Missouri University of Science & Technology — Department of Computer Science
Spring 2021 — **CS 6001: Algorithmic Game Theory**

Homework 2: Markets

Instructor: *Sid Nadendla* **Due:** *March 18, 2021*

Problem 1 Combinatorial Auctions *5 pts.*

Consider combinatorial auctions for M items among N bidders, where each valuation is represented simply as a vector of $2^M - 1$ numbers (a value for each subset of items). Prove that the optimal allocation can be computed in time that is polynomial in the input length: $N(2^M - 1)$.

Hint: Use dynamic programming.

Problem 2 Profit-Maximizing Auctions *5 pts.*

1. Prove that, if a mechanism is truthful in expectation, then for any agent i and any fixed choice of bids by the other agents $\boldsymbol{b}_{-i}$, we have
   - the allocation $x_i(b_i, \boldsymbol{b}_{-i})$ is monotonically increasing, and
   - the payment $p_i(b_i, \boldsymbol{b}_{-i}) = b_i \cdot x_i(b_i, \boldsymbol{b}_{-i}) - \int_0^{b_i} x_i(z, \boldsymbol{b}_{-i}) dz$.
2. Show that the optimal single-item auction for bidders with i.i.d. valuations drawn from any cumulative distribution F is a Vickery's auction with a reservation price[1] $\phi^{-1}(0)$.

Problem 3 Stable Matching *5 pts.*

Prove (using a counter-example) that the deferred acceptance algorithm is not strategy-proof (truthfulness being the dominant strategy at equilibrium) for the females.

Problem 4 Programming Exercise *5 pts.*

- Implement in Python/MATLAB, both VCG auction and profit maximizing auction with $N \geq 1$ bidders, as functions and validate your implementations.
- Implement in Python/MATLAB, the deferred acceptance algorithm with $M \geq 1$ male and $N \geq 1$ female agents, and validate your implementations.

[1] Reservation price is a minimum price below which the item is not sold at all.