CS 601 - Advanced Analysis of Algorithms **Venue: Home**
Time Duration: 12.45 pm to 3.45 pm **Date: May 11,2021**
Instructions: (1) Show complete working on all problems
(2) Answer the questions in the same order as they appear.
(3) Take home exam
Final exam: (Max: 50 points)

1. **[8 points]** Consider Figure 1. Compute the maximum flow that can be sent through the arcs of the network from source node A to the sink node F. The notation (f, c) represents flow for f, and capacity for c.
   (a) Indicate the arcs involved in the minimum cut.
   (b) Let **S** denote the set of nodes such that the set of arcs leaving S comprises the minimal cut. Let **T** denote the remaining set of nodes in the graph. Which nodes belong to S, and which nodes belong to T?
2. **[10 points]** Consider the linear programming problem given by
   Maximize 3x + 2y (profit in thousands of dollars)

   4.4x<100,

   6.67j<100,

   subject to 4x + 2.86 j < 100,

   3x + 6j><100,

   x > 0.

   The two decision variables given by x and y denote the number of cars in thousands, and number of trucks

   X , *y*) to this linear programming problem using the matrix

   method of solving linear equations. Also, determine the optimum value of the cost function in millions of dollars. Draw all the lines and label them. Shade the region which provides the feasible solution. Label the feasible solution boundary vertices.
3. [5 points]

(favorite) **Men's Preference list** (least favorite)

	1st	2nd	3rd	4*	5th
Victor	Bertha	Amy	Diane	Erika	Clare
Wyatt	Diane	Bertha	Amy	Clare	Erika
Xavier	Bertha	Erica	Clare	Diane	Amy
Yancey	Amy	Diane	Clare	Bertha	Erika
Zeus	Bertha	Diane	Amy	Erika	Clare

(favorite) **Women's Preference list** (least favorite)

	|St> >	2nd	3rd	4*	5*
Amy	Zeus	Victor	Wyatt	Yancey	Xavier
Bertha	Xavier	Wyatt	Yancey	Victor	Zeus
Clare	Wyatt	Xavier	Yancey	Zeus	Victor
Diane	Victor	Zeus	Yancey	Xavier	Wyatt
Erika	Yancey	Wyatt	Zeus	Xavier	Victor

Given above the men's and women's preference list. Is the matching perfect? Explain why or why not?

4. **[8 points]**
   (a) Prove that the 3-SAT problem is NP complete.
   (b) Construct a circuit which satisfies the Boolean expression: —>(1 a *x)* a ((x v O) a (v v z)). Show one combination of logic values of (x, *y, z)* which gives the output (i) 0 and (ii) 1.
   (c) When is a problem considered to be NP hard?
   (d) Which of the following graphs in Figure 2 are bipartite? Why and Why not?

5. [8 points] Consider Figure 3.
   (a) Find a Hamiltonian path. Can your path be extended to a Hamiltonian cycle?

(b) Is the graph bipartite? If so, how many vertices are there in each part?
(c) Does the graph have a Hamiltonian cycle?
(d) Suppose you have a bipartite graph in which one part has at least two more vertices than the other. Does G have a Hamiltonian path? Prove or disprove.

6. **[6 points]** Say True of False. Justify your claim.

(a) Let A", and X_2 be decision problems in *NP*. Assume *P* A *NP* . If $X_x <_p X_2$ and $X_2 <_p X_{(}$, then both *X*j and X_2 are *NP* complete.

(b) Let Independent set e *P* . Then Hamiltonian cycle e *P* .

(c) If Vertex cover G *P*, then SAT £ *P* .

7. **IS points]**

(a) Consider Figure 4 representing friendships between a group of students (each vertex is a student and each edge is a friendship). Is it possible for the students to sit around a table in such a way that every student sits between two friends? Justify using known concepts in the class. What does this question have to do with the paths?

(b) Find a matching of the bipartite graphs given in Figure 5 or explain why no matching exists.