Missouri University of Science & Technology	Department of Computer Science
Fall 2020	**CS 5001: Game Theory for Computing**

Homework 2: Basic Models & Solution Concepts

Instructor: *Sid Nadendla* **Due:** *October 4, 2021*

Problem 1 Colonel Blotto Game *7 pts.*

A Colonel Blotto and his adversary, the Folk's Militia each try to occupy two posts by properly distributing their forces simultaneously. Assume that Colonel Blotto has three regiments and Militia has two regiments. If a given player has more regiments than the enemy at a post, then the player receives the enemy's regiments plus one (value for occupying the post). On the other hand, if the player has fewer regiments at a post than the enemy, the player loses one, plus the number of regiments he had at the post. A draw gives both sides zero. The total payoff is the sum of the payoffs at the two posts.

(a) Represent this game in normal-form, when both the players choose their strategies simultaneously.

(b) Find the pure strategy Nash equilibrium of this game.

(c) Write a Python program in Jupyter notebook to compute *Pure Strategy Nash equilibrium* for any general bimatrix game. The program should print "This game has no PSNE," if the bimatrix game does not have a PSNE. At the same time, if the game has multiple PSNE, it should report all the PSNE in the game. Rename your notebook as "<last_name>_FS2021_CS5408_HW2_1c.ipynb".

(d) Validate your findings by comparing your result in (b), using the program you wrote for (c).

Consider an alternative version of this game where (i) Colonel Blotto plays first, followed by Folk's Militia, and (ii) Folk's Militia can only observe which post has greater Colonel Blotto's regiments.

(c) Represent this new game in extensive-form.

(d) Transform the extensive representation of the new game into normal-form.

Problem 2 Iter. Elim. of Dominated Strategies *5 pts.*

(a) Reduce the bimatrix game given below using Iterative Elimination of Dominated Strategies algorithm. Illustrate all the stages of on the example very clearly.

	Left	Center	Right
Up	1,1	2,0	2,2
Middle	0,3	1,5	4,4
Down	2,4	3,6	3,0

(b) Implement *Iterative Elimination of Dominated Strategies* algorithm for any general bimatrix game in Python. Assume that the input to this program has two identical-size matrices A and B. The output comprises of the reduced matrix. Submit your code as a Jupyter notebook under the label "<last_name>_FS2021_CS5408_HW2_2b.ipynb".

(c) Validate your program in (b) on the bimatrix game given in (a).

Problem 3 Rock-Paper-Scissors ***6 pts.***

Rock-Paper-Scissors is a well-known zero-sum game played by most kids, which is described as follows. Say, Alice and Bob simultaneously make a hand signal that represents one of their three pure strategies: *rock*, *paper* and *scissors*. The winner of the game is determined by the following rules:

- Rock *blunts* Scissors
- Scissors *cuts* Paper
- Paper *wraps* Rock

If both the players make the same signal, the result is a draw and obtain nothing. Therefore, Alice's payoff is given by the matrix

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

(a) Prove that there does not exist any pure-strategy equilibria for this game.

(b) Prove that the mixed-strategy minimax equilibrium for this game is $\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}$.

(c) Use LP solvers to implement the algorithm in Python to find mixed-strategy Nash equilibria for any general two-player zero-sum games. Assume that the input to the program is the payoff matrix of one of the players. Submit your code as a Jupyter notebook as "<last_name>_FS2021_CS5408_HW2_3c.ipynb".

(d) Validate your findings by comparing your result in (b), using the program written for (c) on Rock-Paper-Scissors game.

Problem 4 Second-Price Auction ***2 pts.***

Consider a second-price sealed-bid auction with two bidders (players), where players 1 and 2 value the object being auctioned as v_1 and v_2 respectively, such that $v_1 > v_2 > 0$. Assuming that the i^{th} player submits a sealed bid b_i, he/she obtains the object at a price equal to the other player's bid, say b_{-i}, and hence receives the net payoff $v_i - b_{-i}$. Since the other player does not obtain the object, he/she receives the payoff of zero. If there is a tie, then the tie is broken by choosing the winner amongst the two bidders with equal probabilities.

(a) Find the best response functions of both the bidders.

(b) Find all the pure-strategy Nash equilibria in this game.