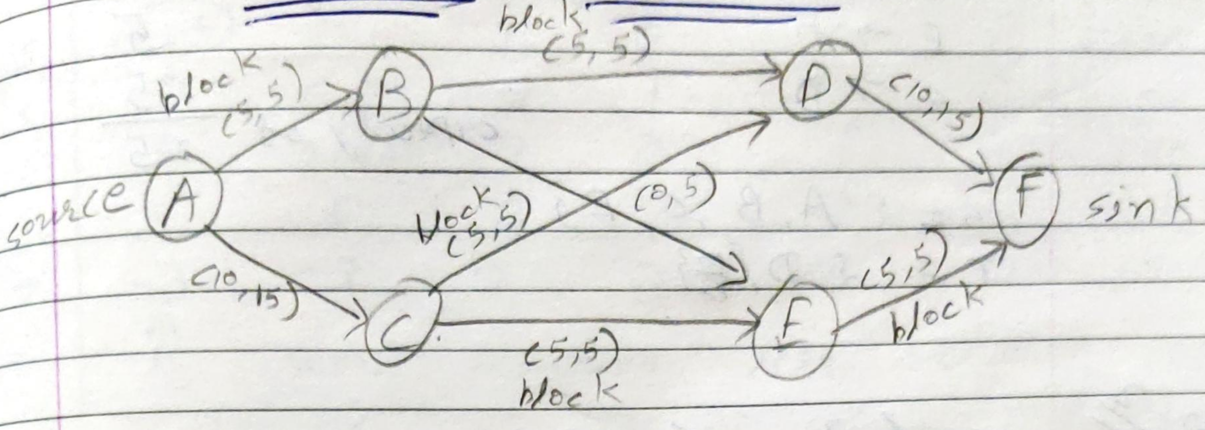
Final Exam

bu7947



Augmenting Path	Bottleneck capacity
A → B → O → F	5
A → C → D → F	5
A → C → E → F	5

∴ Max Flow = 5 + 5 + 5 = 15

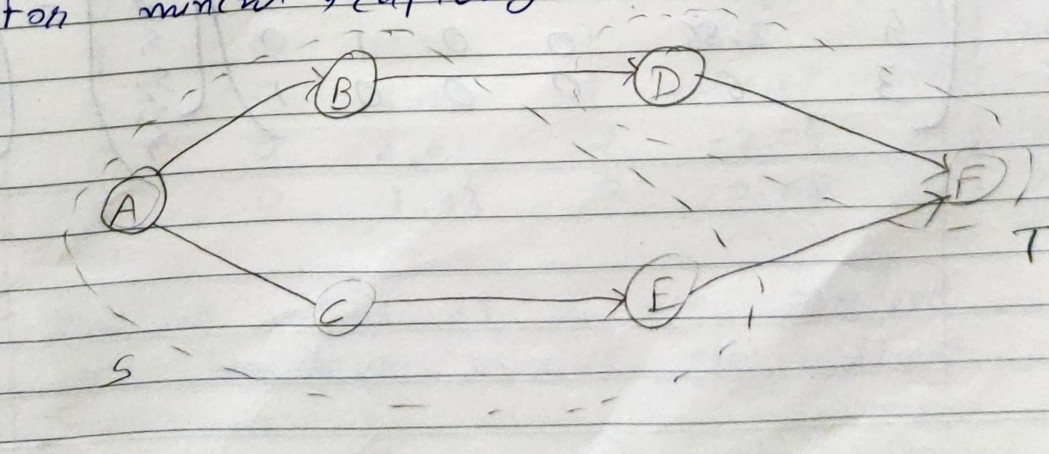
Min - Cut

Set of edges whose removal divides network into two halves X & Y where

S ∈ X

T ∈ Y

For mincut, capacity of cut = Max flow



Remove

$B \to D$ original weight 5

$E \to F$ 5

$C \to D$ 5

capacity of 15

$S = \{A, B, C, E\}$

$T = \{D, F\}$

2.

$4.4x \leq 100$

$6.67y \leq 100$

$4x + 2.86y \leq 100$

$3x + 6y \leq 100$

$x \geq 0$

$4.4x \leq 100 \Rightarrow 4.4x + S_1 = 100$

$6.67y \leq 100 \Rightarrow 6.67y + S_2 = 100$

$4x + 2.86y \leq 100 \Rightarrow 4x + 2.86y + S_3 = 100$

$3x + 6y \leq 100 \Rightarrow 3x + 6y + S_4 = 100$

Matrix form

$$\begin{pmatrix} 4.4 & 0 & 1 & 0 & 0 & 0 \\ 0 & 6.67 & 0 & 1 & 0 & 0 \\ 4 & 2.86 & 0 & 0 & 1 & 0 \\ 3 & 6 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{pmatrix} = \begin{pmatrix} 100 \\ 100 \\ 100 \\ 100 \end{pmatrix}$$

→ Iteration 1

	x	y	s_1	s_2	s_3	s_4	
s_1	4.4	0	1	0	0	0	100 →
s_2	0	6.67	0	1	0	0	100
s_3	4	2.86	0	0	1	0	100
s_4	3	6	0	0	0	1	100
z	-3	-2	0	0	0	0	0

↑ most negative less ratio

x enters and s_1 leaves.

→ Iteration 2

	x	y	s_1	s_2	s_3	s_4	
x	1	0	0.23	0	0	0	22.73
s_2	0	6.67	0	1	0	0	100
s_3	0	2.86	-0.92	0	1	0	9.08 →
s_4	0	6	-0.69	0	0	1	31.81
z	0	-2	0.69	0	0	0	68.19

↑

y enters s_3 leaves.

→ Iteration 3

	x	y	s_1	s_2	s_3	s_4	
y	0	1	0.32	0	0.34	0	3.17
x	1	0	0.23	0	0	0	22.73
s_2	0	0	-2.13	1	-2.27	0	-78.9
s_4	0	0	2.61	0	-2.04	1	12.79
z	0	0	1.33	0	0.68	0	74.53

Now, all values of z are positive
We ~~have~~ have reached the optimum.

Hence, $x = 22.73$
$y = 3.17$

Max. $Z = 3 \times 22.73 + 2 \times 3.17 = 74.53$

Graphical Method

Max $Z = 3x + 2y$

~~4.4x = 100~~ (i) $4.4x = 100$ (iii) $4x + 2.86y = 100$
~~2.67~~ (ii) $6.67y = 100$ (iv) $3x + 6y = 100$

(i)

x	22.73	22.73
y	0	1

(ii)

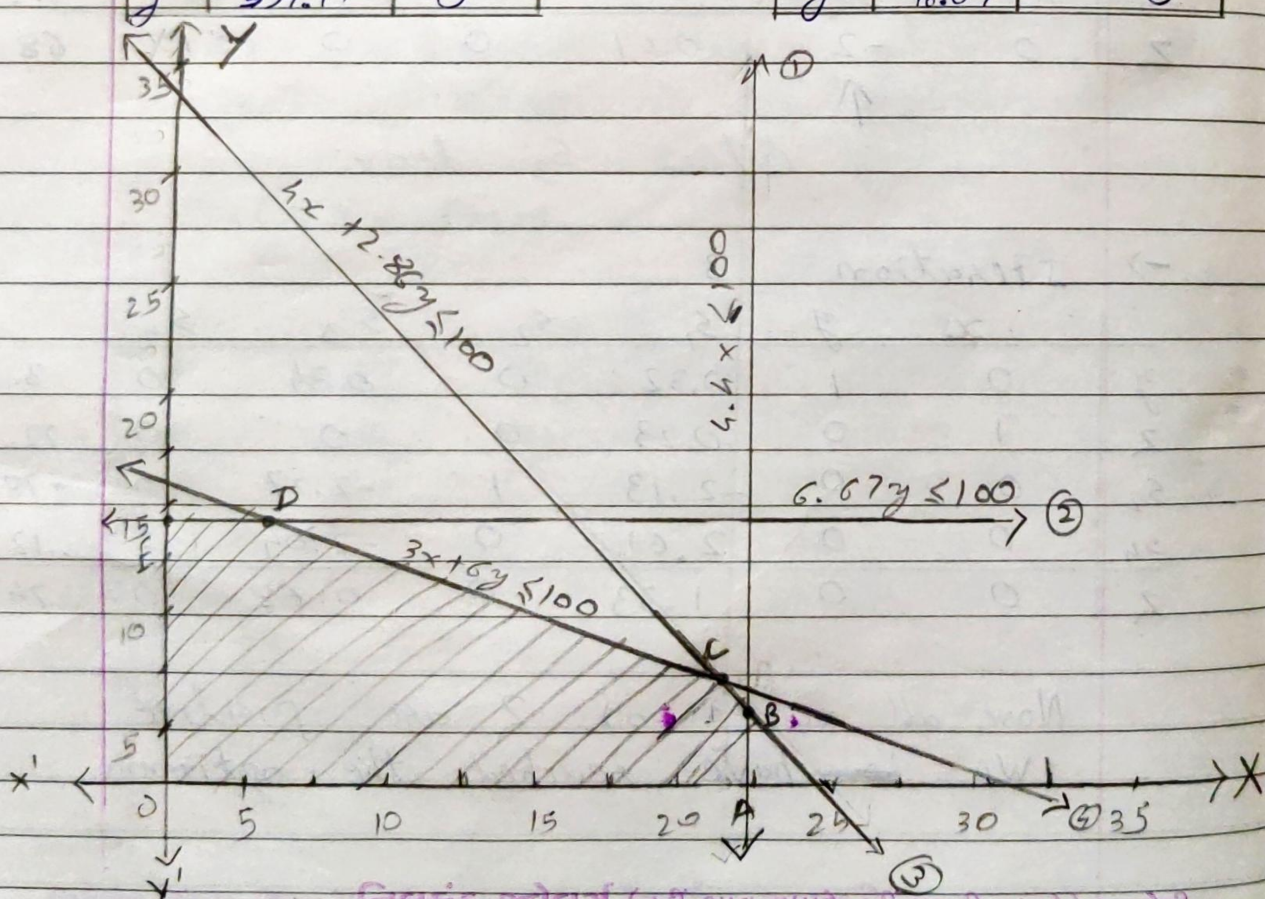
x	0	1
y	14.99	14.99

(iii)

x	0	25
y	34.97	0

(iv)

x	0	33.33
y	16.67	0



Point	Equation	Z
O (0, 0)	(0, 0)	$3(0) + 2(0) = 0$
A (22.73, 3.18)	eq - 1	$3(22.73) + 2(0) = 68.18$
B (22.73, 3.18)	eq-1 + eq-3	$3(22.73) + 2(3.18) = 74.54$
C (20.36, 6.44)	eq-3 + eq-4	$3(20.36) + 2(6.49) = 74.06$
D (3.35, 14.99)	eq - 2 + eq - 4	$3(3.35) + 2(14.99) = 40.03$
E (0, 14.99)	eq - 2	$3(0) + 2(14.99) = 29.99$

=> Hence,

$$\boxed{\begin{array}{l} \text{Max } Z = 74.54 \\ x = 22.73 \;\; \& \;\; y = 3.18 \end{array}}$$

So,

No of cars = 22.73×1000 = 22730 cars

No. of trucks = 3.18×1000 = 3180 trucks

and

Max. Profit = 74.54×1000 = 74540 dollars

Q-3

Men's List

V →	~~B~~	~~A~~	D	E	C
W →	~~D~~	~~B~~	~~A~~	C	E
X →	~~B~~	E	C	D	A
Y →	~~A~~	~~D~~	~~C~~	~~B~~	E
Z →	~~B~~	D	A	E	C

Women's Prefrence List

A →	Z	V	W	Y	X
B →	X	W	Y	V	Z
C →	W	X	Y	Z	V
D →	V	Z	Y	X	W
E →	Y	W	Z	X	V

⇒ V - ~~B~~ A
W - ~~D~~ C
X - B
Y - ~~D~~ E
Z - D

final Pair : V → A
W → C
X → B — 1
Y → E
Z → D

This is set male optimality.

A - Z
B - X
C - W — 2
D - V
E - Y

→ According to women optimality Comparing the figure we see that the pairing is different when male & female optimality is matched.

→ Hence, the matching is not perfect for it to be perfect. All paired should have matched.

Q-4 (a) **Aim** :- To show 3SAT is NP complete
Proof :- Its very clear that 3SAT is in NP, Since SAT is NP to determine whether boolean E in CNF is satisfyble, non deterministically, guess values for all the variables and then evaluate the expression.

→ If E turns out to be tree. then accept this can be carried out in non-deterministic polynomial time. Thus, 3SAT is in NP.

→ To prove NP completeness, we shall reduce CSAT to 3SAT. Let a given CNP expression be $E = e_1 \lambda e_2 \ldots \lambda e_k$, where each e_i is a disjunction of literals.

→ We replace each clause e_i (as shown below) to create a new exp. such that,

(i) f is a clause in 3SAT from

(ii) time taken to construct f is linear in length of E.

(iii) A truth assignment satisfies E if and only if it can be extended to a satisfying truth assignment for f.

The construction of f is as follows,

(1) If an e_i is a single literal, say (x) or $(\bar{x})$ we introduce new variables u & v we replace (x) by the conjunction of four clauses as $(x+u+v)(x+u+\bar{v})(x+\bar{u}+v)(x+\bar{u}+\bar{v})$.

Since, u & v appear in all combinations, the only way to satisfy all 4 clauses is to make x true.

(2) Suppose an e_i is the disjunction of 2 literals, $(x+y)$ we introduce a new variable z and replace e_i by the conjunction of 2 clause. $(x+y+z)(x+y+\bar{z})$. As in case 1, the only way to satisfy both clauses is to satisfy $(x+y)$.

(3) If an e_i, the disjunction of 3 literals, it is already in the form required for 3 CNF.

So, we take e_i as such to construct f.

(4) Suppose, $C_i = x_1 + x_2 \dots + x_m$ for some $m \geq 4$, we introduced new variables,

$y_1, y_2 \dots y_{m-3}$ and replace C_i by the conjunction of clauses.

$$(x_1 + x_2 + y_1)(x_3 + \bar{y}_1 + y_2)(x_4 + \bar{y}_2 + y_3) \dots$$
$$(x_{m-2} + \bar{y}_{m-4} + y_{m-3})(x_{m-1} + x_m + \bar{y}_{m-3}) \quad \text{—} \ (1)$$

If there is a truth assignment, J that satisfy C_i, then atleast one literal in $x_1 + x_2 + \dots x_m$, should be true, then J makes x_j true.

Then in (1) above, if we make $y_1, y_2 \dots y_{j-2}$ true, then k make $y_{j-1} + y_j \dots y_{m-3}$.

False, we satisfy all clauses of eq. no (1) thus, J can be extended to satisfy these clauses.

The reason is that, since x_1 & x_2 are false, then y_1 must be true. in the first is true.

Since x_3 is false $\bar{y}$ is false, y_2 must be true to keep situation alive.

-> The above argument shows how to reduce an instance 'e' of CSAT to an instance F of 3SAT, such that f is satisfyable if E is satisfyable.

(4) linear in the length of E, because none of the 4 cases above expand a clause, by more than a factor 32/3.

And its possible to build F in polynomial time.

Since CSAT in NP complete, it follows that 3SAT is also NP complete.

(b)

$\neg(1 \wedge x) \wedge (x \vee 0) \wedge (y \vee z))$

(i) Output 0

$x = 1, \ y = 0, \ z = 0$

$\neg(1 \wedge 1) \wedge (1 \vee 0) \wedge (0 \vee 0))$

$\Rightarrow \neg(1) \wedge (1 \wedge 0)$

$\Rightarrow 0$

(ii) Output,

Setting value of $x = 0$ makes first clause 1, but second clause 0, which would be output 1.

Also, setting value of $x = 1$, makes first clause 0 & second clause 1, which would be output 1.

Thus, getting output as 1 is not possible.

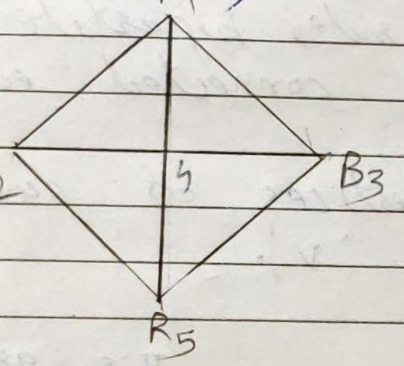
(c) A problem is NP-hard if all problems in polynomial ~~po~~ time reducible to it, even though it may not be NP itself.

If a polynomial time algorithm exists for any of these problems all problems in NP would be polynomial time solvable.

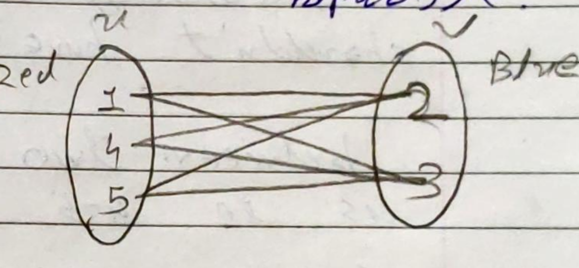
These problems are called NP-complete.

(d) Colouring the graph & finding bipartiteness.

⇒ (figure a)

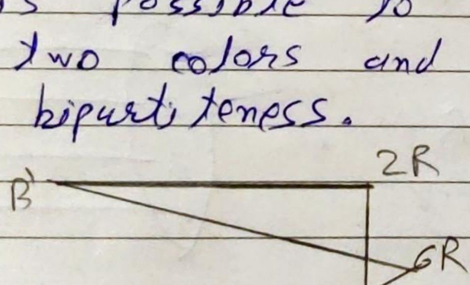


This graph is bipartite.

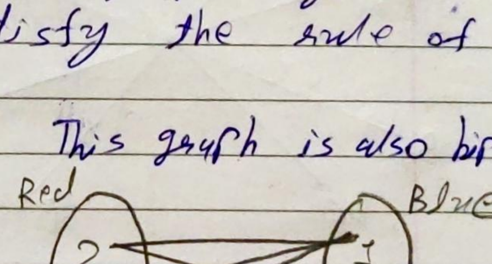


The above graph is bipartite as it is possible to color graph cycle using two colors and satisfy the rule of bipartiteness.

⇒



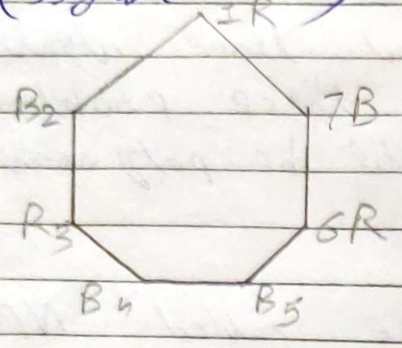
This graph is also bipartite



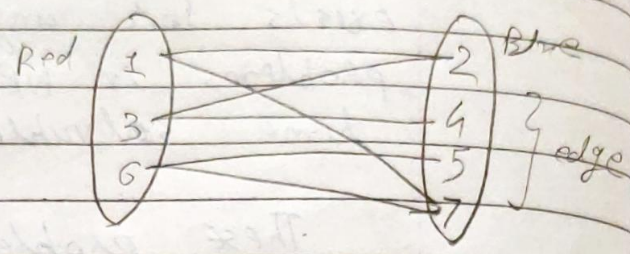
(figure b)

The above graph is bipartite as it can be colored using 2 colors & satisfy the rules.

⇒ (figure c)



This graph is not bipartite.

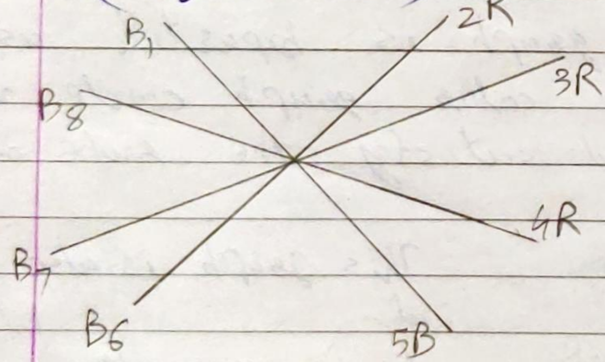


→ It is impossible to color the graph using 2 colors.

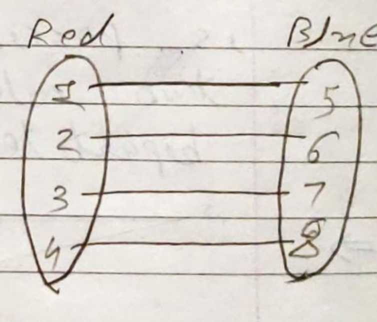
According to rule, bipartite graph shouldn't have 0 connected edge.

between two vertices of same set as in 4-5 in 'v'.

⇒ (figure d)



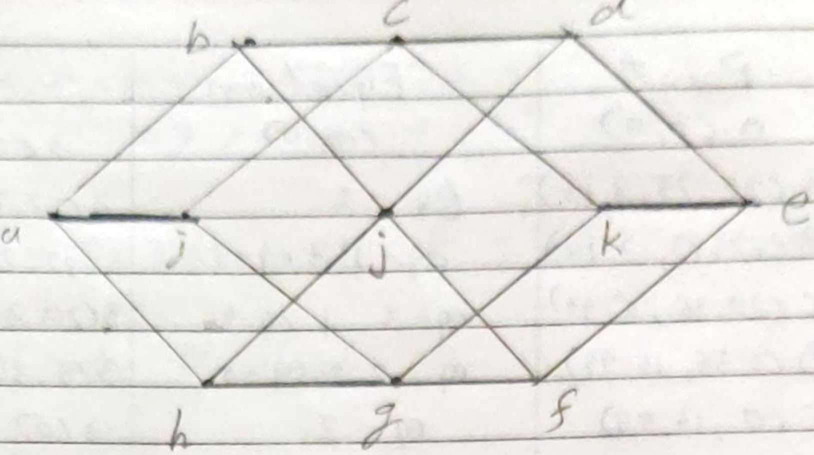
This graph is bipartite.



→ It is possible to color graph using two colors but the given graph is not a cyclic graph.

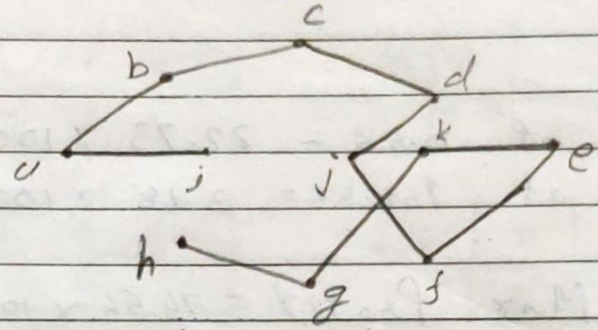
→ Also, it satisfied the rules of bi-partiteness.

2-5



(a) Hamilton path :-

i - a - b - c - d - j - f - e - k - g - h



Hamilton Path

The path cannot be extended a Hamilton Cycle.

(b) Given graph is bipartite.
The bipartition is given by $V = V_1 \cup V_2$

$V_1 = \{a, c, e, g, j\}$
$V_2 = \{b, d, f, h, i, k\}$

In first part, there are 5 vertices & second part there are six vertices.

(c) We know a bipartite Graph $G(V,E)$ is hamiltonian if $|V_1| = |V_2|$ where $V = V_1 \cup V_2$ bipartition of vertex set.

Here,

$$|V_1| = 5 \qquad |V_2| = 6$$

$$\therefore |V_1| \neq |V_2|$$

$\Rightarrow$ P is not a hamilton path.
Hence, graph G cannot have hamilton path.
Hence, proved.

2-6 (a) The answer for this is false.

For a problem to be NP-complete, it must be in NP and it must be NP-hard. X_1, X_2 are in NP and are reducible to each other. But this doesn't prove then they are NP-hard. Proving that they are NP-hard required a reduction from another NP-hard problem.

In fact X_1, X_2 are in NP, but they may even be in P, because P is a subset of NP. In that case, they cannot be NP-complete as otherwise P = NP, which is given to be false.

(b) The answer for this is True. Independent set is NP-complete. Therefore if it is in P, then all problems in NP are in P as well. In particular, Hamilton Cycle which is in NP is also in P.

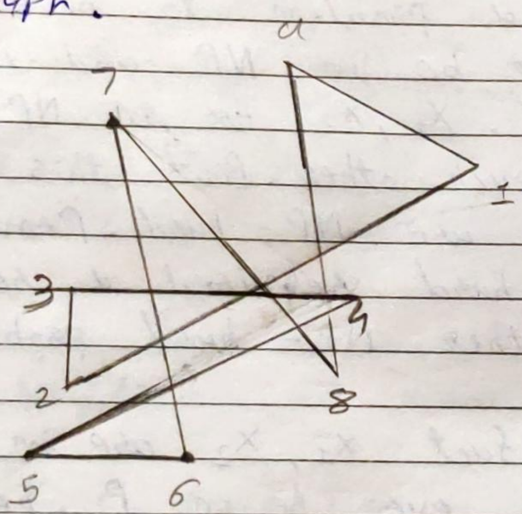
(c) False.

If vertex Cover in P, then as it is NP-complete this implies all problem in NP are in P as well. In particular SAT, which is in NP, is also in P.

Q-7

(a) Yes, its possible to make them sit in a round table. Such that, every student sits between two friends.

In terms of path this is equivalent finding the hamiltonian path in graph.



(b) The first and third graphs are matching (There are other matching as well).

The middle graph does not have a matching. If you, look at the three circled vertices, you see that they only have two neighbours which violets the matching. Condition

$|N(S)| \geq |S|$, the three circled vertices from the set S.