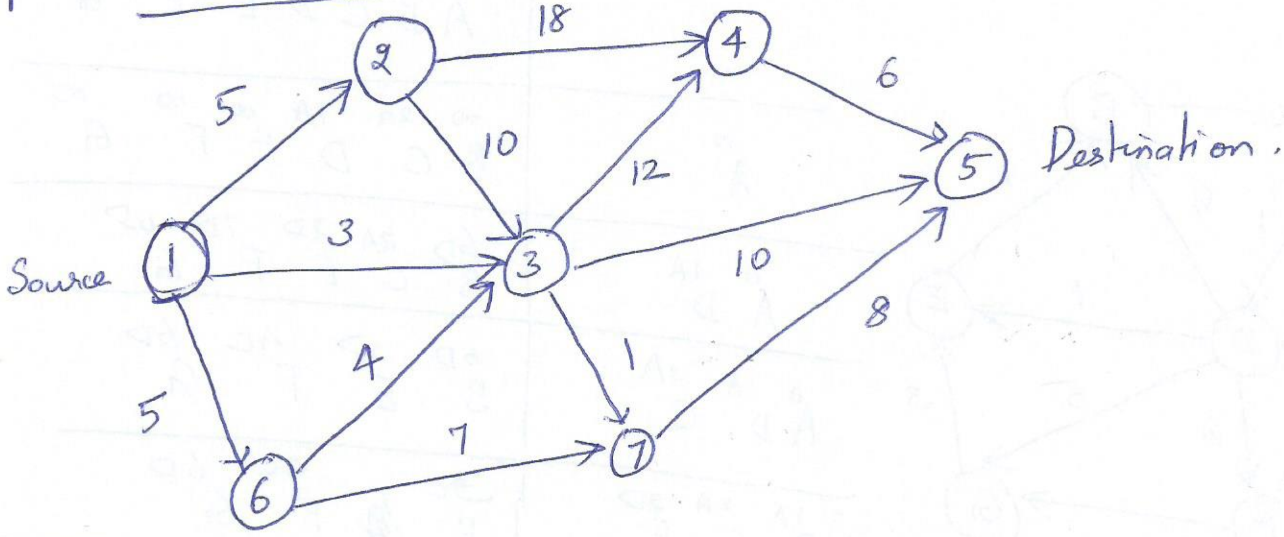
CS-601 Quiz 2 solutions

7. Bell man Ford algorithm:



At the end of iteration 1:

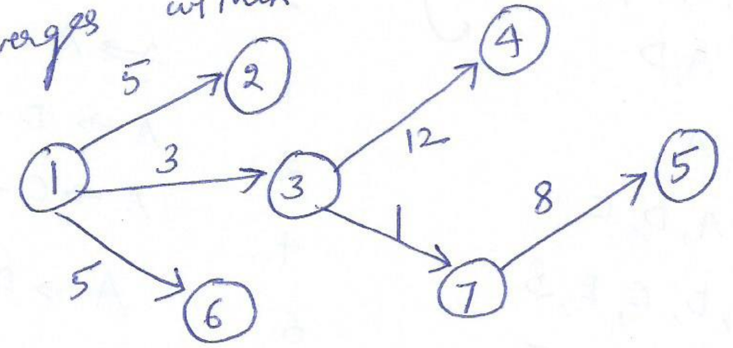
Nodes	1	2	3	4	5	6	7
Weights	0	5	3	15	12	5	4

At the end of iteration 2:

Nodes	1	2	3	4	5	6	7
(Cost) Weight	0	5	3	15	12	5	4

7

The algorithm converges within two iterations and we stop here.



3. FALSE. 1

Consider a graph with nodes s, v_1, v_2, v_3, w, t, edges (s, v_i) and (v_i, w) for each i and an edge (w, t). There is a capacity of 4 on edge (w,t), and a capacity of 1 on all other edges. Then, setting $A = \{s\}$ and $B = V - A$ gives a minimum cut with capacity 3. But, if we add one to every edge, then this cut has capacity 6, more than the capacity of 5 on the cut with $B = \{t\}$ and $A = V - B$.

1

4. (a) Conditions that are necessary:

A matching $M \subseteq E$ is perfect if each node appears in exactly one edge in M.

2

(b) Conditions that are sufficient:

If a bipartite graph $G = (L \cup R, E)$ has a perfect matching, then $|N(S)| \geq |S|$ for all subsets $S \subseteq L$.

2

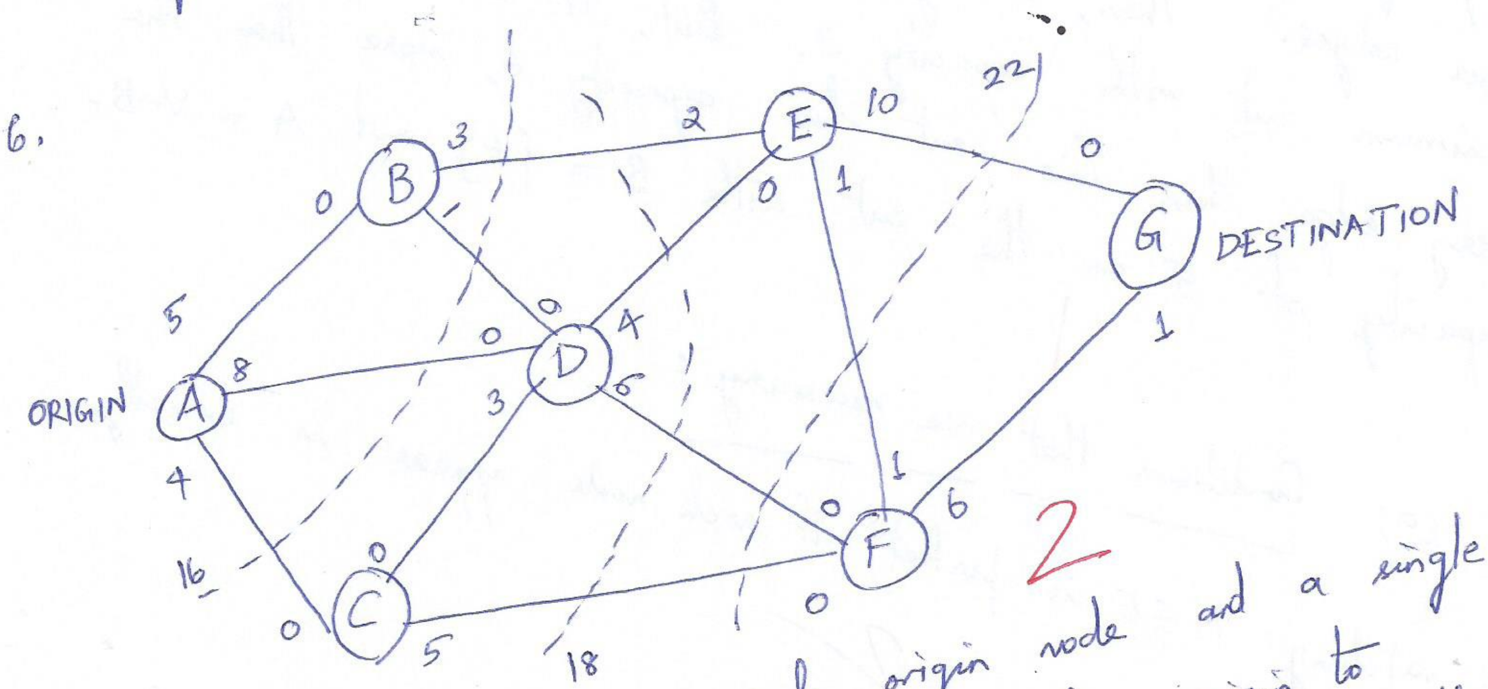
5. (a) (D) 2 Reason: 2 If the final flow is 1, then this means that the minimum cut has only one edge of weight 1 connecting u and v. There are no other edges connecting u in S and another vertex in T, otherwise the flow would be more than 1.

(b) FALSE.

To run in polynomial-time, this algorithm would have to run in polynomial time in ~~log~~ $\log_2 x$.

Our square-root algorithm has $O(\sqrt{x})$ time complexity.

6.



2

For any network having a single origin node and a single destination node, the maximum possible flow from origin to destination equals the minimum cut values for all cuts in the network. The minimum cut is a kind of distributed bottleneck for a whole network as opposed to a simple bottleneck for a series of pipes.

The max-flow/min-cut theorem means that we can determine the minimum-cut value using the Ford and Fulkerson maximum flow algorithm. (5)

To find the minimum-cut, simply mark the arcs that are carrying a flow equal to their maximum flow capacity and look for a cut that consists only of marked arcs and no other arcs.

The cut is the same set of arcs whose flow capacities were forced to zero by the F-F algorithm, hence forcing the termination: B-E, D-E, F-E, and F-G. The cut value in the forward (O-to-D) direction is 14 for these arcs, the same as maximum flow.

So, max flow = 3 + 4 + 1 + 6 = 14 //

3

CS 601 – Advanced Analysis of Algorithms
Time Duration: 8.30 pm – 9.45 pm

Venue: SC-N 120
Date: Nov. 21, 2019

Instructions: (1) Cell phones and other electronic gadgets are prohibited.
(2) Open A4 sheet (2 pages) exam.
(3) Calculators are permitted.
(4) Answer the questions in the same order as they appear.

Quiz 1: [Max: 25 points]

1. [**7 points**] Given the weighted graph (Figure 1) and a destination vertex, find the lowest cost path from every vertex to the destination using the Bellman-ford algorithm. [To get full points, you must show all the individual steps involved in the algorithm, marking the vertices chosen, current edge values, etc]

2. [**8 points**] Given the weighted graph (Figure 2) and a source vertex, what are the shortest paths to each of the other vertices. Use the Dijkstra's algorithm. [To get full points, you must show all the individual steps involved in the algorithm, marking the vertices chosen, current edge values, etc. Also, fill in the Table given below in your answer sheet]

Vertex	Known?	Cost	Path
A			
B			
C			
D			
E			
F			
G			

3. [**2 points**] Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

4. [**4 points**] Regarding the structure of a bipartite graph with perfect matching, we must have $|L| = |R|$.
   (a) What other conditions are necessary?
   (b) Which conditions are sufficient?

5. [**4 points**]

   a) You are given an undirected graph with each edge having a capacity of 1 unit (i.e., a maximum of one unit of water can flow in both directions). Suppose you ran the Ford-Fulkerson algorithm between a pair of vertices s and t on this graph and it terminated with a final flow of 1. Which of the following is the most general statement one can make about the original graph?

   A) There exists only one unique path from s to t in the graph.

B) There exists at least one edge which cannot be removed to disconnect the graph into two pieces, one containing s and the other containing t.
C) The graph is a tree, with s as the root and t as one of the leaves.
D) There exists exactly one edge which can be removed to disconnect the graph into two pieces, one containing s and the other containing t.

IMPORTANT: Please briefly explain your answer to get full credit.

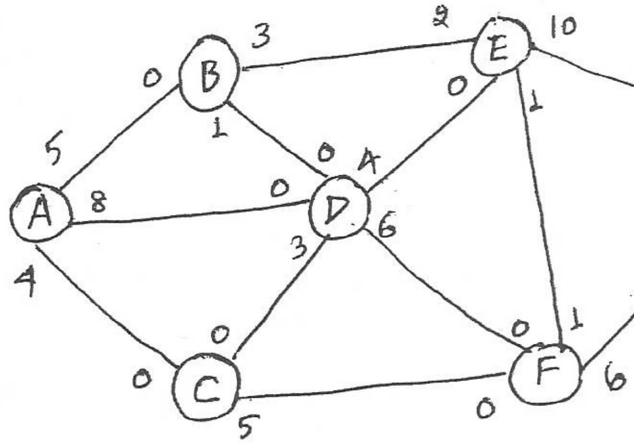
b) Consider the following algorithm for computing the square root of a number:
SQUARE-ROOT(x)
for i = 1, 2, …, x/2
if i*i == x
then output i.

Say True or False. Justify your answer in either case.

"This algorithm runs in polynomial time."

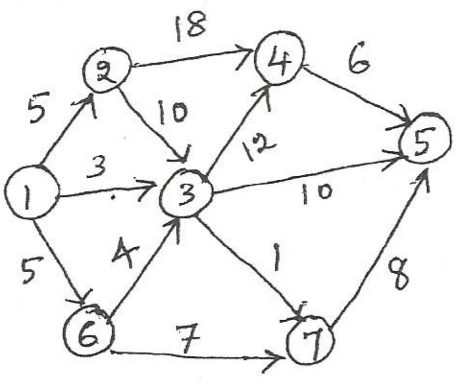
6. **[5 points]** Find the maximum flow from A to G in Figure 3.

Figure 3 :



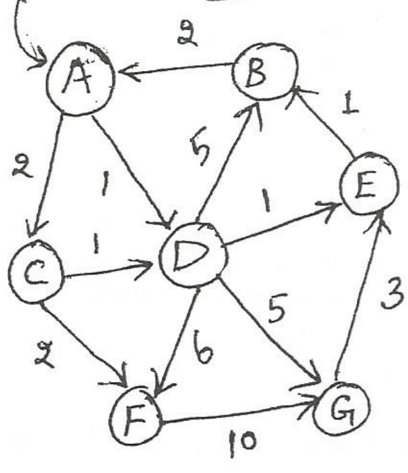
Note: Label near node A on arc A-B is flow capacity in A to B direction. Label near node B on arc A-B is flow capacity in B to A direction.

Figure 1:



Source

Figure 2 :



vertex	known?	cost	path
A			
B			
⋮			
G			