad (4)$$

where  $w_j(a)$  = propensity/weight of action  $a$  for project  $j$ ;  $\emptyset$  = forgetting parameter;  $\varepsilon$  = experimenting parameter; and  $\mathbf{p}_j$  = probability distribution over the entire set of actions. The Roth-Erev learning algorithm was used in this paper by implementing the following pseudo-code:

1. Choose forgetting and experimenting parameters  $\emptyset, \varepsilon \in (0, 1)$ .
2. Initialize the weights  $W(a)$  for each possible action  $a$  (bid value) to 1.
3. For each construction project  $j$ :
  - a. Pick a bid value  $a^*$  proportional to Eq. (4).
  - b. Observe the outcome/reward vector  $R_j(a^*)$  based on the chosen bid value  $a^*$ .
  - c. Calculate the algorithm's reward using Eq. (2).
  - d. Update the individual weights for each action  $a$  using Eq. (3).

According to Sun and Tesfatsion (2007), both the forgetting and experimenting parameters allow the player to explore more actions and to balance the trade-off between exploration and exploitation. Thus, the Roth-Erev learning algorithm allows the player to learn through experience and experimenting different strategies, while continuously weakening the selection probabilities of the poor rewarding strategies and strengthening those of the good rewarding strategies (Eid and El-adaway 2017). Because the Roth-Erev learning algorithm might be sensitive to the values of the forgetting and experimenting parameters (Nallur et al. 2016), different values for these parameters ranging from (0, 1) were used in the simulation with a step of 0.1 similar to the multiplicative weights and exponential weights algorithms.

### Simulation Considerations

This subsection provides all details pertaining to the developed simulation model in this paper.

#### Assumptions

In any developed simulation model, assumptions are needed in order to simplify the inherent complexities, uncertainties, and particularities pertaining to the problem in hand. However, these assumptions need to be reasonable and logical to represent the real-world setting. That said, the following assumptions were made in the developed simulation model:

- Bidders are assumed to be rational, meaning bidders want to win more projects or reduce the winner's curse.
- The number of bidders is assumed to be three ( $n = 3$ ) for simulation purposes. However, any other number of bidders could

be considered when the developed model is used by contractors to represent the true number of contractors submitting bids to the project. In relation to that, the contractors are referred to as Contractor #1, Contractor #2, and Contractor #3 in this paper.

- Contractor #1 is assumed the only bidder utilizing the learning algorithms while the other bidders are not using any learning algorithm. This assumption was made so that to facilitate the comparison between learning and no-learning scenarios and the associated impacts on winning projects and reducing the winner's curse.
- The actual cost of each project  $j$  is assumed to be unknown at time of bid submission, and thus—for each project  $j$ —each contractor has their own estimate about the total cost of the project. Therefore, different contractors possess different estimates for the value of the project. In addition, contractors have a maximum estimation error of  $\delta$  ( $\delta$  was assumed to be 2% in the simulation model based on consultation with experienced contractors in the field; however, contractors can choose their own  $\delta$  value based on their historical bid values when utilizing the developed model in this paper). That said, the  $\delta$  was divided into six symmetrical equal intervals above (positive) and below (negative) the contractor's estimate; thus giving an action set of a total of 13 possible actions or bid values centered at zero and belonging to  $[S_{ij} - \delta, S_{ij} + \delta]$  as specified previously in the "Framework Formulation, Problem Description, and Notations" section. Again, this assumption was considered for simulation purposes only, and contractors can choose any possible action set to use when employing the developed model in this paper.

Note that the winner is determined based on a competitive bidding environment where the project is allocated to the contractor with the lowest submitted bid. In addition, the learning is based on the contractor's as well as the competitors' historical bid decisions. Moreover, the aim is to maximize the total reward on the long-term rather than on the short-term. In this paper, the long-term is defined as bidding on many consecutive construction projects, whereas short-term is defined as bidding on a single project. In simple words, the contractor is interested in maximizing the cumulative profit earned from all the projects that they won rather than maximizing the profit earned from an individual project. That said, the aim is to win more projects on average and to possess the highest average profit among competitors.

### Bidding Strategies

In the simulation model, the authors considered two bidding strategies that can be used by contractors in a competitive bidding environment. The first bidding strategy is winning as many projects as possible as compared to the competitors with little importance given to reducing the winner's curse. This strategy is referred to as Strategy 1 in this paper. This bidding strategy is used by contractors for different reasons including keeping their labor force working and running; forming relationships with new project owners; and depriving competitors from taking extra projects so that to gain a long-term good competitive position within the construction market. This bidding strategy is modeled through defining a binary reward function as shown in Eq. (5). In simple terms, the contractor receives a full reward if they win the project for which they submitted a bid for, whereas the contractor receives no reward if they lose the project. This strategy is usually used by risk-taking bidders

$$R = \begin{cases} 1 & \text{if winning the project} \\ 0 & \text{if loosing the project} \end{cases} \quad (5)$$

where  $R$  = reward received by the contractor.

The second bidding strategy considered in this paper is putting more importance on reducing the winner's curse while sacrificing winning as much projects as possible. This strategy is referred to as Strategy 2 in this paper. This bidding strategy is used by contractors for different reasons including increasing earned profits, minimizing losses as to keep the firm intact during recession periods, and minimizing dealing with unexpected events or risks that could occur during the project execution. Because the winner's curse occurs when the bid value is lower than the actual cost of the project, this bidding strategy is modeled in this paper through defining a reward function that is dependent on the submitted bid as well as the actual value of the project (which is known only after the project is completed) as shown in Eq. (6). In simple terms, the contractor receives a positive reward if they win the project for a bid value higher than the actual value of the project (that is, positive profit), the contractor receives a reward of zero if they did not win the project, and the contractor receives a negative reward (that is, a penalty) if they win the project for a bid value less than the actual value of the project (that is, negative profit). Therefore, contractors following such strategy leverage not winning a project over winning it with negative profits. This strategy is usually used by risk-averse bidders

$$R = \begin{cases} \frac{B_{ij}(S_{ij}, \delta) - C_j}{C_j} & \text{if winning the project} \\ 0 & \text{if loosing the project} \end{cases} \quad (6)$$

Note that the reward functions presented in Eqs. (5) and (6) were used in the three learning algorithms. In addition, a total of four simulation cases, in regard to the learning status of Contractor #1, are considered in this paper: (1) no learning algorithm is used; (2) the multiplicative weight learning algorithm is used; (3) the exponential weight learning algorithm is used; and (4) the Roth-Erev learning algorithm is used. The no learning case was included to act as a reference/benchmark case to understand how learning in construction bidding can benefit contractors. In addition, the other three cases were included to compare between the different learning algorithms considered in this paper.

### Simulation Implementation and Software Packages

The simulation framework considered in this paper was implemented using Python version 3.7.0 (Millman and Aivazis 2011; Oliphant 2007), which is an interpreted, high-level, general-purpose programming language. More specifically, the simulation framework was coded using Project Jupyter, which is a nonprofit organization established to create open-source software, open standards, and services for interactive computing across multiple programming languages including Python. For the software packages, NumPy version 1.17.1 library (Oliphant 2006; Van Der Walt et al. 2011) was used, which is an open-source package that adds support for large and multidimensional matrices, and it provides a large collection of high-level mathematical functions. In addition, Python's version 1.0.3 open-source Pandas package (McKinney 2010) was used for data manipulation and analysis. Moreover, the Random package was used to implement pseudo-random number generators for various distributions (Matsumoto and Nishimura 1998). Finally, Matplotlib version 3.2.1 (Hunter 2007) package was used to plot and visualize the simulation results.

### Results and Analysis

This section presents the results and analysis for the two bidding strategies considered in this paper.



### Strategy 1: Winning More Projects

This subsection presents the results obtained from the developed simulation model for Strategy 1 where the contractor aims to win more projects with no importance given to reducing the winner's curse.

#### Determination of the Parameters for the Algorithms

The performance of any learning algorithm might be sensitive to the values of its parameters: the learning rate in the multiplicative weights and exponential weights learning algorithms, and the forgetting and experimenting rates in the Roth-Erev learning algorithm. In order to account for such sensitivity, the authors experimented different values for these parameters to tune them and

**Table 1.** Average reward for different learning rate values using the multiplicative learning algorithm under Strategy 1

Learning rate	Average reward of Contractor #1
0.10	0.659
0.20	0.673
0.30	0.678
0.40	0.680
0.50	0.680
0.60	0.681
0.70	0.681
0.80	0.682
0.90	0.682

Note: Reward in Strategy 1 is binary as shown in Eq. (5), therefore the average reward  $\in [0, 1]$ .

**Table 2.** Average reward for different learning rate values using the exponential learning algorithm under Strategy 1

Learning rate	Average reward of Contractor #1
0.10	0.659
0.20	0.673
0.30	0.678
0.40	0.680
0.50	0.681
0.60	0.682
0.70	0.683
0.80	0.684
0.90	0.685

Note: Reward in Strategy 1 is binary as shown in Eq. (5), therefore the average reward  $\in [0, 1]$ .

**Table 3.** Average reward of Contractor #1 for different forgetting and experimenting rates using the Roth-Erev algorithm under Strategy 1

Forgetting rate	Experimenting rate								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.524	0.498	0.455	0.419	0.383	0.359	0.33	0.314	0.301
0.2	0.511	0.467	0.429	0.389	0.369	0.349	0.331	0.315	0.301
0.3	0.425	0.448	0.425	0.393	0.369	0.344	0.334	0.313	0.301
0.4	0.434	0.448	0.413	0.400	0.369	0.341	0.325	0.311	0.301
0.5	0.449	0.430	0.406	0.393	0.369	0.338	0.325	0.312	0.301
0.6	0.451	0.434	0.405	0.393	0.367	0.338	0.325	0.312	0.301
0.7	0.445	0.429	0.402	0.391	0.365	0.338	0.326	0.312	0.301
0.8	0.446	0.414	0.400	0.387	0.366	0.339	0.327	0.314	0.301
0.9	0.447	0.412	0.397	0.386	0.364	0.338	0.327	0.314	0.302

Note: Reward in Strategy 1 is binary as shown in Eq. (5), therefore the average reward  $\in [0, 1]$ .

select the best ones with the highest performance under Strategy 1. This was based on the average reward earned. As such, the first step in the simulation model was to determine the best values for the algorithms' parameters that will then be used for a more detailed analysis of the simulation results. That said, Tables 1–3 show the values of the average reward obtained for different values of the parameters using each one of the learning algorithms: multiplicative, exponential, and Roth-Erev, respectively.

As demonstrated in Tables 1 and 2, the multiplicative weights algorithm gives the highest average reward (0.682) for learning rates of 0.8 and 0.9. The exponential weights algorithm gives the highest average reward (0.685) for a learning rate of 0.9. As such, a learning rate of 0.9 was used for both the multiplicative and exponential weights algorithms in the detailed analysis of the simulation's results presented in the next subsection. Conversely, Table 3 shows that the highest average reward (0.524) occurs when the forgetting rate is 0.1 and the experimenting rate is 0.1. As such, these values were used for the Roth-Erev in the detailed analysis of the simulation's results presented in the next subsection.

#### Simulation Results

As specified in the "Methodology" section, four cases were simulated: no learning, learning using the multiplicative weights algorithm, learning using the exponential weights algorithm, and learning using the Roth-Erev learning algorithm. The obtained results for these four cases are presented in Table 4.

As shown in Table 4, in case where no learning algorithm is being used, Contractor #1 won 342 projects out of the 982 projects, whereas Contractor #2 and Contractor #3 won 317 and 323 projects, respectively. The obtained results are logical because it is expected that the winning percentage be divided more or less equally between all three contractors in case none of them is using a learning algorithm: 34.8% for Contractor #1, 32.3% for Contractor #2, and 32.9% for Contractor #3 as shown in Table 4. Comparing the obtained learning results in Table 4 between the different contractors, Contractor #1 secured a pronounced competitive advantage compared to their competitors when using learning algorithms. More specifically, Contractor #1's winning percentage increased to 68.5% when using the exponential weights learning algorithm compared with 16.6% for Contractor #2 and 14.9% for Contractor #3. This competitive advantage was not pronounced when no learning algorithm was used (34.8%, 32.3%, 32.9% for Contractors #1, #2, and #3, respectively).

In addition, Table 4 shows that Contractor #1 won significantly more projects in case of utilizing the learning algorithms. In specific, Contractor #1 won 670, 673, and 515 projects when utilizing the multiplicative weights, exponential weights, and Roth-Erev learning algorithms, respectively. This is equivalent to winning percentages of around 68.2%, 68.5%, and 52.4%, respectively.

**Table 4.** Obtained simulation results for the four simulation cases under Strategy 1

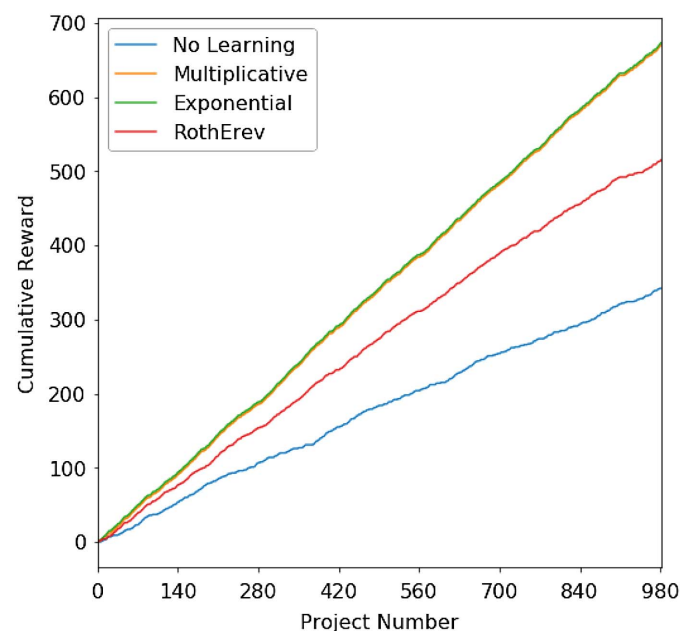
Contractor	Simulation case	No. of projects won	Winning percentage (%)
Contractor #1	No learning	342	34.8
	Multiplicative	670	68.2
	Exponential	673	68.5
	Roth-Erev	515	52.4
Contractor #2	No learning	317	32.3
	Multiplicative <sup>a</sup>	164	16.7
	Exponential <sup>a</sup>	163	16.6
	Roth-Erev <sup>a</sup>	235	23.9
Contractor #3	No learning	323	32.9
	Multiplicative <sup>a</sup>	148	15.1
	Exponential <sup>a</sup>	146	14.9
	Roth-Erev <sup>a</sup>	232	23.6

<sup>a</sup>Indicates that the learning algorithm was used by Contractor #1 only.

The obtained results reflect Contractor #1 can approximately double their chance of winning more projects in the long run by integrating learning algorithms in their bidding decision-making process (68.5% as compared to 34.8% when no learning algorithm was used). Moreover, the results indicated that both the multiplicative and exponential weights learning algorithms have similar outcomes, with the exponential weights algorithm (673 projects) being slightly better than the multiplicative weights algorithm (670 projects) under Strategy 1. In addition, the exponential and multiplicative weights algorithms performed better than the Roth-Erev learning algorithm (515 projects) and no learning algorithm at all (342 projects).

Fig. 2 shows the obtained cumulative long-run reward by Contractor #1 for the different simulation cases.

Because higher reward value means better outcomes, Fig. 2 confirms the obtained results that were reported in Table 4. In other words, Fig. 2 reflects that both the multiplicative and exponential weight learning algorithms have a similar performance under

**Fig. 2.** Cumulative reward for Contractor #1 under the four simulation cases using Strategy 1. Note: the curves for the multiplicative and exponential algorithms are very close to each other.

bidding Strategy 1 with the exponential algorithm being slightly better. In addition, the multiplicative and exponential algorithms have better outcomes than the Roth-Erev learning algorithm and much better performance than the case where no learning is being implemented in the bidding strategy.

Furthermore, the authors compared the four simulation cases from the client's perspective (Caltrans in this paper) and the results are presented in Fig. 3.

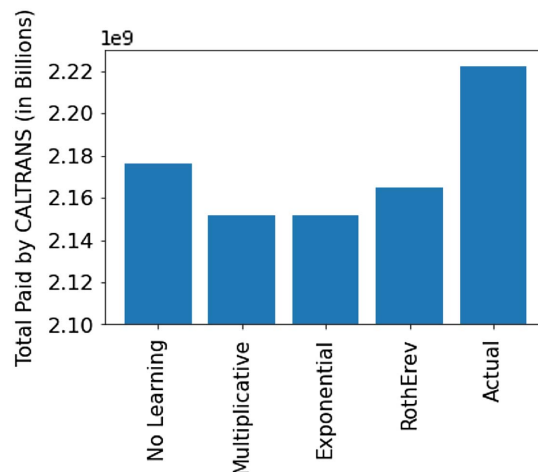
Fig. 3 shows that clients are anticipated to pay a lower total monetary amount to execute their projects in the long run when a contractor is utilizing learning algorithms to increase their probability of winning. This is reflected by Fig. 3 where the total paid by Caltrans is lower when learning algorithms are being used by contractors as compared to the no learning and actual cases. Furthermore, the results shown in Fig. 3 indicate that the minimum monetary value paid by the client is in the cases where the multiplicative and exponential weight learning algorithms were used. The obtained results in Fig. 3 are logical because the use of learning algorithms increases the competition between contractors as to win more projects; which means that contractors might tend to submit lower bid values to improve their competitive position; thus, the client ends up paying less for their projects. As such, it could be concluded that the use of learning algorithms in the construction competitive bidding would benefit both the contractor (because it increases the winning percentage) as well as the project's client (because it decreases the total amount paid by the client).

### Strategy 2: Reducing the Winner's Curse

This subsection presents the results obtained from the developed simulation model for Strategy 2 where the contractor puts more importance on reducing the winner's curse rather than winning as many projects as possible as compared to their competitors.

#### Determination of the Parameters for the Algorithms

Similar to the first strategy, the authors experimented with different values to tune the parameters of the three learning algorithms to select the best ones with the highest outcome with regard to the average profit earned. The first step in the simulation model was to determine the best values for the algorithms' parameters, which will then be used for a more detailed analysis of the simulation's results. Tables 5–7 show the values of the average profit for different values of the parameters using each one of

**Fig. 3.** Actual amount paid by Caltrans versus amount paid under each simulation case using Strategy 1.



**Table 5.** Average profit for different learning rate values using the multiplicative learning algorithm under Strategy 2

Learning rate	Average profit of Contractor #1 (USD)
0.10	−9,477.75
0.20	−5,979.99
0.30	−4,210.64
0.40	−3,165.23
0.50	−2,322.78
0.60	−2,235.69
0.70	−1,959.50
0.80	−1,705.66
0.90	−1,575.11

Note: Profit in Strategy 2 is calculated using Eq. (1).

**Table 6.** Average profit for different learning rate values using the exponential learning algorithm under Strategy 2

Learning rate	Average profit of Contractor #1 (USD)
0.10	−9,477.75
0.20	−5,990.17
0.30	−4,277.38
0.40	−3,165.23
0.50	−2,343.15
0.60	−2,235.69
0.70	−1,979.86
0.80	−1,746.40
0.90	−1,663.28

Note: Profit in Strategy 2 is calculated using Eq. (1).

the learning algorithms: multiplicative, exponential, and Roth-Erev, respectively.

One point worth mentioning in relation to the obtained results shown in Tables 5–7 is that the values reported are the average profit experienced for the entire 982 projects. This means that the contractor experienced positive profits on some projects while experiencing negative profits on other projects; but the net average profit was negative. This negative average profit is attributed to the winner's curse which is experienced by contractors. In relation to that, Dyer et al. (1989) stated that the winner's curse could not be totally avoided in construction projects but it can be reduced. As such, contractors are better off when their negative profits are higher (in negative terms not in absolute value). For instance, in Table 5, the higher average profit value of USD −1,575.11 is better than the lower average profit value of USD −1,705.66. That said,

**Table 8.** Obtained average profit for the four simulation cases using Strategy 2

Contractor	Simulation case	Average profit (USD)
Contractor #1	No learning	−14,913.42
	Multiplicative	−1,575.11
	Exponential	−1,663.28
	Roth-Erev	−11,299.65
Contractor #2	No learning	−15,815.29
	Multiplicative <sup>a</sup>	−17,054.21
	Exponential <sup>a</sup>	−17,030.36
	Roth-Erev <sup>a</sup>	−16,244.50
Contractor #3	No learning	−14,979.50
	Multiplicative <sup>a</sup>	−16,363.18
	Exponential <sup>a</sup>	−16,363.18
	Roth-Erev <sup>a</sup>	−14,984.10

<sup>a</sup>Indicates that the learning algorithm was used by Contractor #1 only.

based on the obtained results in Tables 5–7, the ultimate profit for the multiplicative weight algorithm corresponds to a learning rate of 0.9, the ultimate profit for the exponential weight algorithm corresponds to a learning rate of 0.9, and the ultimate profit for the Roth-Erev algorithm corresponds to forgetting and experimenting rates of 0.1 for both. As such, these values were used for the detailed analysis of the simulation's results that are presented in the next subsection.

### Simulation Results

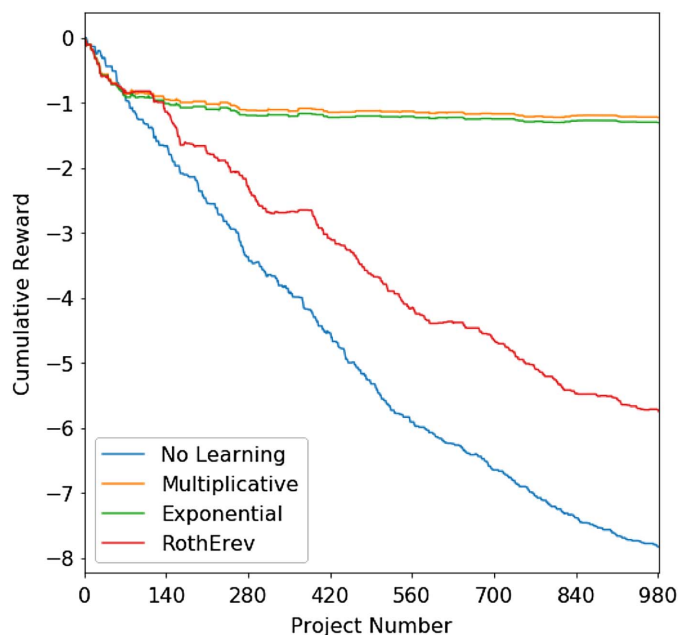
Similar to Strategy 1, four cases were simulated: no learning, learning using the multiplicative weights algorithm, learning using the exponential weights algorithm, and learning using the Roth-Erev learning algorithm. The obtained results for these four cases under Strategy 2 are presented in Table 8.

As shown in Table 8, in case where no learning algorithm is being used, the three contractors suffered approximately equally from the winner's curse: USD −14,913.42 for Contractor #1, USD −15,815.29 for Contractor #2, and USD −14,979.50 for Contractor #3. The results also indicate that the use of learning algorithms has significantly helped Contractor #1 in reducing the winner's curse. More specifically, Contractor #1's average profit increased from USD −14,913.42 to −1,575.11, −1,663.28, and −11,299.65 when utilizing the multiplicative weights, exponential weights, and Roth-Erev learning algorithms, respectively. This is equivalent to a percentage increase in average profit of approximately 89.44%, 88.85%, and 24.23% as compared to the no learning case. In addition, compared to their competitors, Contractor #1 has a better position in reducing the winner's curse. For instance,

**Table 7.** Average profit of Contractor #1 for different forgetting and experimenting rates using the Roth-Erev algorithm under Strategy 2

Forgetting rate	Experimenting rate								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	−11,299.65	−12,554.25	−13,239.15	−14,046.29	−14,162.29	−14,562.59	−15,061.11	−15,573.94	−15,958.98
0.2	−12,049.37	−13,314.96	−13,779.53	−14,169.95	−14,691.16	−15,053.84	−15,390.64	−15,384.31	−15,989.53
0.3	−12,493.12	−13,879.95	−14,298.05	−14,890.16	−15,041.49	−15,133.52	−15,453.52	−15,802.82	−15,989.53
0.4	−13,796.14	−14,233.11	−14,654.72	−14,878.26	−15,191.77	−15,366.74	−15,640.72	−15,844.02	−15,915.11
0.5	−13,513.59	−14,567.50	−14,735.78	−15,016.54	−14,926.85	−15,382.86	−15,575.71	−15,844.02	−15,935.47
0.6	−13,532.38	−14,679.52	−14,897.72	−15,072.93	−15,226.45	−15,434.58	−15,518.06	−15,853.73	−15,979.35
0.7	−13,665.14	−14,574.38	−14,892.41	−15,173.62	−15,325.99	−15,488.94	−15,570.41	−15,843.55	−15,989.53
0.8	−13,704.94	−14,608.36	−14,948.15	−15,183.80	−15,219.48	−15,488.94	−15,580.59	−15,853.73	−15,989.53
0.9	−13,614.43	−14,748.96	−15,003.51	−15,150.04	−15,203.10	−15,509.31	−15,570.41	−15,853.73	−15,989.53

Note: Profit in Strategy 2 is calculated using Eq. (1) and is expressed in USD.



**Fig. 4.** Cumulative reward for Contractor #1 under the four simulation cases using Strategy 2.

using the multiplicative weights learning algorithm, the average loss for Contractor #1 is USD  $-1,575.11$ , the average loss for Contractor #2 is USD  $-17,054.21$ , and the average loss for Contractor #3 is USD  $-16,363.18$ . It could be concluded that the multiplicative weights learning algorithm is the best performing learning algorithm in regard to reducing the winner's curse as it is slightly better than the exponential weights learning algorithm and much better than the Roth-Erev learning algorithm.

Fig. 4 shows the obtained cumulative long-run reward by Contractor #1 for the different four simulation cases under Strategy 2.

Note that the upward increases present in Fig. 4 show that the contractor is making positive profits while the downward decreases reflect that negative profits are being experienced by the contractor. In addition, Fig. 4 shows that the curves for both the multiplicative and exponential learning algorithms approximately flatten over time, which implies that Contractor #1 is learning better from past bidding decisions as they are bidding for more projects, which

results in them minimizing their losses and thus reducing the winner's curse. The obtained simulation results are consistent with what is expected to happen in real-world setting where the learning in recurrent bidding is expected to be composed of two phases: the start-up phase, and the steady-state phase (Fu et al. 2004). In relation to that, Fu et al. (2004) stated that the start-up phase corresponds to "where learning occurs rapidly in the initial bidding attempts of a contractor" while the steady-phase corresponds to "the period when the learning rate flattens out and it indicates the contractor has reached a satisfactory level."

Fig. 4 shows also that both the no learning and Roth-Erev curves are continually decreasing over time, which implies that contractors are continually suffering from the winner's curse. In addition, it is important to highlight that the decreasing slope of the Roth-Erev's curve is less than that of the no learning case. However, the contractor was not able to learn enough from past bidding decisions and the bids of competitors using the Roth-Erev algorithm to the extent of balancing out the initially incurred losses, which was the case for the multiplicative and exponential learning algorithms.

In addition, the authors performed a comparison as related to the total number of projects won by the contractors for each simulation case as well as in relation to the number of times suffering from the winner's curse. The obtained results are presented in Table 9.

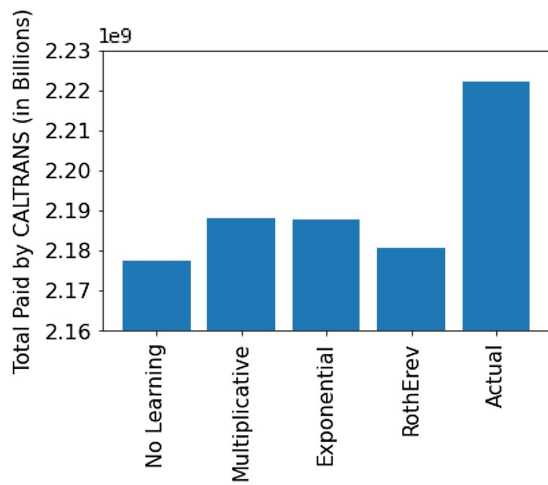
From Table 9, it is shown that Contractor #1 fell prey to the winner's curse in 293 projects, representing approximately 89.3% of the total 328 projects they won when no learning algorithm was used. Conversely, Contractor #1 faced the winner's curse in 78 projects (66.7%) out of the 117 projects they won using the multiplicative weight learning algorithm, 79 projects (66.9%) out of the 118 projects they won using the exponential weight learning algorithm, and 222 projects (81.9%) out of the 271 projects they won using the Roth-Erev learning algorithm. This reflects that the use of learning algorithms in the bidding decision-making process decreases the winner's curse (from 89.3% in the case of no learning to 66.7% in the case of the multiplicative weights algorithm).

In addition, it could be seen from the results obtained in Table 9 that the percentage of winning under Strategy 2 is much less when compared to that under Strategy 1. The reason behind this is that the contractor's primary aim is to reduce the winner's curse under Strategy 2, which means that they need to increase their bid value so that it is as close as possible to—and ultimately slightly higher than—the uncertain actual value of the project. This means that the probability of being the lowest bidder will decrease and thus the probability of winning the project will decrease; this will lead to

**Table 9.** Obtained simulation results for the four simulation cases using Strategy 2

Contractor	Simulation case	Total winning		Winner's curse	
		No. of projects	Percentage (%)	No. of projects	Percentage (%)
Contractor #1	No learning	328	33.4	293	89.3
	Multiplicative	117	11.9	78	66.7
	Exponential	118	12.0	79	66.9
	Roth-Erev	271	27.6	222	81.9
Contractor #2	No learning	328	33.4	290	88.4
	Multiplicative <sup>a</sup>	453	46.1	384	84.8
	Exponential <sup>a</sup>	452	46.0	383	84.7
	Roth-Erev <sup>a</sup>	374	38.1	332	88.8
Contractor #3	No learning	326	33.2	291	89.3
	Multiplicative <sup>a</sup>	412	42.0	346	84.0
	Exponential <sup>a</sup>	412	42.0	346	84.0
	Roth-Erev <sup>a</sup>	337	34.3	291	86.4

<sup>a</sup>Indicates that the learning algorithm was used by Contractor #1 only.



**Fig. 5.** Actual amount paid by Caltrans versus amount paid under each simulation case using Strategy 2.

a lower winning percentage. In fact, balancing both objectives—winning as much projects as possible as well as reducing the winner’s curse—is very hard in the competitive construction bidding environment. As such, it is up to the bidder to choose any strategy they want depending on the overall strategic goal of the company as well as the contractor’s interest in the specific project that they are submitting a bid to. To this end, this paper presented both bidding strategies so that to allow contractors to select the strategy that best suits their interests.

Similar to Strategy 1, the authors compared the four simulation cases from the client’s perspective (Caltrans in this paper) and the results are presented in Fig. 5.

One similarity between the results obtained under Strategy 1 and Strategy 2 is that the actual total amount paid by Caltrans for executing the 982 projects is more than that associated with each one of the four simulation cases. As such, similar to Strategy 1, it could be concluded that the use of learning algorithms in the construction competitive bidding would still benefit both the contractor (because it reduces the winner’s curse) as well as the project’s client (because it decreases the total amount paid by the client). Note that one difference exists between the obtained results in Fig. 5 as compared with those obtained under Strategy 1 (Fig. 3). In relation to that, it was found under Strategy 2 that—in case where the contractor is utilizing a learning algorithm to reduce the winner’s curse—the client is anticipated to pay more on the long run to execute the projects as compared to the case where no learning occurred. The obtained results are logical because the contractor’s main aim is to reduce the winner’s curse under Strategy 2; which means that they need to increase their bid value so that it is as close as possible to—and ultimately slightly higher than—the uncertain actual value of the project. This will decrease the probability of winning and will increase the value paid by the client because higher bid values were submitted by bidders to reduce the winner’s curse.

## Limitations

Any model possesses limitations and potential improvements in the future. As such, the following limitations can be highlighted in relation to the efforts presented in this paper:

- The proposed model is limited to learning in construction bidding in the setting of a competitive bid environment where the winning contractor is the one who submitted the lowest bid.

While this might not be the case for private construction projects, public projects such as those funded and managed by DOTs rely on the competitive bidding environment. The reason behind this is that state and local governments are required to disclose the information related to the submitted bids on public construction projects to protect the public against the squandering of public funds and to prevent abuses such as fraud, waste, and favoritism (Rowles and Cahalan 2020).

- The developed framework investigated only two bidding strategies that could be used by contractors when submitting bids to construction projects.
- To facilitate the comparison between different simulation cases, the developed model focused on the use of learning algorithms by a single agent/bidder rather than the use of the learning algorithms by multiagents.
- The usage of this work is best suited to the cases where enough historical bid values/decisions are available to contractors. While this might not be an easy case for private construction projects, contractors bidding on public construction projects are the ones who are expected to benefit the most from the research presented in this paper.
- The framework was developed, tested, validated, and verified using an actual dataset of 982 infrastructure projects funded and managed by Caltrans. However, to provide further in-depth validation of the results, it would require contractors to use the developed model in their bidding decision-making process for many construction projects and report whether they are gaining more projects or minimizing the winner’s curse compared to their competitors in the long run. This process could take years, and thus it is way beyond the reasonable control of the authors. Hence, it is in the humble opinion of the authors that the dataset used in this paper is deemed sufficient for the purpose of this research; especially realizing the extreme difficulty of putting hands on such datasets involving the projects’ names/description, the winning bid value for each project, the name of the winning contractor, and the actual cost of the project. Nevertheless, the proposed framework could benefit from further validation effort. Therefore, the authors recommend future research efforts to validate the proposed framework by applying it on different project types or similar datasets, if available. Also, future efforts may use simulation experiments and statistical methods to further validate the results, although experiments might not be a fully realistic representation of actual projects.

## Conclusion and Future Work

This paper studies the impact of using different learning algorithms in the construction bidding decision-making process through the use of algorithmic game theory concepts. In relation to that, this paper developed a simulation framework that can be adopted by construction bidders with two bidding strategies: the first strategy (Strategy 1) aims to win more projects while the second strategy (Strategy 2) aims to reduce the winner’s curse. Based on an actual dataset of 982 construction projects, the outcomes of this research demonstrated that learning from past bid sequences benefits contractors in the long-term by winning more projects or reducing the winner’s curse. More specifically, the obtained results reflect that contractors can approximately double their chance of winning more projects in the long run by adopting learning algorithms in their bidding decision-making process. In addition, the use of learning algorithms led to a percentage increase of as high as 89.44% in terms of average profit for the collected dataset as compared to the case where no learning algorithm was used in the bidding decision.



Under Strategy 1, the findings of this paper showed that the exponential weights learning algorithm performed better than the other algorithms, whereas the multiplicative weight algorithm performed better than the other algorithms under Strategy 2. Furthermore, it was found that projects' owners would also benefit when a learning algorithm is being used by bidders as they end up paying less for their projects on the long run. Ultimately, the managerial implications and the paper's contributions include (1) integrating learning algorithms in the bidding decision-making processes, (2) providing practitioners with a model that takes into account two important bidding strategies, and (3) equipping contractors with a practical bidding framework that overcomes the inherent complexities and uncertainties in the competitive construction bidding environment.

There are several opportunities for future research that could address the limitations of the work presented in this paper. First, this research could be extended to consider other methods used for bid evaluation and selection such as the second lowest bid method, the average bid method, and the below average bid method, among others. Second, other bidding strategies could be modeled and formulated by changing the structure of the reward function used by the learning algorithms. Third, this research could be extended by considering the case where all contractors or bidders are using the same or different learning algorithm(s). Fourth, future research work could focus on the development of a comprehensive model that integrates different concepts such as game-theoretic learning algorithms, decision theory, and Bayesian statistics to determine bid decisions under various information settings, especially in the case where historical bids are not available to contractors.

## Data Availability Statement

All data generated or analyzed during the study are included in the published paper.

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