CS 601 – Advanced Analysis of Algorithms **Venue: Take home**
Time Duration: 7 pm – 9 pm **Date: May 01, 2021**
Instructions: (1) Answer the questions in the same order as they appear.
Quiz 2: [Max: 25 points]

1. [**10 points**] Given the weighted graph (Figure 1) and a destination vertex, find the lowest cost path from every vertex to the destination using the Bellman-ford algorithm. [To get full points, you must show all the individual steps involved in the algorithm, marking the vertices chosen, current edge values, etc] [See Figure 1 on next page]

2. [**2 points**] Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have f(e) = c_e).

3. [**4 points**] Regarding the structure of a bipartite graph with perfect matching, we must have |L| = |R|.
   (a) What other conditions are necessary?
   (b) Which conditions are sufficient?

4. [**4 points**]
   a) You are given an undirected graph with each edge having a capacity of 1 unit (i.e., a maximum of one unit of water can flow in both directions). Suppose you ran the Ford-Fulkerson algorithm between a pair of vertices s and t on this graph and it terminated with a final flow of 1. Which of the following is the most general statement one can make about the original graph?

   A) There exists only one unique path from s to t in the graph.
   B) There exists at least one edge which cannot be removed to disconnect the graph into two pieces, one containing s and the other containing t.
   C) The graph is a tree, with s as the root and t as one of the leaves.
   D) There exists exactly one edge which can be removed to disconnect the graph into two pieces, one containing s and the other containing t.
   IMPORTANT: Please briefly explain your answer to get full credit.

   b) Consider the following algorithm for computing the approximate square root of a positive integer:
   SQUARE-ROOT(x)
   for i = 1, 2, …, $\lfloor x/2 \rfloor$
   if i*i == x
   then output i.
   Say True or False. Justify your answer in either case.
   "This algorithm runs in polynomial time."

5. [**5 points**] Find the maximum flow from A to G in Figure 2. [See Figure 2 on next page]

Figure 1:

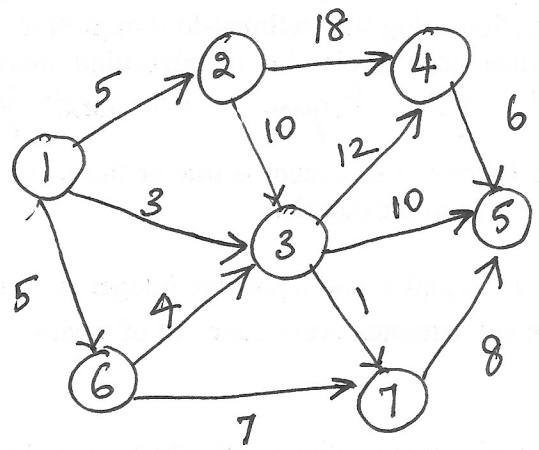

Figure 2: