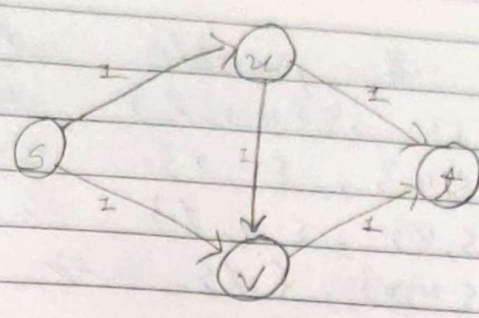
Assignment-4

Q-1 (a)



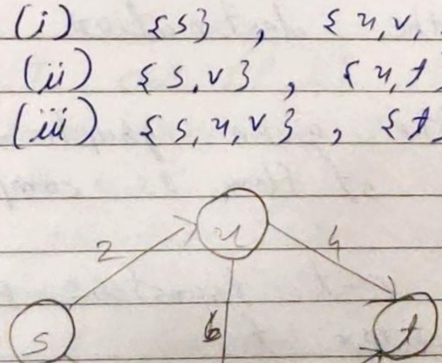
There are exactly 4 possible cuts in the graph.

They are
(a) {s} , {u, v, t}
(b) {s, a} , {v, t}
(c) {s, v} , {u, t}
(d) {s, u, v} , {t}

The capacity of these 4 possible cuts are
{s} , {u, v, t} ⟹ 2
{s, u} , {v, t} ⟹ 3
{s, v} , {u, t} ⟹ 2
{s, u, v} , {t} ⟹ 2

So there are 3 min. cuts in the graph. They are
(i) {s} , {u, v, t}
(ii) {s, v} , {u, t}
(iii) {s, u, v} , {t}

(b)



These are 4 possible cuts :

(a) {s} , {u, v, t} ⟹ 2 + 6 + 2 = 10

(b) {s, u} , {v, t} ⟹ 2 + 2 = 4

(c) {s, v} , {u, t} ⟹ ~~2~~ 4 + 4 = 8

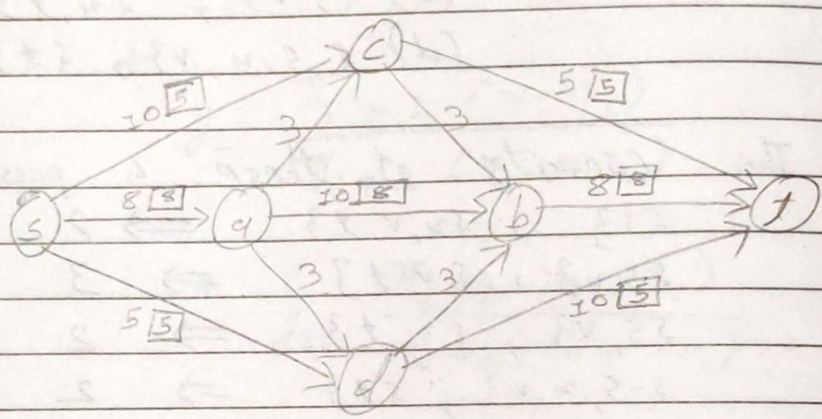
(d) {s, u, v} , {t} ⟹ 2 + 6 + 2 = 10

Min. Possible cut is

(i) {s, u} , {v, t} ⟹ 2 + 2 = 4

2-2

(a) Consider the following graph :



The value of the flow is completed by adding this flow of all the incoming edges of the destination or the ~~t~~ sink node.

In the given graph,
the total st flow is completed a follows:

→ The edge c-t transfers 5 ~~unite~~ unit of flow to the vertex t.

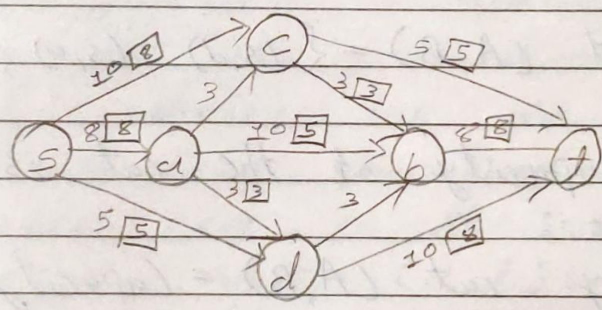
→ The edge b-t transfers 8 ~~unite~~ units of flow to the the vertex t.

→ The edge d-t transfer 5 units of flow to the vertex t.

Hence,
total s-t flow is equal to 5+8+5 = 18 units.

The flow in the given graph is not the max flow since 3 more unit of flow can be transfered to the sink vertex via edge d-t using residual graph.

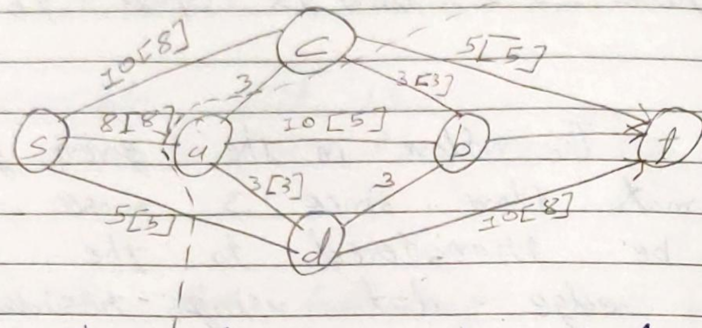
∴ Max flow is shown below:



Hence,
The max. flow of the graph h is 5 + 8 + 8 = 21 units.

(b) The s-t cut (A, B) of a graph G refers to the two subsets A and B such that A union B = V and A intersection B = NULL where V is the vertex set of the graph G, the vertex s belongs to the set A and the vertex t belongs to the set B.

→ The cut where capacity is equal to the max. flow of the graph is known as a min. cut. The minimum cut of the graph is shown below as a dashed arc:



In the above graph, set $A = (s, c)$ and set $B = (a, b, d, t)$

The cut $(A, B) = \{(s,d), (s,a), (c,b), (c,t)\}$

The capacity of the cut is computed as follows:

Capacity cut (A,B) = Capacity(s,d) + Capacity(s,a) + Capacity(c,d) + Capacity(c,t)

$= 5 + 8 + 3 + 5$

$= 21$

equal to the max. flow of the graph.

Q-3 (a) Value of flow is

$$v(f) = \sum_{e \text{ out of } s} f(e)$$

The flow values are represented in square boxes,

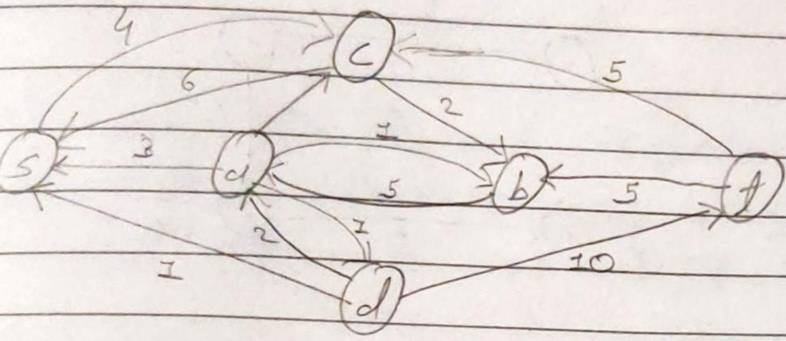
e out of s are 6, 3, 1

So value $= 6 + 3 + 1 = 10.$

The value of the flow is not max.
s-d-t flow is max.

Obtaining a flow of values 1+10+1 = 12

Residual network is shown below.



(b) Starting from s and find which vertices can be reached in the final residual network for are set of the cut, and the remaining vertexs from the other set. so the min cut is {s, a, b, c} and {d, t}.

The forward edges across this cut are {s, a}, {a, c}, {c, b}, {d, t}

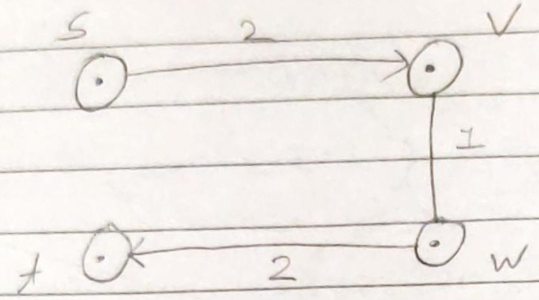
The capacities of those edges are 4, 2, -5, 10.

And the sum of which is 11.

It should be according to the max flow / min cut theorem.

2-4 False

Example consider a graph with vertices s, v, w, t and edges (s, v), (v, w), (w, t) and capacities of 2 on (s, v) and (w, t) and capacity of 1 on (v, w)



Then the maximum flow has value "1" and this does not saturate the edge out of 's'.

2-5 (a) Let us consider a n×n matrix where first row is 1, 1, 1, 1 ... 0 ie each element is 1 and the last element is 0 and the other rows are each 000....1 i.e each element is 1.

The matrix is not re-arrangable. This is true for all integer value n.

Eg :- n = 3 ⇒ the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is not re-arrangable.

(b) We will draw a ~~bap~~ b, P artite graph (P, Q) first

Let P be the vertices corresponding to the rows of the matrix and Q be the n vertices corresponding to the column of the matrix.

→ Join all $p \in P$ and $q \in Q$ with an edge if the corresponding entry in the matrix is 1, ie if $a_{pq} = 1$.

Now, if the constructed bipartity graph has a perfect matching, then the matrix is rearragable.

⇒ A matching of a graph is perfect if every vertex is connected to exactly one edge.

Q.6 Let $G = (V, E)$ be a directed graph with sources $s \in V$ sink $t \in V$ and non-negative edge capacites.

step 1 : Computs a minimum s-t cut C of G, and defines its capacity as $|C|$.

step-2 : Let $C_{e_1}, C_{e_2} \ldots C_{e_n}$ bettler edges in C.

For each C_{e_i}, try of increase in Capacity of

C_{ei} by 1. ie $C_{ei} = C_{ei} + 1$.

step 3: Now compute the minimum cut in the new graph.

Let the new min. cut be C_i Denote its capacity by $|C_i|$

step-4: If $|C| = |C_i|$ for some i, then clearly C_i is also a min. cut in the original graph & C is not equal to C_i.

So the min. cut is not unique conversely if there is a different min. cut C in the original graph h,

there will be some $C_{ei} \in C$ that is not in C, so increasing the capacity of that edge will not change the capacity of c, thus $|C| = |C_i|$

From above, we can conclude that the graph has a unique min. cut if has a unique min. cut if and only if $|C| < |C_i|$ for all. The algorithm takes at most $n+1$ computing of min. cuts and therefore runs in polynomial time.