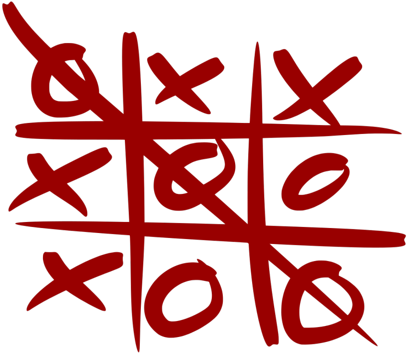
Topic 0: Course Introduction



What is Game Theory?

- Mathematical framework that models interactions between selfish (strategic) agents.
- Distributed Artificial Intelligence (AI): Multiple agents with potentially non-identical rationalities and motives perform autonomous actions.
- The final outcome depends on all the agents' actions.

Game theory focuses on analyzing the final outcome.

On the other hand, mechanism design focuses on designing systems where selfish agents converge to a desired outcome.

1713
Charles Waldegrave presents a minimax mixed strategy solution to a two-person version of the card game *le Her*.

1838
Antoine Augustin Cournot presents a solution (later known as Nash equilibrium) to a market duopoly.

1913
Ernest Zermelo proved that the optimal chess strategy is strictly determined.

1928
Birth of Game Theory: John Von Neumann publishes a paper titled *On the Theory of Games of Strategy* for continuous strategy spaces.

1938
Emile Borel proved a minimax theorem for two-person zero-sum matrix games, when the payoff matrix is symmetric.

1944
John Von Neumann and Oskar Morgenstern co-authors a seminal book titled *Theory of Games and Economic Behavior.*

1947
The second edition of *Theory of Games and Economic Behavior* was published, which presented the derivation of expected utility.

1950-51
John Nash proposes a new solution (later known as *Nash equilibrium*) for n-person games. Merrill Flood and Melvin Dresher investigates *Prisoner's Dilemma* in RAND Corp.

1950s
Lloyd Shapley studies the value of n-person games, investigates stochastic games. Later, in early 1960s, he also investigates potential games and stable-marriage problem.

A Brief History of Game Theory

1960s
Thomas Schelling studies bargaining and introduces the notion of conflict. John C. Harsanyi studies Bayesian games. Reinhard Selten introduced the concept of subgame perfect equilibrium.

1970s
Robert J. Aumann studies repeated games, and defines correlated equilibrium. John Maynard Smith introduces evolutionary stable strategies. Leonid Hurwicz studies incentive compatibility in mechanism design.

1980s
Robert Axelrod wrote computer programs for *tit-for-tat* game tournaments. Eric Maskin studied incentives and dynamic markets. Roger Myerson discovered the connection between optimal allocations and truthful revelation.

1994
John Nash, Reinhard Selten and John Harsanyi won the Nobel Prize.

1990s
Noam Nisan and Amir Ronen have studied algorithmic mechanism design. Elias Koutsoupias and Christos H. Papadimitriou introduced the concept of price of anarchy and studied selfish behavior amongst Internet users.

2000s
Constantinos Daskalakis and Christos H. Papadimitriou studied the computational complexity of finding Nash equilibrium. Tim Roughgarden and Eva Tardos studies price of anarchy and Braess' paradox in selfish routing.

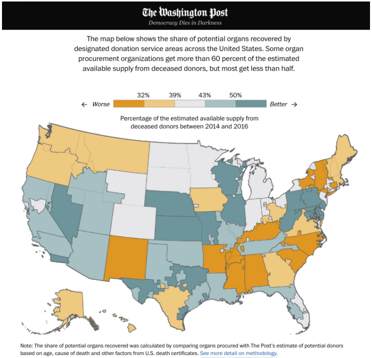
2007
Leonid Hurwicz, Eric Maskin and Roger Myerson won the Nobel prize.

2012
Elias Koutsoupias, Christos Papadimitriou, Noam Nisan, Amir Ronen, Tim Roughgarden and Eva Tardos won the Godel Prize.

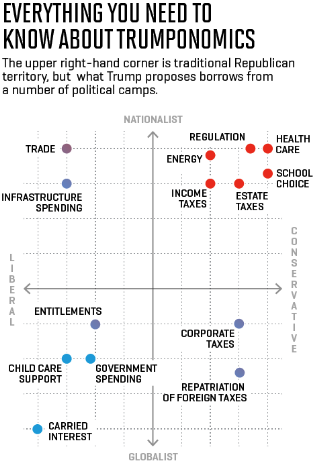
2018
Constantinos Daskalakis won the Rolf Nevanlinna Prize.

Application 1: Economics and Finance

Application 2: Government & Policy Making



Source: Washington Post[1]



Source: Fortune[2]

1 https://www.washingtonpost.com/graphics/2018/national/organ-transplant-shortages/?noredirect=on

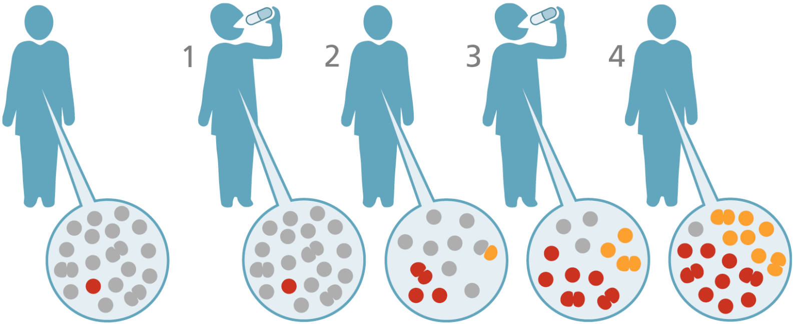
2 https://fortune.com/2016/08/11/trumponomics-chart/

Application 3: Transportation

Application 4: E-Commerce

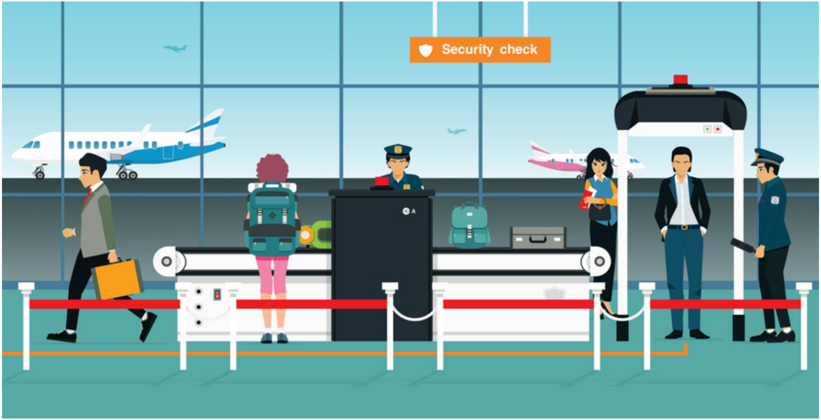
Application 5: Personalization & Targeted Ads

Application 6: Population Dynamics and Evolutionary Biology



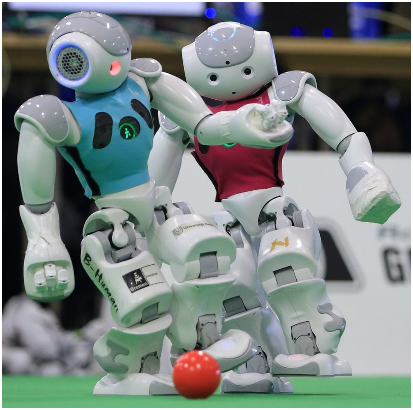
- bacterium sensitive to the antibiotic drug
- antibiotic-resistant bacterium present before initiation of treatment
- antibiotic-resistant bacterium appearing during treatment (by mutation)

Application 7: Airport Security



and many more...

In this course, we will focus[3] on...



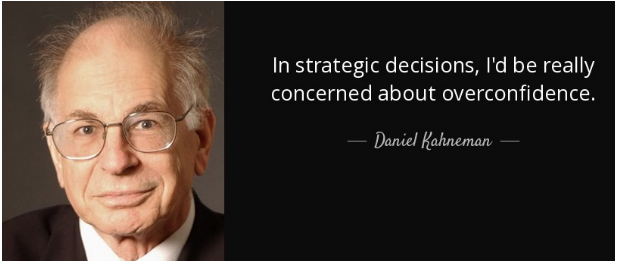
How do emotionless geniuses play games?

— Colin F. Camerer

[3]This course is not about designing video games!

However, in the real world...

Most practical agents (including people) have limitations, emotions and biases. Such players are known as *boundedly rational* agents. The study of game theory in the presence of boundedly rational agents is called *behavioral game theory*, which is out of the scope of this course.



Prerequisites

- Linear Algebra
  - Matrices to organize choice information at competing agents.
  - Matrix operations/reductions (e.g. simplex algorithm) to make informed decisions.
- Probability Theory and Statistics
  - Account for randomized actions at competing agents.
  - Compute the average outcome of the interaction.
- Algorithms
  - Implement AI-based agents
  - Compute the final outcome (solution) algorithmically.
- Calculus
  - Game $\Rightarrow$ Minimax (saddle-point) solutions.
  - Optimization theory??? (no need for this course, but a very powerful tool!)

Textbook Information

This course has **no** *single textbook.*

Instead, we will follow multiple reference books, some being listed below:

- Roger B. Myerson, "Game Theory: Analysis of Conflict," Harvard University Press, 1991.
- Bernhard von Stengel, "Game Theory Basics," Cambridge University Press, 2022.
- Drew Fudenberg, Jean Tirole, "Game Theory," MIT Press, 1991.
- Tamer Başar and Geert Jan Olsder, "Dynamic Noncooperative Game Theory," SIAM, 2nd Ed., 1999.
- Martin J. Osborne, "An Introduction to Game Theory," Oxford University Press, 2003.
- Noam Nisan *et al.* (Editors), "Algorithmic Game Theory, " Cambridge University Press, 2007.
- John von Neumann and Oskar Morgenstern, "Theory of Games and Economic Behavior," 60th Anniversary Commemorative Edition, Princeton University Press, 2007.
- Yoav Shoham, Kevin Leyton-Brown, "Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations," Cambridge University Press, 2008.
- David Easley and Jon Kleinberg, "Networks, Crowds and Markets: Reasoning about a Highly Connected World," Cambridge University Press, 2010.

Resources Available for Free...

S&T Digital Library:

- Roger B. Myerson, "Game Theory: Analysis of Conflict," Harvard University Press, 1991.
- Samson Lasaulce and Hamidou Tembine, "Game Theory and Learning for Wireless Networks," Academic Press, 2011.
- Harold W. Kuhn, "Lectures on the Theory of Games," Annals of Mathematics Studies (Book 166), Princeton University Press, 2003.

Publishers:

- Noam Nisan *et al.* (Editors), "Algorithmic Game Theory, " Cambridge University Press, 2007.
- Yoav Shoham, Kevin Leyton-Brown, "Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations," Cambridge University Press, 2008.
- David Easley and Jon Kleinberg, "Networks, Crowds and Markets: Reasoning about a Highly Connected World," Cambridge University Press, 2010.

Links to free digital copies of these books can be found on the course website for personal use!

Topics

This course is broadly divided into 6 topics:

- **Topic 0:** *Introduction* (1 lecture)
- **Topic 1:** *Decision Theory* (3 lectures)
- **Topic 2:** *Basic Models* (6 lectures)
- **Topic 3:** *Coalitional Games* (3 lectures)
- **Topic 4:** *Dynamic Games* (6 lectures)
- **Topic 5:** *Mechanism Design* (4 lectures)
- **Topic 6:** *Advanced Solution Concepts* (3 lectures)

Tentative Plan

- Submit assignments by assigned due date on GitLab[4].
- Programming language: Python
- In-class quizzes on CANVAS.
- In-class exams[5].
- Grades calculated based on

Type	Grade
Assignments (Top-4 of HWs 1-5 + HW 6)	60% of total grade
Exams (2)	30% of total grade
Quizzes (Top-4 of Quizzess 1-5 + Quiz 6)	10% of total grade
Final Grade for Undergrad Students	$[85-100]$: A, $[75-85)$: B, $[60-75)$: C, $[50-60)$: D, < 50: F
Final Grade for Grad Students	$[85-100]$: A, $[75-85)$: B, $[60-75)$: C, < 60: F

[4] Detailed instructions regarding GitLab are provided on instructor's website

[5] Take home if in-person classes are suspended