Missouri University of Science & Technology — Department of Computer Science

Spring 2023 — **CS 2500: Algorithms**

Sample Questions for Exam 1

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Midterm 1 will be designed as a 1-hour long in-class exam. You may have 3-4 questions to solve, depending on the size of the questions. This handout presents a few review questions for Midterm 1 examination on Mar 14, 2023.

Section 1 Asymptotic Notation

1. Definitions for Θ, O, Ω, o and ω notations.
2. Prove that $\sum_{i=1}^{n} i = \Theta(n^2)$.
3. If $f_1(n) = \Theta(g_1(n))$ and $f_2(n) = \Theta(g_2(n))$, prove that
$$f(n) = f_1(n) + f_2(n) = \Theta(g_1(n) + g_2(n)).$$

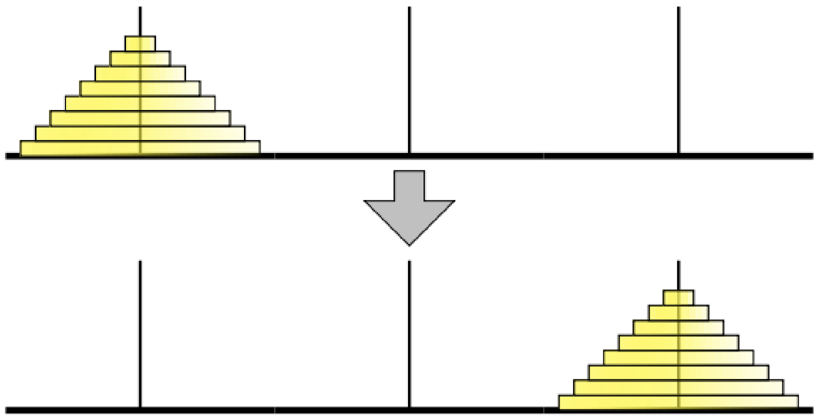
Section 2 Recursion

1. Using substitution method, prove that the recurrence relation $f(n) = 2f(n/2) + n$ results in $f(n) = O(n \log n)$.
2. Solve $T(n) = 2\,T(n/4) + n^2$ using recursion trees, and verify using Master's Theorem.
3. Solve $T(n) = T(n-1) + n^2$ using direct method.

Section 3 Divide & Conquer

1. Say, our goal is to find both maximum and minimum elements of a given input array A of size n efficiently. A naive approach is to find the maximum using $n-1$ comparisons and then find the minimum element over the remaining entries using $n-2$ comparisons. Therefore, this algorithm takes about $2n-3$ comparisons. How can we reduce the number of comparisons by adopting a divide-and-conquer approach?
2. The problem is to compute the product of two n-bit input arrays A and B, where n is significantly larger than the register size in your processor (e.g. Consider the product of 256-bit arrays over a 64-bit processor). In such a case, it is infeasible to run the traditional method for finding the product of two arrays. How would you solve this problem if the size of the register $m = \frac{n}{k}$, for some positive integer k?

3. There are n disks of different sizes arranged on a peg in decreasing order of sizes. There are two other empty pegs. The purpose of the puzzle is to move all the disks, one at a time, from the first peg to another peg in the following way. Disks are moved from the top of one peg to the top of another. A disk can be moved to a peg only if it is smaller than all other disks on that peg. In other words, the ordering of disks by decreasing sizes must be preserved at all times. The goal is to move all the disks in as few moves as possible.



   (a) Design an algorithm to find a minimal sequence of moves that solves the Towers of Hanoi problem for n disks.
   (b) Prove the correctness of your algorithm.
   (c) How many moves are used in your algorithm? Construct a recurrence relation for the number of moves, and solve it to compute the worst-case runtime.

   Note that only one of the above three sub-questions could be given in an in-class exam.

Section 4 Comparison Sorting

1. Write the pseudocode for INSERTION-SORT and prove its correctness.

2. Given two sorted input arrays A and B, how can we merge them into a sorted array? Write the pseudocode for your approach and prove its correctness.

3. Given any input array A, how can we partition it into two subarrays and a pivot element such that every entry in the left subarray is smaller than the pivot, and every entry in the right subarray is larger than the pivot? Write the pseudocode for your approach and analyze its worst-case run-time.

4. Prove that the worst-case run time for HEAP-SORT is $\Theta(n \log n)$.

5. If we were to employ divide and conquer approach to design a comparison sort, demonstrate various sorting algorithms when the pivot element q is always chosen to be

(a) mid-point of the array in each , i.e. MERGE-SORT,

(b) such that $A[1 : q - 1] \leq A[q] \leq A[q + 1 : n]$, i.e. QUICK-SORT,

for a given input array $A = [1,\ 4,\ 7,\ 6,\ 3,\ 9,\ 5,\ 2,\ 4,\ 6]$.

Section 5 Sorting in Linear Time

1. Prove that the running time for comparison sorts is $\Omega(n \log n)$.
2. Demonstrate COUNTING-SORT for the input array $A = [1,\ 4,\ 2,\ 5,\ 3,\ 1,\ 2,\ 4,\ 3,\ 1,\ 4]$.