# Sparse extended information filter using iteration

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***Abstract*—This paper is an attempted extension towards the papers on a novel methodology of implementation of Simultaneous Localization and Mapping using Sparse Extended Information Filters as implemented by Sebastian Thrun. This method of implementation of the SLAM technique aims at reduction of overall error in the system by avoiding linearization error which is cumulative in nature and implementing the state update iteratively. The implementation of this project has been done as a part of understanding the most optimal techniques for implementation of SLAM in autonomous vehicles where the response of the system has to be as low as possible in as the implications of a higher response time could lead to accidents or in severe cases, fatality of the passenger.**

***Keywords—SLAM, Sparsification, Iteration, Filters, posterior, prior, Gaussian.***

## I. INTRODUCTION

Implementation of simultaneous localization and mapping is extremely crucial for the purpose of building efficient decision-making algorithms. In order to create a reference frame from the perspective of the agent, it is important to be able to generate a map of the environment and being able to locate the agent that is gathering the data in the generated map. In the context of Autonomous Vehicles (AVs), the data is gathered using various sensors such as LIDARs, RADARs, Cameras, Proximity sensors, etc[12]. There exist various techniques for implementation of SLAM such as Kalman Filters, Information Filters, Extended Kalman Filters, Extended Information Filters, etc[8].

This paper focuses on a modified version of Extended Information Filters called as Sparse Extended Information Filters as the basis of exploration and performs a modification on it which leads to reduction in error and hence subsequent increase in the overall map accuracy. The major characteristic of this modified implementation is that it retains the ability of the original implementation of extremely fast implementation as well as being scalable. The scalability factor of this paper is extremely crucial as it provides the user with the ability to use it on a distributed system and stitch together a series of local maps to form a global map of the environment and hence reducing the space as well as time overhead considerably.

The generation of a map from the data points contains multiple steps which are based on posterior estimation and approximation. The agent traverses the environment continuously gathering data using either a single sensor or by the implementation of sensor data fusion for higher accuracy and increased spectrum of environmental condition coverage. The agent continuously updates it's state and generates an information map of the environment with detection of important landmarks and noting them. This information is translated into a high dimensional matrix format called as the 'Information Matrix' that undergoes sparsification in order to reduce the amount of data. This minimization of the amount of leads to decrease in the computation time required and hence is deemed feasible for usage in scenarios for time sensitive applications.

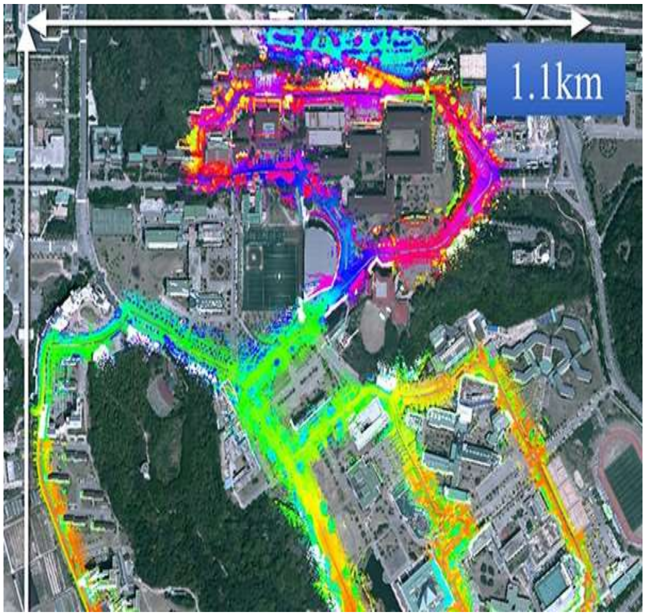
One of the most effective methods for nonlinear estimation is using iteration and has been shown to be effective in SLAM implementation. One of the similar methods from where the inspiration is drawn for this paper is the algorithm for Iterated Extended Kalman Filter. The accuracy of the IEKF is more than that of the standard EKF but it has been observed that the learning rate for IEKF and the speed of implementation of IEKF is even lower than that of EKF[1]. A theoretical analysis will be provided that will support the claims made in the paper with certain assumptions that are fundamental towards the understanding as well as relevance to real-world implementations of the algorithm. The dataset for the implementation has been provided online and has been shown to work on the traditional Sparse Extended Information Filter and hence goes to show that this method retains all the abilities of the SEIF while focusing on the aspect of reduction of linearization errors.

This paper extends the work in a direction that provides a used with a SLAM algorithm with a time complexity of constant time and has the feature of independence from the frequency and count of features in a map that is unknown. This feature is unique to Sparse Extended Information Filters which themselves are derived from the widely utilised Extended Kalman Filters. The implementation of this algorithm shows that the performance measure is comparable in terms of accuracy and computational time to the traditionally used filters used for SLAM implementation.

## II. SLAM IMPLEMENTATION

### *A. Data gathering*

The first step involved in implementation of SLAM is capturing data. A data point is captured using various input sensors that act as the eyes of the system and depending upon the type of sensor used, a two- or three-dimensional model of the environment can be generated. In case of the implementation of data gathering equipment in an autonomous vehicle, multiple sensors are utilized in order to minimize the number of blind spots and generate a map that is as accurate as possible. The quality of the map generated and the reliability of the same is largely dependant on the quality of data gathered and used in order to construct a map which are used to design the decision-making algorithms. The most common hardware used for gathering data are LIDARs, Cameras, Radars, GPSs, proximity sensors, etc[12,13].



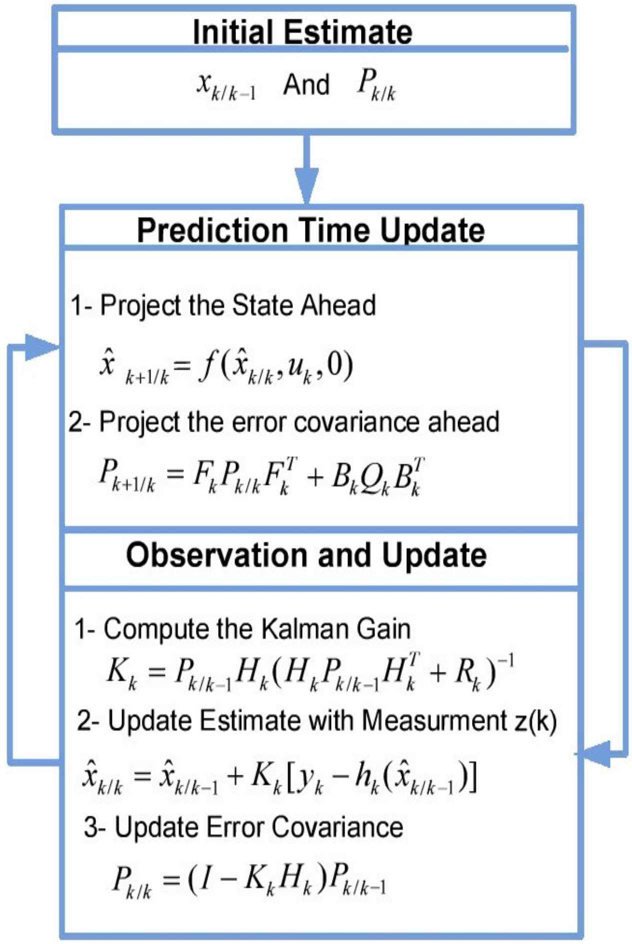

## B. *SLAM*

Simultaneous Localization and Mapping is a technique involving building a software version of the surrounding environment that are understandable by the on-board computers. Practical realisations of probabilistic SLAM have become increasingly impressive in recent years, covering larger areas in more challenging environments. Execution of SLAM protocols and successful and accurate decision making are co-dependant processes that are vital for the future of mobile robotics. SLAM implementation consists of gathering of data points and incrementally constructing a map with consistency while being able to localize the agent and understanding the frame of observation. The most widely used SLAM methodologies have been discussed in the following section of paper focussing on the backbone and modified versions of Kalman Filters and Information filters while discussing their advantages and drawbacks[10].

# III. COMMON SLAM METHODOLOGIES.

## A. *Kalman Filters.*

Kalman filters have been utilised for a long time for their important property of high map accuracy. The major advantage of utilizing Kalman filters is the high accuracy of the posterior estimation which leads to a visualization of data that has very high accuracy. A Kalman filter or even an Extended Kalman Filter are popular choices of map generation in SLAM implementations where the demand is higher accuracy and a trade-off of computation time is accepted. The important characteristic of any SLAM implementation using these common filters is the posterior estimation and approximation. In case of Extended Kalman filters, it approximates the posterior as a Gaussian[10,8]. The basic algorithm for Extended Kalman filters can be given as:




The major drawback of the Extended Kalman filter is the high computational overhead which presents itself as a gridlock for implementation in Autonomous vehicles. Extended Kalman filters have been proven to have a quadratic time complexity. In case of low data, EKFs have high accuracy as well as speed of implementation. But as the complexity of the environment and hence the subsequent data increases, the implementation of Extended Kalman filters becomes proportionally less feasible. Many attempts have been made to reduce the time complexity of the implementation while trying to retain the high quality aspect but have run into many drawbacks that make them impractical for real-world applications. Researchers have attempted to breakdown the environment into individual sub-maps which are traversed by multiple robots simultaneously in order to make the process faster but this technique does not consider marking or identification of pre-visited landmarks and hence can run into redundancy errors. The methodology of Extended Kalman filters does not poses the inherent property of information transfer or translation between individual sub-maps for

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detection of common data points and providing accurate reference data landmarks that can be used for stitching together individual sub-maps and generating a global map of the surrounding in a time small enough to execute corresponding sub-routines for maximizing the global objective at each state iteration. An algorithm that does however overcome this drawback and is able to successfully generate the posterior estimate in a lower time complexity is the Extended Information filter which has been discussed in the following section of the research paper.

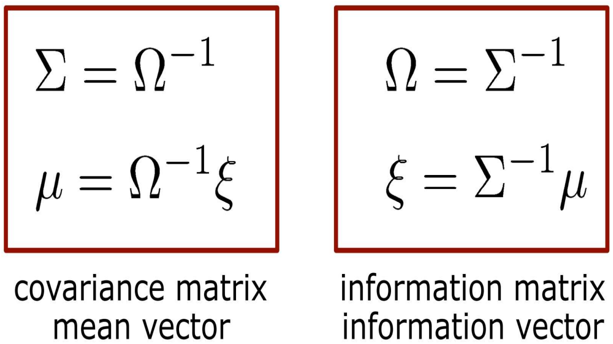
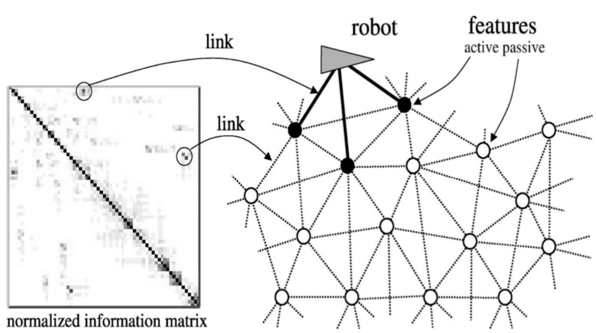



Image credit: Cyrill Stachniss, Uni Frieberg, Sparse Extended Information Filter for SLAM

### B. *Information Filters.*

This part of the paper explains the basic Extended Information filter which has been modified to provide a fast and accurate algorithm. Unlike the Extended Kalman Filter that stores data in the form of a covariance matrix, the Extended Information filter stores the data in the form of an Information matrix which essentially is the inverse of the covariance matrix. Extended Information filters are not as widely used as EKFs although current research and accelerated advancements in the field of mobile robotics with high priority towards response time have led to an increase in the utilization of this technique.




Following the trend of the Extended Kalman filter, the posterior in case of an Extended Information filter is represented as a multivariate Gaussian distribution which is conditioned over the state which is a function of current time. Data representation in an information filter is in the form of an information matrix which has been explained in the next section of the paper. Information filters hold the inherent property of high speed of execution but with the trade-off of lower accuracy. This type of algorithm is hence suitable for mobile agents. The main focus of this paper is to understand the execution of Sparse Extended Information filter which is a modified version of the Extended Information filter with constant time complexity and the unique ability of scalability[14]. The crucial step of sparsification has been explained in the following section of the paper and is responsible for reduction in data and hence a corresponding reduction in computational overhead of the algorithm implementation.

(Insert image about algorithm like in PPT)

## IV. Sparse Extended Information Filter

### A. *Information matrix and landmarks*

As an extension towards the Extended Kalman filter, the format of representation of the Sparse Extended Information Filter is in the form of an information matrix. The information matrix is essentially a translation of the connections between the agent and the landmarks in the surrounding in the format of ones and zeros. The following image is a comparison between the data representation in in the format of a Gaussian estimate and a normalized covariance matrix which is used in Extended Kalman Filters and Normalized Information Matrix which is used in Sparse Extended Information Filters:

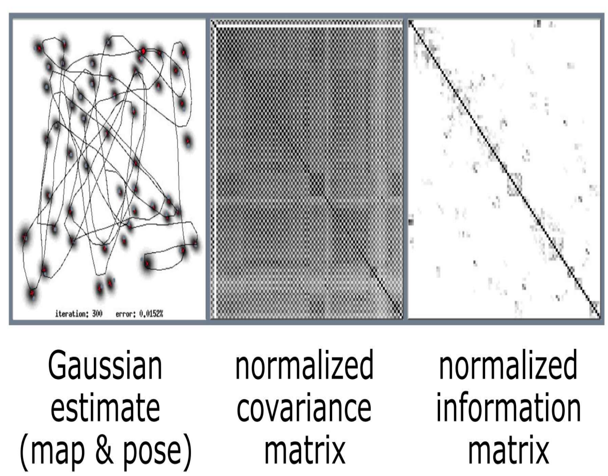
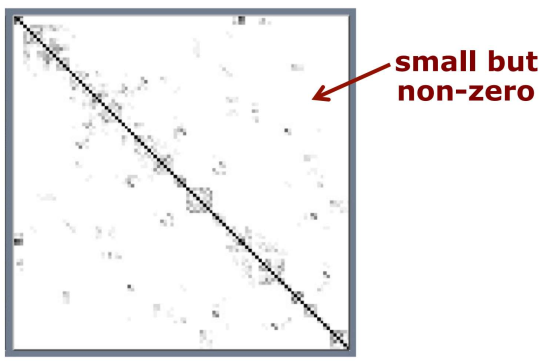



Image credit: Cyrill Stachniss, Uni Frieberg, Sparse Extended Information Filter for SLAM

Information matrix can be interpreted as a graph of constraints or links between landmarks. $\Omega$ij is the strength of the link which is stronger for the landmarks closer to the agent. All the diagonal elements are stronger and are represented by 1 in a normalized matrix whereas most of the non-diagonal elements are close to zero but not completely zero. The landmarks are classified into two types: active and passive. As the agent traverses through the environment, a link is formed between the agent and a landmark. In case of a Sparse Extended Information Filter, this Information matrix undergoes a sparsification step which essentially reduces most of the non-diagonal elements to zeros.

Instead of $\mu_t$ and $\sum_t$, information filters represent the posterior through $H_t$ and $b_t$, which are the information matrix and information vector, respectively. They are defined as follows:




normalized information matrix

Additionally, in case of the SEIF, along with the measurement update step, steps involving motion update, sparsification and recovery are performed which have been explained in the following subsections of the paper.

*B. Motion update*

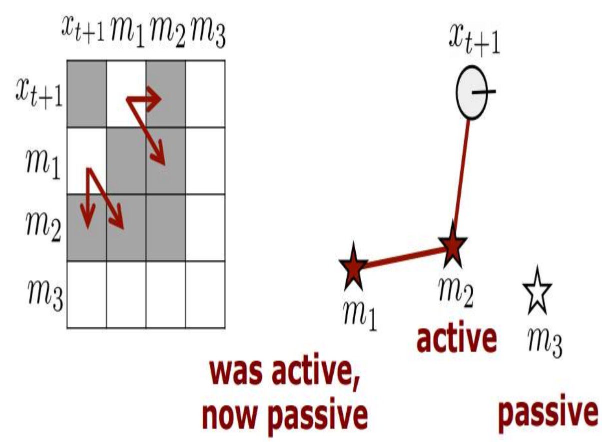
The next major step in the implementation of SLAM in this paper is the continuous update in the filter in reference to the motion of the agent. The environment in this paper is the same as a standard SLAM problem where it is static in nature and only pose changes are considered. The example explained using the following image considers the agent in association with two landmarks[4]. It is also assumed in this implementation that the environment is noise free and hence the link generated is assumed to be between real landmarks and agents only. Any form of noise in the actuation of the robot or movement would in-turn weaken the link between the agent and an active feature. One of the abilities of this algorithm is to retain the information matrix or perform continuous updates on it with accuracy. This means that even though there could be a loss in the information relative to landmarks, there is no loss of information between individual features and hence the landmarks that were active and indirectly linked to each other via the agent become directly linked to each other. This motion update is represented by 1s in the diagonal elements of the information matrix. The equations for motion updates are as following:

$$p\left(\xi_t \middle| z^t, u^t\right) \propto exp\{-\frac{1}{2}\xi_t^T H_t \xi_t + b_t \xi_t\}$$

The above equation explains how to determine the posterior or perform posterior estimation in order to build the information matrix and build the map of the environment.

*C. Sparsification*

The important step of sparsing the information matrix in order to reduce the amount of data contained in it is the key step responsible for reduction in space complexity over the traditional Kalman filters and their extensions. Sparsification is a process in which the links whose intensity is represented as a numeric value is converted to zero. We start by making most of the non-diagonal elements of the information matrix as zero and update the matrix at every iteration. As the motion update takes place, the subsequent landmarks are marked as visited(active) or unvisited(passive). The link between agent and visited landmark is sparsed to zero and a new link between active landmarks is retained. Hence the information matrix as compared to the unsparsed matrix has very few 1s in it and contains only the links that are either between landmarks or are active in nature.




*D. Iteration*

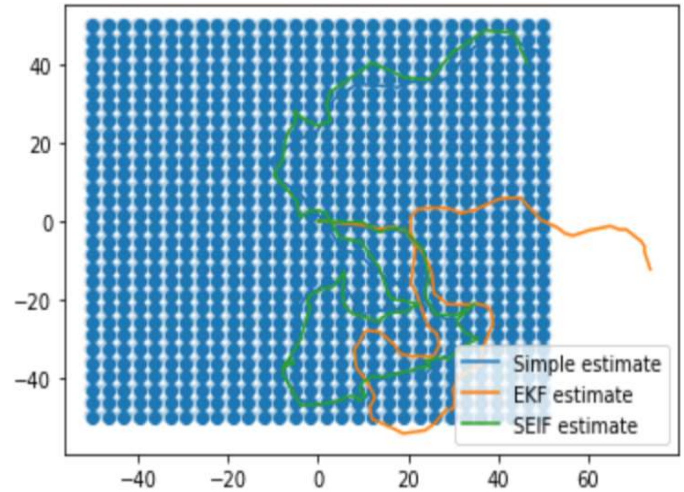
There are three steps in the implementation of the SEIF which can be classified to be the major contributors towards the cumulative error in the posterior. The first one and the focus of the paper is that after posterior estimation, the pose estimate is not equal to the true state and hence an approximation or estimation is done in order to retain it. The second error is due to the novel step in the algorithm of sparsification that leads to loss of data. The computation of the state vector by approximation and recovery leads to errors as well. The approach for reduction of linearization errors is to solve the state update equations in an iterative manner in contrast to the combinatorial linearization estimation over continuous time update. The traditional state equations are maximized and the Jacobian matrix can be expanded according to Taylor series. The approach taken in this technique is to solve it as a Maximimum a posteriori (MAP) problem which is iterative in nature[4]. The posterior state update can be done using the Newton-Rhapson algorithm .Matrix and vector equations in the updated algorithm can be seen as follows:

$$H_t = H_t^i + C_t^{i+1} R_t^{-1} \left(C_t^{i+1}\right)^T$$

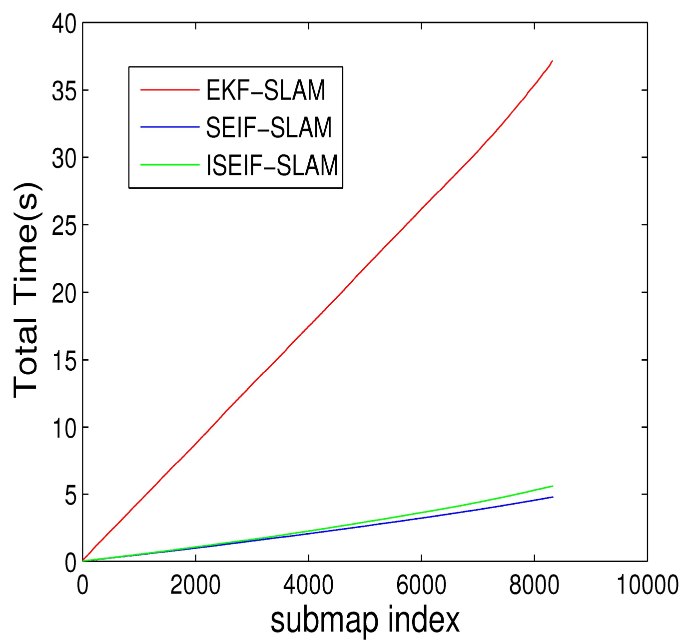
$$b_t^{i+1} = b_t^i + \left(z_t - \hat{z}_t + \left(\left(C_t^{i+1}\right)^T \mu_t^{i+1}\right)^T R_t^{-1} \left(C_t^{i+1}\right)^T\right.$$

IMPLEMENTATION *(Heading 5)*

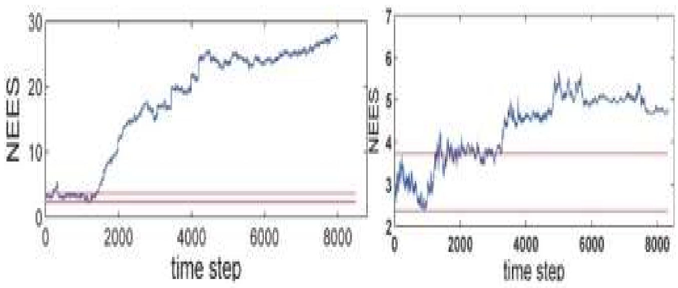
The implementation of this paper involved a modification of the update equations of the Sparse Extended Information Filter and was implemented using python. The data visualization libraries such as NumPy, MatPlotlib and Seaborn were utilized for producing outputs in the form of graphs. The following grapgh depicts an implementation of the traditional Sparse Extended Information Filter and compares the performance to the Extended Kalman Filter. It can be observed that the blue circles in the graph represent simulated landmarks that the simulated agent traverses. The blue line in the graph represents the actual path that has been selected as a reference for performance measure. The green trajectory observed in the graph is the path that has been generated by simulating an agent implementing Sparse Extended Information Filter and the line in green is the path traversed by a simulated agent that implements Extended Kalman Filter for comparison.




The following graph compares an important aspect of SLAM algorithms which is the computational complexity. This performance measure is crucial in the selection of an algorithm specific to an application. It can be observed from the following graph that the Extended Kalman Filter has extremely high computational complexity and hence takes the most amount of time to be executed over the same dataset. The Sparse Extended Information Filter can be observe to be much faster than EKF by a large amount. As discussed previously in this paper, EKF has been shown to have quadratic time complexity and SEIF has linear time complexity. It can also be observed from the graph that the approach in this paper utilizing an iterative update of the state equations can be observed to have a slightly lower time complexity than that of the traditional Sparse Extended Information Filter. Thus it can be assumed that when this algorithm is implemented on multi-robot systems with real-world data sets with dynamic environments, this small reduction in the computational overhead can prove to be vital.



The other major aspect of this algorithm is the minimization of error that is due to linearization combination in case of an Extended Kalman Filter. The following graph is a comparison between SEIF and ISEIF in terms of Normalization Estimation Error Square (NEES). It can be clearly observed that the error reduces considerably in the iterative implementation. Hence this algorithm can be deemed as more suitable for implementation on Autonomous Vehicles for high accuracy and low time complexity. This algorithm is scalable in nature can be implemented in a distributed format on multi-vehicle system which provides a notion of connectedness between cars with the idea of lowering accidents and maximizing the individual global objectives. The following graph shows the NEES comparison:

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