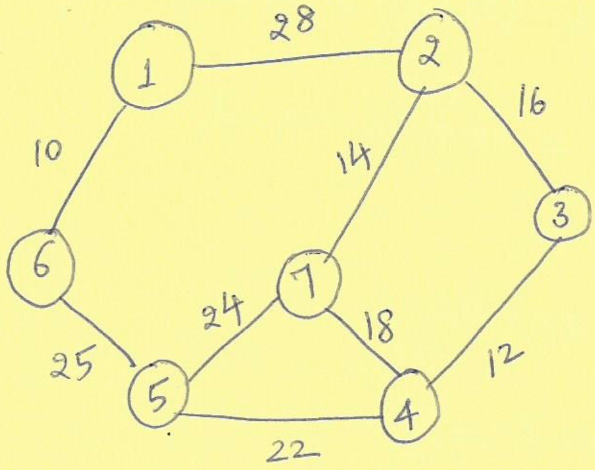
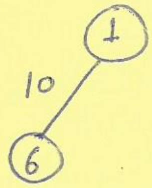
CS-601 Mid-term Solutions

1.

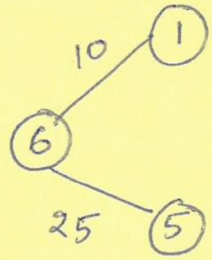


Solution:

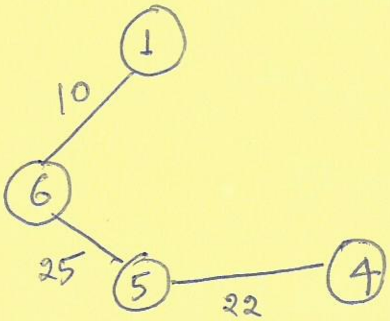
Step 1:



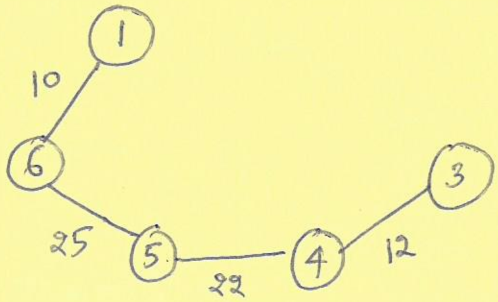
Step 2:



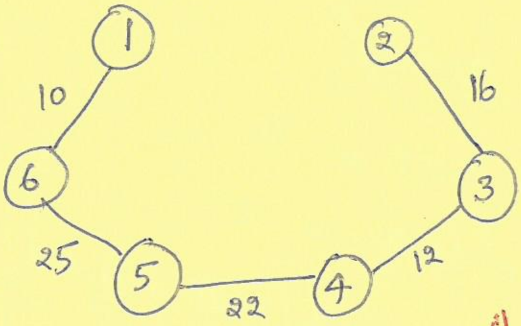
Step 3:



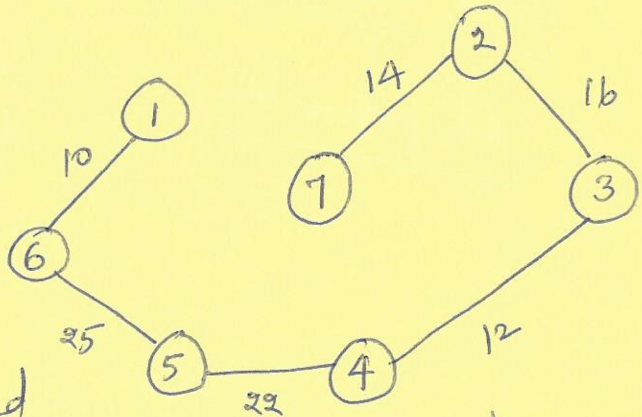
Step 4:



Step 5:



Step 6:



4

Since all vertices have been included in the MST, we stop here. Now, cost of minimum spanning tree

= Sum of all edge weights = 10 + 25 + 22 + 12 + 16 + 14

= 99

(2.)

$a = 2341$ Let $a_1 = 23 * 34$

$b = 3412$ $d_1 = 41 * 12$

Now, $e_1 = (23+41) * (34+12) - a_1 - d_1$

$= 64 * 46 - a_1 - d_1$

Need to recurse!

Now, first sub-problem is $a_1 = 23 * 34$

Sub-problems here: $a_2 = 2 * 3 = 6$

$d_2 = 3 * 4 = 12$

$e_2 = (2+3) * (3+4) - a_2 - d_2$

$= 5 * 7 - 6 - 12$

$= 35 - 18 = 17$

Ans: (first subproblem) $6 * 10^2 + 17 * 10 + 12 = 600 + 170 + 12$

$= 782$

Second sub-problem: $d_1 = 41 * 12$

Sub-problems here: $a_2 = 4 * 1 = 4$

$d_2 = 1 * 2 = 2$

$e_2 = (4+1) * (1+2) - a_2 - d_2$

$= 5 * 3 - 4 - 2 = 15 - 6 = 9$

Ans: (second subproblem) : $4 * 10^2 + 9 * 10 + 2 = 492$

<u>Third sub-problem:</u> $e_1 = 64 * 46 - a_1 - d_1$

<u>Sub-problems:</u>

$$a_2 = 6 * 4 = 24$$
$$d_2 = 4 * 6 = 24$$
$$e_2 = (6+4)(4+6) - a_2 - d_2$$
$$= 10 \cdot 10 - 24 - 24 = 52$$

Ans:
(third sub-problem)

$$24 * 10^2 + 52 * 10 + 24 - 782 - 492$$
$$= 1670$$

$\cancel{2400 + 520 + 24 - 782 - 492}$

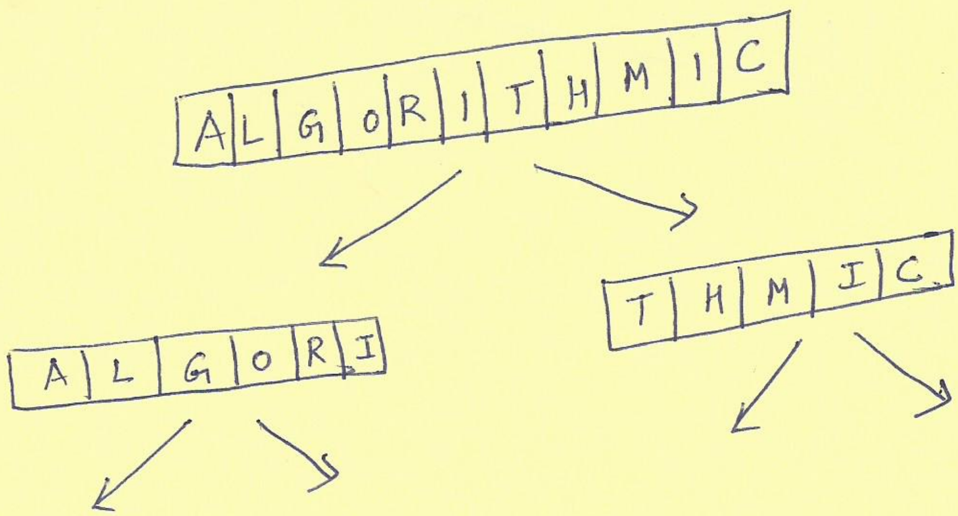
<u>Final Answer:</u>

$$2341 * 3412 = 782 * 10^4 + 1670 * 10^2 + 492$$

$$\begin{array}{r} 7820000 \\ +\ 167000 \\ +\ 492 \\ \hline = 7987492 \end{array}$$

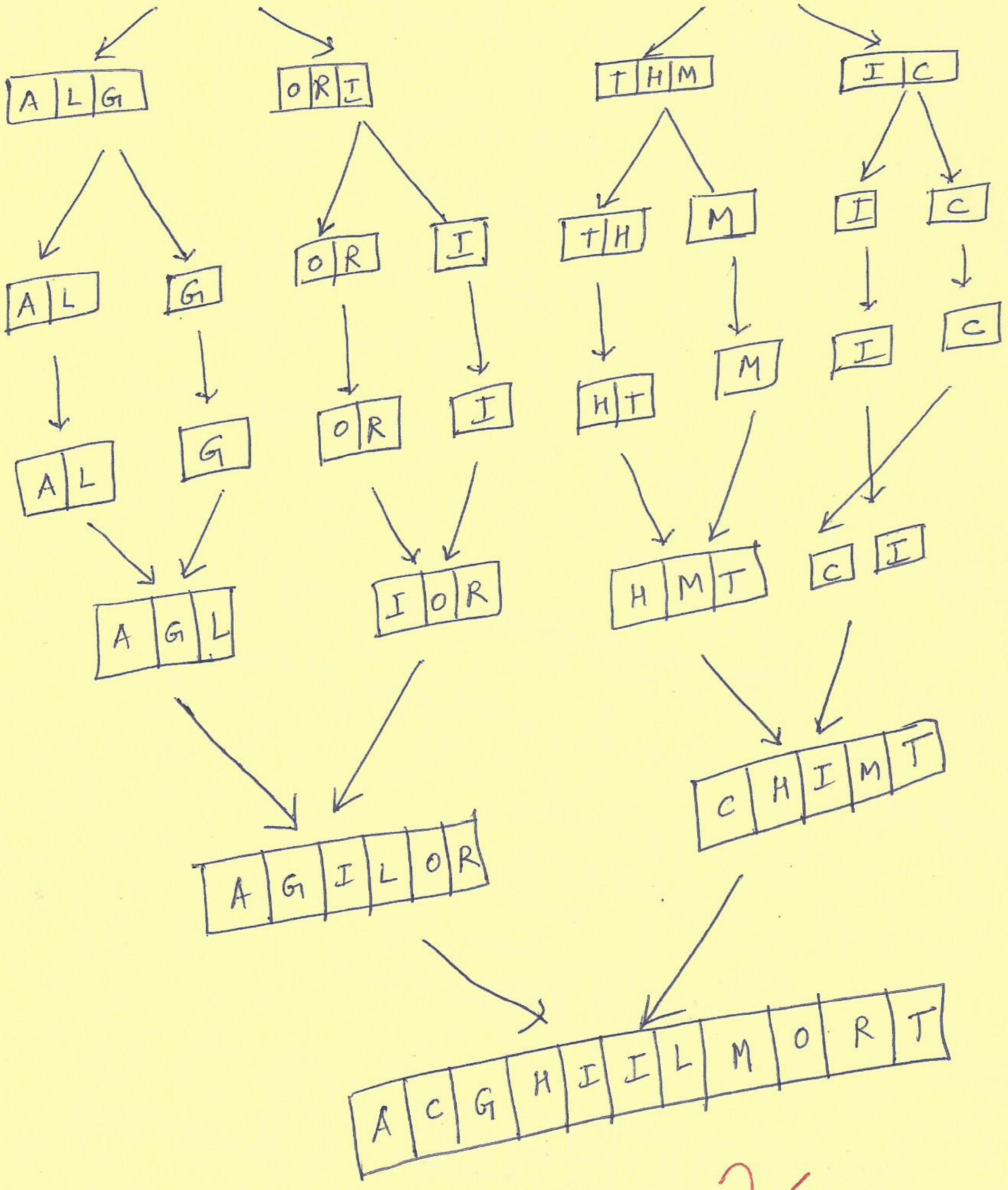
6

5.



divide O(1)

divide O(2)



divide
$O(2^2)$

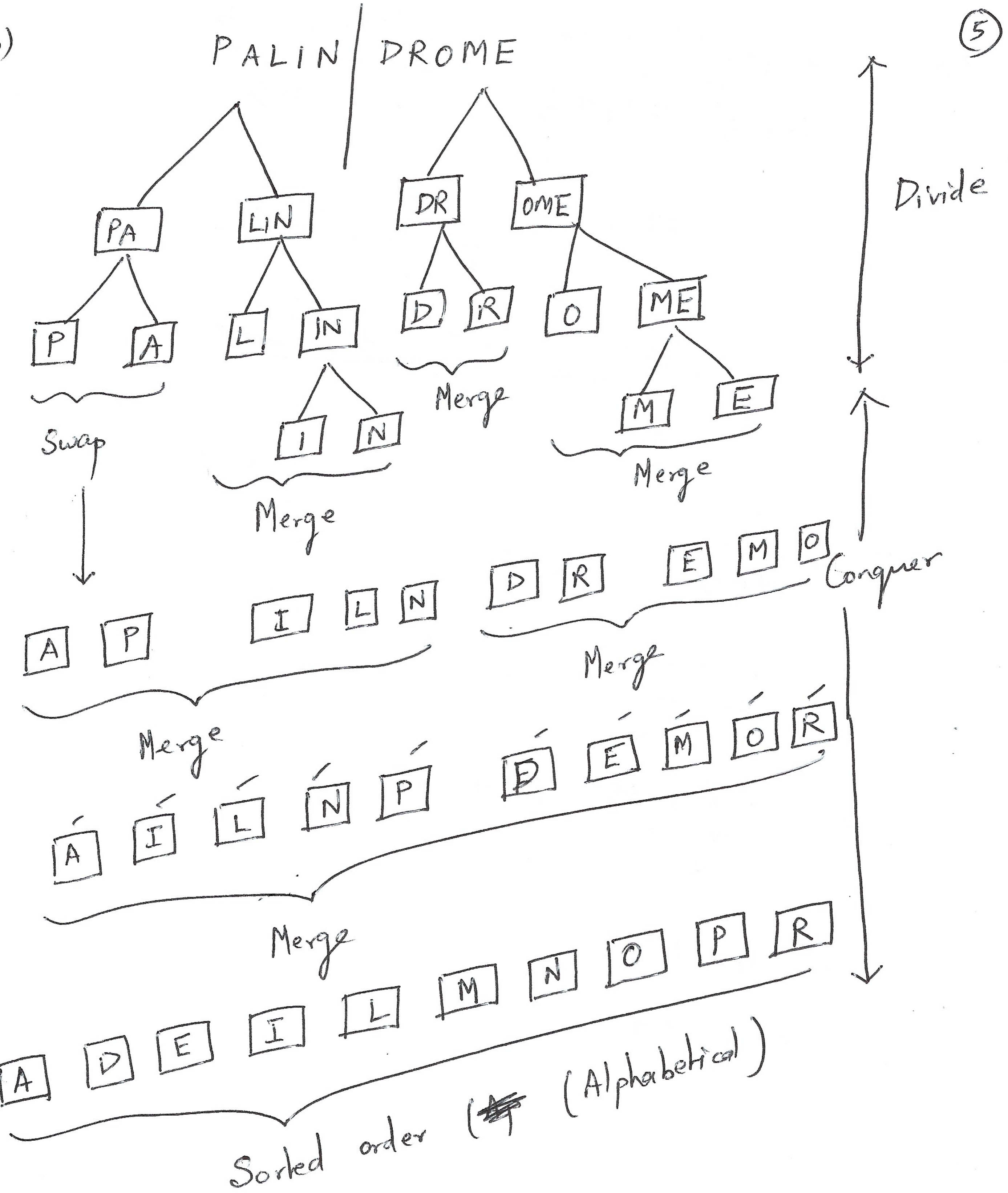
sorting
$2T(n/2)$

merge
$O(n)$

merge
$O(n)$

merge
$O(n)$

(b)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M
E
Swap
Merge
Merge
Merge
Divide
Conquer
A P I L N D R E M O
Merge
Merge
A I L N P D E M O R
Merge
A D E I L M N O P R
Sorted order (Alphabetical)

4. $A_{11} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ ⑥

$A_{12} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$A_{21} = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$A_{22} = \begin{bmatrix} 3 & 1 \\ 3 & 0 \end{bmatrix}$

$B_{11} = \begin{bmatrix} 2 & -3 \\ 0 & 3 \end{bmatrix}$

$B_{12} = \begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}$

$B_{21} = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$

$B_{22} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$

Now, $C = AB$

$$= \left(\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right) \left(\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right)$$

$$= \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}.$$

Conquer: Multiply 8 pairs of $\frac{n}{2} \times \frac{n}{2}$ matrices, recursively.

$$C_{11} = A_{11} \times B_{11} + A_{12} \times B_{21}$$

$$C_{12} = A_{11} \times B_{12} + A_{12} \times B_{22}$$

$$C_{21} = A_{21} \times B_{11} + A_{22} \times B_{21}$$

$$C_{22} = A_{21} \times B_{12} + A_{22} \times B_{22}$$

$$C_{11} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 3 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 6 & 13 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 16 \\ -1 & 7 \end{pmatrix}$$

$$C_{12} = A_{11} \times B_{12} + A_{12} \times B_{22}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 3 & 8 \end{pmatrix} + \begin{pmatrix} 6 & 14 \\ 4 & 7 \end{pmatrix} = \begin{pmatrix} 8 & 18 \\ 7 & 15 \end{pmatrix}$$

$$C_{21} = A_{21} \times B_{11} + A_{22} \times B_{21}$$

$$= \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -12 \\ 4 & -3 \end{pmatrix} + \begin{pmatrix} 9 & 9 \\ 9 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & -3 \\ 13 & 3 \end{pmatrix}$$

$$C_{22} = A_{21} \times B_{12} + A_{22} \times B_{22}$$

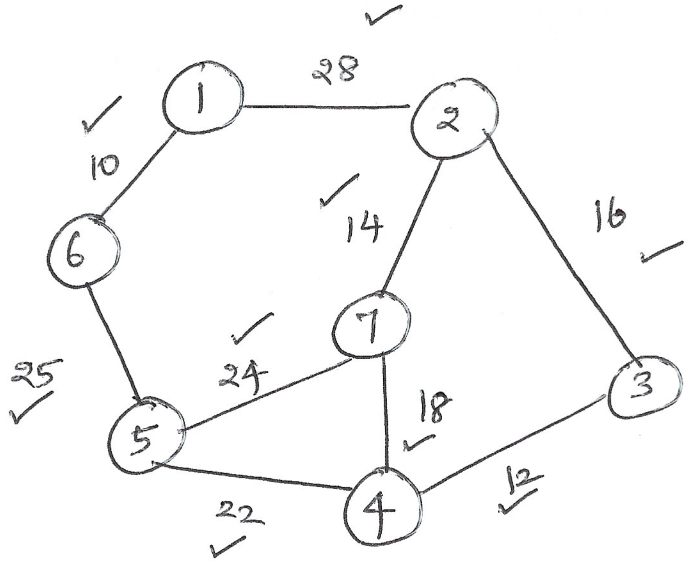
$$= \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -4 \\ 0 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 0 & 7 \end{pmatrix}$$

$$\therefore \; C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} 6 & 16 & 8 & 18 \\ -1 & 7 & 7 & 15 \\ 15 & -3 & -3 & 3 \\ 13 & 3 & 0 & 7 \end{pmatrix}$$

9

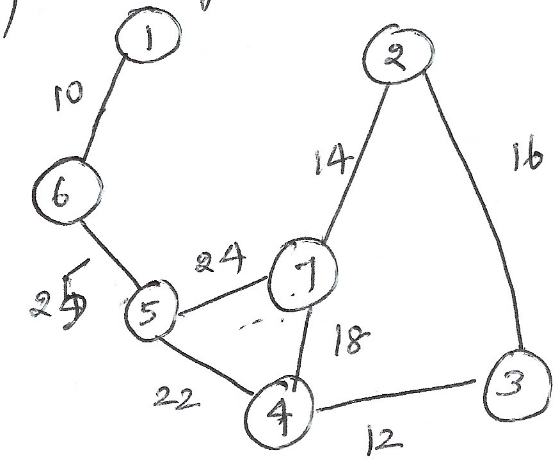
1 (b)



Edge	Weight
1-2	28 ✗
5-6	25 ✓
5-7	24 ✗
5-4	22 ✓
7-4	18
2-3	16
2-7	14
4-3	12
6-1	10

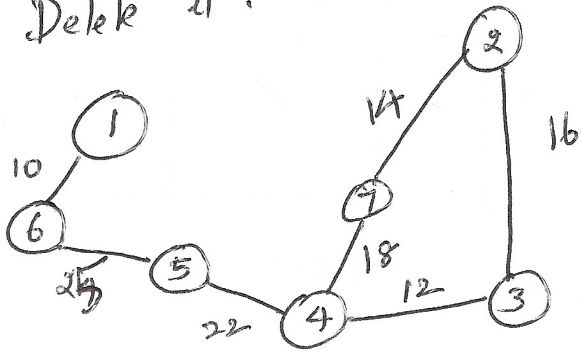
<u>Highest edge is 1-2:</u>

Remove it and see if the remaining nodes keep the graph connected.



<u>Next highest edge is 5-6:</u> If we remove/delete it, then the nodes 1 and 6 will be disconnected from the rest of the nodes in the graph. So, do not delete edge 5-6.

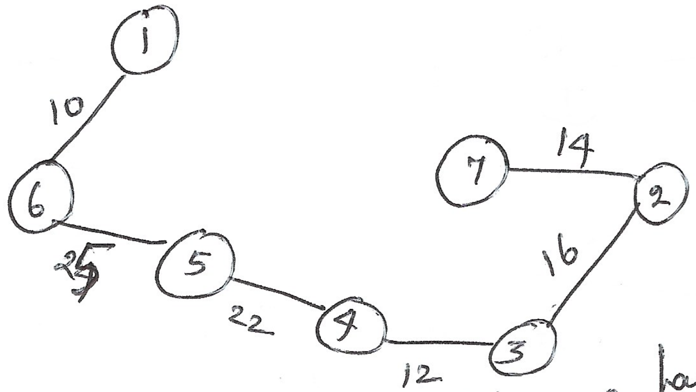
<u>Next highest edge is 5-7:</u> Delete it. This will still keep all nodes connected.



Next highest edge 5-4: This edge cannot be disconnected.

Next highest edge 7-4: This can be disconnected and it can still keep all nodes connected.

Resultant MST:



This MST is the same as what we have using Prim's algorithm.

Sum of all edge weights $= 10 + 25 + 22 + 12 + 16 + 14$

$= 99$

3.

x_j	$A(x_j)$	$B(x_j)$	$C(x_j)$
-1	-1	-1	1
0	1	4	4
1	3	9	27
$\cancel{2}$	$\cancel{5}$	$\cancel{14}$	$\cancel{70}$

of x_j's $= 3$

Since 2 coeffts of $A(x)$ + 2 coeffts of $B(x) - 1$ $= 2 + 2 - 1 = 3$

$A(x) = 1 + 2x$

$B(x) = 4 + 5x$

$A(-1) = 1 - 2 = -1$

$B(-1) = 4 - 5 = -1$

$A(0) = 1$

$B(0) = 4$

$A(1) = 3$

$B(1) = 9$

$\cancel{A(2) = 5}$

$\cancel{B(2) = 14}$

$$C(x) = 1\,\frac{(x-0)(x-1)\cancel{(x-2)}}{(-1-0)(-1-1)\cancel{(-1-2)}} + 4\,\frac{(x+1)(x-1)\cancel{(x-2)}}{1(-1)\cancel{(-2)}}$$

$$+ 27\,\frac{(x+1)(x-0)(1)}{(1+1)(1-0)}$$

$$\cancel{+ 70\,\frac{(\;)(\;)(\;)}{}}$$

$$= \frac{x^2-x}{-1(-2)} + \frac{4(x^2-1)}{(-1)} + \frac{27(x^2+x)}{2}$$

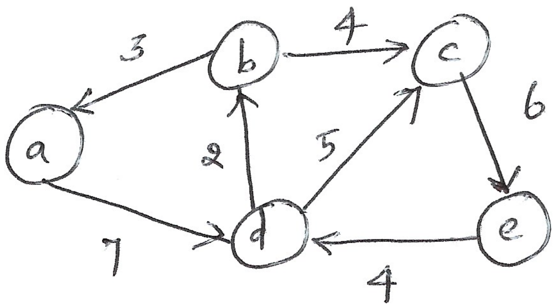
$$= \frac{x^2-x+27x^2+27x}{2} + 4(1-x^2)$$

$$= \frac{28x^2+26x}{2} + 4-4x^2$$

$$= 14x^2+13x+4-4x^2$$

$$= 13x+10x^2+4$$

Q6. Dijkshra's algorithm:



Vertex 'a' is the source:

Tree vertices	Remaining vertices
a(−, 0)	b(−, ∞) c(−, ∞) d(a, 7) e(−, ∞)
d(a, 7)	b(d, 7+2) c(d, 7+5) e(−, ∞)
b(d, 9)	c(d, 12) e(−, ∞)
c(d, 12)	e(c, 12+6)
e(c, 18)	

The shortest paths (identified) and their lengths are:

from a to d: $a \to d$ length = 7

from a to b: $a \to d \to b$ length = 9

from a to c: $a \to d \to c$ length = 12

from a to e: $a \to d \to c \to e$ length = 18