

Final exam: (Max: 50 points)

1. [8 points] Consider Figure 1. Compute the maximum flow that can be sent through the arcs of the network from source node A to the sink node F. The notation (f, c) represents flow for f, and capacity for c.  
(a) Indicate the arcs involved in the minimum cut.  
(b) Let S denote the set of nodes such that the set of arcs leaving S comprises the minimal cut. Let T denote the remaining set of nodes in the graph. Which nodes belong to S, and which nodes belong to T?
2. [10 points] Consider the linear programming problem given by  
Maximize  $3x + 2y$  (profit in thousands of dollars)  
 $4.4x < 100,$   
 $6.67j < 100,$   
subject to  $4x + 2.86j < 100,$   
 $3x + 6j > 100,$   
 $x > 0.$

The two decision variables given by x and y denote the number of cars in thousands, and number of trucks  $(X, y)$  to this linear programming problem using the matrix method of solving linear equations. Also, determine the optimum value of the cost function in millions of dollars. Draw all the lines and label them. Shade the region which provides the feasible solution. Label the feasible solution boundary vertices.

3. [5 points]

|        | (favorite)      | Men's Preference list |                 |                | (least favorite) |
|--------|-----------------|-----------------------|-----------------|----------------|------------------|
|        | 1 <sup>st</sup> | 2 <sup>nd</sup>       | 3 <sup>rd</sup> | 4 <sup>*</sup> | 5 <sup>th</sup>  |
| Victor | Bertha          | Amy                   | Diane           | Erika          | Clare            |
| Wyatt  | Diane           | Bertha                | Amy             | Clare          | Erika            |
| Xavier | Bertha          | Erica                 | Clare           | Diane          | Amy              |
| Yancey | Amy             | Diane                 | Clare           | Bertha         | Erika            |
| Zeus   | Bertha          | Diane                 | Amy             | Erika          | Clare            |

|        | (favorite)        | Women's Preference list |                 |                | (least favorite) |
|--------|-------------------|-------------------------|-----------------|----------------|------------------|
|        | 1 <sup>st</sup> > | 2 <sup>nd</sup>         | 3 <sup>rd</sup> | 4 <sup>*</sup> | 5 <sup>*</sup>   |
| Amy    | Zeus              | Victor                  | Wyatt           | Yancey         | Xavier           |
| Bertha | Xavier            | Wyatt                   | Yancey          | Victor         | Zeus             |
| Clare  | Wyatt             | Xavier                  | Yancey          | Zeus           | Victor           |
| Diane  | Victor            | Zeus                    | Yancey          | Xavier         | Wyatt            |
| Erika  | Yancey            | Wyatt                   | Zeus            | Xavier         | Victor           |

Given above the men's and women's preference list. Is the matching perfect? Explain why or why not?

4. [8 points]  
(a) Prove that the 3-SAT problem is NP complete.  
(b) Construct a circuit which satisfies the Boolean expression:  $\neg((1 \wedge x) \wedge ((x \vee 0) \wedge (\vee \vee z)))$ . Show one combination of logic values of (x, y, z) which gives the output (i) 0 and (ii) 1.  
(c) When is a problem considered to be NP hard?  
(d) Which of the following graphs in Figure 2 are bipartite? Why and Why not?
5. [8 points] Consider Figure 3.  
(a) Find a Hamiltonian path. Can your path be extended to a Hamiltonian cycle?

- (b) Is the graph bipartite? If so, how many vertices are there in each part?  
 (c) Does the graph have a Hamiltonian cycle?  
 (d) Suppose you have a bipartite graph in which one part has at least two more vertices than the other. Does  $G$  have a Hamiltonian path? Prove or disprove.

6. [6 points] Say True or False. Justify your claim.

- (a) Let  $A''$ , and  $X_2$  be decision problems in  $NP$ . Assume  $P \leq_P A''$ . If  $X_1 \leq_P X_2$  and  $X_2 \leq_P X_1$ , then both  $X_1$  and  $X_2$  are  $NP$  complete.  
 (b) Let Independent set  $\in P$ . Then Hamiltonian cycle  $\in P$ .  
 (c) If Vertex cover  $\in P$ , then SAT  $\in P$ .

7. [5 points]

- (a) Consider Figure 4 representing friendships between a group of students (each vertex is a student and each edge is a friendship). Is it possible for the students to sit around a table in such a way that every student sits between two friends? Justify using known concepts in the class. What does this question have to do with the paths?

- (b) Find a matching of the bipartite graphs given in Figure 5 or explain why no matching exists.

Figure 1: (Question 1)

flow, capacity

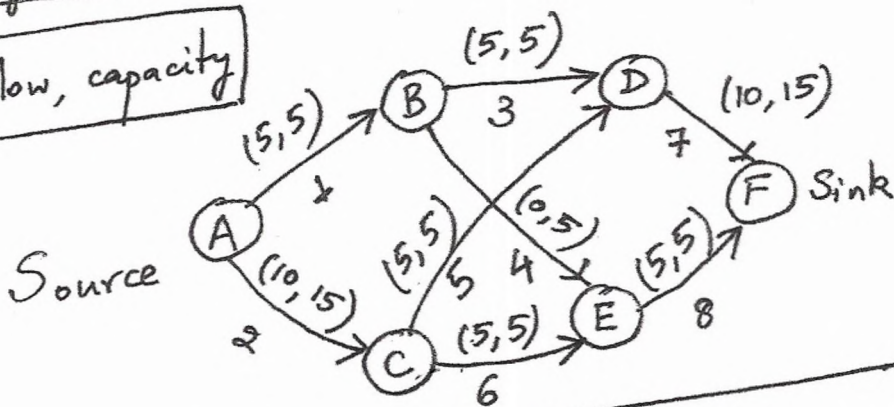


Figure 4: (Question 7a)

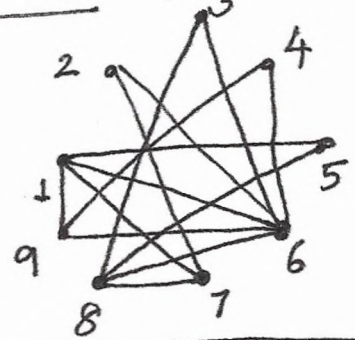


Figure 2: (Question 4d)

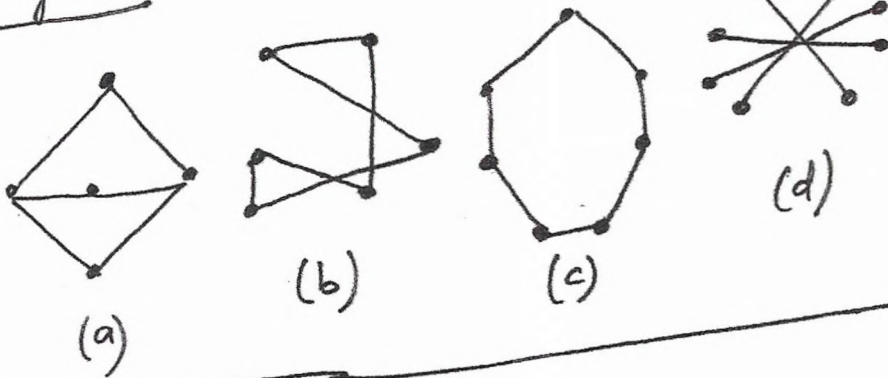


Figure 5: (Question 7b)

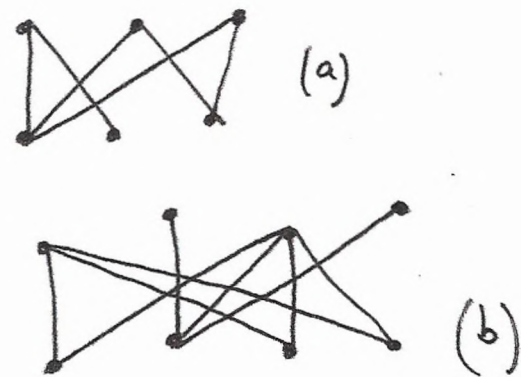


Figure 3: (Question 5)

