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Strategic Information Design in Selfish Routing with Quantum Response Travelers

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***Abstract*—Selfish routing begets inefficiency in multi-agent transportation systems, leading to significant economic losses in our society. Although several powerful techniques (e. g., marginal cost pricing) have been proposed to mitigate price-of-anarchy (a measure of inefficiency), social welfare maximization still remains a huge challenge in selfish routing, especially when travelers deviate from maximizing their own expected utilities. This paper proposes a novel informational intervention to improve the efficiency of selfish routing, especially in the presence of quantal response travelers. Specifically, modeling the interaction between the system and travelers as a Stackelberg game, and develop a novel approximate algorithm, called *LoRI* (which stands for *logit response based information*) to steer the travelers' logit responses towards social welfare using strategically designed information. Simulation results in diverse transportation settings demonstrate that LoRI significantly improves price of anarchy of selfish routing (both in terms of congestion and carbon emissions), even when the travelers use navigation services that recommend optimal shortest-paths according to their selfish interests.**

***Index Terms*—Strategic Information Design, Quantal Response Travelers, Selfish Routing**

I. Introduction

Selfish routing is a strategic framework for which the travelers employ their best-response routes selfishly according to their respective preferences to form an equilibrium. This leads to system inefficiency with respect to social-welfare metrics (e.g., network congestion, carbon emissions), which can be quantified by the price-of-anarchy (PoA) model [1]. Although several smart navigation applications (e.g., GPS devices, navigation applications on mobile/smart devices) present route-recommendations, these technologies continue to serve the travelers' selfish interests by reducing their cognitive overload through shortest path recommendations such as Dijkstra's algorithm [2], Bellman–Ford or Warshall-Floyd, and A^*-algorithms [3]). Consequently, such technologies have little or no impact on improving social welfare metrics such as mitigating congestion [4] and reducing carbon emissions [5]. In fact, travelers often reject route recommendations that involve non-personal transport modalities (e.g., public transportation, ride sharing services and other micro-mobility services [6], [7]), when recommendation systems are redesigned from a social-welfare standpoint.

On the contrary, this paper models the interaction between the system and travelers as a *Stackelberg quantal response equilibrium (QRE) game*, where the system (leader) exhibits expected utility maximization (EUM) behavior, while the travelers (followers) exhibit logit responses [8], thus forming a QRE [9]. Specifically, the system's motive is to reduce both network congestion in terms of travel time and carbon emissions on the entire transportation network, whereas the traveler attempts to minimize travel time and/or carbon emissions along the personal route.

The main contributions of this paper are three-fold. Firstly, to the best of our knowledge, this is the first paper to investigate strategic information design in the presence of multiple logit-response receivers, in contrast to EUM receivers [10], [11], or the setting with a single prospect theoretic receiver [12]. Such a framework is particularly useful in the transportation domain where human travelers often exhibit deviating behaviors from EUM decisions. The second main contribution of this paper is the design of a novel efficient signaling algorithm at the sender called *LoRI* (in short, for ***Lo**git **R**esponse based **I**nformation*) that reveals information strategically to the receivers to steer their behavior towards system optimum (be it in terms of traffic congestion, carbon emissions, or a combination of the two attributes.) Thirdly, the paper also validates the performance superiority of LoRI over shortest-path algorithms (in terms of both network congestion and carbon emissions) on diverse simulation environments, built with heterogeneous traveler motives and various network topologies such as mulit-modal Wheatstone network (inspired by Braess' Paradox in selfish routing) and a real transportation network derived from a segment of Manhattan, New York.

Specifically, two versions of *LoRI* are proposed to approximate *Stackelberg-QRE equilibrium*: (i) *LoRI-v1*, produces near-optimal strategic information at the cost of computational complexity, and (ii) *LoRI-v2* trades off computational complexity in the first version of LoRI for a minimal compromise in costs at both system and traveler, by decoupling weakly correlated logit responses based on travelers' interactions on the network at the moment. We demonstrate that LoRI-v2 significantly outperforms LoRI-v1 in terms of scalability with increasing number of travelers.

The remainder of the paper is organized as follows. In Section II, a brief survey of relevant literature on the design of strategic interactions in selfish routing is presented. The technical gaps in the literature are highlighted in order to clearly justify the contributions of this paper. Then, the traveler-system interaction model and problem statement is

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formally presented in Section III. Upon investigating the network state dynamics and the traveler's belief dynamics in Section IV, the design of the proposed algorithm LoRI is presented formally using backward induction principles in Section V. The validation methodology and the associated datasets used in this paper are presented in detail in Section VI. Upon discussing the results of various simulation experiments in Section VII, concluding remarks and some ideas for future work are presented in Section VIII.

II. Strategic Route Interventions: A Brief Survey

In the literature, several techniques have been proposed to drive PoA in selfish routing towards unity, i.e., drive the equilibrium outcome towards system optimal. These techniques can be categorized into three broad classes based on: (i) marginal cost pricing, (ii) Stackelberg routing, and (iii) infromation revelation.

In *marginal cost pricing* techniques, selfish travelers are imposed taxes based on their marginal contribution to the system's objective [13]. However, the impact of underestimating marginal costs on the optimality in terms of system objectives was shown to be at least as good as having no taxes [14], making it practically infeasible for real-world adoption. Although attempts have been made to implement such solutions by authoritarian regimes [15], the friction to adopt marginal cost pricing continues to persist due to various political reasons in democratic nations.

In *Stackelberg routing* that influences traveler behavior, a fraction of agents are routed centrally (probably using autonomous fleets), while the remaining travelers are allowed to choose their routes selfishly [16]. A similar routing algorithm proposed in [17] is based on multi-objective A* with a goal to design routes that decrease the overall network congestion. These techniques leverage the leader's advantage in Stackelberg games to improve the price of anarchy. However, the practical limitation is that decision autonomy cannot be scaled across the entire multi-agent system in practical selfish routing.

In *information-revelation* techniques [18], [19], [20], the traffic state of the entire network is revealed to the travelers as opposed to recommending shortest routes. Although such systems do not mitigate travelers' cognitive-overload, they generate a positive impact on traffic congestion and other global objectives even in non-strategic settings [18]. However, these systems suffer from poor persuasive ability to induce behavior modification among travelers.

Inspired by the seminal paper on Bayesian persuasion [21], strategic information design has been investigated in the context of intelligent transportation systems when the network congestion state is uncertainly available at the travelers. For example, in a simple Wheatstone network, best-response signals have been computed under first-best, full information, public-signal and optimal information structure scenarios [22]. In the case of optimal information structures, partial state information is only revealed to the travelers, leading towards *information framing*. Similar results have been found in the case of Pigou networks (graphs with parallel routes between a single-source and a single-destination) in the presence of state uncertainty on one of the routes [23]. Optimal information structures have been found using Bayesian persuasion framework to reduce average traffic spillover on a specific route in a Pigou network. Despite these initial attempts, existing works in strategic information design in transportation settings has several limitations, as mentioned above.

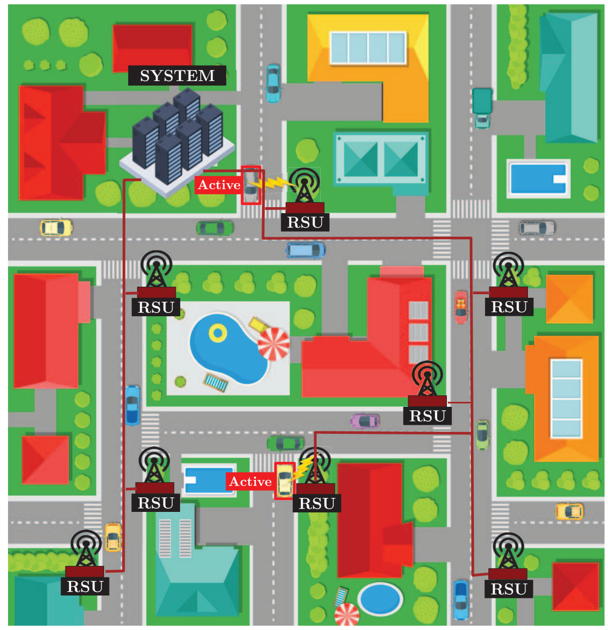

Fig. 1: Transportation System Model

III. Model and Problem Formulation

Consider a transportation network[1] $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{0, 1, \cdots, N\}$ represents the set of physical locations (vertices), and $\mathcal{E}$ represents the transport interconnections (edges) between various locations in $\mathcal{V}$. Assume that there are Λ_t travelers at time t, where each traveler $\ell \in \{1, \cdots, \Lambda_t\}$ is characterized by their origin $o_\ell \in \mathcal{V}$ and their target destination $d_\ell \in \mathcal{V}$. Without any loss of generality, let $\mathcal{P}_\ell$ denote the set of all paths from o_ℓ to d_ℓ on $\mathcal{G}$. Assume that the ℓ^{th} traveler picks a route $p_{\ell,t} \in \mathcal{P}_\ell$, and has traveled along a sequence of edges $p_{\ell,1:t}$ until time t. For simplicity, assume that the travelers can update their route plans only when they arrived at some intermediate vertex along the chosen route plan. Such travelers are labeled as *active*, and are illustrated with a red box in Figure 1. Let $\alpha_{\ell,t}$ denote the state of the ℓ^{th} traveler, where

$$\alpha_{\ell,t} = \begin{cases} 1 & \text{if the } \ell^{th} \text{ traveler is active,} \\ 0 & \text{if the } \ell^{th} \text{ traveler is inactive.} \end{cases} \tag{1}$$

[1] Note that this network model also supports a multi-modal transportation network which is typically modeled as a multi-layered graph, where the m^{th} layer consists of the m^{th} unimodal subgraph $\mathcal{G}_m$, and switch edge sets $\mathcal{E}_{i,j}$ interconnect i^{th} unimodal subgraph to the j^{th} unimodal subgraph.

Assume that vertices located at intersections are equipped with road side units (RSUs), which are all managed by a centralized traffic manager (a.k.a. system), as shown in Figure 1. Active travelers report their current location and path choices to the system through the nearest RSU, as well as obtain strategic information in return. Based on reported locations from all the travelers, the system determines the congestion state $c_{e,t}$ by evaluating the total number of inactive travelers present on edge $e \in \mathcal{E}$ at time t. Let $s_t = \{c_{e,t}\}_{e \in \mathcal{E}}$ represent the overall network state at time t. Given its current state $c_{e,t}$, the edge e returns a K-attribute cost $\boldsymbol{x}(c_{e,t}) = [x_1(c_{e,t}), \cdots, x_K(c_{e,t})]$. Examples of such attributes include *travel time* and *carbon emissions*. Specifically, given any edge e with $c_{e,t}$ travelers on it, travel time $T_{e,t}$ on edge e at time t can be estimated using Bureau of Public Roads (BPR) formula [24]:

$$T_{e,t}(c_{e,t}) = f_e \left[1 + \alpha \left(\frac{c_{e,t}}{c_e^*} \right)^{\beta} \right], \tag{2}$$

where c_e^* is the capacity[2] of the edge e, f_e denotes the free-flow travel time of edge e, and α, β are constants which usually take the values of 0.15 and 4 respectively. On the contrary, the rate of carbon emissions per vehicle can be estimated using a static emission model proposed in as follows:

$$C_{e,t}(T_{e,t}) = 0.2038 \cdot T_{e,t} \cdot \exp\left(\frac{0.7962 \cdot L_e}{T_{e,t}} \right), \tag{3}$$

where L_e is the length (in kilometers) of edge e and $T_{e,t}$ is the estimated travel time (in minutes) of edge e as defined in Equation (2). In the case of multi-modal transportation networks, certain cost attributes are not valid for some edges that support specific transport modalities. For example, carbon emissions is not applicable to walking trails. In such cases, assume that the edge cost with respect to such attributes is zero without any loss of generality.

Note that each cost attribute is measured using a different unit. For example, travel time $T_{e,t}$ is measured in minutes, while carbon emissions $C_{e,t}$ is measured in grams per vehicle per hour. Therefore, for simplicity, assume that each traveler combines attribute-wise costs linearly using an appropriate coefficient such that all the costs are equivalently measured in ***dollars***, as shown in [25]. In other words, the ℓ^{th} traveler evaluates a cost on edge e at time t as

$$z_\ell(c_{e,t}) = \sum_{k=1}^{K} b_{\ell,k} \cdot x_k(c_e, t), \tag{4}$$

where $b_{\ell,k}$ is the appropriate unit-conversion coefficient that enables the travelers to subjectively fuse indivudual attribute costs into an aggregate monetary cost. On the contrary, government agencies (e.g. transportation departments) typically measure the social cost differently than travelers' subjective aggregates. Therefore, assume that there exists a central authority (*a.k.a.* system) whose goal is to mitigate inefficiencies present within the transportation system. Therefore, the system also measures the social cost of edge e at time t in a heterogeneous manner as

$$y(c_{e,t}) = \sum_{k=1}^{K} a_k \cdot x_k(c_{e,t}), \tag{5}$$

where a_k is the appropriate unit-conversion coefficient used by the system. In practice, the system (e.g. transportation department) has access to sensing infrastructure across the network to measure the network state in real-time. Therefore, this paper assumes that the system has perfect knowledge about the current network state s_t.

Without any loss of generality, let $\psi_{e,t}(\eta, \lambda | \Lambda_t)$ denote the system's belief regarding the state of edge e being $c_{e,t} = \lambda$ at time t, which can be evaluated recursively as shown in Section IV. However, if the ℓ^{th} traveler is deemed active, assume that the system presents a tensor signal $\boldsymbol{\mu}_{\ell,t} = [\mu_{\ell,e,t}]_{e \in \mathcal{E}}$ where

$$\mu_{\ell,e,t}(\eta, \lambda) = \Big[\mathbb{P}_\ell(c_{e,t+1} = \lambda | c_{e,t} = \eta) \Big]_{\lambda,\eta=0}^{c_e}, \tag{6}$$

is of the form of a state transition probability matrix. The main goal of the system is to design best-response tensor signals $\boldsymbol{\mu}_T = \{\boldsymbol{\mu}_{1,T}, \cdots, \boldsymbol{\mu}_{\Lambda_T,T}\}$ to all active travelers such that the social cost of the overall network at time T, defined as

$$U_{0,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}_T\right) \;=\; \sum_{t=1}^{T} \sum_{e \in \mathcal{E}} \sum_{\lambda=0}^{|\Lambda_t|} \psi_{e,t}(\eta, \lambda | \Lambda_t) \cdot y(\lambda), \tag{7}$$

is minimized. Here, $\boldsymbol{p}_T = \{p_{1,T}, \cdots, p_{\Lambda_T,T}\}$ denotes the path profile chosen by the travelers, and $\delta_{e,t}(\lambda)$ denotes the *a priori* system's belief probability regarding the state of edge e being $c_{e,t} = \lambda$ at time t.

On the other hand, let $\phi_{\ell,e,t}(c)$ denote the traveler's belief probability regarding the state that edge $e \in \mathcal{E}$ has $c_{e,t} = c$ travelers at time t, i.e., $\phi_{\ell,e,t}(c) = \mathbb{P}_\ell(c_{e,t} = c)$. Then, the ℓ^{th} traveler's expected cost is given by

$$\begin{aligned} U_{\ell,T}\left(\boldsymbol{\mu}_{\ell,T}, \boldsymbol{p}_T\right) = & \sum_{e \in \boldsymbol{p}_{\ell,1:T}} z_\ell(c_{e,t_{\ell,e}}) \\ & + \sum_{e \in \boldsymbol{p}_{\ell,T} - \boldsymbol{p}_{\ell,1:T}} \left(\sum_{\lambda=1}^{|\Lambda_t|} \phi_{\ell,e,t}(\lambda) \cdot z_\ell(\lambda) \right), \end{aligned} \tag{8}$$

where $t_{\ell,e}$ is the time at which the ℓ^{th} traveler is at the tail of edge e, and $\boldsymbol{p}_{\ell,T} - \boldsymbol{p}_{\ell,1:T}$ represents the sequence of edges that the traveler will travel in the future, if he/she continues to stay on the same decision policy $\boldsymbol{p}_{\ell,T}$. Note that the first term in $U_{\ell,T}\left(\boldsymbol{\mu}_{\ell,T}, \boldsymbol{p}_{\ell,T}\right)$ is the incurred (deterministic) cost from traversed, while the latter term is the future (unknown) cost from the remaining path to be traversed. In order to also capture the deviations observed in human decision making, assume that the ℓ^{th} traveler minimizes a stochastic cost

$$V_{\ell,T}(\boldsymbol{\mu}_{\ell,T}, \boldsymbol{\pi}_T) = \mathbb{E}_{\boldsymbol{\pi}_{\ell,T}}\left[U_{\ell,T}\left(\boldsymbol{\mu}_{\ell,T}, \boldsymbol{p}_T\right) \right] + \epsilon_{\boldsymbol{p}_{\ell,T}}, \tag{9}$$

where $\boldsymbol{\pi}_{\ell,T}$ is a mixed probability distribution over the set of all paths $\mathcal{P}_\ell$, and $\epsilon_{\boldsymbol{p}_{\ell,T}}$ is independently and identically distributed Gumbel noise to capture extreme-value uncertainties.

[2]Capacity of an edge is the maximum number of vehicles it can support at any time, beyond which congestion occurs.

Putting together the decision rationalities, the interaction between the system and the travelers can be modeled as a one-shot Stackelberg-Quantal-Response (SQR) game [26], which is formally defined below.

Problem 1. *The equilibrium of an SQR game between the system and travellers is defined as the pair* $(\mu^*_{\ell,t}, \pi^*_{\ell,t})$*, where*

$$\begin{aligned} \boldsymbol{\mu}^*_T &= \arg\min_{\boldsymbol{\mu}_T} U_{0,T}\left(\boldsymbol{\mu}_T, \boldsymbol{p}^*_T(\boldsymbol{\mu}_T)\right), \\ \boldsymbol{\pi}^*_{\ell,t}(\boldsymbol{\mu}_{\ell,T}) &= \arg\min_{\boldsymbol{\pi}_{\ell,t}} V_{\ell,T}(\boldsymbol{\mu}_{\ell,T}, \boldsymbol{\pi}_{\ell,T}), \\ &\qquad \text{for all } \ell \in \{1, \cdots, \Lambda_T\}, \end{aligned} \tag{10}$$

where $\boldsymbol{p}^*_T(\boldsymbol{\mu}_T)$ *is the profile of paths sampled and chosen by travelers at time* T *using their respective quantal response strategies,* $\boldsymbol{\pi}^*_{\ell,t}(\boldsymbol{\mu}_{\ell,T})$ *for all* $\ell \in \{1, \cdots, \Lambda_T\}$*.*

The complexity of solving Problem 1 stems from both stages of the game, as shown in Equation (10). While the latter stage involves computing a logit equilibrium which is known to be NP-Hard [27] in general, the first stage is a non-linear optimization problem (NP-Hard as well) over the set of row-stochastic matrices whose size is proportional to the size of the transportation network.

IV. System Dynamics and Belief Updating

This section focuses on two different dynamics: (i) how the system updates its state transition probability $\psi_{e,t}(\eta, \lambda)$, and (ii) how travelers update their respective beliefs $\phi_{l,t}$.

A. State Transition Dynamics at the System

Let $\rho_{\ell,t}(e, e')$ denote the probability that the ℓ^{th} traveler is present on edge e at time t given that he/she is on edge e' at time $t-1$. In other words, we have

$$\rho_{\ell,t+1}(e, e', \alpha_{\ell,t}) = \sum_{p_{\ell,t} \in \mathcal{P}_{l,t}} \pi_{\ell,t}(p_{\ell,t} | \mu_{\ell,t}, e \in p_{\ell,t}, e' \in p_{\ell,t-1}, \alpha_{\ell,t} = 1). \tag{11}$$

Given $c_{e,t}$ at time t on every edge $e \in \mathcal{E}$, we can now compute the state transition probability $\psi_{e,t+1}$ recursively as follows:

$$\begin{aligned} \psi_{e,t+1}(\eta, \lambda | \Lambda_t) &= \mathbb{P}\left(c_{e,t+1} = \lambda \mid c_{e,t} = \eta\right) \\ &= \mathbb{P}\left(\sum_{\ell=1}^{\Lambda_{t+1}} \mathbb{1}(e_{\ell,t+1} = e) = \lambda \;\middle|\; \sum_{\ell=1}^{\Lambda_t} \mathbb{1}(e_{\ell,t} = e) = \eta \right) \\ &= \rho_{\ell,t+1}(e, e') \cdot \psi_{e,t+1}(\eta, \lambda - 1 | \Lambda_t - 1) \\ &\qquad + \left(1 - \rho_{\ell,t+1}(e, e')\right) \cdot \psi_{e,t+1}(\eta, \lambda | \Lambda_t - 1). \end{aligned} \tag{12}$$

B. Traveler's Belief Update

The ℓ^{th} traveler infers the network state s_t at time t via constructing a row-stochastic belief

$$\phi_{\ell,t} = \left[\phi_{\ell,e,t}\right]_{e \in \mathcal{E}},$$

Algorithm 1 Full Cost Matrix Computation (for LoRI-v1)

ℓ.FindTravelerCostMatrix_Full(G, Λ_t)

// G = road network object in adjacency list repr.
// Λ_t = list of all travelers on the network at time t
// ℓ = traveler id for which cost matrix is computed

// Compute all available paths from current location to destination at each traveler
for each traveler $i \in \Lambda_t$
  $\mathcal{P}_i = i$.FindAllPaths(G)

// Compute path profiles based on all travelers except ℓ
$\mathbb{P}_\ell = \ell$.PathProfile(Λ_t)

// Predict the number of travelers on G at $t = 1, \cdots$
for each $i \in \Lambda_t$
  i.PredictTravelerLocationInTime($\mathcal{P}_\ell, \mathbb{P}_\ell$)
G.SetPredictedNetworkState(Λ_t)

// Evaluate multi-attribute costs at each edge in G
G.EdgeCostListInTime

// Compute $\boldsymbol{Z}_\ell = \{z_{\ell,1}, z_{\ell,2}, \cdots\}$ using multi-attribute costs in G
for each path index $p \in \mathcal{P}_\ell$
  for each profile index $q \in \mathbb{P}_\ell$
    $\boldsymbol{Z}_\ell[p, q] = \ell$.SetCostMatrix($G, p, q$)
return $\boldsymbol{Z}_\ell$

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where $\phi_{\ell,e,t} = \left\{\phi_{\ell,e,t}(c)\right\}_{c=0}^{\infty}$ is the traveler's belief vector regarding the state of the edge $e \in \mathcal{E}$ at time t, i.e., $\phi_{\ell,e,t}(c) = \mathbb{P}_\ell(c_{e,t} = c)$. In other words, given the system's signal $\boldsymbol{\mu}_{\ell,t}$, the ℓ^{th} traveler updates his/her prior belief using Bayes rule to obtain their posterior belief, as shown below:

$$\phi_{\ell,e,t+1}(\lambda) = \frac{\phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}{\sum_{\lambda=0}^{\infty} \phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}. \tag{13}$$

Assuming that the denominator in Equation (13) always converges to some value in the region $[0, 1]$, every traveler's belief regarding the future state of the network remains stationary until the system presents a signal.

V. Proposed Strategic Information Design

The equilibrium of the proposed SQR game is evaluated using backward induction, i.e. evaluate travelers' QRE as a function of system's signal, and then compute the best-response signal at the system.

A. Traveler's Quantal Response Analysis

The cost that the traveler attains by choosing a path $p_{\ell,t} \in \mathcal{P}_{\ell,t}$ is decomposed into (i) known (nominal) cost at the traveler, and (ii) an unknown random cost $\epsilon_{p_{\ell,T}}$, as shown in Equation (9). In this paper, we assume that the noise term $\epsilon_{p_{\ell,T}}$

Algorithm 2 Partial Cost Matrix Computation (for LoRI-v2)

ℓ.FINDTRAVELERCOSTMATRIX_PARTIAL(G, Λ_t)

// G = road network object in adjacency list repr.
// Λ_t = list of all travelers on the network at time t
// ℓ = traveler id for which cost matrix is computed

// Compute all available paths from current location to destination at each traveler
for each traveler $i \in \Lambda_t$
  $\mathcal{P}_i = i$.FINDALLPATHS(G)

// Compute path profiles based on all travelers except ℓ
$\mathbb{P}_\ell = \ell$.PATHPROFILE(Λ_t)

// Predict the number of travelers on G at $t = 1, \cdots$
for each $i \in \Lambda_t$
  i.PREDICTTRAVELERLOCATIONINTIME($\mathcal{P}_\ell, \mathbb{P}_\ell$)
G.SETPREDICTEDNETWORKSTATE(Λ_t)

// Evaluate multi-attribute costs at each edge in G
G.EDGECOSTLISTINTIME

// Compute current incident edge sets for all travelers
$S(\ell) = G.\mathit{IncidentEdges}(\ell)$
$\tilde{\Lambda}_{\ell,t} = [\ ]$ // Initialize list of influential active travelers
for $i \neq \ell$
  if $S(\ell) \cap G.\mathit{IncidentEdges}(i) \neq \emptyset$
    $\tilde{\Lambda}_{\ell,t}$.APPEND(i)

// Compute path profiles of all influential travelers to ℓ
$\tilde{\mathbb{P}}_\ell = \ell$.PATHPROFILE($\tilde{\Lambda}_{\ell,t}$)

// Compute reduced $\tilde{\boldsymbol{Z}}_\ell = \{z_{\ell,1}, z_{\ell,2}, \cdots\}$ using multi-attribute costs in G
for each path index $p \in \mathcal{P}_\ell$
  for each profile index $q \in \tilde{\mathbb{P}}_\ell$
    $\tilde{\boldsymbol{Z}}_\ell[p, q] = \ell$.SETCOSTMATRIX($G, p, q$)
return $\tilde{\boldsymbol{Z}}_\ell$

in the traveler's expected cost is independently, identically distributed extreme value, also known as Gumbell distribution. Consequently, the travelers evaluate quantal path probabilities (as opposed to best responses), which is formally stated in the following theorem.

Theorem 1 ([8]). *The ℓ^{th} traveler's logit choice probability for the path $p_{\ell,t}$ at time t is given by:*

$$\pi_{\ell,T}(p_{\ell,T}) = \frac{\exp\left(\alpha \cdot U_{\ell,T}(\mu_{\ell,T}, p_{\ell,T})\right)}{\sum\limits_{p'_{\ell,T} \in \mathcal{P}_{\ell,T}} \exp\left(\alpha \cdot U_{\ell,T}(\mu_{\ell,T}, p'_{\ell,T})\right)}, \quad (14)$$

where $\alpha \geq 0$ is the parameter of the quantal response model.

In this paper, QRE amongst active travelers is evaluated using a software package called Gambit [28], a library of game theory software and tools for the construction and analysis of finite extensive and strategic games. The strategic game played amongst all the active travellers in the second stage in the SQR game stated in Problem (1) is computed using GAMBIT-LOGIT function, which uses a predictor-corrector method to compute QRE in normal-form games, as proposed in [9]. In the prediction stage, this method generates a choice prediction using differential equations describing the principle branch of the logit response correspondence in the first stage. In the next stage, the corrector step refines the choice prediction using Newton's method to find the zero of Equation (14).

Note that the computation of logit probabilities defined in Theorem 1 is a well-known NP-Hard problem. Therefore, novel approximation methods are necessary for scalable signaling algorithms, particularly when there are large number of travelers on a large road network. This paper presents two different approaches based on how nominal cost matrices are computed at the travelers. However, note that both these approaches are used to compute cost matrices of active travelers alone for the sake of mitigating computational complexity.

1) Full Cost-Matrix Computation: In the first approach, the full cost-matrix of each traveler is computed for a given profile of paths selected by all the travelers. This matrix is formatted in such a way that $U_{\ell,t}$ has $|\mathcal{P}_{\ell,t}|$ rows and $\prod\limits_{i \neq \ell} |\mathcal{P}_{i,t}|$ columns. In other words, given a path choice made by the ℓ^{th} traveler and the residual profile of all the remaining inactive travelers in Λ_t, each cost entry in the $U_{\ell,t}$ matrix is the computed via predicting the travelers' locations for all $t = 1, \cdots, T$. Given the location predictions of all the travelers at each time t, the total number of travelers on edge $e \in \mathcal{E}$ at time t is predicted over the entire simulation time. Based on the predicted network state, various cost attributes (e.g. travel time, CO emissions) are predicted for different edges at each time $t = 1, \cdots, T$ and then combined together into the ℓ^{th} traveler cost matrices for each time $t = 1, \cdots, T$, as stated in Equation (4). These matrices are compiled as a list and returned by Algorithm 1. If the traveler's cost matrix is computed using Algorithm 1 in LoRI (Algorithm 3), we label it as ***LoRI-v1***.

2) Partial Cost-Matrix Computation: Note that the full cost-matrix causes a significant bottleneck in the computation of cost matrices in Algorithm 1 due to its exponential increase in size with the number of active travelers. Specifically, if there are Λ active travelers, each with p route choices, then the size of the cost matrix will be $p \times p^\Lambda$. In an attempt to alleviate this complexity, a new method is proposed to evaluate a reduced cost matrix that considers all other active travelers who can only *influence* the ℓ^{th} traveler at that moment. Formally, if $S(\ell)$ represents the set of all edges that are incident to the ℓ^{th} traveler's current vertex (assuming that the ℓ^{th} traveler is active), then the reduced cost matrix is constructed using path profiles compiled by all other travelers such that

$$S(i) \cap S(\ell) \neq \emptyset, \quad \text{for all } i \neq \ell.$$

$$
\begin{aligned}
U_{0,T} &= \sum_{t=1}^{\tau}\sum_{e\in\mathcal{E}} y(c_{e,t}) + \sum_{t=\tau+1}^{T}\sum_{e\in\mathcal{E}}\sum_{\lambda=1}^{\infty}\Big[\rho_{\ell,t}(e,e')\cdot\psi_{e,t}(\eta,\lambda-1|\Lambda_{t-1}-1) \\
&\qquad\qquad +\Big(1-\rho_{\ell,t}(e,e')\Big)\cdot\psi_{e,t}(\eta,\lambda|\Lambda_{t-1}-1)y(\lambda)\Big] \\
&= \sum_{t=1}^{\tau}\sum_{e\in\mathcal{E}} y(c_{e,t}) + \sum_{t=\tau+1}^{T}\sum_{e\in\mathcal{E}}\sum_{\lambda=1}^{\infty}\left[\left(\sum_{p_{\ell,t}\in\mathcal{P}_{l,t}}\pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t})\right)\cdot\psi_{e,t}(\eta,\lambda-1|\Lambda_{t-1}-1)\right. \\
&\qquad\qquad \left.+\left(1-\sum_{p_{\ell,t}\in\mathcal{P}_{l,t}}\pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t})\right)\cdot\psi_{e,t}(\eta,\lambda|\Lambda_{t-1}-1)y(\lambda)\right]
\end{aligned} \tag{15}
$$

This method is formally presented in Algorithm 2. If the traveler's cost matrix is computed using Algorithm 2 in LoRI (Algorithm 3), we label it as ***LoRI-v2***.

B. Best-Response Signaling

LoRI is the proposed approach that finds approximate best-response signaling scheme at the sender, which is summarized in Algorithm 3. Note that Algorithm 3 is called *LoRI-v1*, if Line 7 is implemented using $i.\mathit{FindTravelerCostMatrix\_Full}(G,\Lambda_t)$ found in Algorithm 1. On the other hand, if Line 7 in Algorithm 3 is implemented using $i.\mathit{FindTravelerCostMatrix\_Partial}(G,\Lambda_t)$ found in Algorithm 2, we have *LoRI-v2*.

The crux of LoRI is to compute the leader's optimal strategy via solving the following problem:

$$
\min_{\mu_{\ell,T}} \quad U_{0,T}\left(\mu_\ell,\mu_{-\ell},\pi_{\ell,T}(p_{\ell,T}|\mu_{\ell,T}),p_{-\ell,T}\right) \tag{P1}
$$

Using Equation (12), we write the term $\psi_{e,t}(\lambda)$ and expand $U_{0,T}$ as shown in Equation (15), and further expand this using Equation (11). For better representation, we write $\rho_{\ell,t+1}(e,e',\alpha_{\ell,t}) = \sum_{p_{\ell,t}\in\mathcal{P}_{l,t}}\pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t})$. By Definition 6, $\mu_{\ell,t}=[\mu_{\ell,e,t}]_{e\in\mathcal{E}}$ is a vector of $\mu_{\ell,e,t}$ for every edge $e\in\mathcal{E}$ in the network. $\mu_{\ell,e,t}=[\mathbb{P}_\ell(c_{e,t+1}=\lambda|c_{e,t}=\eta)]_{\lambda=0}^{c_e}$ is a vector of probabilities $\mathbb{P}_\ell(c_{e,t+1}=\lambda|c_{e,t}=\eta)$ for all possible values of λ. We assume that the upper bound of λ is the capacity c_e of the edge e. Since the system has a cost minimization rationality, it will send a signal $\mu_{\ell,t}$ at time t to the ℓ^{th} traveler such that the path chosen by the traveler minimizes the system's overall cost. Since we have a leader-follower game, we use backward induction to solve for the optimal leader strategy, i.e., the optimal signal at the system. System cannot send signal to every traveller at the same time as every traveller's decision depend on all the other travellers as well. Therefore, for every time step, the system sends signals $\mu_{\ell,t}$ to travelers in a round-robin fashion.

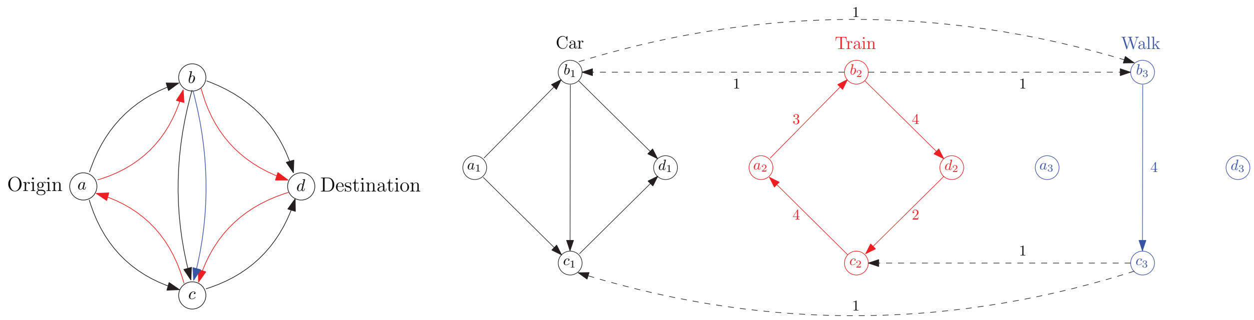
The search space in this optimization problem comprises of all right stochastic matrices which can be shown as a convex set. However, it is analytically hard to verify whether or not, the objective function $U_{0,T}$ stated in Equation (15) is convex in μ. Note that the term $\pi_{\ell,t}$ represents logit probabilities which are known to be non-convex. Equation (15) comprises of convex combination of sum of logit probabilities whose convexity properties are hard to verify. Therefore, we employ interior point algorithms to compute the approximate signal. In our simulation experiments, we use CVXPY [29] package to implement interior point search in Algorithm 3.

Algorithm 3 LoRI (for both versions)

```
LoRI(G, Λ_t)
1  // G = game object with road network in adj. list repr.
2  // Λ_t = list of all travelers on the network at time t

3  // Compute the QRE amongst all active travelers
4  ℂ = [ ] // Initialize ℂ as an empty list
5  for i ∈ Λ_t
6      if i.IsActive
7          C_i = i.FindTravelerCostMatrix(G, Λ_t)
8          ℂ.append(C_i)
9  π*_t = QRE(ℂ)

10 // Sample a path for each traveler from the quantal
       response π*_t
11 p*_t = π*_t.sample()

12 // Compile all the travelers paths in a list ℙ
13 ℙ = G.GetInactiveTravelerPaths()
14 ℙ.append(p*_t)

15 // Compute the system's utilities for different signals,
       given a path profile
16 U_{0,t} = G.SetSystemUtility(G, Λ_t, ℙ)

17 // Compute the best response signal for each traveler
18 μ*_t = U_{0,t}.min.key()

19 return μ*_t
```

VI. Validation Methodology and Datasets

In this paper, two types of transportation networks are simulated to empirically evaluate the performance of the following four algorithms: (i) LoRI-v1, (ii) LoRI-v2, (iii) Full Information Disclosure (labeled *Full_Info*) (iv) Dijkstra's



(a) Network with Nodes as Physical Locations

(b) Multi-Layered Expansion Model of Figure 2a

Fig. 2: Multi-Modal Wheatstone Transportation Network

shortest path algorithm[3] (labeled *SSSP*). In both networks, LoRI is designed based on two cost attributes, namely *Travel Time* (ref. to Eqn. (2)), and *CO Emissions* (ref. to Eqn. (3)).

A. Multi-Modal Wheatstone Network

Given its significance in the selfish routing literature due to Braess' Paradox, a multi-modal Wheatstone network is considered for validation in this paper. Assume that the network consists of four location vertices and ten interlinking edges, as illustrated in Figure 2a. Let there be $M = 3$ transport modalities supported by this network, and $\mathcal{M}$ ={*Car* (colored black), *Train* (colored red) and *Walk* (colored blue)}. Using unimodal subgraphs and switch edges (depicted using dashed lines), we expand the example network into a multi-layered graph $\mathcal{G}_{exp.}$, as shown in Figure 2b. The travel time for edges that support *train* mode can be extracted from their arrival and departures time. In our example network, as illustrated in Figure 2b, travel times for the train mode are assumed as $\{a \to b : 3, b \to d : 4, d \to c : 2, c \to a : 4\}$. For simplicity, we assume CO emissions per traveler on the *train* to be $0.5 * travel\_time$, since the train consumes the same amount of fuel irrespective of the number of travelers availing that service. For the edge corresponding to *walking* modality, we assume the travel time $\{b \to c : 4\}$ and CO emissions to be simply 0. In this example, we consider the total cost of traveling on a switch edge to be 1. On the contrary, we model edge costs that support *car* mode of transportation using two attributes, namely travel time (as defined in Equation (2)) and CO emissions (as defined in Equation (3)) which are evaluated using the number of travelers $c_{e,t}$ on edge e at time t.

On this network, two different simulation scenarios are considered. In *Scenario 1*, three travelers are simulated with unique origin-destination pairs: $\{(a,d),(d,b),(c,b)\}$, on LoRI, Full-Info and SSSP algorithms. On the contrary, *Scenario 2* simulates 35 random travelers across the entire multi-modal transportation network with x number of travelers interacting directly with LoRI, while all the other travelers $(35 - x)$ interact with SSSP. We assume the system's weight for travel time to be 0.7. Any specific details about the two scenarios will be provided in Section VII, where results from these experiments are presented in discussed in detail.

[3]Note that variants of Dijkstra's algorithm are currently used in most state-of-the-art navigation systems to minimize travelers' costs in a selfish manner

Fig. 3: Section of Manhattan Network

B. Manhattan Network

In order to reliably study the scalability of both *LoRI-v1* and *LoRI-v2* on a real transportation network, simulation experiments are also conducted on a small portion of Eastern portion of Downtown Manhattan network in New York City, as shown in Figure 3. The network itself is obtained from Open Street Maps and converted to graphml format using a Python package called *igraph*. This network supports two transport modes $\{Car, Walk\}$, and consists of 4410 vertices and 15089 edges of car, walk and switch modes. Similar to *Scenario 1* in the multi-modal Wheatstone network, 10 different travelers with varying motives and origin-destination pairs are simulated on the chosen Manhattan sub-network in such a way that the potential paths between these 10 travelers overlap at some point. More specifics are provided in Section VII, alongside the discussion of experimental results.

Both simulation experiments are implemented in Python using the following packages: python-igraph v0.9.6 (to maintain the network graph and run Dijkstra's algorithm), gambit v16.0.1 (to compute QRE), cvxpy v1.1.14 (to solve the nonlinear minimization problem in LoRI algorithm), numpy v1.21.1 (for supporting basic mathematical operations), matplotlib v3.4.3 (for plotting) and all their dependencies.

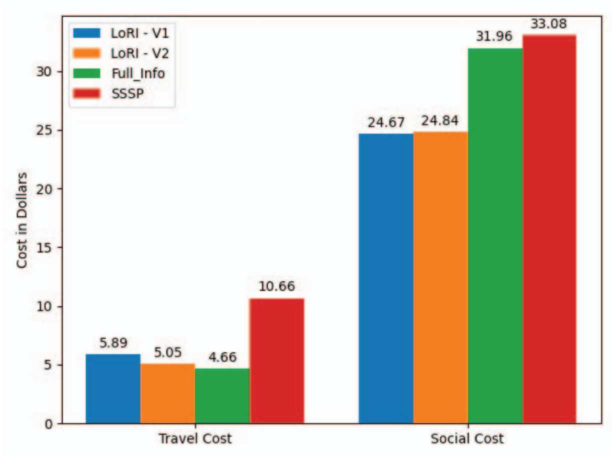


Fig. 4: Average Costs at the System and Traveler due to LoRI, Full Information Disclosure and Dijkstra's Algorithm in Experiment 1.1

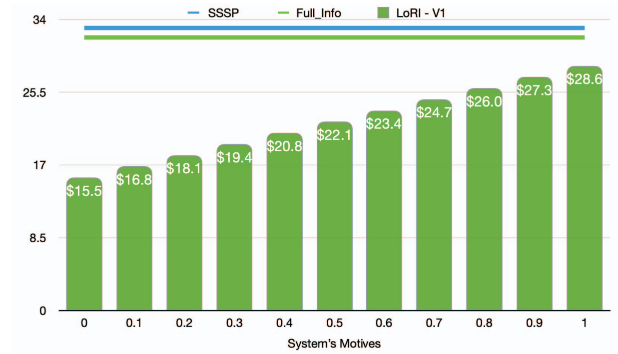


Fig. 5: System's Social Cost across Different Motives due to LoRI, Full_Info and SSSP in Experiment 1.2 on Wheatstone Network

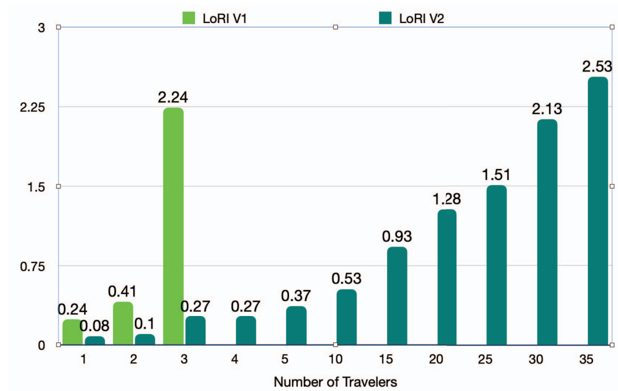


Fig. 6: Runtime Comparison of LoRI-v1 and LoRI-v2 for Varying Number of Travelers in Scenario 2 on Wheatstone Network

VII. Results and Discussion

A. Results on Multi-Modal Wheatstone Network

Two experiments are performed in *Scenario 1*: (a) In Experiment 1.1, the system's weight for travel time is assumed to be 0.7 within the context of simulating the LoRI-v1 and LoRI-v2 algorithms. We compute the empirical average costs across different traveler motives at both traveler and system, and plot them in Fig. 4. Note that the travelers experience a significant cost reduction since LoRI considers multi-modal costs, as opposed to SSSP (Dijkstra's algorithm) which is typically run on networks with travel-time based edge weights. Another important observation is that full information disclosure is also harmful for the system, since it could lead travelers towards selfish decisions, as opposed to socially optimal choices. However, LoRI-v1 and LoRI-v2 produces significant reduction in costs at both the system and the travelers. Although there is little loss in performance due to LoRI-v2, the gap in costs due to LoRI-v2 when compared to full information disclosure and Dijsktra's algorithm is quite significant. (b) In Experiment 1.2, social costs are evaluated by the system for all the three algorithms using Eqn. (7) under different motives, which are represented by varying travel time weights from 0 to 1.0, in steps of 0.1. Fig. 5 depicts the system's social cost of LoRI in comparison to that of *Full_Info* and *SSSP*. Note that LoRI consistently outperforms both *Full_Info* as well as *SSSP* across varying system motives. Specifically, since both *Full_Info* and *SSSP* algorithms choose paths that minimize travel time selfishly at each traveler, the path choices remain fixed irrespective of the system's motive, thus resulting in a constant, but high system cost. On the contrary, if the system gives a higher weight to carbon emissions (i.e. move left on the x-axis in Fig. 5), then LoRI offers a significant gain over *Full_Info* and *SSSP* by reducing the system cost drastically. In the case where the weight on travel time is zero, system cost reductions are as high as about 50% in comparison to that of *Full_Info* and *SSSP*.

LoRI-v1 and/or LoRI-v2 outperforms full information disclosure and Dijkstra's algorithm in terms of system costs, at the expense of computational efficiency. This is demonstrated with our experiment results in *Scenario 2*. As shown in Fig. 6, both LoRI-v1 and LoRI-v2 are computationally expensive, and does not scale very well on networks with large number of travelers. However, LoRI-v2 scales significantly better in runtime than LoRI-v1 with increasing number of travelers in the network. The experiment is limited to 35 travelers on a multi-modal Wheatstone bridge since the network becomes completely congested beyond this point.

B. Results on Manhattan Network

The results obtained on the Manhattan network simulation (labeled Experiment 2) is very similar to the ones found on multi-modal Wheatstone network network. Firstly, Table I presents the average costs at both the travelers and the system due to LoRI, Full Information and Dijkstra's algorithm. Note that the system costs reduce by almost 14% when the travelers interact with LoRI when compared to Dijkstra's algorithm or full information disclosure. In Fig. 7, the system's social costs are evaluated across different system motives for all the three algorithms: LoRI, *Full_Info* and *SSSP*. Similar to the results found in the multi-modal Wheatstone network, LoRI outperforms *Full_Info* and *SSSP* by a significant factor,

Algorithm	Traveler Cost	System Cost
SSSP	\$3.276	\$32250.03
Full_Info	**\$1.26**	\$31122.4
LoRI	\$1.350	**\$27572.46**

TABLE I: Comparison of System and Traveler Costs due to SSSP, Full-Info and LoRI on Manhattan Network in Experiment 2

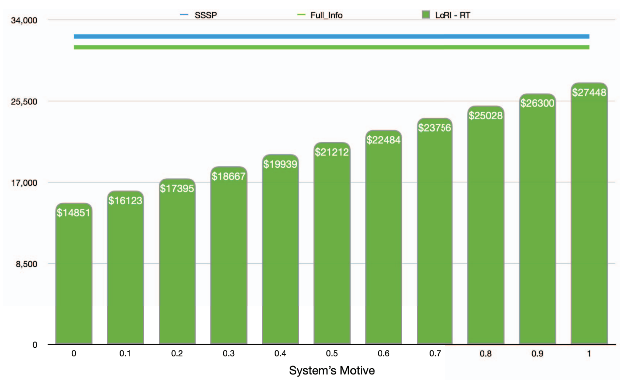


Fig. 7: System's Social Cost across Different Motives due to LoRI, Full_Info and SSSP in Experiment 2 on the Manhattan Network

especially when the system motives are not aligned with travel time. A tremendous gain of 50% improvement in system costs is observed in LoRI when the system motive is perfectly aligned with carbon emissions.

VIII. Conclusion and Future Work

In summary, we proposed a novel Stackelberg signaling framework to improve the inefficiency of selfish routing in the presence of behavioral agents. We modeled the interaction between the system and quantal response travelers as a Stackelberg game, and developed two novel approximate algorithms *LoRI-v1* and *LoRI-v2* that construct strategic, personalized information regarding the network state to steer travelers' route decisions towards social optimum. We demonstrated the performance of LoRI and compared with that of a Dijkstra's algorithm as well as full information disclosure on both multi-modal Wheatstone network and a segment of Manhattan network. We presented both the performance gain as well the tradeoff between system's costs and runtime due to LoRI. In the future, we will design computationally efficient, approximate algorithms that scale well on urban networks with large number of travelers. We will also investigate the impact of strategic information design when agents exhibit diverse rationalities and intervention preferences (e.g. route recommendations as in Dijkstra's algorithm, network state information as in full information disclosure and LoRI).

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